Tuning magnetic confinement of spin-triplet superconductivity: Supplementary Information

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SUPPLEMENTARY NOTE

Details of the Ginzburg–Landau theory

In the main text, we have focused on a single-component superconducting order parameter for simplicity. At the same time, it has been suggested that the order parameter in UTe_2 may be two-component and have a non-zero magnetic moment of the Cooper pair aligned along the field direction – the so-called nonunitary triplet superconductor. This section analyzes the theoretical predictions for the upper critical field in this case.

Two-component order parameter

The odd-wave superconductor, such as believed to be the case in UTe_2, is described by an order parameter $\hat{\Delta}(\mathbf{k}) = i(\vec{d}_{\mathbf{k}} \cdot \vec{\sigma})\sigma_2$ expressed in terms of the $\vec{d}$-vector. The superconducting order parameter in a crystal can be decomposed into a linear superposition of the irreducible representations $\Gamma_i(\hat{\mathbf{k}})$ of the point-group symmetry:

$$\vec{d}(\mathbf{k}) = \sum_i \vec{\eta_i}\Gamma_i(\hat{\mathbf{k}}). \tag{1}$$

In the case of UTe_2, the point group D_{2h} contains only the one-dimensional representations, which would generically have different transition temperatures. As a result, the single-component order parameter, for which the Ginzburg–Landau (GL) theory has been presented in the main text, is expected to provide a sufficient description.

However, it has been pointed out that in a strong magnetic field, along for instance the crystallographic $\hat{b}$ direction, the point group symmetry is lowered to C_{2h}, allowing for a linear superposition of the two irreducible representations, such as B_{1u} and B_{3u} [1]. Therefore, we shall consider a two-component order parameter which encodes the expression for the $\vec{d}$-vector as a linear combination of these two irreducible representations [2]:

$$\vec{d}(\mathbf{k}) = \eta_x \hat{d}_{B_{3u}}(\mathbf{k}) + \eta_y \hat{d}_{B_{1u}}(\mathbf{k}) \tag{2}$$

Given the field in the $\hat{b}$ direction, it is convenient to choose the coordinate system $(a, b, c) \to (y, z, x)$. In this notation, it was shown in Ref. 2 that the leading components of the B_{1u} and B_{3u} irreducible representations take the form:

$$\vec{d}_{B_{3u}}(\mathbf{k}) \sim \sin(k_b b)\, \hat{c} \;\propto\; (1, 0, 0) \tag{3}$$

$$\vec{d}_{B_{1u}}(\mathbf{k}) \sim \sin(k_b b)\, \hat{a} \;\propto\; (0, 1, 0), \tag{4}$$

so that the two-component order parameter transforms like the $\boldsymbol{\eta} = (\eta_x, \eta_y)$ vector in the xy-plane. Written explicitly as a matrix in the spin space, the superconducting order parameters takes on the following form:

$$\hat{\Delta} = i(\vec{d} \cdot \sigma)\sigma_2 = \Delta_0(\mathbf{k}) \begin{pmatrix} -\eta_x + i\eta_y & 0 \\ 0 & \eta_x + i\eta_y \end{pmatrix}. \tag{5}$$

Notice that the absence of the z-component of the d-vector (along the applied field direction) means that the order parameter matrix is diagonal, describing *equal-spin pairing* [2].

The most general Ginzburg–Landau theory allowed by orthorhombic symmetry can be split into two contributions: $F[\boldsymbol{\eta}] = F_0[\boldsymbol{\eta}] + F_B[\boldsymbol{\eta}]$, the first term being the zero-field Landau free energy

$$F_0[\boldsymbol{\eta}] = \alpha_x|\eta_x|^2 + \alpha_y|\eta_y|^2 + \frac{\beta_1}{2}(\boldsymbol{\eta} \cdot \boldsymbol{\eta}^*)^2 + \frac{\beta_2}{2}|\boldsymbol{\eta} \cdot \boldsymbol{\eta}|^2 + \frac{\beta_x}{2}|\eta_x|^4 + \frac{\beta_y}{2}|\eta_y|^4, \tag{6}$$

where we have allowed for the xy anisotropy due to the orthorhombicity. In particular, $\alpha_x(T) \neq \alpha_y(T)$ allows, in principle, for different critical temperatures for the two components of $\boldsymbol{\eta}$, corroborated by two distinct signatures in the temperature dependence of the specific heat data [3].

The gradient terms [4] also contribute to the free energy, and become relevant when computing the upper critical field:

$$F_B[\boldsymbol{\eta}] = \frac{B^2}{8\pi} + K_1(D_i\eta_j)^*(D_i\eta_j) + K_2(D_i\eta_i)^*(D_j\eta_j) + K_3(D_i\eta_j)^*(D_j\eta_i) + F_{\rm ani}[\boldsymbol{\eta}] \quad (7)$$

where the Einstein summation is implied over repeated indices $i,j=\{x,y\}$ and $D_i = -i\nabla_i + \frac{2\pi}{\Phi_0}A_i$ denotes the covariant derivative in the presence of the external vector potential $\mathbf{A}$. In addition to the above terms, the orthorhombic symmetry of the crystal allows to write down three more terms which we grouped into $F_{\rm ani}$ [4]:

$$F_{\rm ani}[\boldsymbol{\eta}] = K_x(D_x\eta_1)^*(D_x\eta_1) + K_y(D_y\eta_2)^*(D_y\eta_2) + K_z(D_z\eta_i)^*(D_z\eta_i). \quad (8)$$

Note that the coefficients K_a play the role of $\hbar^2/2m_a$ in the traditional form often used to write down the GL equations, and the orthorhombic symmetry means that the effective mass tensor is anisotropic.

The applied magnetic field polarizes the electron spins in the normal state, resulting in the internal magnetization $\mathbf{M}(H)$. The free energy now contains additional terms coupling the superconducting order parameter to the magnetization:

$$F_{\rm coupl}[\boldsymbol{\eta},\mathbf{M}] = g\mathbf{M}^2(\boldsymbol{\eta}\cdot\boldsymbol{\eta}^*) - w(\mathbf{M}+\mathbf{H})\cdot(i\boldsymbol{\eta}\times\boldsymbol{\eta}^*), \quad (9)$$

The first term is the same as in the main text, whereas the second term describes the interaction of the total magnetic field $\mathbf{B}=\mu_0(\mathbf{M}+\mathbf{H})$ with the internal moment of the Cooper pair. This last term only appears in the *nonunitary* state where $\boldsymbol{\eta}^*$ is not collinear with $\boldsymbol{\eta}$, such as for instance

$$\boldsymbol{\eta}_{\rm chiral} = (1,\, i\delta), \quad (10)$$

where δ is taken to be real (generically, $|\delta|\neq 1$ because of the absence of C_4 rotation in the xy-plane). Note that we do not need to consider the pair-breaking effect of the internal Zeeman field, due to the equal-spin pairing nature of the order parameter in Eq. (5). Indeed, this is consistent with the experimental evidence that H_{c2} exceeds the Pauli–Clogston limit in UTe_2 for field $H \parallel \hat{b}$.

We consider now the field along the $\hat{z}$ (i.e. crystalographic $\hat{b}$) direction, as in the experiment, and choose the gauge where $\mathbf{A}=(-Hy,0,0)$. Differentiating with respect to η_j^* ($j=x,y$) and dropping the quartic terms in the free energy, we obtain the linearized GL equation in the form of two differential equations for η_x and η_y:

$$\begin{aligned}(K_{123}+K_x)D_x^2\eta_x + K_1D_y^2\eta_x + \tilde{\alpha}_x(T)\eta_x + (K_2D_xD_y + K_3D_yD_x)\eta_y - iw(M+H)\eta_y = 0\\ (K_{123}+K_y)D_y^2\eta_y + K_1D_x^2\eta_y + \tilde{\alpha}_y(T)\eta_y + (K_2D_xD_y + K_3D_yD_x)\eta_x + iw(M+H)\eta_x = 0\end{aligned} \quad (11)$$

where we have denoted $K_{123}\equiv K_1+K_2+K_3$ and

$$\tilde{\alpha}_i(T) = -\alpha_{0i}\left(\frac{T_c^{(i)}-T}{T_c^{(i)}}\right) + gM^2, \qquad i=\{x,y\} \quad (12)$$

denotes the renormalized quadratic coefficients in the GL expansion due to the effect of coupling to the internal magnetization. Note that generically, the two transition temperatures T_c^x and T_c^y are not necessarily equal to each other, although the quartic coupling between the components of $\boldsymbol{\eta}$ means that in an experiment, both components become non-zero below the same physical T_c.

Unfortunately, the coupled differential equations (11) do not admit an analytical solution. Some progress can be made by searching for the solution in the form

$$\eta_i(x,y,z) \sim \tilde{\eta}(y)e^{ik_xd}e^{ik_zb}, \quad (13)$$

and ignoring the derivatives ∂_x in the Eqs. (11). Then, the equations simplify:

$$-K_1\partial_y^2\eta_x(y) + (K_{123}+K_x)\left(\frac{2\pi H}{\Phi_0}\right)^2 y^2\eta_x(y) + \tilde{\alpha}_x(T)\eta_x - iw(M+H)\eta_y = 0 \quad (14)$$

$$-(K_{123}+K_y)\partial_y^2\eta_y(y) + K_1\left(\frac{2\pi H}{\Phi_0}\right)^2 y^2\eta_y(y) + \tilde{\alpha}_y(T)\eta_y + iw(M+H)\eta_x = 0 \quad (15)$$

In order to allow for an analytical solution, let us limit the discussion to the case when $w=0$ (i.e. the case of a unitary order parameter). Then, it is straightforward to find the solution for the upper critical field similar to the procedure in the main text:

$$\begin{aligned}H_{c2}^{(x)} &= \frac{\Phi_0}{2\pi\sqrt{K_1(K_{123}+K_x)}}\left[\alpha_{0x}\left(\frac{T_c^x-T}{T_c^x}\right) - gM^2(H)\right]\\ H_{c2}^{(y)} &= \frac{\Phi_0}{2\pi\sqrt{K_1(K_{123}+K_y)}}\left[\alpha_{0y}\left(\frac{T_c^y-T}{T_c^y}\right) - gM^2(H)\right].\end{aligned} \quad (16)$$

Without carrying out the detailed calculation, which would depend on the phenomenological parameters of the GL theory and the details of the $M(H)$ dependence, it is clear that the H_{c2} is reduced for both components of the order parameter because of the presence of magnetization (g is positive in the free energy Eq. (9)). In particular, when $M(H)$ undergoes a metamagentic transition at field H^* as in UTe_2, the magnetization suddenly increases, and this has the potential of significantly reducing H_{c2}. We conclude that the behavior is qualitatively the same as described in the main text.

The physical H_{c2} is the larger of the two expressions in Eq. (16) above, and we further argue that $H_{c2}^x > H_{c2}^y$ in UTe_2. Indeed, using the effective mass approximation to the electron dispersion Eq. (7) in the main text, we conclude that $K_x \sim t_c d^2/\hbar^2$ is proportional to the interlayer hopping strength t_c. That hopping is presumably very small due to the quasi-2D nature of the Fermi surfaces of UTe_2, as remarked in the main text, and hence $K_x \ll K_y$, making $H_{c2}^x > H_{c2}^y$ in the limit of zero temperature, where we can neglect the difference between α_x and α_y. This is entirely consistent with the presence of two superconducting regions, denoted "SC1" and "SC2" in Figs. 3 and 4 in the main text, with SC1 having a higher H_{c2}. In our notation,

$$\mathrm{SC1}: \quad \boldsymbol{\eta}_1 \sim (1,0) \propto B_{3u} \tag{17}$$

$$\mathrm{SC2}: \quad \boldsymbol{\eta}_2 \sim (1,\delta) \text{ or } (1,i\delta) \propto B_{3u} + B_{1u} \tag{18}$$

SUPPLEMENTARY FIGURES

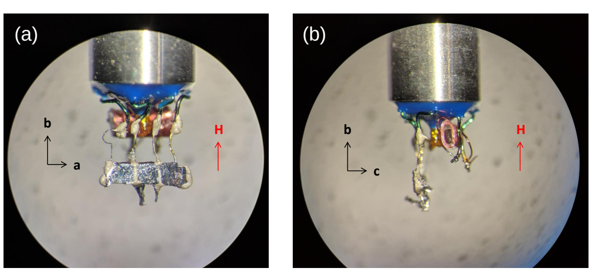

FIG. 1. (a) The front view (b) side view of UTe_2 samples mounted in the pressure cell. The lower sample was set up for four-probe resistance measurements and the upper sample was mounted inside a coil for TDO measurements. The crystallographic axis of the two samples are aligned in the same orientation as indicated by black arrows, with applied magnetic field directed vertically along the crystallographic b-axis.

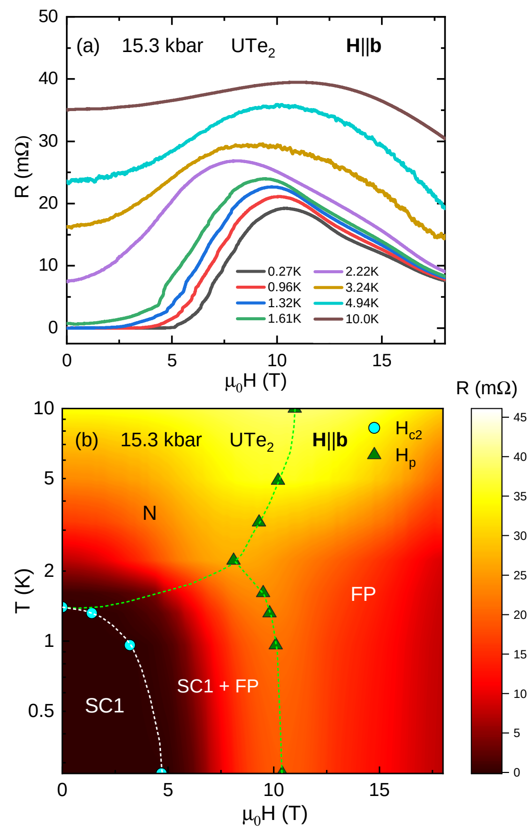

FIG. 2. (a) Magnetoresistance and (b) field-temperature phase diagram of a UTe_2 single crystal for fields applied along the crystallographic b-axis at 15.3 kbar. The cyan circles indicate the T_c transition into the SC1 superconducting phase obtained by field sweeps which are determined by zero-resistance criteria. Green triangles label the position H_p of the peak in magnetoresistance in panels (a). Under this pressure which is close to the critical pressure P_c, the transition between the SC1 phase and FP phase broadens, resulting in a wide regime of coexistence.

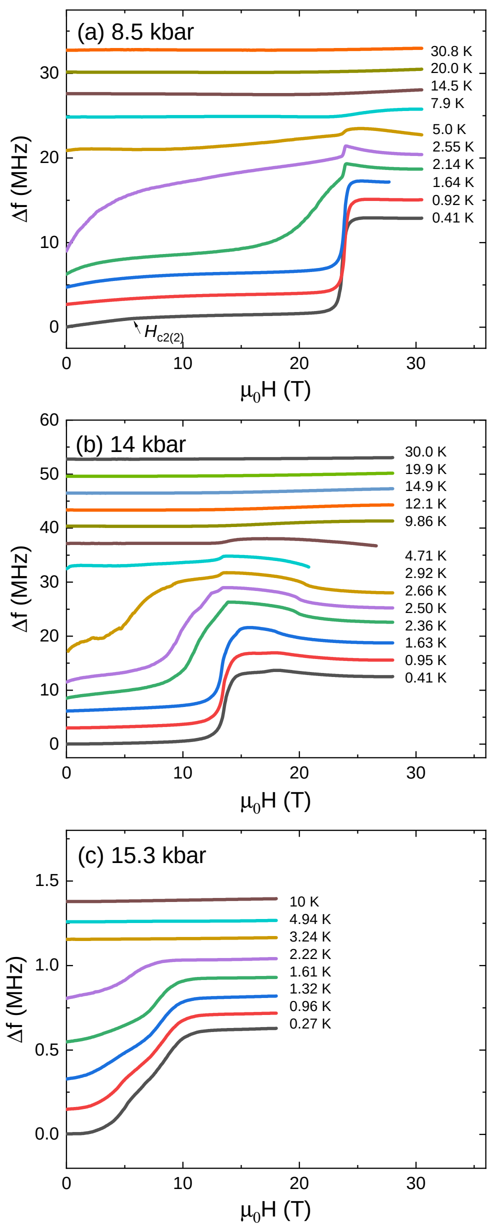

FIG. 3. Tunnel diode oscillator frequency variation of UTe_2 as a function of magnetic fields applied along the crystallographic b-axis, under applied pressures of (a) 8.5 kbar, (b) 14 kbar and (c) 15.3 kbar. All curves are vertically shifted for presentation.

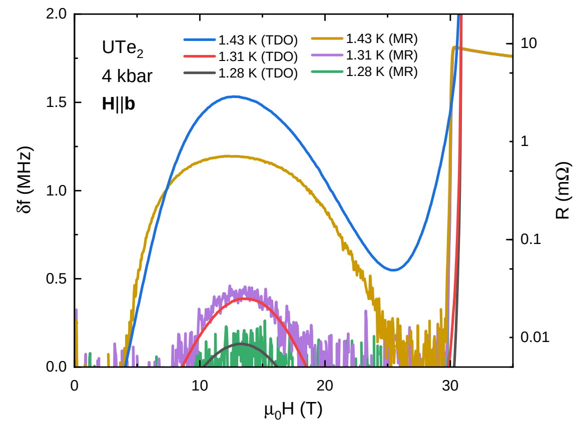

FIG. 4. Tunnel diode oscillator (TDO) frequency variation and magnetoresistance (MR) of a single crystal UTe_2 at 4 kbar with field parallel to the b-axis, where the latter is presented in log scale. As field increases, the MR data indicates reentrant superconductivity as the resistance changes from zero to nonzero and then back to zero. Simultaneously, the sign of TDO frequency variation shows consistent results going from negative (diamagnetism) to positive (paramagnetism), and then back to negative (diamagnetism). Near 30 T, the frequency variation changes sign again as the sample enters the field-polarized phase. Note that for the TDO data, an offset as well as a small linear background (0.1MHz/T) have been applied to match the MR results.

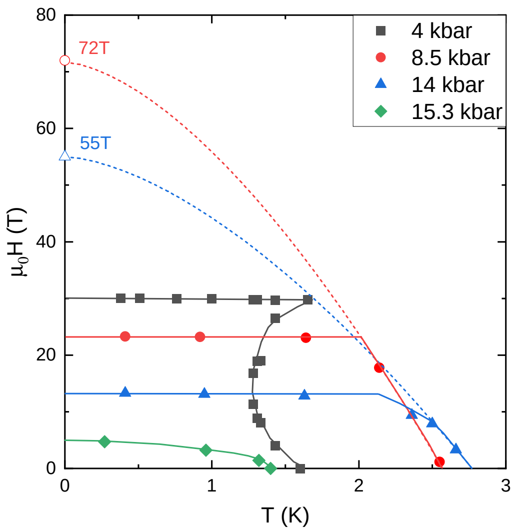

FIG. 5. The SC1 phase boundaries at different pressures with labelled putative upper critical fields. The uncommon shape of the phase boundary (i.e. flat top) suggests that the SC1 phase is truncated by the FP phase at the cutoff field H_c. The open labels indicate the putative upper critical fields which are calculated using the WHH equation, $H_{c2}(0) = -0.69 \times s \times T_c$. s represents the slope of phase boundary at T_c (i.e. $\frac{\mathrm{d}H_{c2}}{\mathrm{d}T}|_{T_c}$) which is estimated by considering the two closest data points to T_c. The resulting upper critical fields are 72 T and 55 T for 8.5 kbar and 14 kbar. The dashed-curved lines serve as guides to the eye.

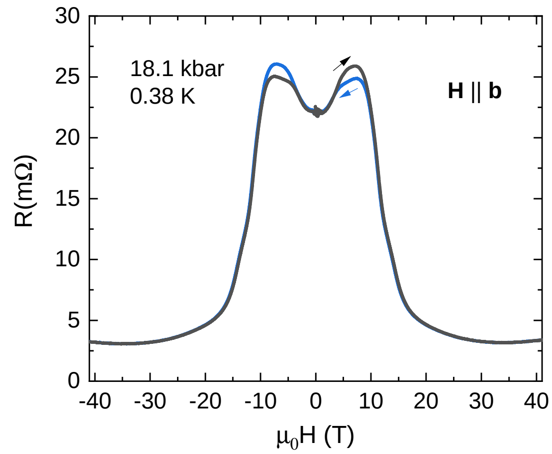

FIG. 6. The magnetoresistance of UTe_2 from 41 T to -41 T at 18.1 kbar. The hysteresis loops in both positive and negative field regions is consistent with the ferromagnetic-like ground state.

SUPPLEMENTARY REFERENCE

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