Supplementary Information

Supplementary Note 1

We compute the one- and two-hole propagators approximately for the case of two-dimensional antiferromagnets and exactly for the mixed-dimensional case. Here we provide a brief overview of the approach in the mixed-dimensional case.

One-hole propagator: The equation of motion for the one-hole propagator $G_{1h}(k,\omega) = \langle\Psi_{\rm GS}|\, h_k \hat{G}(\omega) h_k^\dagger \,|\Psi_{\rm GS}\rangle$ is exactly solvable in the self-consistent non-crossing scheme since crossing diagrams identically vanish as the hole must always retrace its path back to the origin. Here, $\hat{G}(\omega) = [\omega - H]^{-1}$, $|\Psi_{\rm GS}\rangle$ represents the Ising AFM, and H is in the mixed-dimensional limit. One then finds

$$G_{1h}(k,\omega) = \left\{ \left[G_{1h}^0(k,\omega)\right]^{-1} - \Sigma(k,\omega) \right\}^{-1}, \tag{1}$$

where $G_{1h}^0(k,\omega) = \omega^{-1}$ is the static hole propagator governed by $H_0 \equiv H_{t=0}$ and the self-energy

$$\begin{aligned} \Sigma(k,\omega) &= 2t^2 F_{1h,1b}(k,\omega - 3J/2), \\ \text{with } F_{1h,1b}(k,\omega-3J/2) &= \left\{ \left[G_{1h}^0(k,\omega)\right]^{-1} - \Sigma_F(k,\omega) \right\}^{-1} \end{aligned} \tag{2}$$

is a generalized hole–one-boson propagator that is solved self-consistently:

$$\Sigma_F(k,\omega) = t^2 F_{1h,1b}(k,\omega - J). \tag{3}$$

Note the different t^2 factors and shifts in ω in the definitions of the self-energies. These reflect the following. The first magnon created by the hole has an energy $3J/2$, corresponding to the breaking of three antiferromagnetic bonds, and can be generated in two different directions along x. However, subsequent magnons each cost an energy J, corresponding to breaking two bonds, and can only be generated unidirectionally. As such, all processes involving more than a single magnon are summed up to obtain $F_{1h,1b}$, from which one finds $G_{1h}(k,\omega)$. This approach gives for the lowest pole $G_{1h}(k,\omega) \sim [\omega - E_{\rm p}(k)]^{-1}$, where $E_{\rm p}(k)$ is the energy dispersion of a magnetic polaron formed of the hole dressed by magnons.

Two-hole propagator: The two-hole propagator in the $\sigma^z_{\rm Total} = 0$ sector $G_{2h}(K,\omega) = \langle\Psi_{\rm GS}|\, h_{k_\sigma} h_{k_{-\sigma}} \hat{G}(\omega) h^\dagger_{k_{-\sigma}} h^\dagger_{k_\sigma} \,|\Psi_{\rm GS}\rangle$ is also found exactly via a non-crossing self-consistent approach. Here, $K = k_\sigma + k_{-\sigma}$ is the total momentum of the pair. To see the exactness of the non-crossing scheme for two-hole states, note that one hole of spin σ always first emits a string of magnons before they are absorbed by the second hole of spin $-\sigma$; this represents the only possible magnon-mediated interaction between the holes, and thus all crossed boson lines connecting the two fermion lines vanish. One then sums all diagrams self-consistently (all crossed boson lines vanish) to find the $\mathcal{T}$-matrix describing the interaction between two holes. A discrete pole in $G_{2h}(K,\omega)$ signals the formation of a bound state of two holes, with a dispersion $E_{\rm BP}(K)$. Note that such a bound state has an infinite lifetime even if it is not the lowest energy state, since there are no matrix elements to couple the bound pair to a configuration of two isolated particles.