

Supplementary Information — Spatially Referenced Watershed Models for the Binational Red–
Assiniboine River Basin: Bayesian vs Frequentist Comparison

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SPARROW (Spatially Referenced Regression on Watershed Attributes) model

In this study, we used the standard SPARROW framework that separates the nutrient fate and transport within a watershed context into three conceptual processes, which ultimately describe the loading downstream: 1) nutrient export from different sources and land uses, 2) land-to-water delivery, and 3) nutrient attenuation rates during their transportation through streams and reservoirs¹. SPARROW estimates mean annual nutrient flux F_i (μ in natural logarithmic space) in a headwater sub-watershed i with the following governing equation:

$$\mu_i = \ln(F_i) = \left[\ln \left\{ \sum_{j=1}^{J_i} \left(\sum_{n=1}^{N_c} \beta_n S_{n,j} \right) R_{i,j}^{lwd} + \beta_{SC} T L Q_{bool} + \beta_{PT} S_{PT} + F_{QA} \right\} H_{i,j}^R \right] + \varepsilon_i \quad (1)$$

where the subscripts i and j refer to monitored stations with tributary measurements and upstream reach catchments, respectively. For each water quality station i , mean annual TP load was estimated using Fluxmaster²; for details see Jenkinson and Benoy³. For each scenario calibrated using the Bayesian approach, the number of area sources N is denoted by the vector $[N_1, N_2, \dots, N_c]$. Scenarios 1, 2, and 3 have five sources: agricultural sources, forests/wetlands, stream channels, WWTPs, and Qu'Appelle River. For Scenarios 4, 7, and 5, one source was added in each case for a total of six sources, where agricultural sources are split into fertilizers and manure for Scenarios 4 and 7, and forests/wetlands were split into forests and wetlands for Scenario 5. For Scenario 6, one source was added, which was urban area, for a total of six sources. For Scenario 8, one source was added for a total of six sources, where agricultural sources were split into being located in Canada and the United States (U.S.). For Scenario 9, two sources are added for a total of seven sources, which corresponds to the combination of splits of Scenarios 7 and 8. $[J_1, J_2, \dots, J_i]$ refers to the total number of stream reaches in monitored station catchments i ; ε_i represents a random and normally distributed error term. β_n is the export coefficient from a particular nutrient area source n (agriculture, agriculture fertilizer, agriculture manure, agriculture manure – Canada, stream channels...); S_n is a measurable quantity for the nutrient area source n , such as the area occupied by a specific land cover, or fertilizer and manure inputs to agricultural areas. β_{SC} represents the export coefficient for stream channels, where L represents the length of the stream, T selects only stream elements (not reservoirs), and Q_{bool} is a boolean that selects stream flows between 0.03 m³/s and 0.21 m³/s. β_{PT} is the export coefficient of wastewater treatment plants, and S_{PT} is the wastewater treatment plant input load. F_{QA} is the point source for the Qu'Appelle River, expressed as a load.

The catchment transfer function $R_{i,j}^{lwd}$ accounts for the land-to-water delivery process at the catchment scale, which can be modulated by a suite of factors (or land-surface characteristics) associated with drainage, such as catchment slope, areal extent of wetlands, soil hydraulic conductivity, precipitation, and soil bulk density:

$$R_{i,j}^{lwd} = e^{(\alpha(Z_j - Avg(Z_j)))} \quad (2)$$

where Z refers to a measurable quantity of a factor, α represents the land-to-water delivery coefficient for a factor, which is represented by precipitation in our model.

61 The attenuation in connected reservoirs and lakes $H_{i,j}^R$ followed:

$$H_{i,j}^R = \prod_{j=1}^j \frac{1}{\left(1 + \frac{k_R}{V_{h_i}}\right)} \quad (3)$$

62 Where instances of j included in multiplication were only applicable to reaches on the path of
 63 catchment j to water quality station, and where V_h represents hydraulic load (annual outflow rate
 64 divided by the surface lake/reservoir area, m/yr); k_R refers to the first order rate coefficient for
 65 net in-lake total phosphorus (TP) settling (m/yr).

66 Nutrient loading accumulation in nested station catchments is estimated by adding upstream
 67 loading⁴. During typical model calibration, downstream loads are computed after all upstream
 68 simulated loads are replaced with their measured loads to prevent errors from propagating
 69 downstream¹. This may overestimate the “true” performance of the model compared to if the
 70 model completely simulated all of the loads^{5,6}. In this study in all Bayesian simulations, we used
 71 the “unconditioned” approach to estimate nutrient loading in downstream nested station
 72 catchments, F_i^* , with simulated loading inputs F_i from upstream station catchments:

$$\mu_i = \ln(F_i^*) = \left[\ln \left\{ \sum_{j=1}^{J_i} \left(\sum_{n=1}^{N_c} \beta_n S_{n,j} \right) R_{i,j}^{lwd} + \beta_{SC} T L Q_{booi} + \beta_{PT} S_{PT} + F_{QA} H_{i,j}^R + \left(\sum_{i=1}^{U_i} F_i \right) H_i^R \right\} \right] + \varepsilon_i \quad (4)$$

73 where U_i is a number of upstream monitored station catchments, which flow into the station
 74 catchment i . H_i^R is delivery fraction between the upstream and downstream water quality
 75 stations. F_i^* and F_i are referred to as $Load_{SPARROWi}$ in the manuscript.

76 Several evaluation metrics were used to assess model performance and robustness. These metrics
 77 provide complementary diagnostics: Root Mean Square Error (RMSE) quantifies predictive
 78 accuracy by measuring the discrepancy between observed and modeled loads; Deviance
 79 Information Criterion (DIC) evaluates model performance within the Bayesian framework by
 80 balancing goodness-of-fit against model complexity; and Normalized Difference Index (NDI)
 81 quantifies the degree of prior-to-posterior updating, thereby indicating the extent to which
 82 parameter estimates are informed by the observational data during calibration.

83 RMSE was selected because it directly quantifies the magnitude of prediction error between
 84 observed and modelled total phosphorus (TP) loads and is widely used in watershed modeling
 85 and previous SPARROW applications to evaluate model predictive accuracy. It provides an
 86 intuitive measure of how closely predicted loads match monitoring observations in the units of
 87 the response variable and has been used to assess previous SPARROW models e.g., Robertson et
 88 al.⁴.

89 DIC was used because the model is estimated within a Bayesian framework using MCMC
 90 sampling. DIC is specifically designed for Bayesian model comparison and balances goodness of
 91 fit with model complexity, thereby penalizing models that achieve improved fit through
 92 excessive parameterization. This is particularly important in spatial watershed models where
 93 increasing the number of parameters or source partitions can otherwise lead to overfitting. Lower

94 DIC values indicate a better trade-off between goodness-of-fit and model complexity. For further
95 details and equations please refer to Spiegelhalter et al. ⁷.

96 NDI provides a diagnostic of parameter identifiability, with high values indicating parameters
97 strongly informed by the calibration data, and low values indicating parameters largely
98 constrained by prior knowledge. We assessed changes between priors and posteriors and
99 between Scenario 1 and posteriors.

100 Values near zero indicate limited data influence, while larger values indicate stronger updating.
101 For further details and equations please refer to Neumann et al. ⁸.

102 Calibration results

103 Table S1. Model Fit RMSE, R^2 , and DIC values for all scenarios. RMSE percent differences
 104 (representing improvement) for Scenarios 2-9 are calculated in comparison with Scenario 1.

Scenario	Description	RMSE (Calibrated)		RMSE (Predicted)		R^2 (Calibrated)		R^2 (Predicted)	DIC
		Value	Diff.	Value	Diff.	Value	Value		Value
Scenario 1	Benoy et al. ⁹	0.489		0.648		0.915		0.858	
Scenario 2	Variability on WWTPs	0.329	33%	0.572	12%	0.964		0.890	154.2
Scenario 3a-MARB	Priors from MARB	0.325	34%	0.639	1%	0.965		0.863	163.0
Scenario 3b-MI	Priors from Bayesian SPARROW	0.333	32%	0.573	12%	0.963		0.890	155.7
Scenario 3c-Extr	Priors from Non-Bayesian SPARROW (Extremes)	0.335	31%	0.564	13%	0.962		0.893	154.5
Scenario 3d-FS	Priors from Non-SPARROW Field Studies	0.331	32%	0.577	11%	0.963		0.888	154.3
Scenario 4	Split Fertilizers vs Manure	0.331	32%	0.556	14%	0.964		0.897	152.5
Scenario 5	Split Forests vs Wetlands	0.332	32%	0.574	11%	0.964		0.890	153.0
Scenario 6	Addition of Urban	0.321	34%	0.604	7%	0.966		0.879	156.7
Scenario 7	Scenario 3d-FS & Scenario 4	0.333	32%	0.562	13%	0.963		0.895	153.5
Scenario 8	Scenario 3d-FS & Country Hierarchy (Agr. Inputs)	0.346	29%	0.568	12%	0.960		0.893	156.0
Scenario 9	Scenario 7 & Country Hierarchy (Manure)	0.347	29%	0.555	14%	0.960		0.899	151.7

105

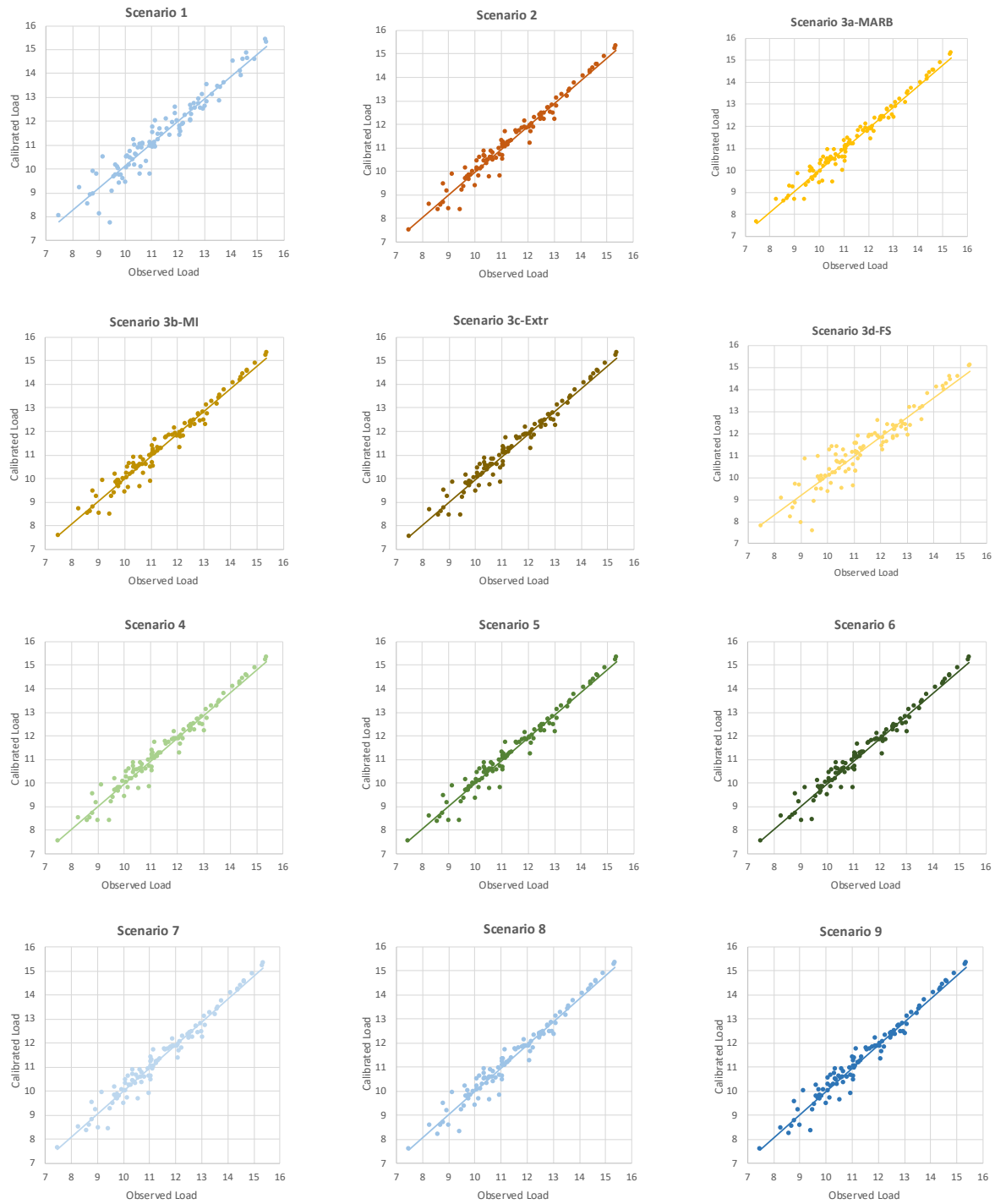


Figure S1. Calibration of loads in log scale – Benoy et al.⁹ model (Scenario 1) and Bayesian main scenarios (Scenarios 2-9).

Table S2. Observed vs calibrated loads (kg/yr) for Scenario 1, and the four key scenarios.

SPARROW ID	Station Catchment ID	Observed Load	Scenario 1 Cal. Load	Scenario 2 Cal. Load	Scenario 3d-FS Cal. Load	Scenario 4 Cal. Load	Scenario 8 Cal. Load
62906	1	5,941	7,479	5,464	5,763	5,146	5,417
62792	2	34,123	39,728	32,600	33,910	30,850	31,430
62659	3	1,767	3,164	1,959	2,146	1,988	2,068
62617	4	68,782	55,371	64,020	65,720	63,390	64,050
62457	5	7,516	18,071	10,110	10,670	10,090	10,330
62437	6	17,478	17,570	19,420	20,110	19,120	19,230
62412	7	76,599	96,472	78,330	80,620	76,220	76,070
62081	8	182,359	105,039	123,900	128,900	121,100	119,400
62004	9	145,573	229,565	145,100	145,300	144,300	142,900
69251	10	264,024	166,661	236,600	236,000	237,100	244,000
69020	11	12,355	2,326	4,639	4,679	4,788	4,263
68621	12	308,015	280,137	214,100	216,400	215,700	236,200
68543	13	455,645	311,572	213,400	213,700	216,000	247,000
68143	14	13,247	8,743	10,110	11,030	10,200	10,470
67406	15	22,195	12,849	12,840	13,520	13,470	13,050
66912	16	38,052	17,653	18,760	17,430	18,770	16,770
66893	17	62,911	54,845	39,690	38,630	39,500	36,990
66888	18	62,587	63,135	45,580	43,940	44,740	42,530
66271	19	17,330	12,395	16,130	16,330	16,140	16,390
65229	20	16,570	24,128	17,690	17,770	17,220	17,740
65221	21	18,643	16,993	19,470	19,290	18,640	19,350
65193	22	23,490	23,372	23,920	23,930	23,440	23,990
65190	23	43,584	65,974	50,890	51,330	51,090	51,160
66157	24	82,302	117,586	81,260	80,650	82,520	80,430
65171	25	62,691	55,811	65,660	67,150	65,910	66,550
65041	26	133,100	157,344	142,700	143,900	142,100	142,300
65016	27	191,335	145,388	178,000	179,900	177,700	176,700
64915	28	271,161	219,535	233,900	238,000	234,000	232,200
64911	29	428,174	273,066	279,300	280,500	271,600	268,600
64574	30	6,598	19,908	13,840	13,350	15,130	13,800
64545	31	20,388	15,063	23,260	22,830	25,080	22,830
64540	32	49,244	30,305	40,400	39,870	41,500	39,450
63575	33	61,609	127,572	88,030	91,320	92,200	88,520
64248	34	57,284	17,988	19,290	20,370	20,130	19,450
63487	35	145,076	297,614	192,100	194,800	200,700	191,200
61081	36	38,648	61,979	41,160	39,910	40,300	40,670
61069	37	32,430	40,525	43,110	39,690	40,230	40,010
61002	38	66,065	59,692	76,840	67,130	69,370	69,490
60999	39	66,493	68,194	72,430	67,740	69,430	68,960
60271	40	172,389	134,984	149,600	145,300	149,700	141,600
60997	41	55,450	66,497	60,200	57,900	58,890	59,360
60206	42	274,716	268,804	265,100	256,500	262,800	258,400
60182	43	258,367	320,516	258,000	248,900	257,000	246,100
59638	44	202,322	151,052	150,400	141,800	149,900	139,300
59602	45	299,412	346,923	274,000	266,600	278,400	266,900
60174	46	379,031	281,631	294,000	275,800	292,600	277,300
59573	47	725,368	711,022	587,000	552,000	606,900	549,000
59570	48	486,020	752,227	508,600	499,600	516,500	499,200
59556	49	587,049	501,165	584,000	581,300	591,900	577,400
59077	50	156,005	166,673	146,600	145,300	148,500	148,100
59073	51	69,379	166,985	131,100	130,800	133,400	134,300
59052	52	938,444	826,348	975,200	977,700	998,300	966,100
58159	53	30,745	76,665	45,380	45,040	46,180	46,760
57848	54	175,504	120,797	159,900	160,500	160,400	159,100
57847	55	103,419	178,404	135,400	133,300	139,200	136,000
57839	56	1,753,564	1,104,999	1,609,000	1,595,000	1,615,000	1,585,000
57537	57	6,529	7,783	6,281	6,793	6,537	6,457
57291	58	176,498	89,971	79,880	87,170	85,940	81,320
57257	59	1,326,990	2,039,085	1,371,000	1,367,000	1,433,000	1,346,000
57168	60	1,724,903	1,366,327	1,528,000	1,529,000	1,540,000	1,490,000
57167	61	1,917,699	2,157,984	1,851,000	1,848,000	1,864,000	1,825,000
56869	62	14,361	16,280	12,410	13,460	13,000	12,570
56847	63	23,459	37,063	27,360	30,220	29,980	28,550
56540	64	26,822	35,651	26,310	27,950	28,510	25,830
55756	65	35,812	53,636	38,270	41,700	41,780	39,260
55305	66	2,255,472	2,254,632	2,128,000	2,140,000	2,154,000	2,091,000
54198	67	25,247	25,818	19,630	18,330	19,650	17,310
54062	68	42,048	53,317	43,480	42,880	43,530	41,880
53575	69	2,209,305	2,869,000	2,198,000	2,200,000	2,249,000	2,193,000
53560	70	3,005,922	2,212,409	2,945,000	2,928,000	2,938,000	2,920,000
51912	71	87,491	96,641	85,100	89,710	84,330	91,320
51726	72	62,884	101,196	73,350	76,900	74,160	78,530
51724	73	64,419	63,199	79,760	83,320	81,340	86,510
50647	74	8,184	3,360	4,703	5,115	4,632	5,667
48920	75	16,743	22,268	17,750	19,030	17,990	24,250
34261	76	75,773	74,338	68,830	71,360	69,210	74,230
32553	77	43,516	63,545	38,800	33,420	38,120	33,700
31396	78	57,026	57,692	48,440	44,420	48,610	46,090
28909	79	263,378	173,859	212,000	208,000	207,700	208,700
22936	80	29,210	33,224	27,460	28,650	27,470	30,110
22710	81	350,829	348,687	327,700	329,200	327,600	331,600
22261	82	492,785	377,384	362,500	358,000	354,300	367,900
21854	83	414,766	506,285	374,300	375,700	371,400	381,000
17386	84	15,405	25,739	17,310	20,360	16,970	18,960
10445	85	30,971	60,818	35,230	35,650	35,580	37,660
10115	86	9,375	37,022	20,390	20,330	21,560	21,900
9196	87	5,349	5,116	4,557	4,431	4,631	3,866
9010	88	15,242	16,962	26,590	26,450	27,590	28,040
8542	89	23,767	38,482	35,150	35,870	36,400	36,720
8530	90	40,308	23,915	53,140	54,010	53,790	55,020
8035	91	3,856	10,236	5,640	5,777	5,215	5,461
7994	92	26,240	46,338	41,180	41,870	41,890	43,060
7985	93	30,997	26,666	53,910	55,920	55,250	56,490
7382	94	118,435	90,386	132,000	140,400	129,500	136,500
7292	95	106,587	120,720	121,100	128,300	121,200	124,900
1964	96	210,785	212,498	218,500	219,700	215,800	226,500
807	97	802,425	689,216	741,700	745,100	743,200	765,000
702	98	353,610	410,826	332,500	336,400	331,900	344,600
433	99	775,108	378,638	680,100	681,200	666,400	685,400
222	100	4,505,002	4,974,967	4,217,000	4,243,000	4,277,000	4,333,000
104	101	4,649,398	4,547,900	4,557,000	4,574,000	4,566,000	4,598,000

Priors vs posteriors

Table S3. Characteristics of Parameters prior probability distributions for SPARROW parameters in Red-Assiniboine River Basin (RARB) with descriptions, units, means, standard deviations (S.D.), and interpretations. Interpretation column shows that 90% of the probability lies between the two values shown.

Parameters/ Priors	Units	Mean	S.D.	Interpretation
<i>Sources (β_{FM}): Fertilizers and Manure</i>				
Scenario 1 ⁹	Fraction	0.011	0.004	0.005 - 0.018
Scenario 2 ⁹	Fraction	0.011	0.004	0.005 - 0.018
Scenario 3a-MARB ¹⁰	Fraction	0.027	0.005	0.019 - 0.035
Scenario 3b-MI ^{† 4,9-11}	Fraction	0.026	0.005	0.018 - 0.034
Scenario 3c-Extr ^{‡ 4,9-11}	Fraction	0.050	0.050	0.003 - 0.040
Scenario 3d-FS ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
Scenario 4-Fertilizers ⁹	Fraction	0.011	0.004	0.005 - 0.018
-Manure ⁹	Fraction	0.011	0.004	0.005 - 0.018
Scenario 5 ⁹	Fraction	0.011	0.004	0.005 - 0.018
Scenario 6 ⁹	Fraction	0.011	0.004	0.005 - 0.018
Scenario 7-Fertilizers ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
-Manure ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
Scenario 8 -Canada ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
-U.S. ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
Scenario 9-Fertilizers ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
-Manure-Canada ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
-Manure-U.S. ^{§ 12,13}	Fraction	0.050	0.030	0.011 - 0.100
<i>Sources (β_{FW}): Forests and Wetlands</i>				
Scenario 1 ⁹	kg/km ² /yr	6.02	3.2	1 - 11
Scenario 2 ⁹	kg/km ² /yr	6.02	3.2	1 - 11
Scenario 3a-MARB ¹⁰	kg/km ² /yr	4.22	1.3	2 - 6
Scenario 3b-MI ¹⁴	kg/km ² /yr	6.00	4.3	1 - 13
Scenario 3c-Extr ^{‡ 9,10}	kg/km ² /yr	10.00	10.0	2 - 27
Scenario 3d-FS ¹⁵	kg/km ² /yr	7.18	3.8	2 - 14
Scenario 4 ⁹	kg/km ² /yr	6.02	3.2	1 - 11
Scenario 5-Forests ⁹	kg/km ² /yr	6.02	3.2	1 - 11
-Wetlands ⁹	kg/km ² /yr	6.02	3.2	1 - 11
Scenario 6 ⁹	kg/km ² /yr	6.02	3.2	1 - 11
Scenario 7 ¹⁵	kg/km ² /yr	7.18	3.8	2 - 14
Scenario 8 ¹⁵	kg/km ² /yr	7.18	3.8	2 - 14
Scenario 9 ¹⁵	kg/km ² /yr	7.18	3.8	2 - 14
<i>Sources (β_U): Urban Areas including cottages and roads</i>				
Scenario 6 ¹⁰	kg/km ² /yr	103	23	65 - 141

Parameters/ Priors	Units	Mean	S.D.	Interpretation
<i>Sources (β_{sc}): Stream Channels</i>				
Scenario 1 ⁹	kg/m/yr	0.070	0.04	0.02 - 0.14
Scenario 2 ⁹	kg/m/yr	0.070	0.04	0.02 - 0.14
Scenario 3a-MARB ¹⁰	kg/m/yr	0.093	0.02	0.06 - 0.13
Scenario 3b-MI ^{† 9,10}	kg/m/yr	0.081	0.03	0.03 - 0.13
Scenario 3c-Extr ^{‡ 9,10}	kg/m/yr	0.200	0.20	0.03 - 0.54
Scenario 3d-FS ^{§ 9,10}	kg/m/yr	0.082	0.02	0.05 - 0.11
Scenario 4 ⁹	kg/m/yr	0.070	0.04	0.02 - 0.14
Scenario 5 ⁹	kg/m/yr	0.070	0.04	0.02 - 0.14
Scenario 6 ⁹	kg/m/yr	0.070	0.04	0.02 - 0.14
Scenario 7 ^{§ 9,10}	kg/m/yr	0.082	0.02	0.05 - 0.11
Scenario 8 ^{§ 9,10}	kg/m/yr	0.082	0.02	0.05 - 0.11
Scenario 9 ^{§ 9,10}	kg/m/yr	0.082	0.02	0.05 - 0.11
<i>Sources (β_{PT}): Wastewater Treatment Plants</i>				
Scenario 1 ⁹	Fraction	1.000	—	—
Scenario 2 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 3a-MARB ¹⁰	Fraction	1.000	0.19	0.7 - 1.4
Scenario 3b-MI ^{8,16}	Fraction	1.000	0.30	0.5 - 1.5
Scenario 3c-Extr ⁹	Fraction	1.000	1.00	0.2 - 2.7
Scenario 3d-FS ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 4 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 5 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 6 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 7 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 8 ⁹	Fraction	1.000	0.06	0.9 - 1.1
Scenario 9 ⁹	Fraction	1.000	0.06	0.9 - 1.1
<i>Land-to-water delivery (a_{PPT})</i>				
Scenario 1 ⁹	mm/yr	0.0093	0.002	0.006 - 0.013
Scenario 2 ⁹	mm/yr	0.0093	0.002	0.006 - 0.013
Scenario 3a-MARB ¹⁰	mm/yr	0.0020	0.001	0.001 - 0.004
Scenario 3b-MI ^{† 9,10}	mm/yr	0.0060	0.002	0.003 - 0.009
Scenario 3c-Extr ^{‡ 9,10}	mm/yr	0.0100	0.010	0.002 - 0.027
Scenario 3d-FS ^{§ 9,10}	mm/yr	0.0055	0.003	0.001 - 0.010
Scenario 4 ⁹	mm/yr	0.0093	0.002	0.006 - 0.013
Scenario 5 ⁹	mm/yr	0.0093	0.002	0.006 - 0.013
Scenario 6 ⁹	mm/yr	0.0093	0.002	0.006 - 0.013
Scenario 7 ^{§ 9,10}	mm/yr	0.0055	0.003	0.001 - 0.010
Scenario 8 ^{§ 9,10}	mm/yr	0.0055	0.003	0.001 - 0.010
Scenario 9 ^{§ 9,10}	mm/yr	0.0055	0.003	0.001 - 0.010
<i>Losses in reservoirs (k_R)</i>				
Scenario 1 ⁹	m/yr	5.73	2.6	2 - 10
Scenario 2 ⁹	m/yr	5.73	2.6	2 - 10
Scenario 3a-MARB ¹⁰	m/yr	16.77	3.3	11 - 22
Scenario 3b-MI ¹⁴	m/yr	12.84	4.8	5 - 21
Scenario 3c-Extr ^{‡ 4,9-11}	m/yr	20.00	20.0	3 - 54
Scenario 3d-FS ^{§ 4,9-11,17}	m/yr	8.42	4.3	2 - 16
Scenario 4 ⁹	m/yr	5.73	2.6	2 - 10
Scenario 5 ⁹	m/yr	5.73	2.6	2 - 10
Scenario 6 ⁹	m/yr	5.73	2.6	2 - 10
Scenario 7 ^{§ 4,9-11,17}	m/yr	8.42	4.3	2 - 16
Scenario 8 ^{§ 4,9-11,17}	m/yr	8.42	4.3	2 - 16
Scenario 9 ^{§ 4,9-11,17}	m/yr	8.42	4.3	2 - 16

Robertson et al.⁴ (Binational Great Lakes); Neumann et al.^{8,16} (Georgian Bay); Benoy et al.⁹ (Binational RARB); Robertson et al.¹⁰ (Mississippi/Atchafalaya (MARB)); Robertson et al.¹¹ (MRB3 (USA Laurentian Great Lakes)); Rattan et al.¹² (Northern Great Plains streams); Harmel et al.¹³ (MANAGE database); Kim et al. (Bay of Quinte)¹⁴; Bourne et al.¹⁵ (Manitoba streams); Baulch et al.¹⁷ (Northern Great Plains surface waters); § Based on distributions from published values; † Average of published values; ‡ Extreme values based on published values.

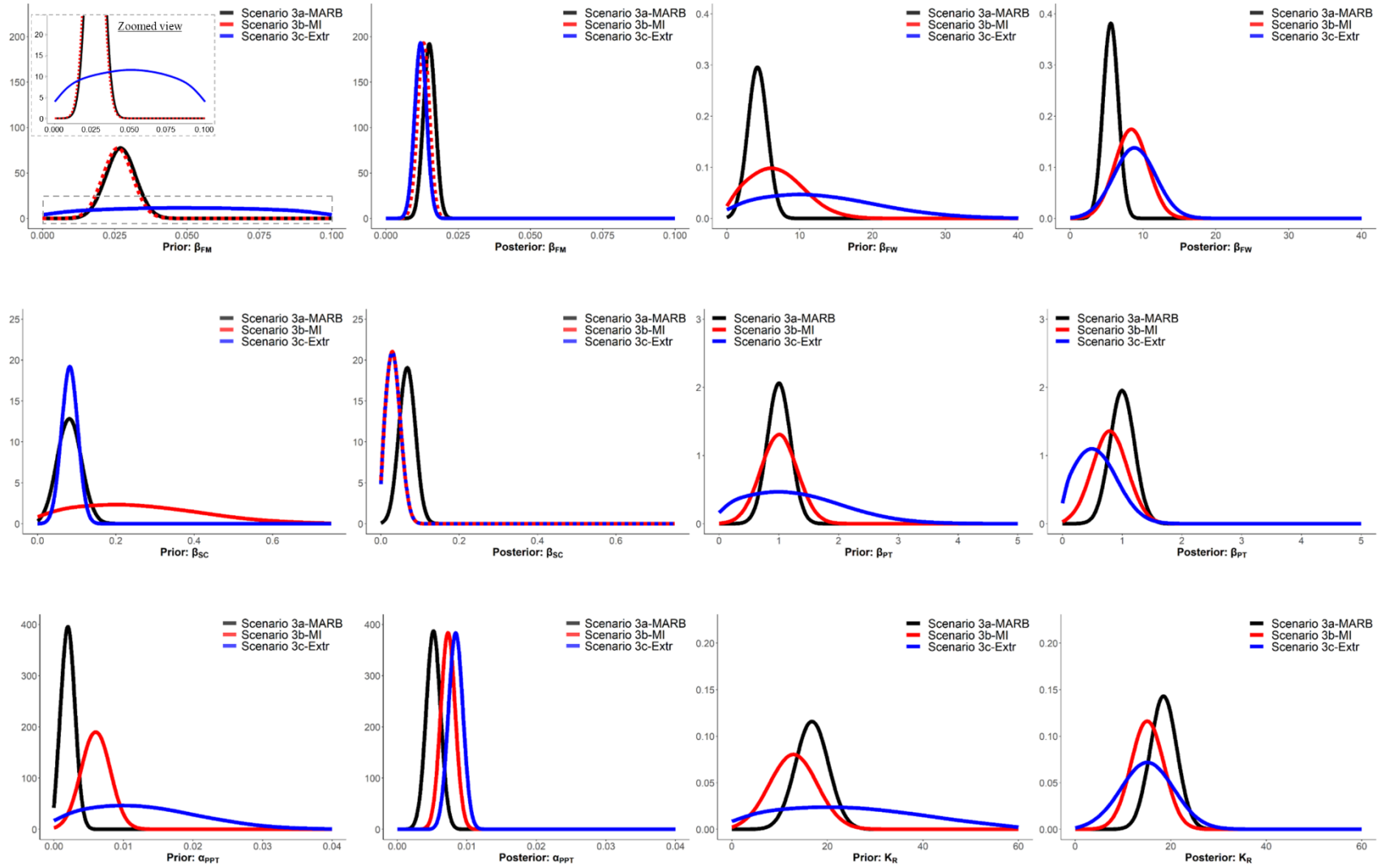
Priors informed by field studies were used for: (1) Fertilizers and Manure and (2) Forests and Wetlands. For field-based estimates of fertilizer and manure nutrient exports, we used the publicly available MANAGE (Measured Annual Nutrient Loads from Agricultural Environments) database described by Harmel et al.¹³. This database compiles field-scale annual nitrogen (N) and phosphorus (P) runoff measurements from agricultural sites across North America. Runoff volumes are measured using flumes, weirs, or flow gages, and water samples are analyzed in laboratories to determine total phosphorus (TP) and other nutrient concentrations. The database reports annual nutrient yields (kg/ha/yr) along with associated precipitation, runoff, and erosion data. Importantly, MANAGE provides measured export coefficients derived from monitored field runoff rather than coefficients estimated from regression analyses or watershed-scale models. For this study, relevant field sites in Manitoba and Minnesota were identified and converted to TP export ratios (e.g., percent of TP exported). Means and standard deviations were calculated and used as priors. These estimates were compared with fertilizer application rates reported in Rattan et al.¹², which evaluated seven small sub-watersheds in the Red River Valley, Manitoba, Canada, to assess consistency between observed export rates and reported nutrient inputs.

Empirical measurements of TP runoff from forests and wetlands were drawn from Bourne et al.¹⁵. Manitoba-specific TP loading coefficients were derived from Chambers and Dale¹⁸ to represent pastureland and forest sources. Chambers and Dale¹⁸ compiled a database of 277 forest export coefficients across North America and reported averages and standard deviations for (1) North American watersheds, (2) Canadian watersheds, and (3) Northern/Central Alberta. Their study synthesized previously published field investigations rather than conducting new measurements.

Table S4. Posteriors: Comparison of the posterior SPARROW parameter estimates for Red-Assiniboine River Basin (RARB).

Parameters/ Posteriors	Units	Mean	S.D.	Interpretation
<i>Sources (β_{FM}): Fertilizers and Manure</i>				% of TP is exported
Scenario 1	Fraction	0.011	0.004	1.1%
Scenario 2	Fraction	0.010	0.001	1.0%
Scenario 3a-MARB	Fraction	0.015	0.002	1.5%
Scenario 3b-MI	Fraction	0.013	0.002	1.3%
Scenario 3c-Extr	Fraction	0.012	0.002	1.2%
Scenario 3d-FS	Fraction	0.009	0.001	0.9%
Scenario 4-Fertilizers	Fraction	0.012	0.002	1.2%
-Manure	Fraction	0.005	0.003	0.5%
Scenario 5	Fraction	0.010	0.002	1.0%
Scenario 6	Fraction	0.008	0.002	0.8%
Scenario 7-Fertilizers	Fraction	0.011	0.002	1.1%
-Manure	Fraction	0.003	0.003	0.3%
Scenario 8 -Canada	Fraction	0.014	0.003	1.4%
-U.S.	Fraction	0.008	0.002	0.8%
Scenario 9-Fertilizers	Fraction	0.010	0.002	1.0%
-Manure-Canada	Fraction	0.035	0.020	3.5%
-Manure-U.S.	Fraction	0.002	0.002	0.2%
<i>Sources (β_{FW}): Forests and Wetlands</i>				kg of TP is exported per km ² per year
Scenario 1	kg/km ² /yr	6.02	3.2	6
Scenario 2	kg/km ² /yr	6.65	1.8	7
Scenario 3a-MARB	kg/km ² /yr	5.56	1.0	6
Scenario 3b-MI	kg/km ² /yr	8.39	2.2	8
Scenario 3c-Extr	kg/km ² /yr	8.86	2.8	9
Scenario 3d-FS	kg/km ² /yr	7.31	2.0	7
Scenario 4	kg/km ² /yr	6.83	1.7	7
Scenario 5-Forests	kg/km ² /yr	5.40	2.7	5
-Wetlands	kg/km ² /yr	7.14	2.0	7
Scenario 6	kg/km ² /yr	6.35	1.9	6
Scenario 7	kg/km ² /yr	7.33	1.9	7
Scenario 8	kg/km ² /yr	6.52	1.9	7
Scenario 9	kg/km ² /yr	6.63	1.8	7
<i>Sources (β_U): Urban Areas including cottages and roads</i>				kg of TP is exported per km ² per year
Scenario 6	kg/km ² /yr	65	22	65

Parameters/ Posteriors		Units	Mean	S.D.	Interpretation
<i>Sources (β_{sc}): Stream Channels</i>					TP export per linear unit of stream per year
Scenario 1	kg/m/yr	0.070	0.04		0.07
Scenario 2	kg/m/yr	0.030	0.02		0.03
Scenario 3a-MARB	kg/m/yr	0.067	0.02		0.07
Scenario 3b-MI	kg/m/yr	0.029	0.02		0.03
Scenario 3c-Extr	kg/m/yr	0.029	0.02		0.03
Scenario 3d-FS	kg/m/yr	0.062	0.02		0.06
Scenario 4	kg/m/yr	0.023	0.02		0.02
Scenario 5	kg/m/yr	0.031	0.02		0.03
Scenario 6	kg/m/yr	0.027	0.02		0.03
Scenario 7	kg/m/yr	0.056	0.02		0.06
Scenario 8	kg/m/yr	0.059	0.02		0.06
Scenario 9	kg/m/yr	0.053	0.01		0.05
<i>Sources (β_{pt}): Wastewater Treatment Plants</i>					Percentage (%) of TP from WWTP is exported
Scenario 1	Fraction	1.000	–		100.00%
Scenario 2	Fraction	0.992	0.06		99.19%
Scenario 3a-MARB	Fraction	0.997	0.20		99.70%
Scenario 3b-MI	Fraction	0.791	0.29		79.06%
Scenario 3c-Extr	Fraction	0.496	0.40		49.61%
Scenario 3d-FS	Fraction	0.992	0.06		99.16%
Scenario 4	Fraction	0.991	0.06		99.12%
Scenario 5	Fraction	0.990	0.06		99.02%
Scenario 6	Fraction	0.991	0.06		99.06%
Scenario 7	Fraction	0.990	0.06		99.00%
Scenario 8	Fraction	0.990	0.06		99.02%
Scenario 9	Fraction	0.988	0.06		98.77%
<i>Land-to-water delivery (a_{ppt})</i>					Precipitation enhanced TP delivery by %
Scenario 1	mm/yr	0.0093	0.002		44%
Scenario 2	mm/yr	0.0091	0.001		42%
Scenario 3a-MARB	mm/yr	0.0052	0.001		14%
Scenario 3b-MI	mm/yr	0.0073	0.001		27%
Scenario 3c-Extr	mm/yr	0.0084	0.001		36%
Scenario 3d-FS	mm/yr	0.0090	0.001		41%
Scenario 4	mm/yr	0.0093	0.001		44%
Scenario 5	mm/yr	0.0092	0.001		43%
Scenario 6	mm/yr	0.0083	0.001		35%
Scenario 7	mm/yr	0.0093	0.001		44%
Scenario 8	mm/yr	0.0102	0.001		54%
Scenario 9	mm/yr	0.0103	0.001		56%
<i>Losses in reservoirs (k_R)</i>					TP settles at about X m/year
Scenario 1	m/yr	5.73	2.6		6
Scenario 2	m/yr	8.42	1.9		8
Scenario 3a-MARB	m/yr	18.48	2.7		18
Scenario 3b-MI	m/yr	15.10	3.3		15
Scenario 3c-Extr	m/yr	15.06	5.4		15
Scenario 3d-FS	m/yr	12.22	2.8		12
Scenario 4	m/yr	8.40	1.9		8
Scenario 5	m/yr	8.26	1.9		8
Scenario 6	m/yr	8.30	1.9		8
Scenario 7	m/yr	12.16	2.8		12
Scenario 8	m/yr	13.33	3.0		13
Scenario 9	m/yr	13.04	2.9		13



160

161

Figure S2. Probability density plots for parameter priors and posteriors of Scenarios 3a-MARB, 3b-MI, and 3c-Extr.

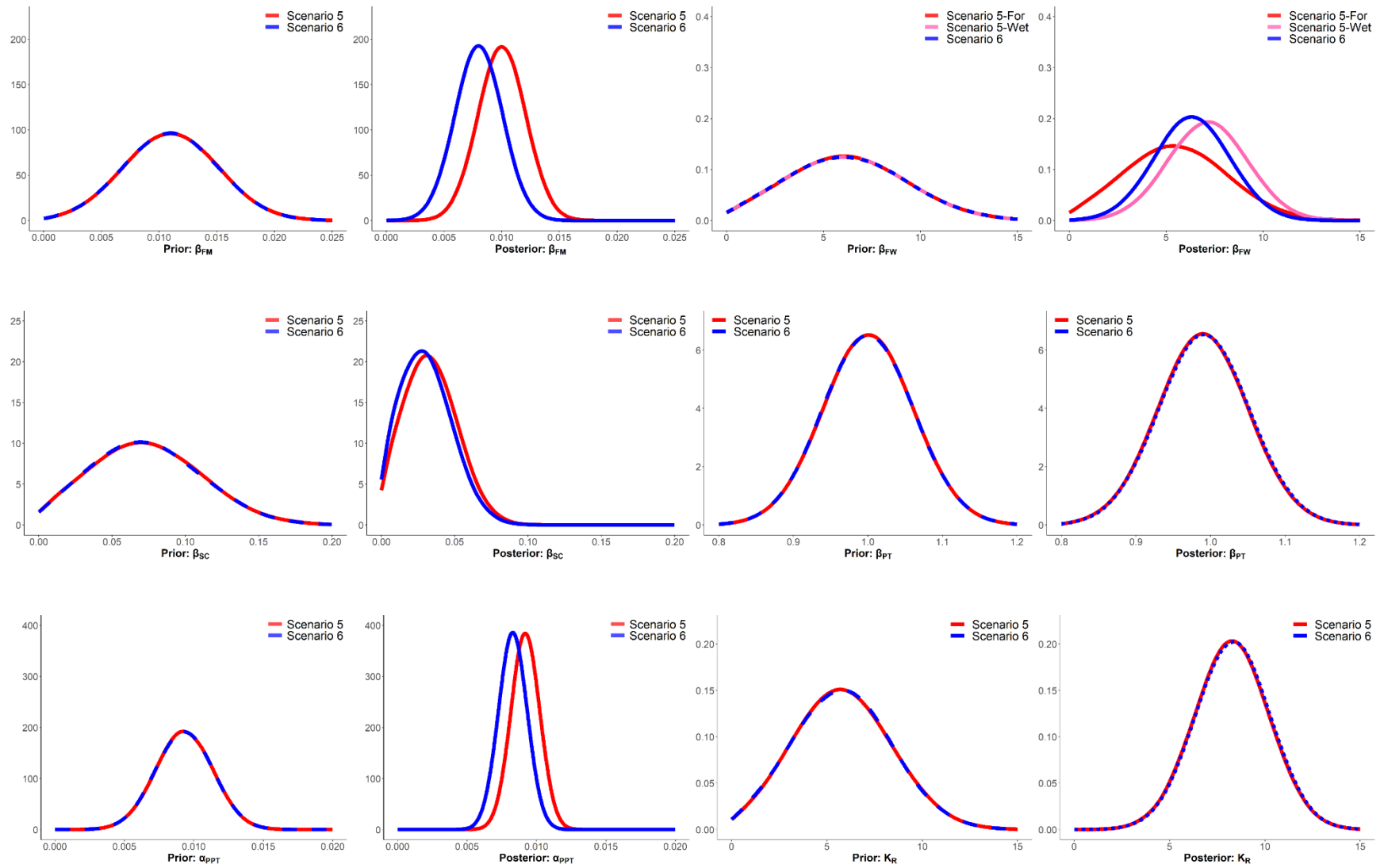
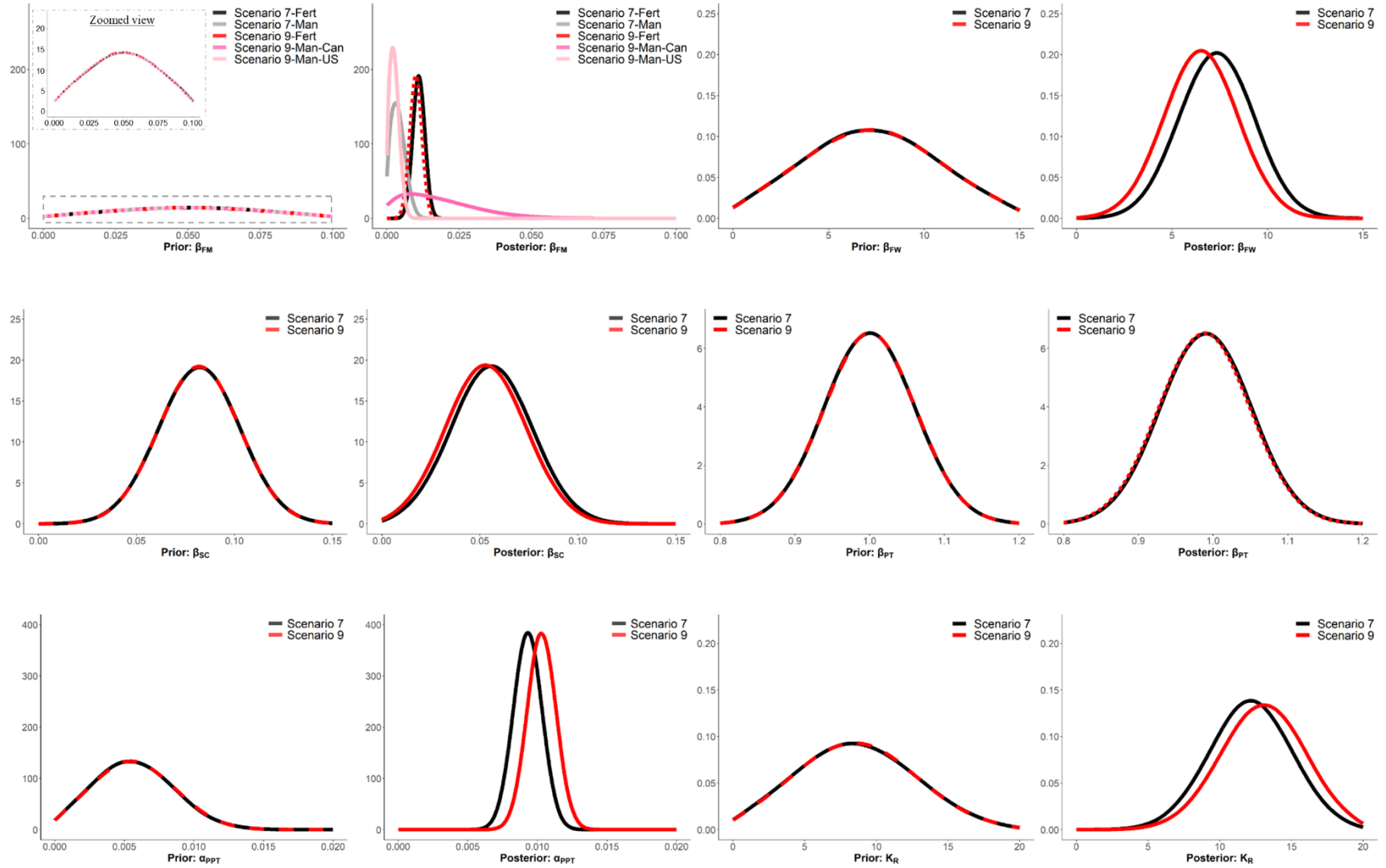


Figure S3. Probability density plots for parameter priors and posteriors of Scenarios 5 and 6.



164

165 Figure S4. Probability density plots for parameter priors and posteriors of Scenarios 7 and 9.

Loads and Yields for all non-key scenarios

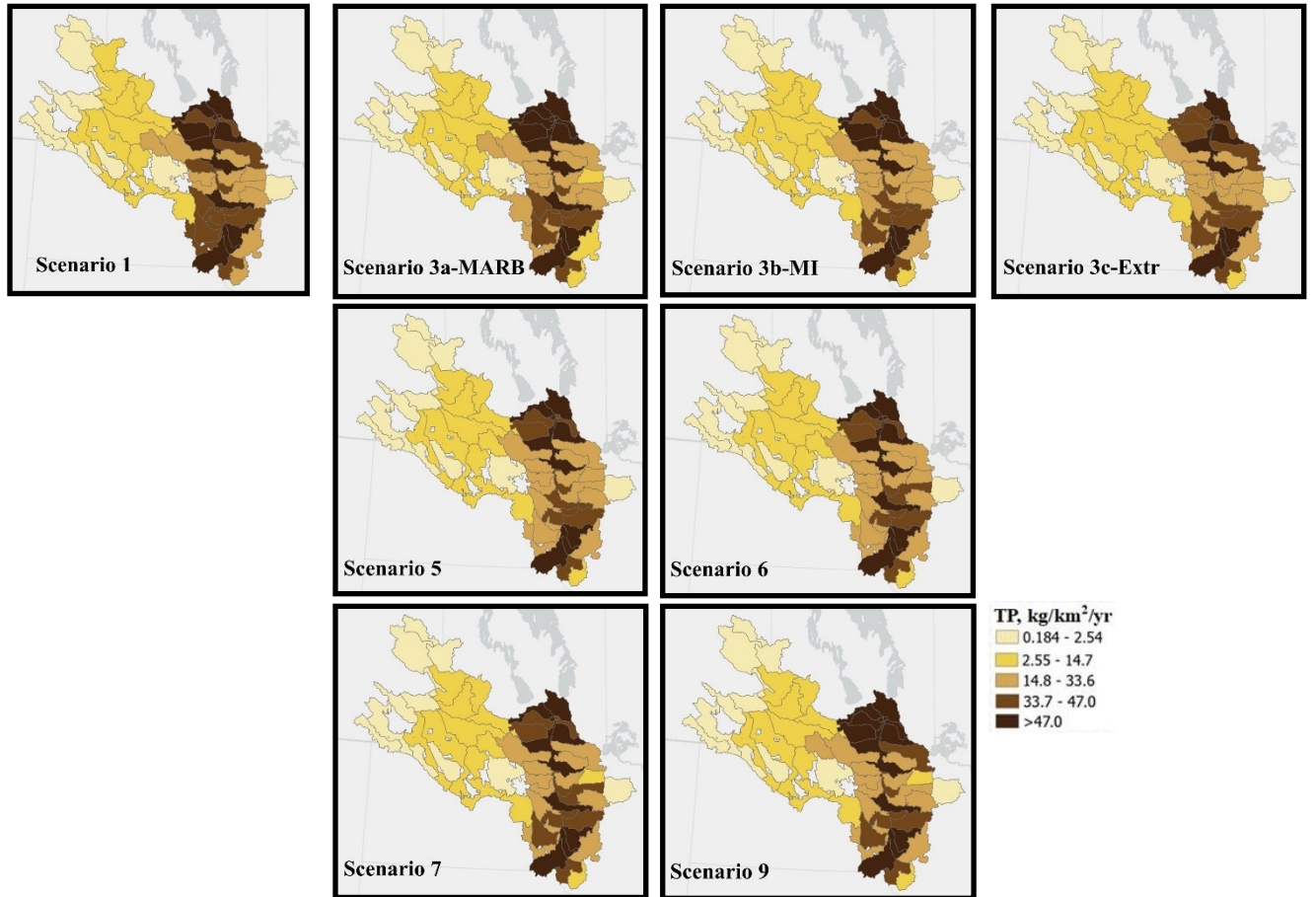
Table S5. Summary of loads and yields per major basin, province /state, and country for non-key scenarios.

		Aggregated (Non-Decayed) TP Load (MT/yr)			Delivered Aggregated TP Load (MT/yr)			Aggregated TP Yield (kg/km ²)			Delivered Aggregated TP Yield (kg/km ²)			TP Delivery Fraction		
		Scen 3a-MARB	Scen 3b-MI	Scen 3c-Extr	Scen 3a-MARB	Scen 3b-MI	Scen 3c-Extr	Scen 3a-MARB	Scen 3b-MI	Scen 3c-Extr	Scen 3a-MARB	Scen 3b-MI	Scen 3c-Extr	Scen 3a-MARB	Scen 3b-MI	Scen 3c-Extr
Major (River) Basin	Red	5,283	4,992	4,828	4,222	3,991	3,810	43.0	40.7	39.3	34.4	32.5	31.0	0.80	0.80	0.79
	Souris	841	603	518	182	141	122	15.4	11.1	9.5	3.3	2.6	2.2	0.22	0.23	0.24
	Assiniboine	1,082	825	716	362	286	241	25.4	19.4	16.8	8.5	6.7	5.6	0.33	0.35	0.34
Province /State	Manitoba	2,738	2,249	1,955	2,028	1,711	1,476	45.7	37.6	32.7	33.9	28.6	24.7	0.74	0.76	0.75
	North Dakota	2,062	1,832	1,756	1,483	1,379	1,336	29.7	26.4	25.3	21.4	19.9	19.2	0.72	0.75	0.76
	Minnesota	1,548	1,684	1,769	1,144	1,222	1,263	34.0	37.0	38.9	25.1	26.8	27.7	0.74	0.73	0.71
	Saskatchewan	791	582	507	85	76	67	18.1	13.3	11.6	1.9	1.7	1.5	0.11	0.13	0.13
	South Dakota	67	74	76	26	30	31	49.0	53.7	55.2	18.9	21.6	22.3	0.39	0.40	0.40
Country	Canada	3,529	2,831	2,462	2,113	1,787	1,543	34.1	27.3	23.8	20.4	17.3	14.9	0.60	0.63	0.63
	US	3,677	3,589	3,600	2,653	2,630	2,629	31.6	30.8	30.9	22.8	22.6	22.6	0.72	0.73	0.73
RARB		7,206	6,420	6,063	4,766	4,417	4,172	32.8	29.2	27.6	21.7	20.1	19.0	0.66	0.69	0.69

		Aggregated (Non-Decayed) TP Load (MT/yr)		Delivered Aggregated TP Load (MT/yr)		Aggregated TP Yield (kg/km ²)		Delivered Aggregated TP Yield (kg/km ²)		TP Delivery Fraction	
		Scen 5	Scen 6	Scen 5	Scen 6	Scen 5	Scen 6	Scen 5	Scen 6	Scen 5	Scen 6
Major (River) Basin	Red	4,725	4,759	3,916	3,934	38.5	38.8	32.0	32.1	0.83	0.83
	Souris	460	473	128	135	8.4	8.7	2.4	2.5	0.28	0.29
	Assiniboine	681	656	277	273	16.0	15.4	6.5	6.4	0.41	0.42
Province /State	Manitoba	1,971	1,871	1,601	1,519	33.0	31.3	26.7	25.4	0.81	0.81
	North Dakota	1,678	1,773	1,333	1,392	24.1	25.6	19.2	20.0	0.79	0.79
	Minnesota	1,680	1,750	1,262	1,316	36.9	38.5	27.8	29.0	0.75	0.75
	Saskatchewan	465	432	90	85	10.6	9.9	2.1	2.0	0.19	0.20
	South Dakota	72	64	34	30	52.8	47.0	25.1	21.8	0.48	0.46
Country	Canada	2,436	2,303	1,691	1,604	23.5	22.2	16.3	15.5	0.69	0.70
	US	3,430	3,586	2,630	2,738	29.5	30.9	22.6	23.5	0.77	0.76
RARB		5,866	5,888	4,322	4,342	26.6	26.8	19.6	19.7	0.74	0.74

		Aggregated (Non-Decayed) TP Load (MT/yr)		Delivered Aggregated TP Load (MT/yr)		Aggregated TP Yield (kg/km ²)		Delivered Aggregated TP Yield (kg/km ²)		TP Delivery Fraction	
		Scen 7	Scen 9	Scen 7	Scen 9	Scen 7	Scen 9	Scen 7	Scen 9	Scen 7	Scen 9
Major (River) Basin	Red	5,018	5,588	4,087	4,593	40.9	45.5	33.3	37.5	0.81	0.82
	Souris	514	578	128	138	9.4	10.6	2.4	2.5	0.25	0.24
	Assiniboine	757	864	282	325	17.8	20.3	6.6	7.6	0.37	0.38
Province /State	Manitoba	2,092	2,840	1,642	2,257	35.0	47.5	27.4	37.7	0.79	0.79
	North Dakota	1,894	1,813	1,472	1,415	27.2	26.1	21.1	20.3	0.78	0.78
	Minnesota	1,724	1,730	1,281	1,274	37.9	38.1	28.1	28.0	0.74	0.74
	Saskatchewan	522	590	78	84	11.9	13.5	1.8	1.9	0.15	0.14
	South Dakota	58	57	25	25	42.2	41.8	18.4	18.0	0.44	0.43
Country	Canada	2,614	3,430	1,720	2,341	25.2	33.2	16.6	22.6	0.66	0.68
	US	3,676	3,600	2,778	2,715	31.6	31.0	23.8	23.3	0.76	0.75
RARB		6,290	7,030	4,498	5,056	28.6	32.0	20.4	23.0	0.72	0.72

172 **Delivered aggregated yields maps per sub-sub-drainage area for all non-key scenarios**



173 Figure S5. Delivered aggregated yields for sub-sub-drainage areas for non-key scenarios.

Hotspots – Canada and U.S. – Yields per Sub-Sub-Drainage Area

Table S6. Canadian delivered yield hotspot values (kg/km²/yr). Rows sorted using Scenario 8.

Major Basin	Sub-Sub-Drainage Area	Scenario 1	Scenario 2	Scenario 3d-FS	Scenario 4	Scenario 8
Red	Lower Red	89.5	87.3	86.8	88.2	97.6
Red	Rat and Tourond	62.8	58.1	56.8	51.3	82.6
Red	Upper Red (Canada)	65.2	59.4	58.7	62.9	71.6
Red	Seine	47.0	43.0	41.6	37.4	59.4
Assiniboine	Lower Assiniboine - Mouth	51.2	48.8	49.7	49.1	58.4
Red	Morris	47.5	40.5	42.0	43.3	58.4
Red	La Salle	43.3	37.4	38.3	39.5	52.6

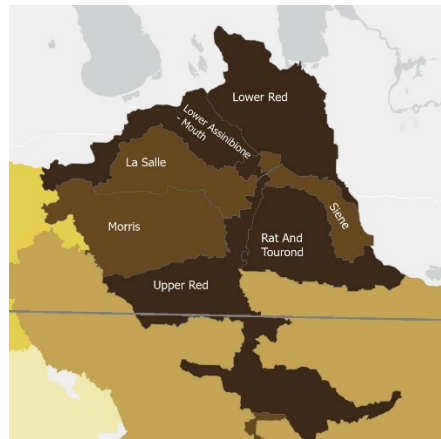


Figure S6. Delivered yield hotspots in Canada. Colors represent the average yield across the four key scenarios, with darker brown indicating higher yields. The Canada–U.S. border is indicated by the grey line.

Table S7. U.S. delivered yield hotspot values (kg/km²/yr). Rows sorted using Scenario 8.

Major Basin	Sub-Sub-Drainage Area	Scenario 1	Scenario 2	Scenario 3d-FS	Scenario 4	Scenario 8
Red	Upper Red (US)	96.8	89.8	89.3	93.0	90.3
Red	Buffalo	69.3	61.8	60.7	61.5	62.9
Red	Western Wild Rice	66.9	58.8	55.1	62.6	55.6
Red	Sand Hill-Wilson	50.3	44.9	46.5	48.0	45.0
Red	Eastern Wild Rice	45.2	39.9	39.7	40.2	40.4
Red	Bois De Sioux	35.2	40.4	39.0	42.5	38.8
Red	Elm Marsh	44.0	37.7	38.2	42.0	36.8
Red	Grand Marais-Red	40.3	35.0	36.1	37.0	34.1

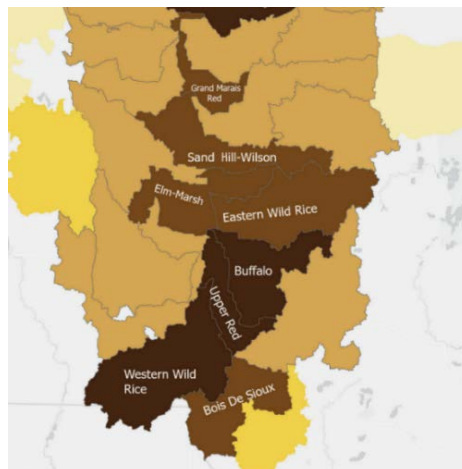


Figure S7. Delivered yield hotspots in the US only. Colors represent the average yield across the four key scenarios, with darker brown indicating higher yields.

CODA Example

Table S8. CODA extract for chain 1 – iterations 10,010 to 10,180 (with thin 10 therefore below 1,001 to 1,018) for all coefficients, sigma, and log scale loads.

[illegible]

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