

Radial-velocity discovery of a second planet in the TOI-1338/BEBOP-1 circumbinary system

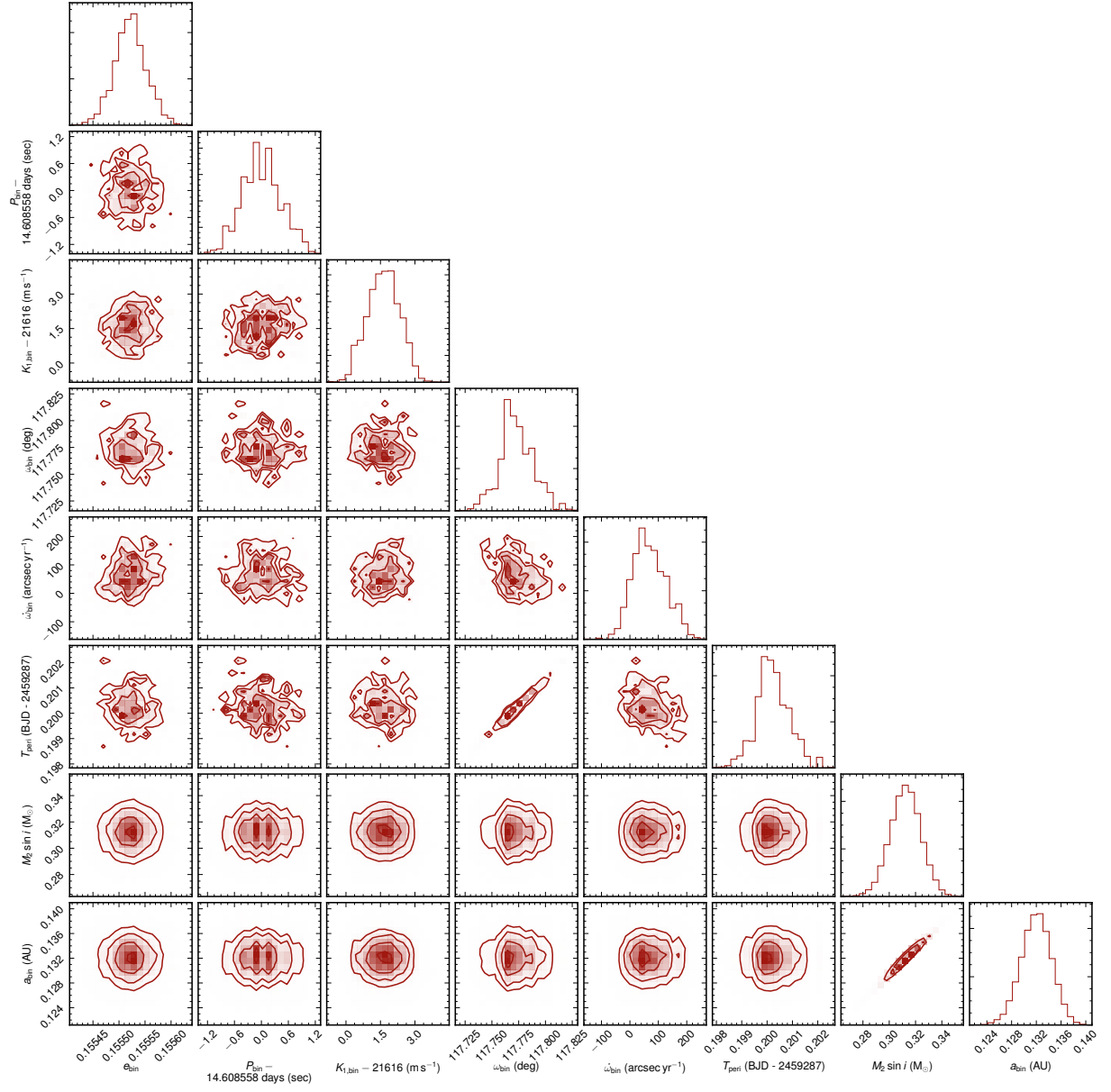
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Supplementary Information

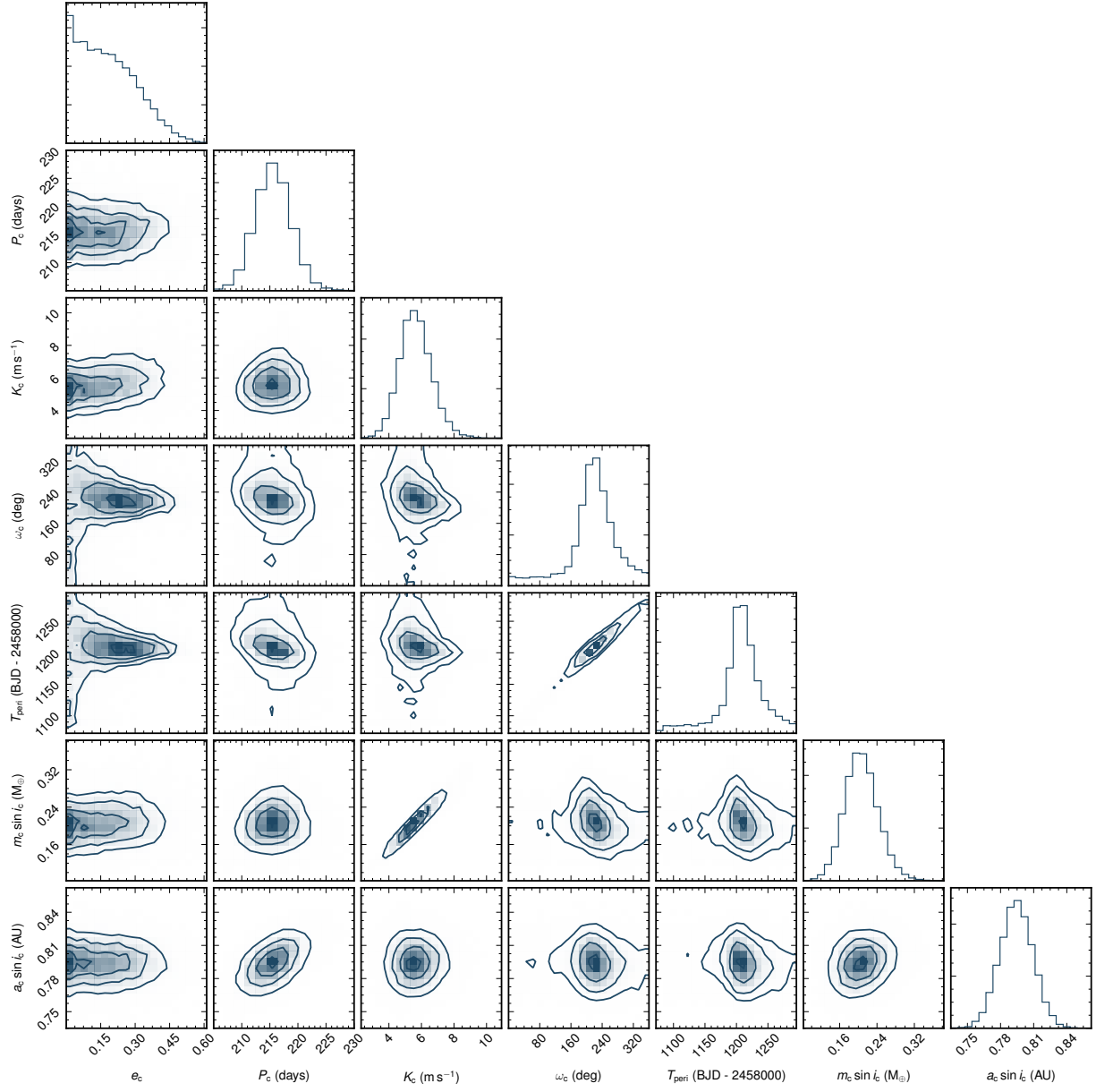
Supplementary Table 1: Prior distributions for the binary and planetary RV model signals in kima

Parameter	Unit	Prior distribution	
		Binary	Planet
N_p		1	$\mathcal{U}(0, 3)$
P	days	$\mathcal{U}(14.61 \pm 0.01)$	$\mathcal{LU}(4 \times P_{\text{bin}}, 2000)$
K	m s^{-1}	$\mathcal{U}(21617 \pm 15)$	$\mathcal{MLU}(0.1, 100)$
e		$\mathcal{U}(0.1556 \pm 0.001)$	$\mathcal{K}(0.867, 3.03)$
ϕ		$\mathcal{U}(0, 2\pi)$	
ω		$\mathcal{U}(0, 2\pi)$	
$\dot{\omega}_{\text{bin}}$	arcsec year^{-1}	$\mathcal{N}(0, 1000)$	-
σ_{jit}	m s^{-1}	$\mathcal{LU}(0.01, 67)$	
γ	m s^{-1}	$\mathcal{U}(30761 \pm 100)$	

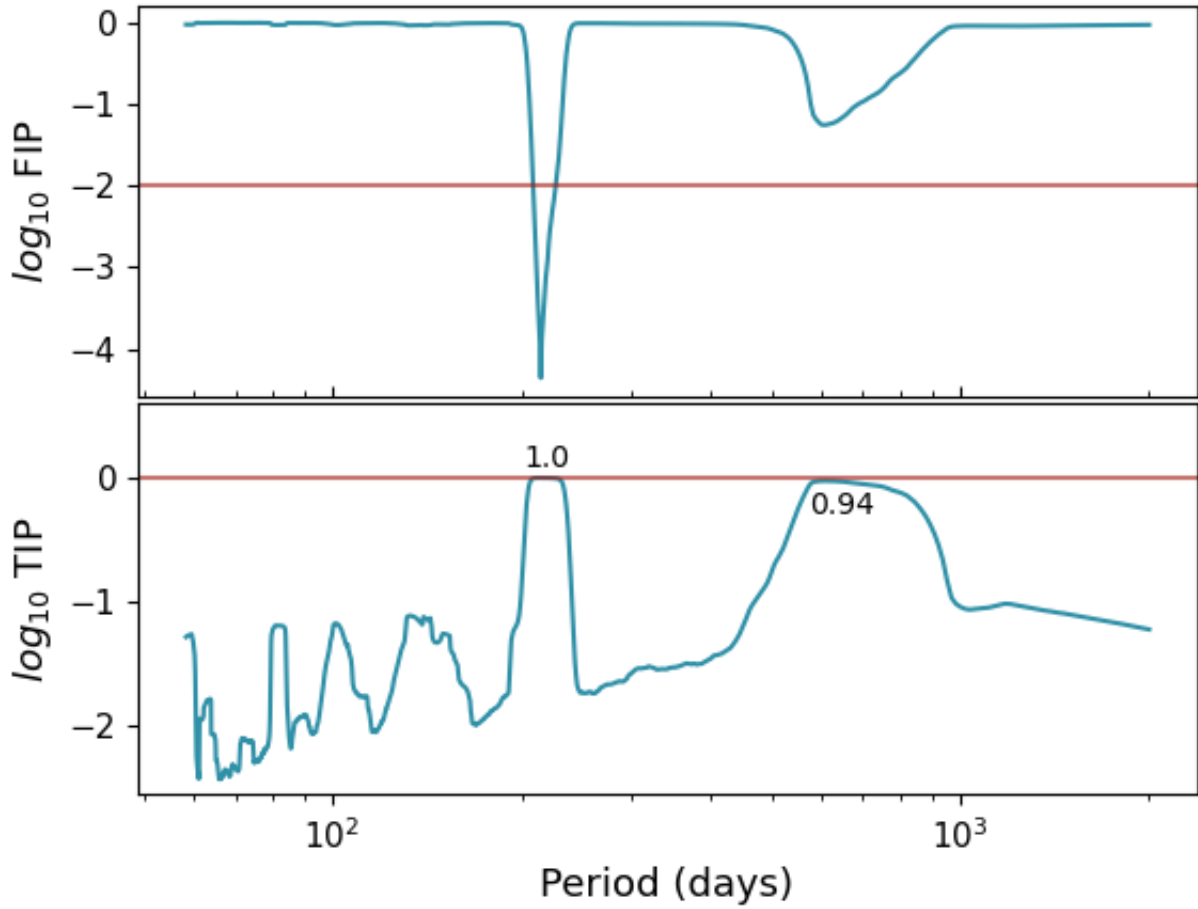
Notes: N_p denotes the Number of Planetary Keplerian signals to fit to the data. P_{bin} denotes the Period of the binary. $\mathcal{U}$ denotes a uniform prior with an upper and lower limit, $\mathcal{LU}$ is a log-uniform (Jeffreys) prior with upper and lower limits, $\mathcal{MLU}$ is a modified log-uniform prior with a knee and upper limit, $\mathcal{N}$ is a gaussian prior with a mean and standard deviation, and $\mathcal{K}$ is a Kumaraswamy prior (97) which takes two shape parameters (98).



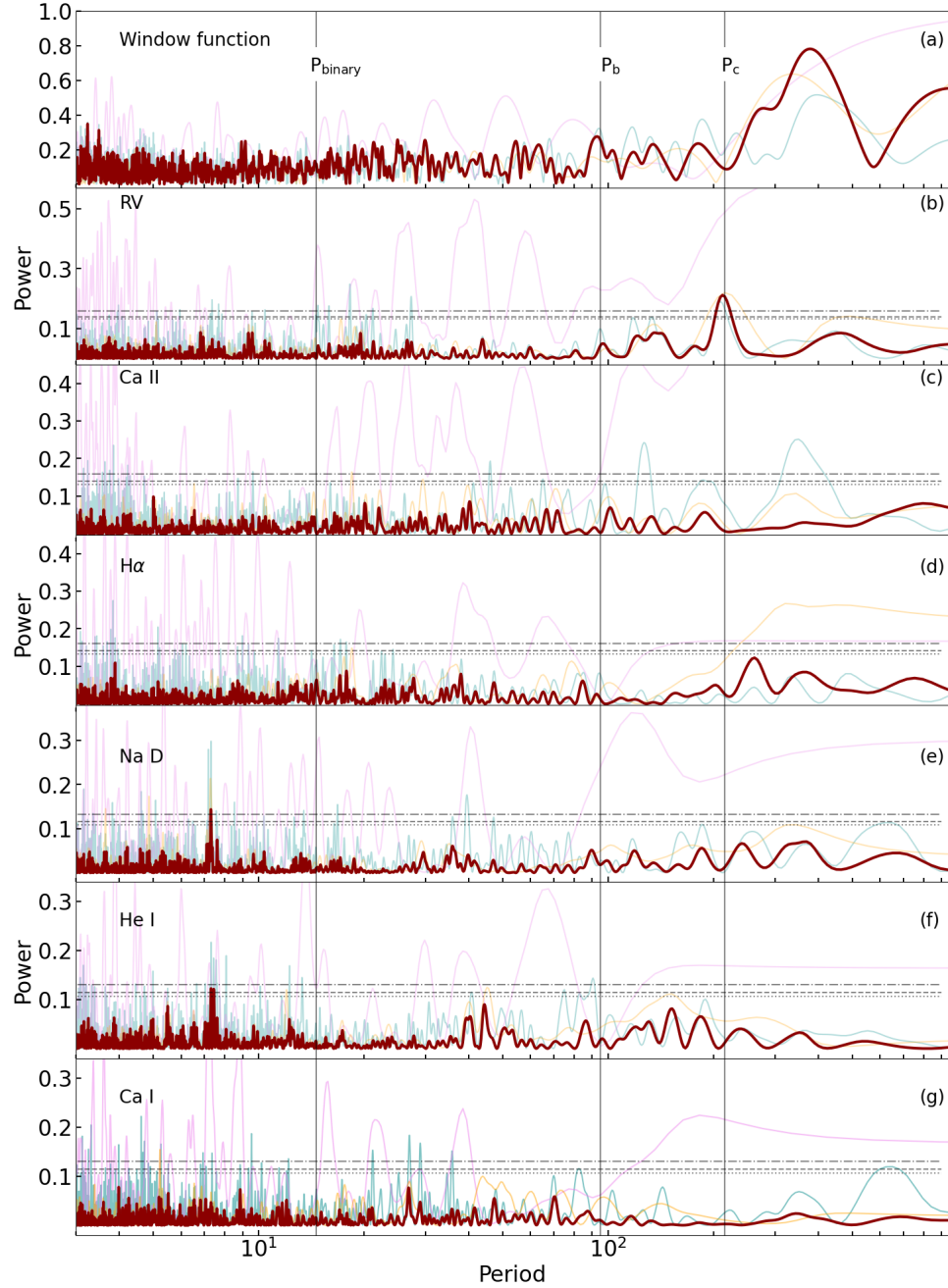
Supplementary Figure 1: Corner plot of posterior sample distributions for the Binary stars' orbital parameters; eccentricity, orbital period, semi-amplitude, argument of periastron, binary apsidal precession, time of periastron passage, $M \sin i$ in Solar masses, and semi-major axis.



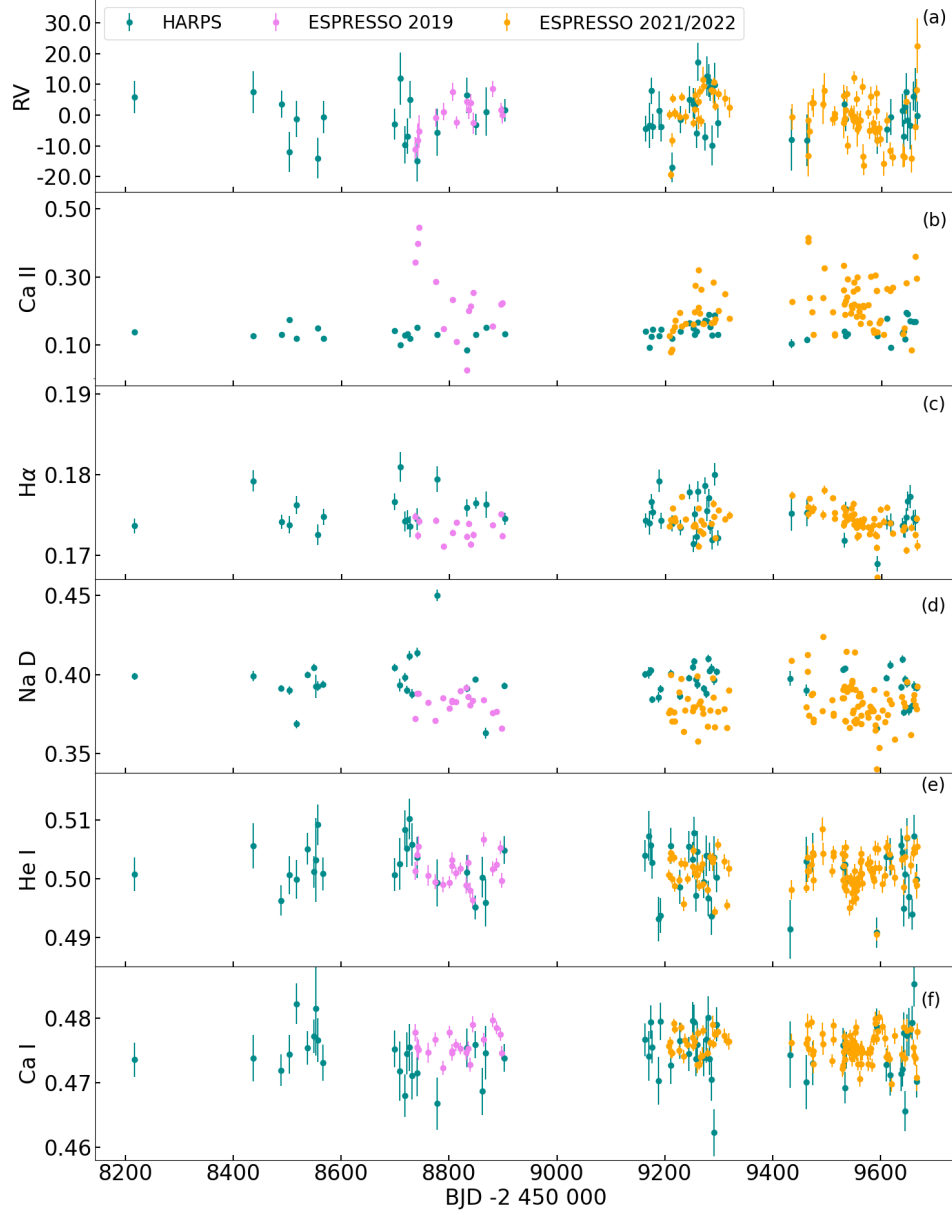
Supplementary Figure 2: Corner plot of posterior sample distribution for BEBOP-1c orbital parameters; eccentricity, orbital period, semi-amplitude, argument of periastron, time of periastron passage, $M \sin i$ in Earth masses, and semi-major axis.



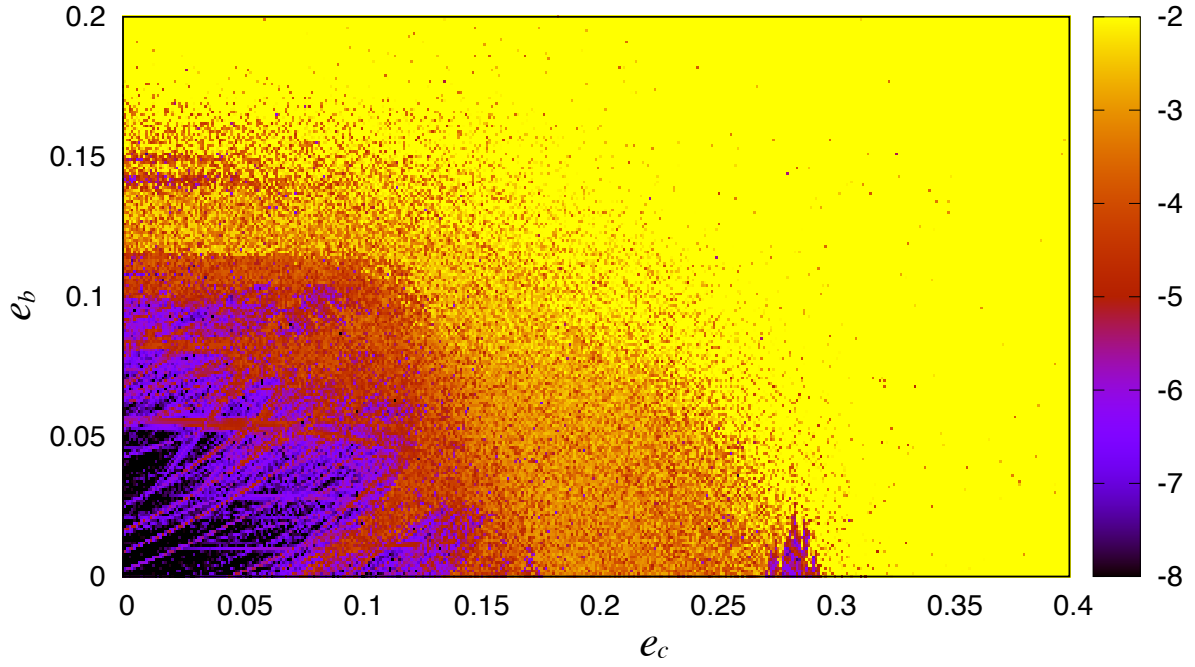
Supplementary Figure 3: Top: False Inclusion Probability (FIP) periodogram showing the clear detection of BEBOP-1c at ≈ 215 days. Bottom: True Inclusion Probability (TIP) periodogram showing the significance of any other peaks with $\text{TIP} > 0.2$. The second highest peak occurs at ≈ 700 days with a significance of 94%.



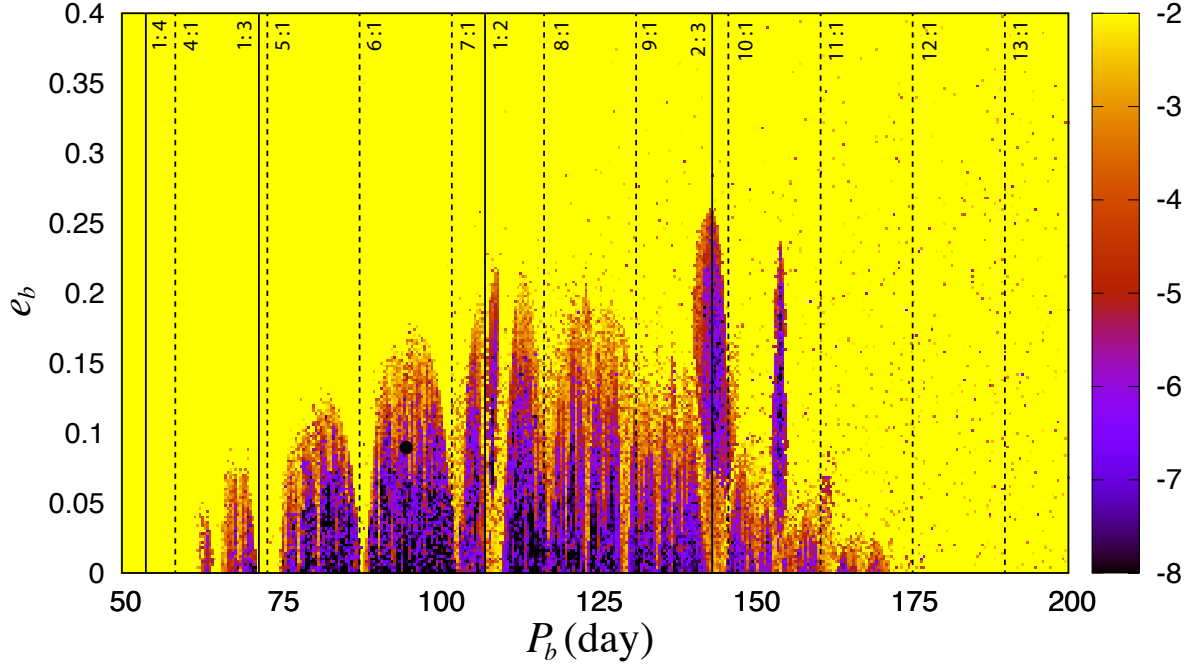
Supplementary Figure 4: Generalised Lomb-Scargle periodogram of BEBOP-1 spectroscopic data. The top panel shows the window function of the combined dataset (dark red). b, Periodogram of the radial-velocity measurements. c-g, Periodogram for Ca II, H α , Na D, He I and Ca I index. The cyan, violet and orange curves corresponds to HARPS, ESPRESSO 2019 and ESPRESSO 2021/2022, respectively, and the dark red curves correspond to the combined dataset. The horizontal lines represent the bootstrapped false alarm probability levels of 10% (dotted), 1% (dashed), and 0.1% (dotted dash). The vertical solid lines indicate the binary period and planetary periods (P_b and P_c).



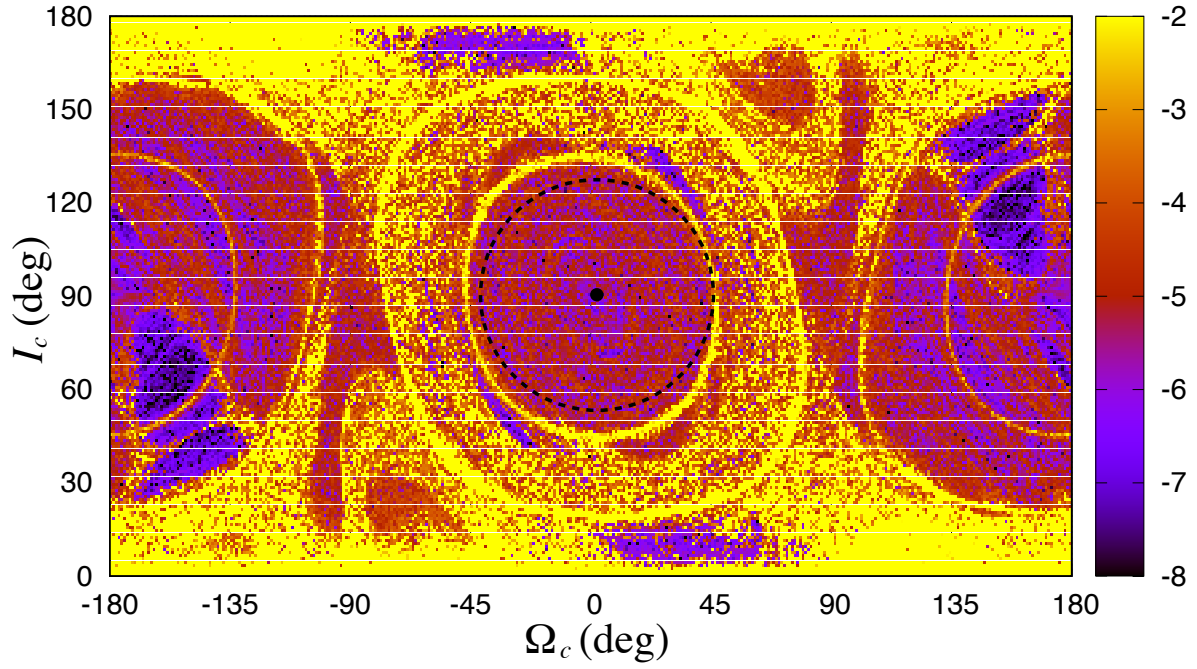
Supplementary Figure 5: Radial velocity, Ca II, H α , Na D, He I and Ca I timeseries for BEBOP-1. The estimated errors for activity indicators are determined through error propagation of integrated fluxes and the signal-to-noise ratios of spectral lines utilized in computing the activity indices.



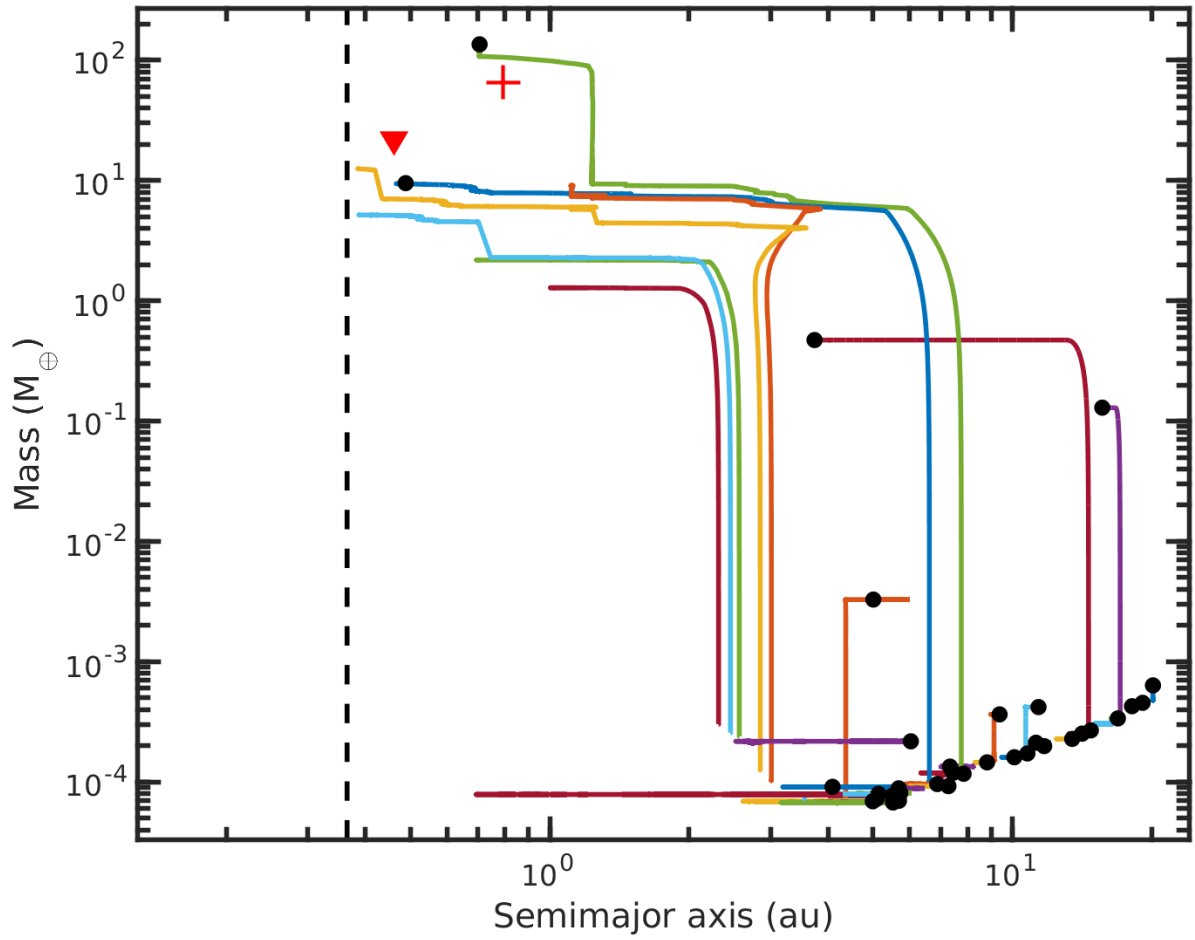
Supplementary Figure 6: Stability analysis of the BEBOP-1 system in the plane (e_c, e_b) , assuming coplanar orbits. For fixed initial conditions (Table 1 main text), the parameter space of the system is explored by varying the eccentricities of both planets, with a step size of 0.001. For each initial condition, the system is integrated over 5000 yr and a stability indicator is calculated which involved a frequency analysis of the mean longitude of the inner planet (see text). Chaotic diffusion is indicated when the main frequency of the mean longitude varies in time. Yellow points correspond to highly unstable orbits, while purple points correspond to orbits which are likely to be stable on a billion-years timescales.



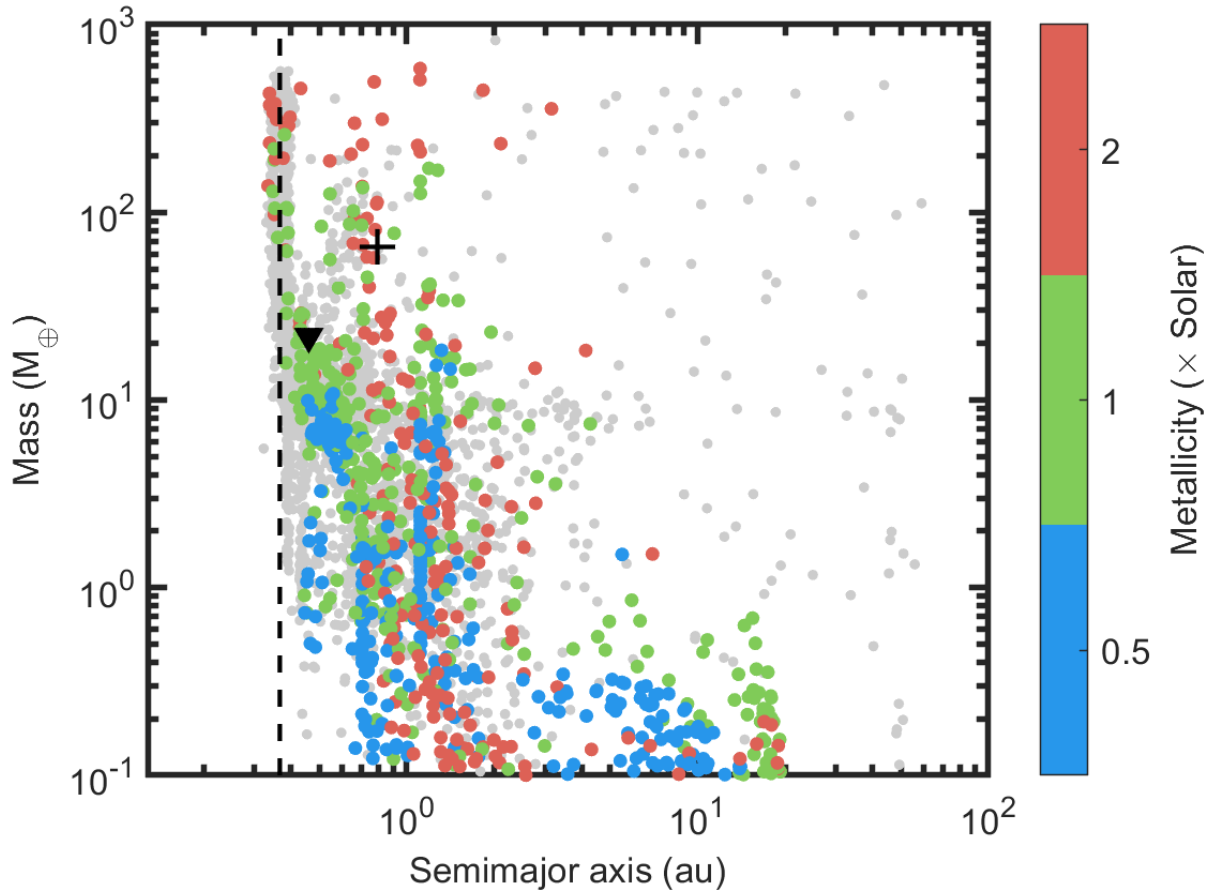
Supplementary Figure 7: Stability analysis of the BEBOP-1 system in the plane (P_b, e_b) , assuming coplanar orbits. For fixed initial conditions (Table 1 main text), the parameter space of the system is explored by varying the orbital period P_b and the eccentricity e_b of the inner planet. The step size is 0.4 day in orbital period and 0.002 in eccentricity. The black dot gives the best fit parameters for the inner planet (17). The color codes are the same from Figure 7, the dashed black lines indicate resonances with the binary, and the solid black lines resonances with the outer planet BEBOP-1c.



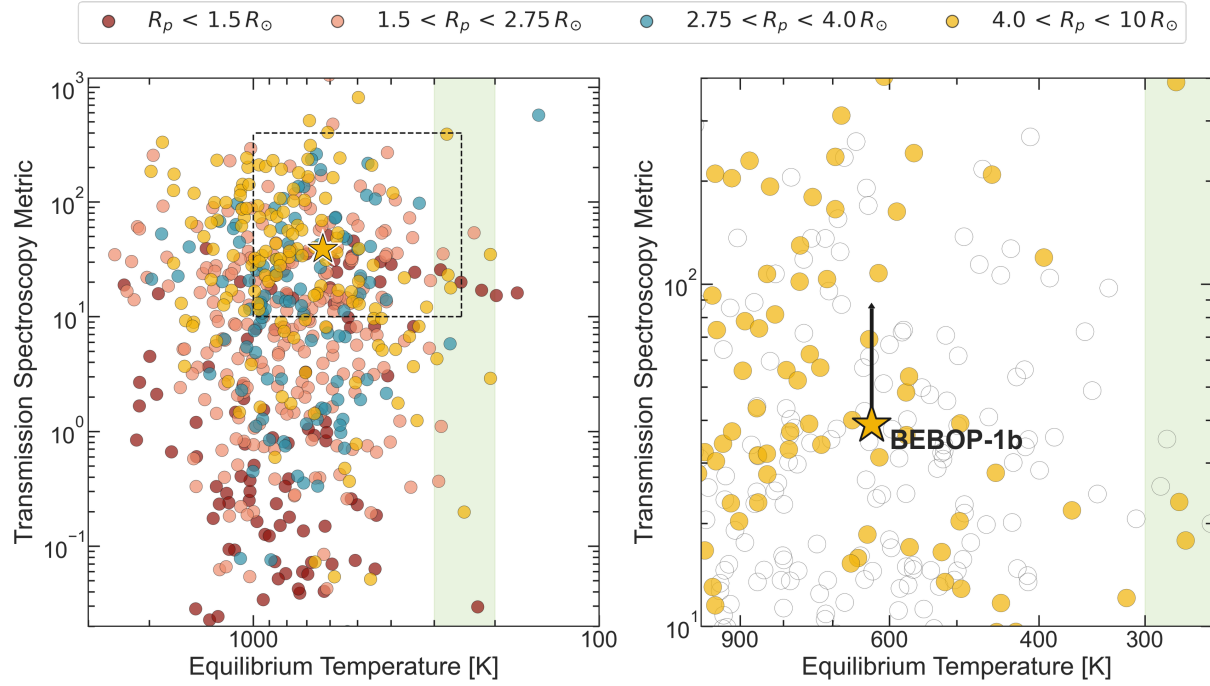
Supplementary Figure 8: Stability analysis of the BEBOP-1 system in the plane (Ω_c, I_c) . For fixed initial conditions (Table 1 main text), the parameter space of the system is explored by varying the inclination I_c and the longitude of the node Ω_c of the outer planet, with a step size of 1° . The black dot gives the best fit parameters, while the dashed circle corresponds to mutual inclinations equal to 40° . The color codes are the same from Figure 7.



Supplementary Figure 9: Evolution of the mass versus semimajor axes for the best matching simulated planetary system formed through pebble accretion to BEBOP-1. Black dots represent the final masses and locations of the planets, whilst the red triangle and plus show the observed locations of TOI-1338b and BEBOP-1c respectively. The black dashed line denotes the stability limit for the system (92).



Supplementary Figure 10: A mass versus semimajor axes plot showing all planets formed in simulations. Different colour points show the initial metallicity of the systems, whilst the grey points show planets that were lost, either through accretion into other planets, or ejection from the system. TOI-1338b and BEBOP-1c are denoted by the black triangle and plus symbols. The black dashed line denotes the stability limit for the system (92).



Supplementary Figure 11: Transmission spectroscopy metric as a function of equilibrium temperature for planets with measured masses. Planets are coloured according to their size range, with TOI-1338b plotted as a yellow star as it falls into the sub-Jovian regime of $4.0 < R_p < 10 R_{\oplus}$. In both panels we show in light green the position of the temperate zone between 200 – 300 K. In the right hand panel we zoom in on the vicinity of TOI-1338b and indicate with a black arrow the range of possible TSMs for a mass down to $10 M_{\oplus}$. Data for this plot was retrieved from the NASA Exoplanet Archive (95) on May 18th 2022.

References

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Journal Of Radial-Velocity Observations

Supplementary Table 2: Journal of Observations containing our HARPS data for BEBOP-1. Flags indicate whether the measurement is identified as a potential outlier for the following reasons: W, wrong star; B, bisector outlier; F, FWHM outlier; R, during binary transit; P, during planetary transit; S, identified by student-t. Dates are given in BJD - 2,400,000. V_{rad} are the measured radial velocities with their uncertainties $\sigma_{V_{\text{rad}}}$. FWHM is the Full Width at Half Maximum of the Gaussian fitted to the cross correlation function, and *contrast* is its amplitude. Bis. span is the span of the bisector slope. Uncertainties on FWHM and bis. span are $2 \times \sigma_{V_{\text{rad}}}$.

flag	BJD-2,400,000 [days]	V_{rad} [km s ⁻¹]	$\sigma_{V_{\text{rad}}}$ [km s ⁻¹]	FWHM [km s ⁻¹]	contrast	bis. span [km s ⁻¹]
	58216.529441	50.78673	0.00527	8.62517	33.284	0.04984
	58436.821943	48.95425	0.00683	8.60645	33.510	0.01450
	58488.758689	23.57766	0.00433	8.61047	33.532	0.02913
	58503.731900	26.22766	0.00648	8.62748	32.803	0.04815
	58516.712301	14.71036	0.00592	8.62249	33.393	0.03321
	58556.646009	20.57355	0.00664	8.61148	33.356	0.02785
	58566.604454	50.14641	0.00526	8.65706	33.177	0.06616
	58698.935537	50.75479	0.00501	8.63891	33.166	0.04375
	58708.900586	30.92087	0.00844	8.62618	33.307	0.03230
	58717.869356	14.86234	0.00608	8.64292	33.309	0.03386
	58721.884268	19.14344	0.00577	8.63451	33.503	0.03270
	58726.880800	49.14747	0.00606	8.63034	33.580	0.02617
	58740.815851	46.73639	0.00673	8.65363	33.015	0.02828
	58777.865197	7.57447	0.00775	8.62193	33.480	0.04777
	58832.810367	37.48155	0.00573	8.63670	33.207	0.03620
	58848.722188	21.71885	0.00350	8.64730	33.414	0.02062
F	58861.667840	41.08977	0.00616	8.55803	33.640	0.03357
	58867.644660	16.88223	0.00783	8.64317	33.465	0.02665
	58902.582483	50.11217	0.00377	8.60631	33.474	0.04012
	59163.705508	43.13564	0.00417	8.60739	33.669	0.04133
	59170.792366	14.32580	0.00741	8.59433	33.516	0.02690
	59173.721395	12.42927	0.00439	8.61091	33.624	0.02420
	59176.668061	32.95458	0.00417	8.61199	33.595	0.01304
	59188.634700	14.25913	0.00636	8.62851	33.702	0.04388

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flag	BJD	V_{rad}	$\sigma_{V_{\text{rad}}}$	FWHM	contrast	bis. span
	59191.651879	35.49221	0.00486	8.59691	33.561	0.02025
	59211.662441	45.90963	0.00487	8.61660	33.531	0.04283
	59227.640383	32.91789	0.00448	8.61194	33.672	0.03462
	59244.662941	8.79879	0.00444	8.64343	33.221	0.01978
	59251.644571	44.61290	0.00520	8.63634	33.503	0.02609
	59254.602429	49.90968	0.00448	8.63231	33.600	0.03055
	59257.658434	22.97524	0.00496	8.62586	33.461	0.02828
	59260.590560	8.80567	0.00614	8.63341	33.413	0.06138
	59272.597942	19.04391	0.00456	8.61572	33.388	0.02352
	59276.545689	15.95709	0.00645	8.65730	33.152	0.05425
	59280.570701	43.13379	0.00535	8.65045	33.420	0.05059
	59282.587090	50.43272	0.00429	8.66531	33.226	0.03972
	59286.640886	25.88951	0.00658	8.61449	33.288	0.04448
	59291.599947	19.03116	0.00717	8.67435	32.989	0.01348
	59296.574262	49.02190	0.00485	8.65725	33.296	0.05447
	59432.915546	23.53480	0.01006	8.60847	33.470	-0.00621
	59461.840683	27.18387	0.00837	8.63513	33.182	0.02817
F	59474.875056	44.09898	0.00608	8.55833	33.404	0.05304
	59530.787352	50.18431	0.00435	8.59874	33.507	0.04165
	59533.768571	40.07132	0.00398	8.59069	33.524	0.03166
W/S	59536.865577	8.58978	0.00536	8.78151	32.610	-0.05729
	59592.644734	35.36664	0.00442	8.58328	33.543	0.02524
	59610.628629	7.60999	0.00424	8.60636	33.508	0.03719
	59617.633579	47.93738	0.00585	8.62591	33.392	0.04730
	59638.669783	11.07926	0.00542	8.61874	33.062	0.02959
	59640.662829	9.73189	0.00551	8.62356	33.143	0.02188
	59643.549232	28.77716	0.00566	8.60709	33.489	0.04687
	59645.617218	42.22367	0.00624	8.62776	32.944	0.05350
	59648.619602	50.65155	0.00662	8.64878	32.875	0.01530
	59652.605979	17.00512	0.00764	8.58887	32.870	0.03801
	59658.559334	31.63510	0.00474	8.60176	33.121	0.01684
	59662.578359	50.64357	0.00709	8.61825	32.884	0.04475
	59666.562593	24.65893	0.00460	8.62847	33.322	0.02856
	59678.558881	48.91383	0.00613	8.63485	33.275	0.04387
	59682.528521	10.85815	0.00699	8.61124	33.206	0.07145
	59686.540476	22.71545	0.00689	8.60983	32.928	0.05059
	59688.495789	36.51633	0.00633	8.61060	33.306	0.04156

Supplementary Table 3: Journal of Observations containing our 2019 ESPRESSO data for BEBOP-1 pre-calibration lamp change. Flags indicate whether the measurement is identified as a potential outlier for the following reasons: W, wrong star; B, bisector outlier; F, FWHM outlier; R, during binary transit; P, during planetary transit; S, identified by student-t. Columns are as in Table 2.

flag	BJD-2,400,000 [days]	V_{rad} [km s ⁻¹]	$\sigma_{V_{\text{rad}}}$ [km s ⁻¹]	FWHM [km s ⁻¹]	contrast	bis. span [km s ⁻¹]
F	58736.877282	21.76153	0.00386	9.35270	45.08543	-0.02995
	58740.853942	46.75387	0.00508	9.37866	43.97280	-0.02901
	58743.882022	47.54869	0.00525	9.34604	44.67442	-0.02851
	58760.868104	24.05023	0.00525	9.40855	44.55251	-0.00934
	58774.797532	32.50228	0.00288	9.36375	45.12590	-0.00107
	58788.804499	39.20782	0.00281	9.35552	46.13411	-0.01414
	58800.687988	50.49684	0.00211	9.33485	46.21587	0.00094
	58805.840799	11.96096	0.00293	9.34411	46.18027	-0.02645
	58805.853819	11.86026	0.00276	9.32303	46.18795	-0.02347
	58812.832857	41.52667	0.00202	9.34686	46.40748	-0.00833
	58820.719656	10.11904	0.00191	9.33895	46.26380	-0.01669
	58831.662527	46.86291	0.00312	9.34385	45.91930	-0.00895
	58835.596434	8.75913	0.00302	9.35211	45.82418	-0.03332
	58839.611578	25.24170	0.00281	9.34291	45.97437	0.00664
	58843.641422	48.67236	0.00338	9.36552	45.89060	-0.03628
	58864.576277	9.94548	0.00245	9.33974	46.43191	-0.03564
	58880.582185	8.10059	0.00260	9.36042	46.02511	-0.03986
	58887.547492	48.91687	0.00226	9.34008	46.16192	-0.03410
	58895.570898	9.36537	0.00245	9.35557	45.63188	-0.02719
	58897.568595	21.74950	0.00285	9.33254	45.99687	-0.00162

Supplementary Table 4: Journal of Observations containing our 2021/22 ESPRESSO data for BEBOP-1 post-calibration lamp change. Flags indicate whether the measurement is identified as a potential outlier for the following reasons: W, wrong star; B, bisector outlier; F, FWHM outlier; R, during binary transit; P, during planetary transit; S, identified by student-t. Columns are as in Table 2.

flag	BJD-2,400,000 [days]	V_{rad} [km s ⁻¹]	$\sigma_{V_{\text{rad}}}$ [km s ⁻¹]	FWHM [km s ⁻¹]	contrast	bis. span [km s ⁻¹]
S	59207.687480	43.80224	0.00156	9.37761	46.31836	-0.01166
	59209.628417	50.37108	0.00159	9.37580	46.43171	-0.02890
	59211.660755	45.77950	0.00217	9.35553	46.17093	-0.03101
	59213.608767	25.62042	0.00147	9.35216	45.85679	-0.01713
	59216.617681	8.18971	0.00153	9.37519	46.34949	-0.03437
	59218.740865	20.18316	0.00159	9.36935	46.11973	-0.01806
	59226.586386	43.49514	0.00155	9.36541	46.26637	-0.03917
	59230.661220	7.43254	0.00144	9.36048	46.40324	-0.04251
F	59234.763082	30.43838	0.00253	9.38958	45.42103	-0.02132
	59237.675552	47.27860	0.00239	9.38360	45.80074	-0.01717
R	59242.594322	28.37619	0.00158	9.35821	46.07464	0.02823
	59251.647333	44.47071	0.00156	9.36148	46.26004	-0.00822
	59253.579655	50.52710	0.00157	9.36675	46.24797	-0.02805
	59255.616689	44.90922	0.00231	9.36382	45.55454	-0.01475
	59260.656954	8.86909	0.00349	9.38242	45.03587	-0.02082
	59261.570641	13.44921	0.00147	9.36278	46.13836	-0.01415
	59263.543084	27.30447	0.00174	9.35886	46.22777	-0.02054
	59265.633180	41.15591	0.00192	9.37520	45.80166	-0.03367
	59269.686062	48.06284	0.00402	9.36591	45.33246	-0.02726
	59270.617600	41.76652	0.00167	9.36544	46.13202	-0.00691
	59279.615948	37.38595	0.00148	9.32690	46.62544	-0.02920
	59284.600155	46.44112	0.00154	9.37661	46.35578	-0.00420
	59288.601602	8.24637	0.00393	9.39004	45.29095	-0.01497
	59289.558302	7.94851	0.00140	9.36444	46.36742	-0.01240
	59292.617105	26.25967	0.00160	9.36279	46.38443	-0.01914
	59298.529197	49.47896	0.00180	9.36071	46.01858	-0.01272
	59310.557405	46.63806	0.00277	9.35520	45.72558	0.01329
	59315.529230	30.05916	0.00161	9.22765	46.59440	-0.01299
	59318.493789	7.49658	0.00334	9.37612	46.25952	0.00924

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flag	BJD	V_{rad}	$\sigma_{V_{\text{rad}}}$	FWHM	contrast	bis. span
	59434.886191	7.72835	0.00406	9.38789	45.20792	-0.02862
	59461.875388	26.60478	0.00179	9.38405	46.00120	-0.00395
F	59464.797631	7.78951	0.00610	9.38198	44.24505	-0.03947
	59464.852082	7.91591	0.00562	9.43539	44.26999	-0.00213
	59466.891173	18.78338	0.00248	9.33472	45.93830	-0.02929
	59471.888348	48.91225	0.00257	9.35499	46.10455	-0.00382
	59473.770470	49.65291	0.00441	9.37446	45.85213	-0.02673
F	59475.781757	35.16473	0.00544	9.40688	44.61897	-0.03005
F	59475.793768	35.03023	0.00471	9.40944	45.39922	-0.02857
	59491.728251	18.88142	0.00603	9.31472	46.12284	-0.07435
	59493.720410	7.44845	0.00559	9.35603	45.28000	-0.04729
	59509.854678	13.07129	0.00236	9.34211	46.23157	-0.02061
	59512.842954	34.02253	0.00245	9.35205	46.87258	-0.00193
	59512.855927	34.11283	0.00263	9.35547	46.69308	-0.00158
	59529.677831	46.60567	0.00361	9.32951	45.79053	-0.02134
F	59530.618883	49.68125	0.00494	9.40343	44.94001	-0.02776
	59531.802019	50.44588	0.00226	9.33524	46.23785	-0.03232
	59532.720289	47.66931	0.00204	9.33130	46.39628	-0.01964
	59533.679007	40.78062	0.00228	9.34486	46.03277	-0.00715
R	59534.615899	30.40381	0.00337	9.35282	45.33094	0.01244
	59535.766807	16.55441	0.00255	9.34206	45.89073	-0.03407
F	59536.619244	9.64249	0.00294	9.41335	45.01306	-0.03384
F/S	59537.738391	7.59953	0.00371	9.41515	46.07402	-0.03767
	59542.613295	37.64360	0.00311	9.37607	45.61895	-0.04708
	59543.758054	44.17420	0.00190	9.33951	46.37724	-0.02434
	59544.665713	48.05618	0.00205	9.34654	46.12688	-0.01813
	59545.669437	50.44361	0.00231	9.35663	46.08663	-0.04244
	59546.842367	49.57590	0.00287	9.32741	46.09767	-0.03066
	59547.702845	45.48948	0.00291	9.36807	45.72661	-0.01271
	59548.622872	37.39873	0.00351	9.38229	45.23370	0.00187
	59549.670272	24.81471	0.00212	9.34777	45.99796	-0.00368
	59550.722645	13.21611	0.00169	9.32627	46.16202	-0.00265
	59551.671532	7.91486	0.00293	9.35368	45.91887	0.03150
	59552.697168	8.43029	0.00194	9.33663	46.34639	-0.00803
	59553.676730	13.05334	0.00250	9.34812	46.18860	-0.02586
	59554.576962	19.02977	0.00331	9.36551	45.58520	-0.02356
	59555.605078	26.50216	0.00557	9.37734	45.04094	-0.06264
	59557.592836	39.92466	0.00235	9.35799	45.96443	-0.03400
	59558.690961	45.71291	0.00203	9.33884	46.20128	-0.02282

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flag	BJD	V_{rad}	$\sigma_{V_{\text{rad}}}$	FWHM	contrast	bis. span
	59561.597912	49.09863	0.00317	9.37203	46.28033	-0.02749
F	59562.588878	43.45288	0.00405	9.40179	46.07450	-0.00592
	59563.570834	33.56211	0.00269	9.37255	46.56322	0.00182
	59564.605166	20.81757	0.00255	9.35978	46.22947	-0.01380
F/S	59565.620260	10.95610	0.00398	9.39488	45.62137	-0.05093
B/F	59566.626009	7.41922	0.00318	9.39870	45.77566	-0.05742
	59575.684564	50.37154	0.00214	9.34620	46.22991	-0.02944
	59577.752093	38.32622	0.00270	9.37025	45.73892	-0.00015
	59580.674743	8.59139	0.00317	9.35019	45.25616	-0.02305
	59582.578623	11.25863	0.00264	9.33852	45.84375	-0.01116
	59584.601711	24.89331	0.00227	9.34412	46.15903	-0.00402
	59586.580398	38.52810	0.00200	9.34233	46.24767	-0.01804
	59588.561475	48.28672	0.00347	9.35869	45.42404	0.00701
	59590.718141	49.42114	0.00288	9.37818	45.52608	0.00617
	59590.763503	49.27649	0.00257	9.34866	45.75520	-0.03599
	59592.683281	34.77366	0.00219	9.34450	46.11654	-0.01601
	59595.638420	7.62069	0.00233	9.36676	46.04454	-0.05363
	59597.574782	13.47948	0.00244	9.34960	45.45612	-0.00326
F	59603.794993	49.94906	0.00394	9.40307	44.85825	-0.00935
B/F/S	59607.733856	29.58589	0.00438	9.43764	44.46652	-0.07771
	59610.557055	7.43040	0.00243	9.37618	45.63957	-0.00768
P	59613.678124	23.84446	0.00488	9.35368	45.36720	0.00387
	59616.558505	42.91328	0.00269	9.37147	45.71280	-0.01982
	59620.543282	46.73256	0.00303	9.33914	45.78754	0.02492
	59626.642810	12.57945	0.00368	9.33877	45.58173	-0.01760
	59639.611804	7.44974	0.00384	9.36696	45.70465	-0.01814
	59642.523956	21.14343	0.00325	9.35484	45.06757	-0.03741
	59645.600842	41.97985	0.00386	9.35632	46.48695	0.00010
S	59648.534020	50.54759	0.00650	9.36679	46.66851	-0.06742
	59655.659961	11.45834	0.00449	9.35032	46.25790	-0.02203
F	59660.602305	44.05437	0.00372	9.40772	44.79431	-0.02115
	59662.591082	50.49499	0.00427	9.34963	44.80618	-0.01204
	59664.650017	44.95555	0.00370	9.36024	45.49735	-0.02922
	59666.625811	23.74319	0.00842	9.38995	43.39249	-0.04636
	59668.519892	7.97514	0.00368	9.34184	44.44794	-0.00950