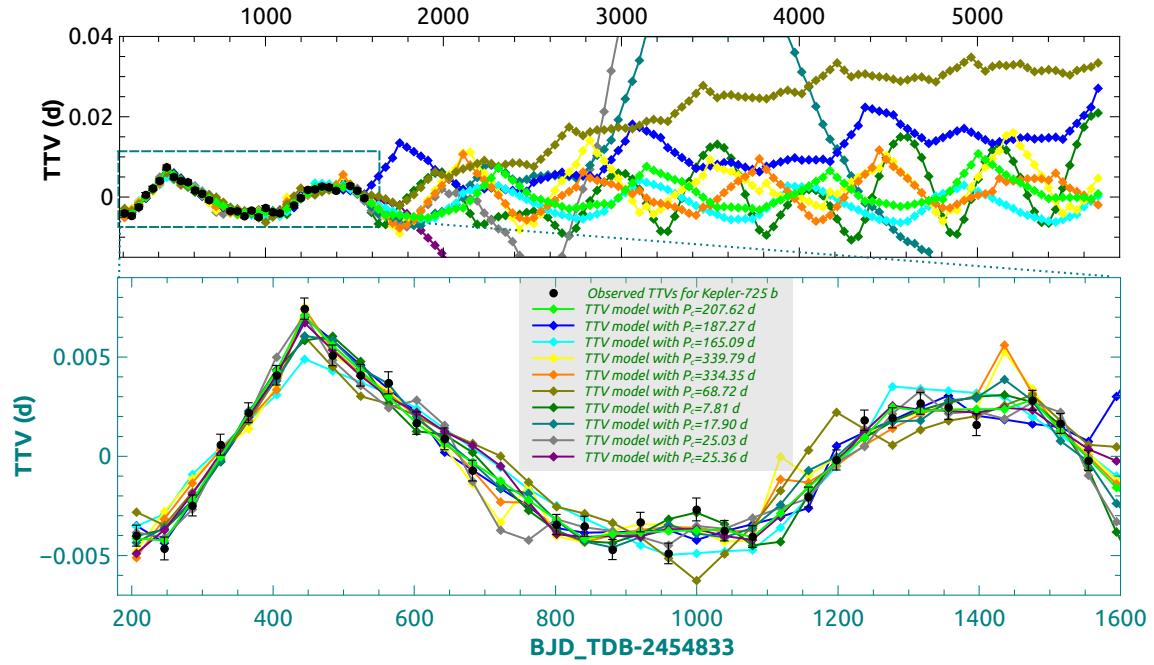
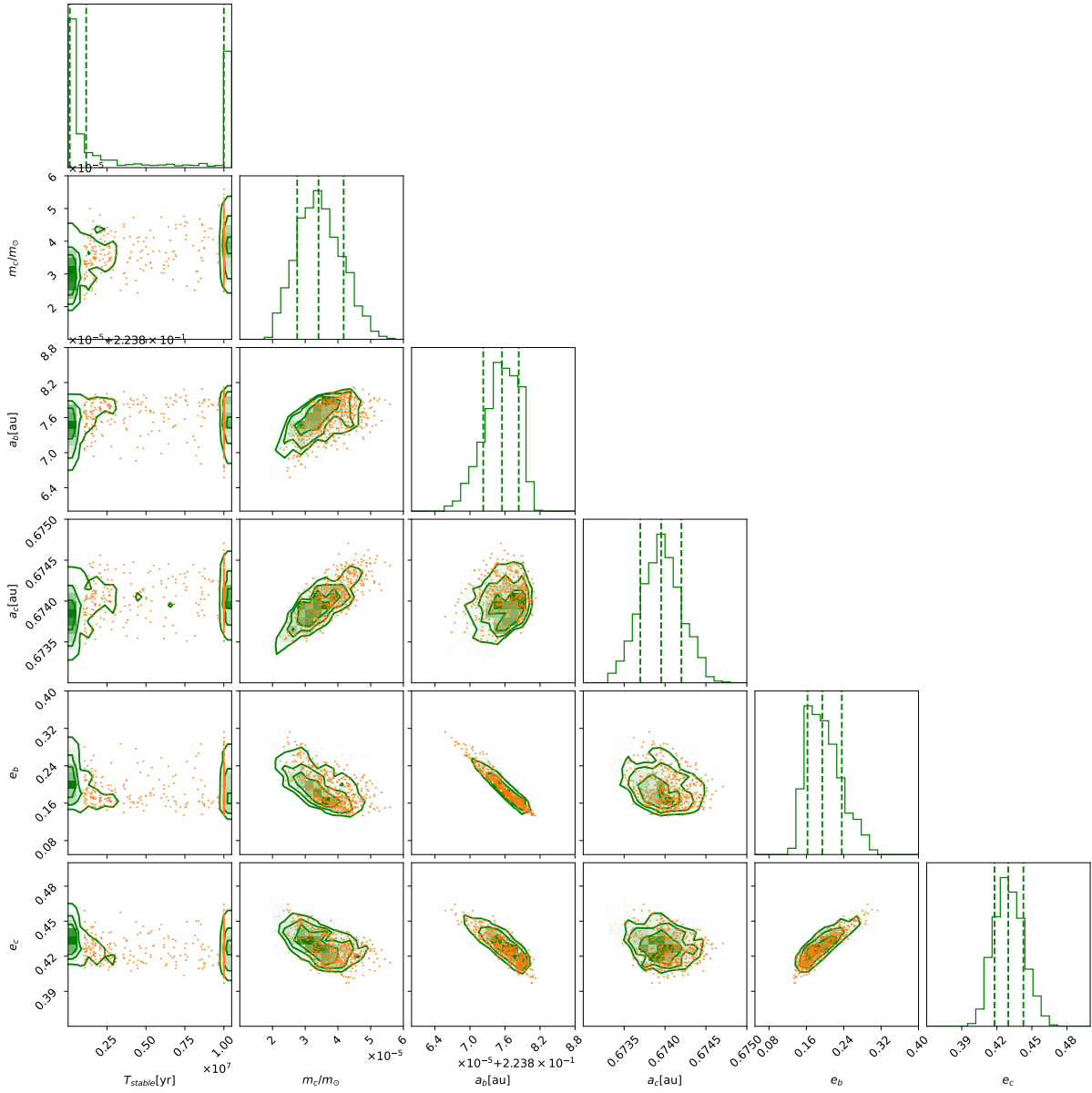

A temperate 10-Earth-mass exoplanet around the Sun-like star Kepler-725

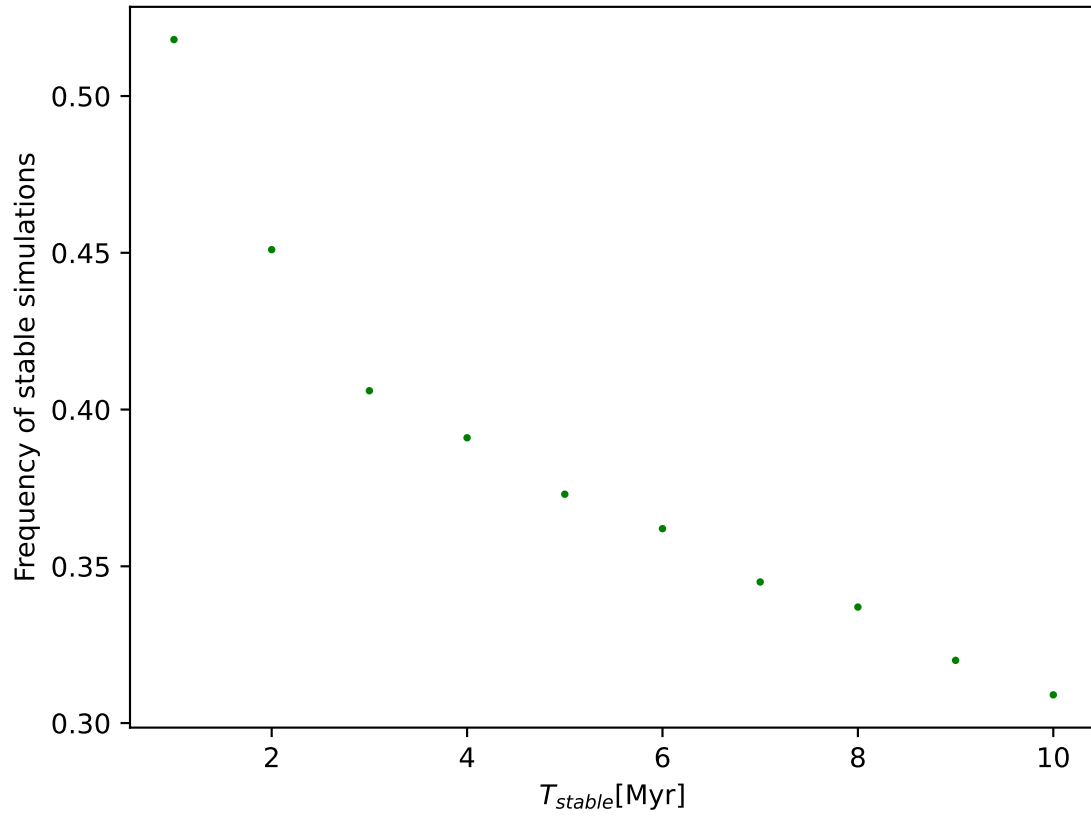
In the format provided by the
authors and unedited



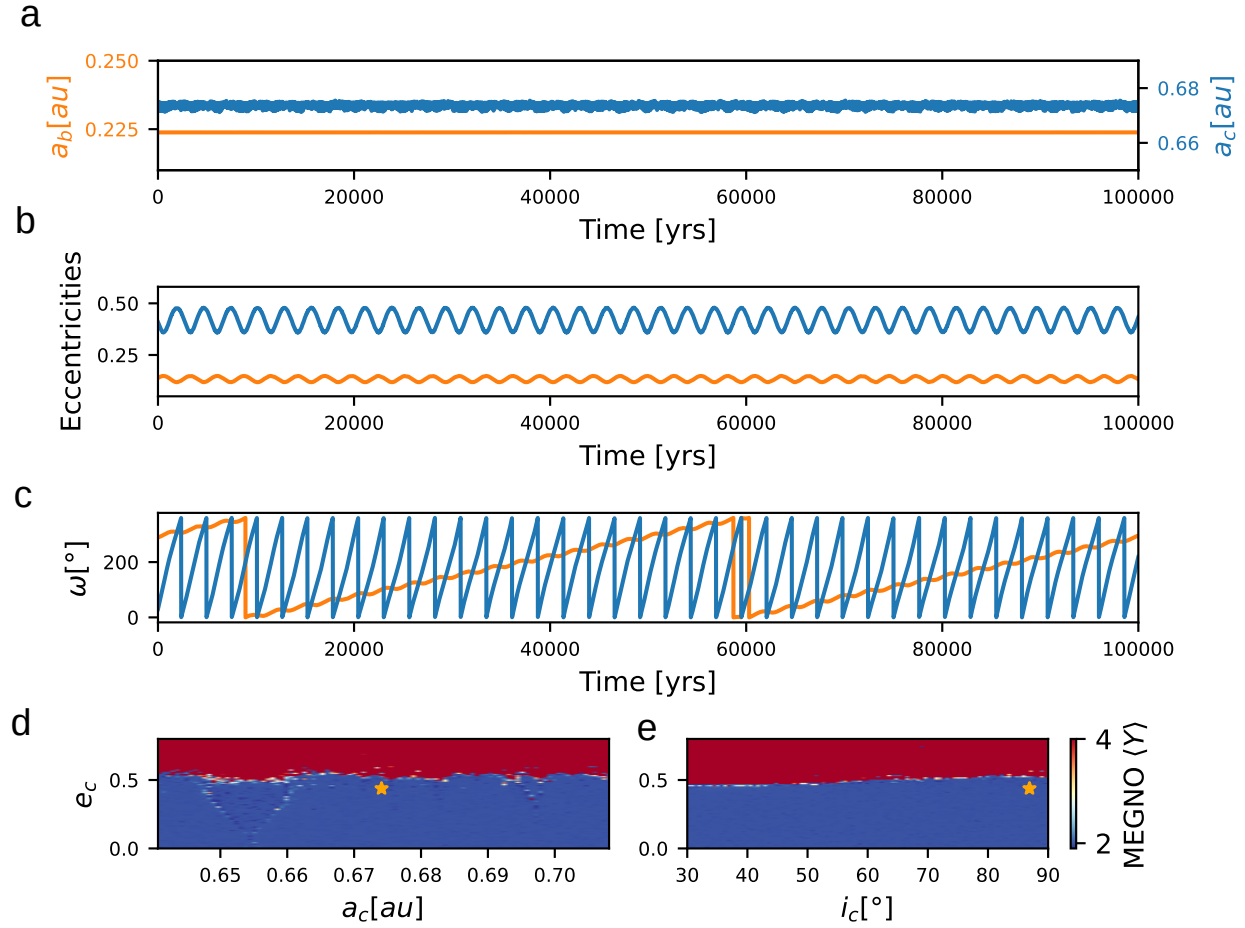
Supplementary Figure 1: **The global solution of TTVs alongside some of the other local optimum solutions as well as the long-term TTV predictions.** The green diamond line represents the global optimal TTV model, while the black solid dots are the observed TTVs for Kepler-725 *b*. All error bars represent 1-standard deviation.



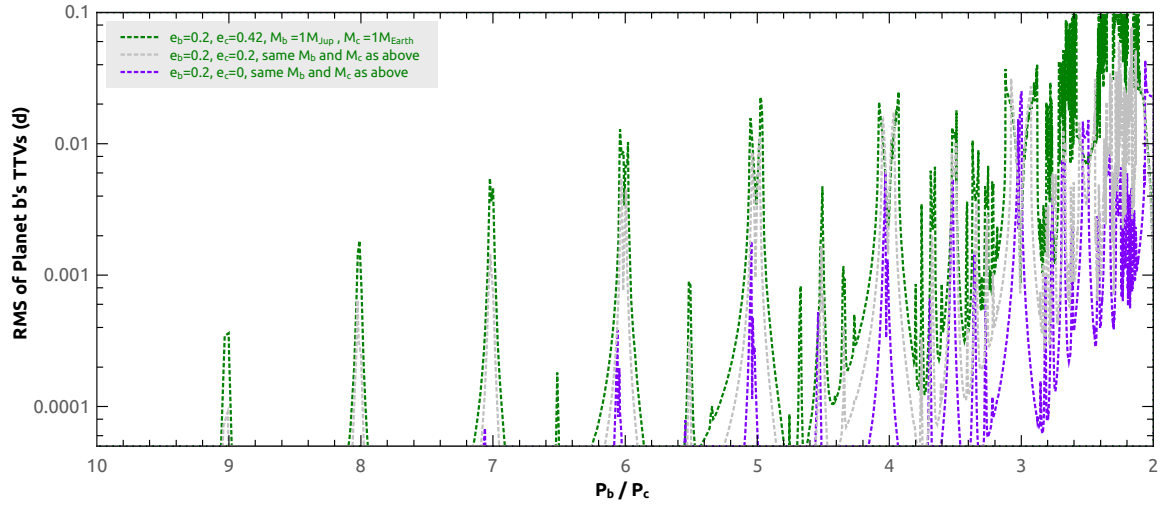
Supplementary Figure 2: **Initial conditions adopted in long-term dynamical simulations for Kepler-725 system in two-planet architectures.** Stable orbital configurations with stable time scale T_{stable} longer than 1 Myr are depicted with orange.



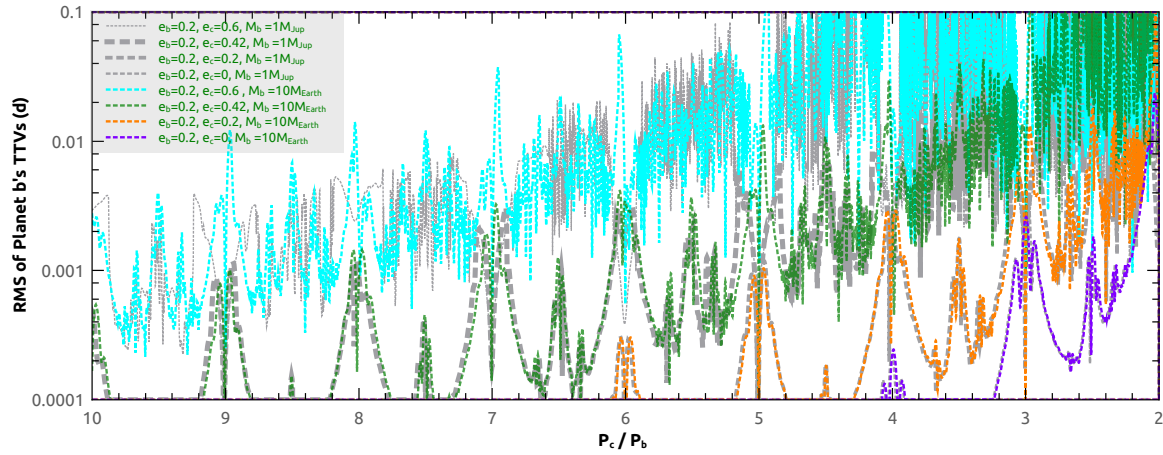
Supplementary Figure 3: **The frequency of stable dynamical simulations for Kepler-725 system in two-planet architectures along with the duration of orbital integrations.** The frequency of stable dynamical simulations are represented with green dots.



Supplementary Figure 4: **The long-term dynamical evolution of Kepler-725 *b* and *c*'s orbits for optimal transit and TTV models, as well as MEGNO maps.** (a) The orange and blue curves label semi-major axes of Kepler-725 *b* and *c*, respectively. (b) The orange and blue curves are orbital eccentricities of Kepler-725 *b* and *c*, respectively. (c) The orange and blue curves represent the arguments of periastron of Kepler-725 *b* and *c*, respectively. (d) The MEGNO map of Kepler-725 system with different values between Kepler-725 *c*'s semi-major axis (a_c) and eccentricity (e_c). The blue areas with MEGNO values $Y \leq 2$ conservatively show regular orbits, while the red regions represent unstable orbits. (e) Similar MEGNO map as in left panel but with various combinations between Kepler-725 *c*'s inclination (i_c) and eccentricity (e_c). In addition, the optimal solution of TTV modelling is marked by orange stars in both panels **d** and **e**.



Supplementary Figure 5: TTV signals of a warm Jupiter gravitationally induced by an inner terrestrial exoplanet around a Sun-like star. This figure shows the TTV amplitude of a synthetic planet b with $M_b = 1.0 M_{Jup}$ induced by an inner terrestrial planet c (different from the outer perturbing terrestrial planet c as in Fig. 3). In this case, the outer planet b has same orbital parameters as Kepler-725 b except $i_b = 90^\circ$, $M_{b0} = 0^\circ$ (i.e., mean anomaly at BJD_TDB=2455039), and the inner planet c has various orbital periods with $e_c = 0, 0.2, 0.42$ and mean anomaly $M_{c0} = 180^\circ$ at the same reference time. Due to possibly less stable orbital configurations with $e_c = 0.6$ for the inner planet c , we didn't compute the relative TTV signals.



Supplementary Figure 6: **Comparison between TTVs of an inner $10 M_{\oplus}$ sub-Neptune and those of an inner gas giant induced both by an outer terrestrial planet.** This figure shows the TTV amplitudes of synthetic planet b with $M_b = 1.0 M_{Jup}$ (same as in Fig. 3) and $10 M_{\oplus}$, respectively, induced by an outer terrestrial planet c with $M_c = 1 M_{\oplus}$. For all cases, the outer planet c has same orbital parameters as Kepler-725 c except $i_c = 90^\circ$, $M_{c0} = 180^\circ$ (i.e., mean anomaly at BJD_TDB=2455039) and $e_c = 0, 0.2, 0.42, 0.6$, and the inner planet b has same orbital parameters as Kepler-725 b except various orbital periods and mean anomaly $M_{b0} = 0^\circ$ at the same reference time.