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Valley formation on early Mars by subglacial and fluvial erosion

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1 A brief note on glacial hydrology

The cartoon in figure 1 shows the subglacial environment of a large ice sheet (adapted from Rennermalm and co-authors [40]), with the different glacial hydrology environments. This cartoon is not, and does not intend to resemble, a description of the Late Noachian Icy Highlands ice sheet hydrological characteristics, and only aims to provide a visual guide to describe glacial and subglacial hydrology. The information below is extracted from chapter 6 in Cuffey and Paterson [6].

Ice sheets gain mass in form of snow in the upper elevation areas that conform the accumulation zone (figure 1), and lose mass through melting/sublimation in the lower elevation areas found in the ablation zone. The Equilibrium Line Altitude (ELA) is the imaginary line that separates areas of net gain and areas of net loss. Whereas surface ice does not start melting until the ELA, meltwater may pond beneath the ice and form subglacial cavities within the accumulation zone. If the bed is frozen, subglacial meltwater will not be present or will be extremely localized and associated with, i.e., hypersaline deposits [44, 34] .

Surface glacial meltwater constitutes the supraglacial regime. When surface ice melts in the ablation zone (below the ELA, see figure 1), meltwater ponds in supraglacial lakes (1A) or flows downstream over the ice in networks of channels very much like ordinary surface runoff (1B). While supraglacial streams may transport meltwater to the ice margin [9], most of the water disappears into the ice through cracks (crevasses, 1D) or vertical passageways (moulins, 1E) into the subglacial and englacial environments.

Crevasses and moulins (1D and 1E) provide pathways for meltwater to penetrate into the ice. Most supraglacial streams flow into moulins, contributing to maintaining the passage open with the frictional dissipation produced when turbulent meltwater cascades down. Within the ice itself, there exist a number of channels, fractures and water pockets constituting the englacial system (1C). These networks of fractures connect the surface and subglacial drainage systems, and account for meltwater storage and transport within the ice.

Beneath the ice is the subglacial environment, where diverse drainage passageways store and transport meltwater [6]: (1) ice-walled conduits; (2) channels incised into the rock but roofed and walled by ice; (3) thin films of water distributed irregularly; (4) meter-scale interconnected cavities; (5) sediment or bedrock saturated with water; and (6) subglacial lakes. Channels incised solely into the ice are called Röthlisberger- or R-channels [43], whereas subglacial channels incised in the bedrock are called Nye- or N-channels [33]. The primary source of meltwater to the subglacial hydrology comes from the surface, below the ELA. However, in upper zones of the accumulation zone or in the central areas of large ice sheets (Antarctica and Greenland) where surface melting is

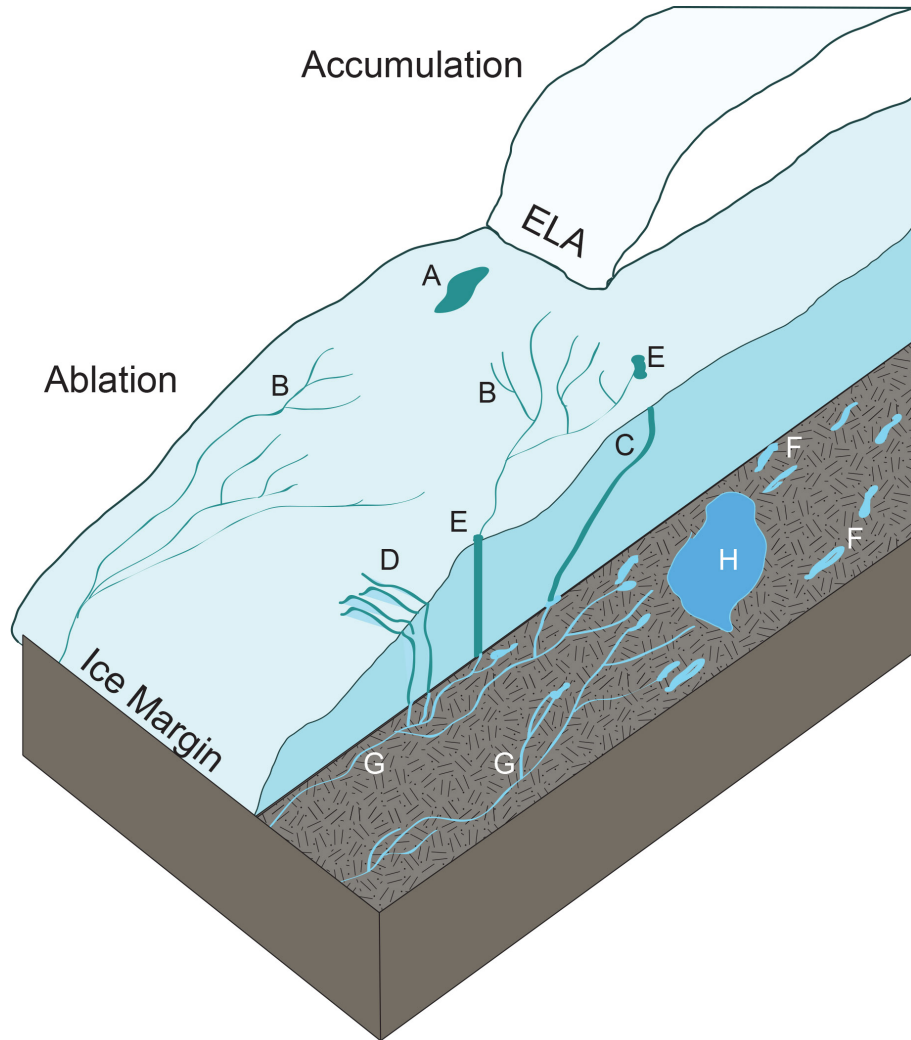


Figure 1: Cartoon depicting the glacial hydrology environment of a large ice sheet in the context of the equilibrium line altitude (ELA) and ice margin. (A) Supraglacial lake. (B) Supraglacial channels. (C) Englacial channel. (D) Crevasses receiving water. (E) Moulins draining into subglacial channels, which coalesce and emerge at the margin. (F) Subglacial cavities (water may also concentrate in films and the pore space, not pictured here). (G) Subglacial channel networks. (H) subglacial lake. Figure adapted from Rennermalm and co-authors [40] and Cuffey and Paterson [6].

nearly non-existent, subglacial meltwater originates entirely from basal melting [22, 6, 44, 34].

At the ice margin, meltwater in supraglacial streams (top-down melting) [9] merges with subglacial runoff, originating fluvio-glacial streams in the ice

foreland (not pictured).

2 Detailed workflow for metrics measurement

This section complements the manuscript and Methods section by providing more details on the measuring process of the 6 morphometric parameters. Additionally, in section 4 “Error estimation and propagation” we detail the distinct sources of error, quantify the uncertainty, and calculate the error propagated to the 1st, 2nd, and 3rd principal components. Panels (a) through (f) in figure 2 show the measurement workflow followed for each metric. The dataset including all valley networks analyzed, with metrics and error associated, is presented in supplementary table 1 (dataset).

2.1 Extraction of longitudinal and cross sectional profiles

2.1.1 Longitudinal profiles and undulations

The presence of positive slope sections in the longitudinal profiles of valley networks is a key distinctive characteristic of subglacial erosion, both on Mars and Earth [48, 13, 24, 11]. Given the importance of reliably identifying and accurately interpreting subglacial profile undulations, here we carefully detail the process of extraction and interpretation of the longitudinal profiles. It is important to note that, despite our efforts, the evaluation of reversals in the profile is the most subjective part of the study, and that our interpretations (listed extensively in supplementary table 4, observations) are open to debate.

We first start by producing high resolution, manually mapped streamlines that follow the lower elevation points at the bottom of the valley (bottom line) of the main stem in each valley network, by using a combination of CTX and THEMIS mosaics overlain with MOLA colored relief data. This mapping effort improves the quality of the longitudinal profile extracted by reducing the data scatter and the sections where the mapped line deviates into the side walls (refer to figure 4), which may create artificial uphill sections.

We then extract elevation values for the mapped streamlines by using MOLA altimetry points instead of the gridded MOLA DEM. Although the nominal resolution of the blended MOLA/ HRSC DEM is close to 200 m/pixel, the real spacing of MOLA laser altimeter points may be as close as a few meters, and as far as tens of kilometers (black dots in figure 3, also section 4). To address the possible interpolation problems, we obtain the MOLA laser altimetry points (figure 3 black dots) from the PDS server (<http://ode.rsl.wustl.edu/mars/indextools.aspx>) and extract those that intersect with the mapped streamline or are within 100 m of it, accounting for the ~ 100 m MOLA laser footprint [54, 32]. Altimeter points in the profiles are shown in figures 3 and 10 together with their error bars (red crosses), calculated as explained in section 4. From the altimeter points, we produce a filtered profile with a moving mean filter with a window of 5 points (dashed black line, bottom figure 3).

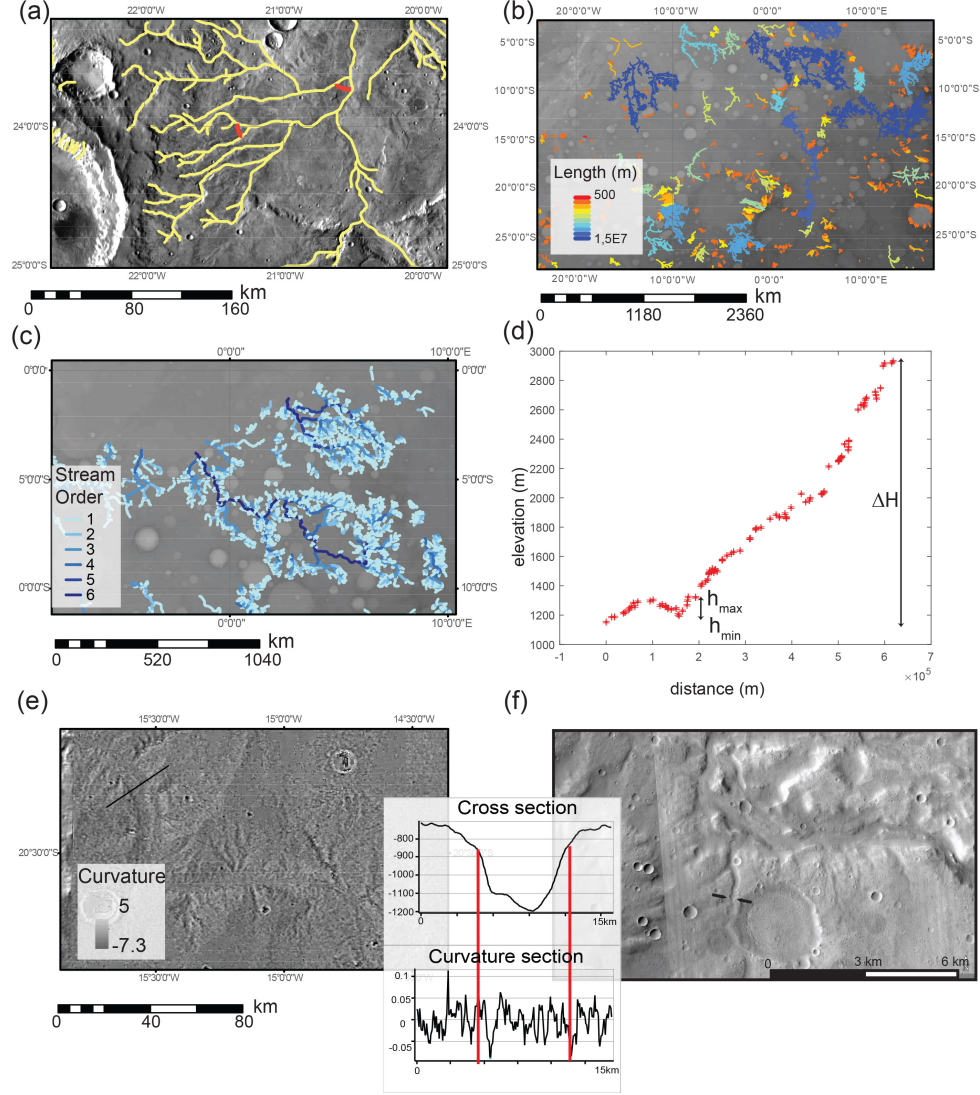


Figure 2: (a) Examples of angles at a junction between two streamlines γ (red). (b) Color coded stream line length L . (c) Stream lines color coded to their stream order S_n . (d) MOLA altimeter points and error bars from a longitudinal profile, indicating the total topographic drop (ΔH) to the uphill section at an undulation ($h_{max} - h_{min}$) Ω . (e) Valley network width W with elevation and curvature profiles. Length over width makes the aspect ratio R . (f) CTX image with resolved first order tributaries, from which we extract the width of tributaries comparable to MOLA resolution λ .

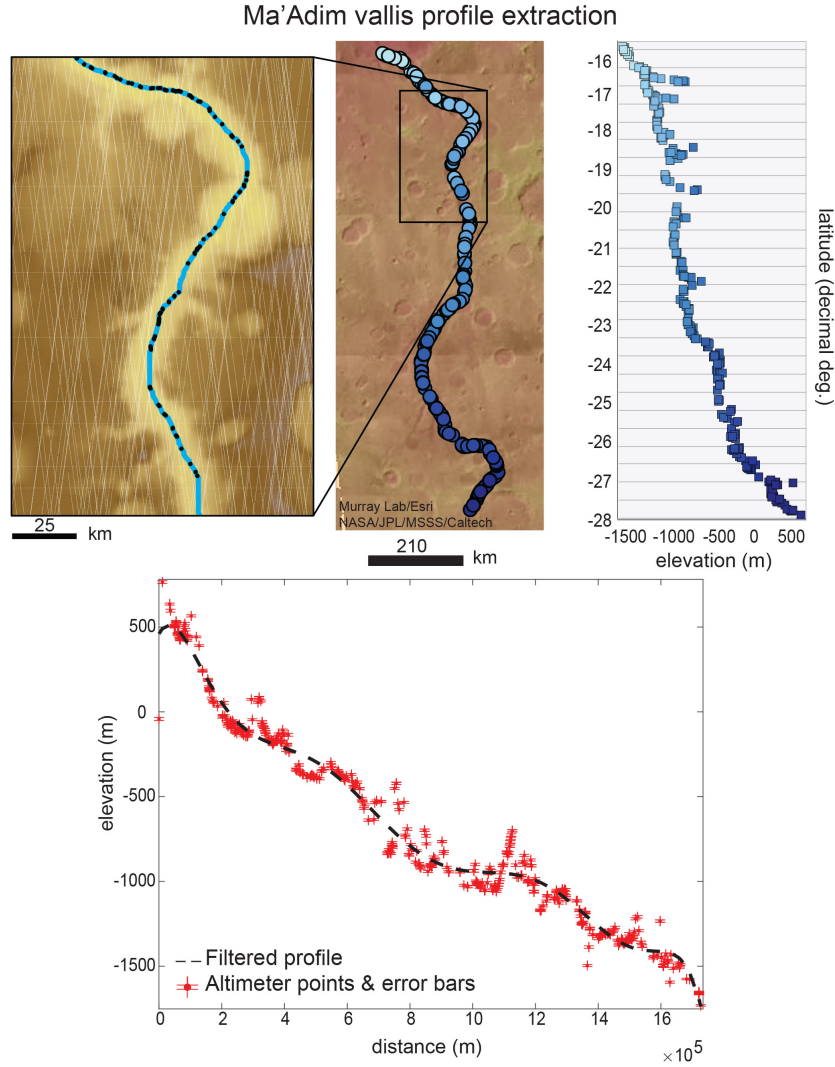


Figure 3: Upper left: Streamline (blue), MOLA tracks (white), and extracted individual altimeter points (black) for Ma'adim vallis (background is MOLA/ HRSC blended DEM). Upper right: Ma'Adim altimeter points overlapping the valley next to the matched longitudinal profile (latitude vs. topography). Bottom: Longitudinal profile after filtering (black) together with the individual altimeter points and corresponding error bars (red).

Undulations are finally measured following the sketch of figure 2(d) at a region of positive slope by dividing the local maxima $h_{max} - h_{min}$ at the undulation by the total profile topographic drop ΔH . The results are then evaluated, and kept only if the elevation of the maxima $h_{max} - h_{min}$ is larger than the error bars, calculated as detailed in section 4.

2.1.2 Discerning true subglacial undulations

Undulations in the longitudinal profile of a valley network may exist either as primary, when pressure gradients drive the flow uphill, or as a consequence of post-emplacement mechanisms. Processes inducing non-primary undulations include: (1) post/syn-emplacement surface tectonics, such as faulting and folding, observed e.g., in Warrego valles [1]; (2) aeolian or glacial infill, observed e.g. in Mamers and Clasia valles [26]; (3) MOLA altimeter data interpolation issues or data spikes, observed e.g., in Ma'Adim valles; (4) sediment infill from a tributary, observed in e.g., Al-Qahira vallis; (5) post/syn-emplacement cratering and ejecta blanketing superimposed on the valley, observed in nearly all valley networks [20]; (6) mismatch between the mapped streamlines and the real valley bottom line, observed e.g., in Nanedi valles (figure 4); (7) volcanic or landslide deposits inside the valley, found e.g., in M68; (8) chaos, subsidence or collapse, observed in e.g., Shalbatana vallis [41]; and (9) differential erosion, leading to terrain deflation and potential terrain inversion, found e.g., in Indus valles [31, 21]. We often find primary undulations in spatial association with changes in cross-sectional size or presence of inverted ridges (see supplementary table 4, observations).

To discern undulations attributed to the primary formation of the valley network from other post-emplacement origins (secondary), we first characterize the reversals in the context of the valley planform, where we delineate the location of craters and ejecta that intersect the valley (figure 5, panels a and d). Ejecta blankets eroded by the valley, as well as breached craters (shown in dashed lines), are older than the valley and therefore contribute to create pre-existing topography, but do not cause an alteration of the profile. Undulations located within eroded cratering signatures are likely primary. In contrast, craters and ejecta younger than the valley have disrupted the profile (shown in solid lines), and any related undulations are not primary. After identifying craters and ejecta, we focus in interpreting the detail of each reversal using the CTX global mosaic overlain with MOLA/HRSC colorized terrain. Here we can assess the presence of infill (aeolian, lineated infill, blocky terrain, landslides, sedimentary supply, etc.), the presence of tectonic structures such as faults and folds, smaller craters and ejecta within the valley, the possibility of the streamlines climbing into steep walls, spikes in the MOLA data, etc. A full description and assessment of the origin of each undulation above detection in the longitudinal profile of all valley networks under study is included in the supplementary table 4 (observations). Here, we present the case studies of Surius and Loire valles, from profile extraction to full undulation assessment.

Surius valles: Surius valles is centered at 48° W, 60° S, and drains into

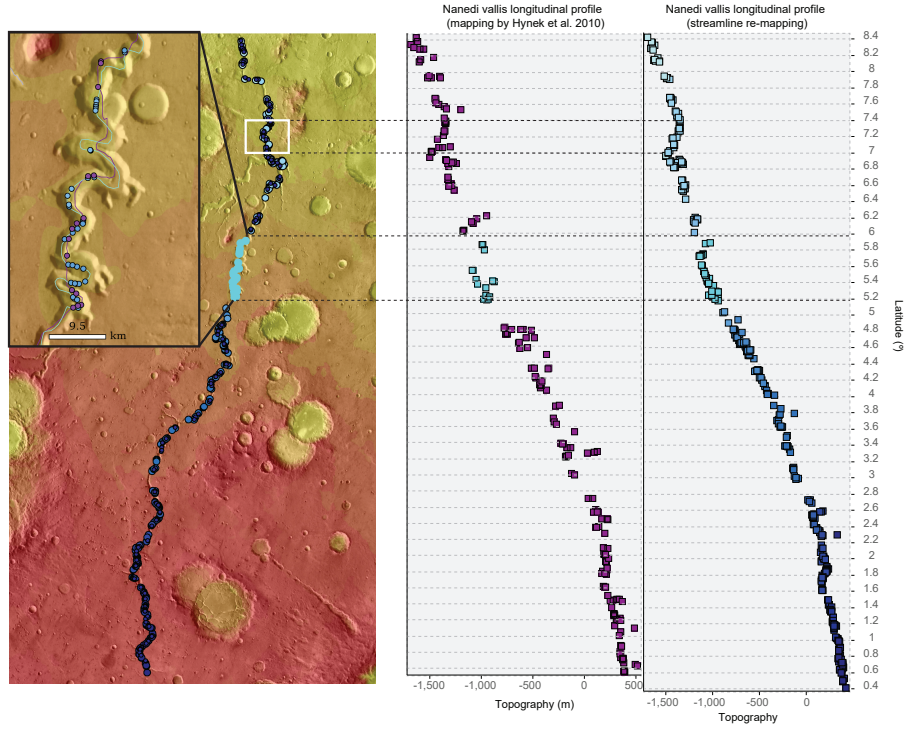


Figure 4: Nanedi valles planform and longitudinal profile. Left: THEMIS overlay with MOLA shaded relief and MOLA altimetry points following the streamlines by Hynek et al., 2010 [18] colored in purple and our high resolution streamlines in blue. Closeup: CTX mosaic overlay with MOLA shaded relief showing the detail of the Hynek et al., 2010 streamline (purple) and our streamline (blue). Right panels: longitudinal profile of Nanedi valles obtained from the Hynek et al., 2010 streamline (purple) and our streamline (blue). Blue boxes in the planform map highlight areas with reversals found in the longitudinal profile.

the Argyre impact basin with a S-N paleoflow direction (figure 5a). We start our analysis by carefully manually mapping the valley bottom line as detailed above and shown in figure 4, using the ESRI ArcGIS editor toolbox. With the mapped streamline, we project and extract the MOLA altimeter points located 100 m or less from it, accounting for the ~ 100 m MOLA laser footprint [54, 32] (figure 3). We then calculate and project the error bars (section 4, result in 5b), and perform a detailed visual assessment of all sections where the valley bottom is ascending beyond the error bars, which are indicated with the numbers 1-4 in figure 5c.

Surius contains four undulations in its longitudinal profile above the detection limit set by the size of the errorbars. Lineated valley infill is visible at CTX resolution in numerous parts of this valley, making the interpretation of undulations challenging in some locations. From upstream to downstream, the

first reversal (1, refer to the numbers in the bottom profile panel c) is located in a meandering section of the valley, and not visibly related to tectonic structures, superimposed craters/ejecta, sedimentary or lineated valley infill, or other post-emplacement processes. We interpret it to be primary. The second reversal (2) is again not visibly associated to post-emplacement processes, including lineated valley infill. However, the valley is hard to visualize here and it is possible that the undulation is associated with a mismatch between the valley bottom and the streamline. We discard this undulation. The next reversal (3) is associated with a narrowing of Surius valles' cross-section, without signs of infill (LVF or aeolian) or other post-emplacement mechanisms, so we interpret it to be primary. At the rim of Argyre, the valley becomes harder to trace as fingerprints of other processes of likely glacial origin appear (e.g., cirques and crevasses). Lineated valley infill and possibly a glacial tongue are present 12 km before the termination of the valley. The last reversal (4), shown with the CTX mosaic closeup at the bottom right of panel (a) (highlighted blue dots), starts before this zone of glacial overprint. Here Surius climbs 830 m in 31 km, up the Argyre rim, to then flow down and into Argyre. A fraction of the elevation gained in this reversal (630 m) is not visibly associated with any post-emplacement process visible at CTX resolution, but the last 200 m uphill are associated with the presence of lineated valley fill, as the valley enters the region with evident glacial overprint. We interpret the first part to be primary, and discard the elevation gained in association with the glacial bodies, but we acknowledge the possibility for differing interpretations for this reversal.

At the termination of Surius valles a complex of anastomosing inverted ridges appear, which have been interpreted to be an esker system [23, 2, 10]. The presence of eskers and the wet-based glaciation overprint of the rim region add to the case of a subglacial origin for Surius. Figure 5 shows the planform of Surius valles, the high resolution main stem streamline, and the MOLA altimeter points extracted, as well as the delineated craters and ejecta blankets that intersect the valley (dashed lines for craters older than the valley, solid lines for younger). The value of the four undulations in Surius valles is of 3%, 3%, 1.8%, and 14.3%, but we acknowledge that these values are subject to revision as better topography and imagery of the region become available, in particular with respect to identifying lineated valley infill. The largest reversal is recorded in supplementary table 1 (dataset).

Loire valles: Loire valles is centered around 18° S, 17° W, and flows following an overall E-SE to W-NW direction into the Loire/Parana basin, as shown in figure 5d. The longitudinal profile of Loire valles is complex and often affected by cratering, as shown in panels d, e, and f, and shows up to 8 reversals. From up to downstream, the first (1) is associated with a region where the valley blurs, east of the eroded ejecta blanket of a crater. The second undulation (2) is very gentle, ≈ 50 km long and 50 m high, and is not visibly related to infill, mismatch of streamlines with valley bottom line, structures or cratering, so we interpret its origin to be primarily associated with valley formation. The next reversal (3) is ≈ 11 km long, ≈ 200 m high, and is again not clearly associated with post-emplacement modification processes. Although there are two

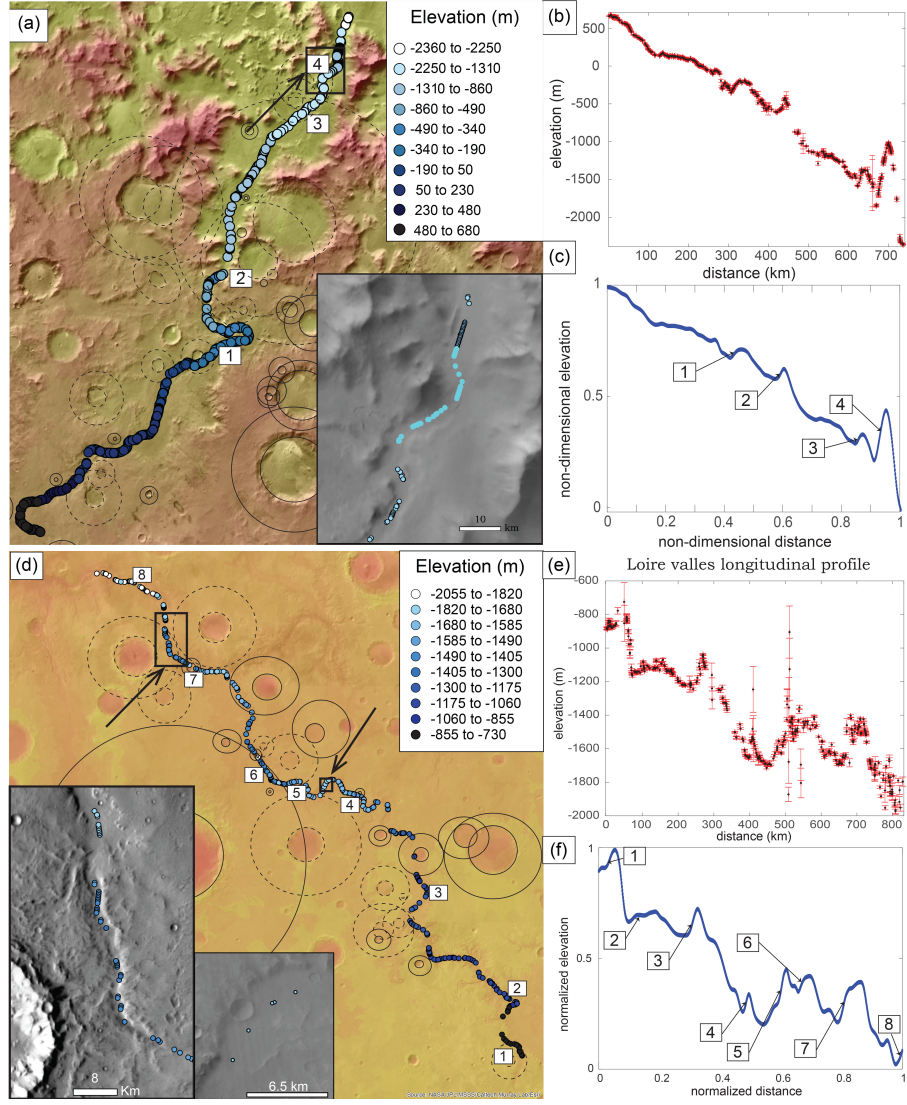


Figure 5: Planform, closeup mosaic, and longitudinal profiles of Surius valles (upper panel) and Loire valles (bottom panel). Left: valley network planforms (THEMIS and MOLA colored relief) showing the MOLA altimetry data (blue dots, color coded by elevation). Craters and ejecta are shown with two concentric solid circles if superimposed (younger) and dashed if eroded by the valley. CTX mosaic close-ups show the detail of the reversals indicated with black arrows. Right: longitudinal profiles showing error bars (upper panel) and indicating the profile reversals above detection limit (bottom), with numbers indicated also in the left panels for cross-referencing

craters in the proximity to the west and one to the north, the valley incises

into the ejecta of both western craters (the smaller, closer one to the reversal has lost the ejecta to erosion almost entirely), whereas the ejecta blanket of the northern crater terminates very visibly much before the reversal. We interpret this undulation to be primary. The following undulation (4) is associated with a valley meander, and with a partially eroded crater to the north. The next reversal (5) is important (≈ 55 km, 200 m high) and it is spatially correlated with a significant widening of the channel cross section, as well as with the end of the meandering section and the disappearance of Loire. Located to the west, the ejecta blanket of Jones crater is partly responsible for the coverage and erasure of the valley. However, Loire starts climbing well before the possible ejecta influence, and the valley is clearly traceable as it flows uphill until its disappearance. In this section, there is no infill, ejecta/cratering, visible surface tectonics, mismatches in streamline mapping into the side walls, etc. We interpret this undulation up until the disappearance of Loire into Jones ejecta to be primary, and point out that changes in channel cross section (even disappearance) are commonly related with undulating sections in subglacial streams. Loire valley disappears abruptly for about 60 km, in a similar manner to other similarly sinuous and discontinuous valleys in the area. Immediately after the meandering section, the cross section of Loire valley is nearly 14 km wide and ≈ 450 m deep. After the Jones crater ejecta blanket, located to the west and partly overlapping the valley during its disappearance, the cross-section of Loire valley is ≈ 145 m deep and 7 km wide. The initial, pre-Jones cross-section is never recovered, and the closer Loire valley comes back to these values is ≈ 100 km after the disappearance, where it also recovers elevation it recorded before Jones' ejecta. After the massive Jones ejecta blanket, the next undulation (6) is located spatially correlated with an ejecta blanket incised by Loire. We interpret this to be a primary undulation, although the cratering process may have affected this topographic signature. The next undulation (7) is associated with the ejecta and rim of two craters superimposed on the valley. The last undulation (8) also shows the valley incising into ejecta from a western nearby crater. Although the crater may have imposed the initial topographic uphill, the valley has nonetheless excavated the ejecta uphill and produced a reversal. We interpret this undulation to be primary. At this point, the valley widens and the bottom becomes covered in blocky infill. It is not until this point that Loire recovers most of the cross section, and most importantly the elevation, it had before disappearing into the Jones ejecta. The valley finishes shortly after.

2.1.3 Width measurements

Cross-sectional elevation profiles were extracted from HRSC DEM data when available (acquired from <http://hrscview.fu-berlin.de/mex4/> at variable resolutions, ranging from 25 to 200 m/pixel), and else from the global MOLA/HRSC blended DEM described above. For each valley network under analysis, we digitized 10 lines including the channel and inter valley fragments on each side (figure 2d), and starting after the valley network reached the highest stream order. We then extracted the elevation profiles using a tool for ArcGIS (3D

analyst – Functional Surface – Stack profile), with the result shown with blue crosses in figure 6, upper panel.

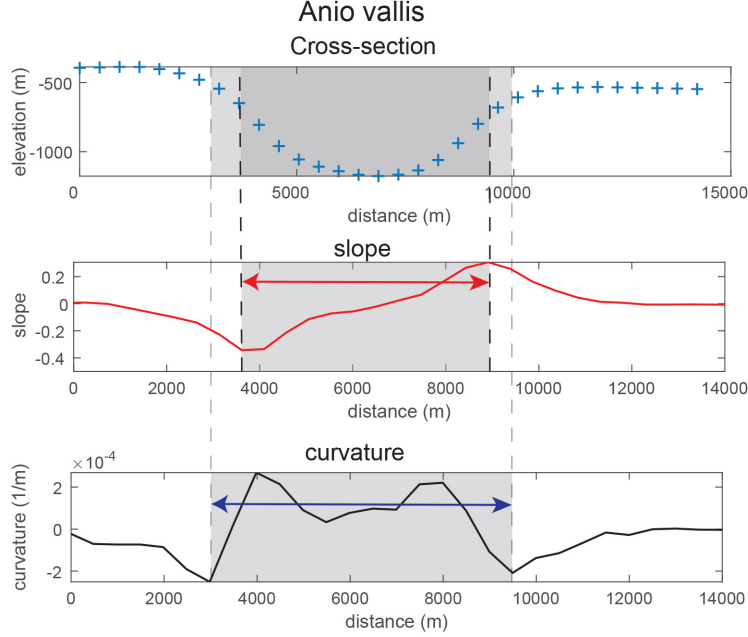


Figure 6: Measurement of the top cross sectional width, Anio Vallis. Top panel shows the cross sectional profile (elevation vs. distance), middle panel is the slope profile (slope vs. distance), and bottom panel is the curvature profile (curvature vs. distance). Shaded ranges in the slope and curvature profiles indicate regions between minimum and maximum slope (middle), and between two curvature minimums (bottom panel). Both areas are superposed in the cross sectional upper panel, showing the top width estimation (blue arrow) and the lower bound (red arrow).

The automatic extraction of the top width consisted in the derivation of slope and curvature profiles (figure 6) by calculating, respectively, the first and second derivatives of elevation with distance. For each profile, top width is given by the distance between the points of absolute minimum curvature (Fig. 6, blue arrow in the bottom panel). To correct for irregularities, width has to be equal or larger than the distance between the minimum and maximum slopes (figure 6 middle panel, red arrow). For each valley network, the top width is then the mean of the 10 measurements obtained as outlined above, with error attributed to mean standard deviation (see section 4).

2.1.4 Minimum width measurements

First order tributaries (i.e., valley networks with no tributaries of their own) yield the smallest channel widths. For Mars, the smallest, ‘fine-scale’ tributaries are in the range 100–500 m wide [53], that is, approaching or below DEM

resolution. In the majority of systems under analysis, first order widths are well above this limit (1–2 km across, see supplementary table 1, dataset), making the methodology outlined above applicable. For these valley networks, we digitized 5 lines across the width of 10 first order tributaries and proceeded with profile analysis in the same manner outlined above. In those systems where first order valley width was comparable to DEM resolution, we used CTX imagery at 12 m/pixel (available at <https://viewer.mars.asu.edu/viewer/ctx#T=0>) to manually measure top width. In this case, we took 5 profiles across the channel for each valley network, and repeated the measurement, again, for 10 first order tributaries or as many as existing in the valley network. For measuring purposes, we used a sinusoidal projection centered in each valley network (latitude and longitude given in supplementary table 1, dataset) to minimize projection error.

2.2 Fractal dimension measurements

Fractal (Hausdorff) dimensions were calculated using the valley network streamlines as an image input for a box counting algorithm (boxcount), which is a built-in function of MATLAB (<https://www.mathworks.com/matlabcentral/fileexchange/13063-boxcount?focused=5083247&tab=example>).

2.3 Junction angle measurements

Junction angles were extracted automatically from valley network stream lines [18]. To provide a meaningful measurement and take into account the variability in stream line direction between successive tributary junctions, we simplified the stream lines at different length baselines, as shown in figure 7 below. The two baselines used correspond to 200 m (purple line), twice the resolution of THEMIS at which valley networks were delineated [5, 18], and 5000 m (red line), larger than valley median top width. We identified the locations of the junctions between tributaries automatically using the ‘Intersection Analysis’ ArcGIS built-in toolbox.

Stream lines were then buffered and clipped at the junction between tributaries using the ArcGIS toolbox, as shown in figure 7 (zoomed in upper left panel). We extracted measurements of the junction angle (acute angle between tributaries at each junction) by taking the dot product between the direction of both tributaries and solving for the angle cosine.

3 PCA technical details

3.1 Contributions of each metric to the valley network variance

Each Principal Component (PC) is by definition of the transformation a known linear combination of the set of 6 metrics defined in the text (R , λ , γ , S_n , D_f , and Ω), with coefficients scaled to the variance captured by each metric. Figure

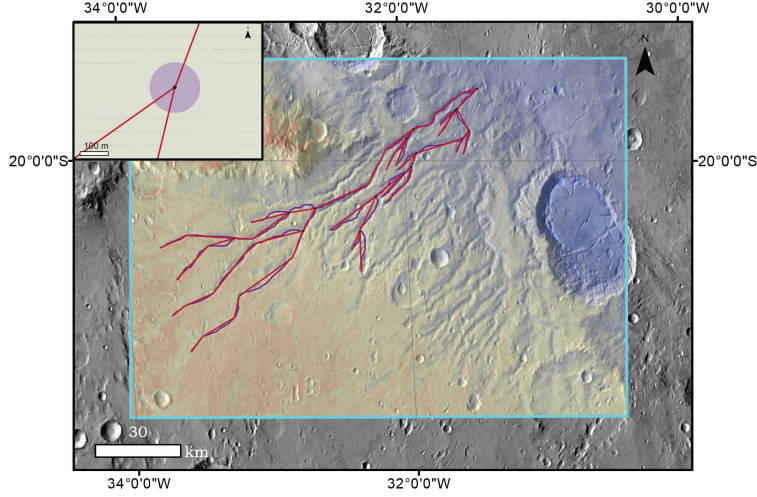


Figure 7: Example of the junction angle measurement for Arda Valles, with streamlines overlaying a colorized elevation map (MOLA) and a THEMIS mosaic. The panel shows the simplified streamlines (purple and red lines) the automatically identified points at the junctions (black points). Upper left panel zooms in a junction showing the 1 km buffer circle, from which the angle between the tributaries is extracted.

8 shows the dependence on each of the metrics of the first, second and third PC (dots), which capture up to 82% of the variance.

The first PC captures 40.5% of the data variance, and it is dominated by the stream order, and followed by the width of 1st order tributaries and aspect ratio. Width of order 1 tributaries, stream order and undulations dominate the 2nd PC, in turn, which captures 25.3% of the total dataset variance. The third PC is largely dominated by the undulations and captures 16.5% of the total variance. Individually, the metric that explains a larger portion of valley network variability is the stream order S_n (27.8%), followed by the width of order 1 tributaries λ (27.4%), the magnitude of undulations Ω (19.5%), length to valley network width aspect ratio R (15.5%), junction angle γ (6.9%), and fractal dimension D_f (2.7%), in turn.

3.2 Sensitivity analysis

We assess the robustness of the PCA-based valley network classification, and therefore the dependence of the manuscript results on our choice of valley networks, by performing a sensitivity analysis on different subsets of Martian valleys. From the suite of 66 systems presented in this study, we first analyzed the variance captured by each metric in subsets of all fluvial, all glacial, all subglacial, and all sapping networks as inferred from the results of the valley network classification. Then we performed similar evaluations of the metric relative contribution to the variance in 210 different runs with random data

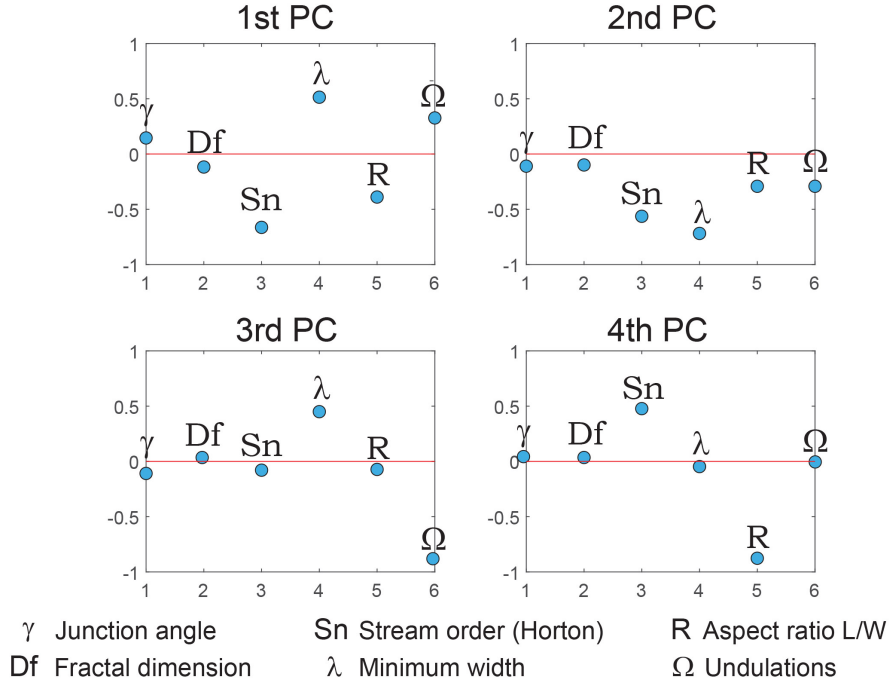


Figure 8: Linear decomposition of the Principal Components, scaled to the variance that each of the metrics captures in that direction. From left to right, symbols are junction angle γ , fractal dimension D_f , stream order S_n , width of 1st order tributaries λ , aspect ratio R , and undulations Ω . X-label shows the order in the PC vector, whereas y-label shows the magnitude.

subsets.

Table 1 below shows the results of the sensitivity analysis in terms of the relative difference between the variance captured by a particular metric in a given run and the variance captured by that metric considering the entire dataset. Columns show the different metrics, and rows show the sample size of the evaluation. The relative difference between the variance captured by each metric in all the data and random subset diminishes in all metrics as subset selections grow larger, and becomes larger for smaller subsets, as expected.

All metrics but the aspect ratio show average relative differences below 20%. Aspect ratio, however, shows a high relative difference up to 39%. We relate this higher difference to the presence of two valley networks with substantially larger R than the rest, which makes the relative difference a large function of whether these two data points are part of the subset or not. This is however not a data bias, since these are the longest valley networks on Mars [18], and also relatively narrow. For the complete results including analysis of each subsample of valley networks, please see supplementary table 5, sensitivity. The results of the PCA classification, including the distance of each valley network to fluvial, glacial,

sapping, and subglacial subsets of synthetic valley networks, are reported in supplementary table 3, results.

4 Error estimation and propagation

Each of the measurements contains a measure of uncertainty, which propagates into the quantification of the metrics and eventually to the three principal components used to classify the valleys.

4.1 Sources of error

We will treat each of the following sources of error as independent sources, such that the final error associated with e.g., the length measurement is:

$$\delta L = \sqrt{\sum \delta_{L_i}^2} \text{ with } \delta_{L_i} \text{ being the individual sources of error as listed below.}$$

4.1.1 Map projection and associated Tissot’s distortion

The local distortion associated with using any map projection can be calculated with Tissot’s Indicatrix distortion tensor (also known as the distortion ellipse), and it is latitude and longitude dependent. The angular distortion affects junction angle, the areal inflation affects fractal dimension, and width, minimum width (minimally), and length vary according to Tissot’s scale factors given by the semiaxis of the distortion ellipse. We calculate the errors introduced with Tissot’s distortion applied to the Sinusoidal projection with center in the latitude/longitude coordinates given in supplementary table 1 (dataset) for each valley network. To do so, we follow the mathematical definition of the projection and the differential algebra required to calculate Tissot’s indicatrix [46, 19]. Below we will use $[\delta L_{Tiss}, \delta A_{Tiss}, \delta \gamma_{Tiss}]$ to refer to the error introduced by length, area, and angular Tissot’s distortion respectively. Given the wide distribution of the valley networks analyzed, we chose to use a Sinusoidal projection centered at each valley instead of using a global projection to minimize error.

4.1.2 Statistical error

Three measurements have statistical uncertainties: junction angle γ , minimum width λ , and width. For each valley network we calculate the statistical error of the mean measurement as the standard deviation of all measurements taken divided by the square root of the number of samples. For example, the statistical error associated with the junction angle is:

$$\delta \gamma_{stat} = \sigma_{gamma} / \sqrt{N_{gamma}}$$

with N_{gamma} being the number of junction angles in the network, and σ_{gamma} the standard deviation. Whereas the number of junction angles in a network varies from valley network to valley network, we took $N_{width} = 5$ for our width measurement, yielding the error δW_{stat} . For the minimum width, $N_{\lambda} = 5N_t$

with N_t the number of tributaries selected (ranging from ten to all in the network), yielding the error $\delta\lambda_{stat}$.

4.1.3 Dataset resolution

Length, width, minimum width, and elevation are affected by the resolution of CTX, THEMIS, and the blended MOLA/ HRSC DEM as well as the instrumental error from the MOLA altimeter. The uncertainty related to the length measurement is THEMIS resolution $\delta L_{res} = 100$ m because valley networks are mapped with THEMIS data; to width is the MOLA/ HRSC DEM resolution as width is measured from elevation cross-sectional profiles $\delta W_{res} = 256$ m (lat/long dependent); and to minimum width is the resolution of CTX $\delta\lambda_{res} = 12$ m.

4.1.4 Error associated with longitudinal profile elevation

The identification of subglacial channels within the valley networks relies partly on the undulations metric, and therefore the estimation of the uncertainty in longitudinal profile elevation is crucial to the results of this paper. Our measurement of profile elevation is affected by three sources of systematic error:

1. The instrumental error (footprint size of MOLA and vertical error).
2. The shift between MOLA and THEMIS mosaics used to map the streamlines, in each valley network location.
3. The error of valley network mapping (THEMIS resolution + deviations from the center line).

They can essentially be combined in two: the three-dimensional distance between the point where the MOLA measurement is done and the point where the valley network really is (THEMIS-MOLA shift + THEMIS resolution and human mapping error) and what the laser altimeter value gives compared to the real elevation value (MOLA instrumental resolution). First we quantify the difference between the real valley network elevation and elevation of the valley network streamlines mapped here using the THEMIS dataset complemented with CTX mosaics. The shift in THEMIS to MOLA data is such that there exist a small displacement between a point in MOLA and the same point in the THEMIS mapping 9:

$$x_{THEMIS} = x_{MOLA} + \Delta x, \quad y_{THEMIS} = y_{MOLA} + \Delta y \quad (1)$$

The shift between a point in the mapped streamlines and the real location of that point in MOLA is a combination of THEMIS resolution and the MOLA-THEMIS shift:

$$\Delta x = \sqrt{\Delta x_{TH-MOL}^2 + \Delta x_{TH}^2}, \quad \Delta y = \sqrt{\Delta y_{TH-MOL}^2 + \Delta y_{TH}^2}. \quad (2)$$

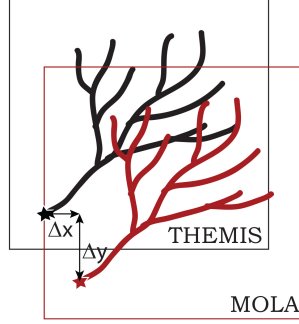


Figure 9: Shift between the MOLA dataset, where profile measurements are extracted from, and THEMIS data where valley networks are mapped from.

The real elevation of a point in the valley network (red star in the image) is given by means of a Taylor expansion around the equivalent THEMIS point in the valley network (black star in the image):

$$z_{VN} = z_{MOLA} + \frac{\Delta z}{\Delta x_{x,y_{MOLA}}} \Delta x + \frac{\Delta z}{\Delta y_{x,y_{MOLA}}} \Delta y. \quad (3)$$

The associated uncertainty in elevation is therefore:

$$\delta z_{TH-MOL} = S_x \Delta x + S_y \Delta y, \quad (4)$$

Where S_x and S_y are the local slopes:

$$S_x = \frac{\Delta z}{\Delta x_{x,y_{MOLA}}}, \quad S_y = \frac{\Delta z}{\Delta y_{x,y_{MOLA}}}. \quad (5)$$

In addition to the THEMIS-MOLA shift, there is a source of systematic error in the size of the MOLA laser altimeter footprint, which also has associated a vertical uncertainty δz_i :

$$\delta z_{MOLA} = \frac{\delta z_i}{\delta x_i} \delta x_i + \frac{\delta z_i}{\delta y_i} \delta y_i + \delta z_i \quad (6)$$

Which can again be written in terms of local slopes ($S_{i,x}, S_{i,y}$) around the MOLA measured point:

$$\delta z_{MOLA} = S_{i,x} \delta x_i + S_{i,y} \delta y_i + \delta z_i \quad (7)$$

Assuming both sources of error are independent, the total uncertainty associated with each altimeter point within the valley network profile is:

$$\delta z = \sqrt{\delta z_{TH-MOL}^2 + \delta z_{MOLA}^2}. \quad (8)$$

To constrain δz_{TH-MOL} and δz_{MOLA} we need to estimate the local slopes $S_x, S_{i,x}, S_y, S_{i,y}$ at the baselines of $\Delta x, \delta x_i, \Delta y$, and δy_i respectively. The

longitudinal slopes S_x , $S_{i,x}$ are calculated using the polynomial best fit to the altimeter points and then estimating, from this fit, the values $S_x = \delta z / \Delta x$ and $S_{i,x} = \delta z / \delta x_i$.

We then need to estimate the cross-sectional slopes $S_y, S_{i,y}$. Starting from the cross-sectional profiles extracted to measure the width of the valley, we randomly sample a number N of slope values in each cross-section using a Gaussian probability distribution centered around the bottom of the channel. A Gaussian probability tends to select slope values close to the bottom line (most of the streamlines are close to the valley bottom), but selects with decreasing probabilities slopes located on the valley flanks (some streamlines depart from the center into the side walls). Without measuring S_y or $S_{i,y}$ directly, we are constraining their variance and treating them like a probability distribution variable, with its average set around the valley bottom, its standard deviation set to Δy and δy_i respectively. After calculating the local slopes, the uncertainty in the vertical coordinate of each MOLA point is:

$$\delta z = \sqrt{\delta z_{TH-MOL}^2 + \delta z_{MOLA}^2} \quad (9)$$

The final error in elevation is then propagated into the undulations metric, as detailed below. Figure 10 below shows the longitudinal profile of two valley networks including the altimeter points with vertical error bars.

4.2 Error propagated to the metrics

4.2.1 Aspect ratio error

The uncertainty in the aspect ratio propagates from width and length:

$$\delta R = \sqrt{\left(\frac{L\delta W}{W^2}\right)^2 + \left(\frac{\delta L}{W}\right)^2}, \quad (10)$$

with width error

$$\delta W = \sqrt{\delta L_{Tiss}^2 + \delta W_{stat}^2 + \delta W_{res}^2} \quad (11)$$

and length error

$$\delta L = \sqrt{\delta L_{Tiss}^2 + \delta L_{res}^2} \quad (12)$$

calculated for each valley network.

4.2.2 Junction angle error

The junction angle error is a combination of statistical error and angular distortion due to the map projection:

$$\delta \gamma = \sqrt{\delta \gamma_{stat}^2 + \delta \gamma_{Tiss}^2} \quad (13)$$

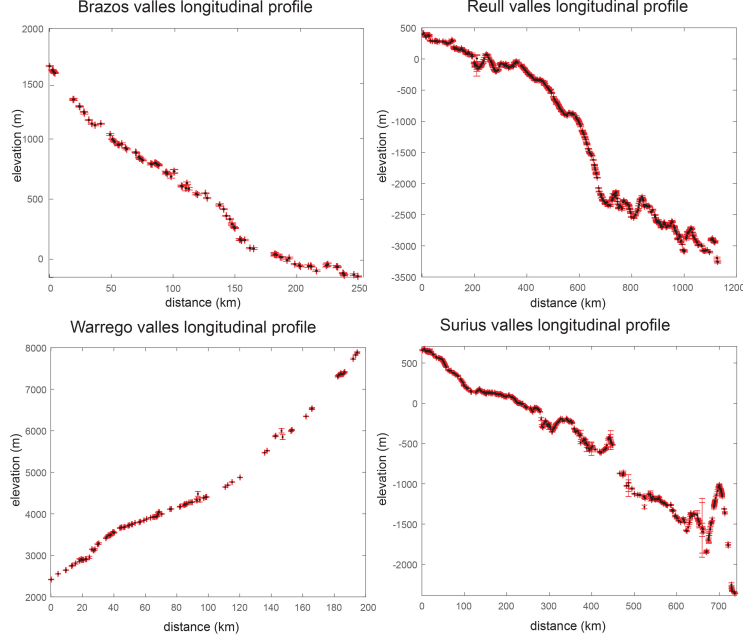


Figure 10: Longitudinal profiles of four valley networks showing elevation errorbars. Examples (a) and (c) have no undulations, or undulations smaller than errorbars, thus below the capability of detection. Examples (b) and (d) show real undulations larger than the extent of the error bars.

4.2.3 Minimum width error

The error in the minimum width metric is again a linearly independent combination of resolution and distortion.

$$\delta\lambda = \sqrt{\delta\lambda_{res}^2 + \delta L_{Tiss}^2} \quad (14)$$

4.2.4 Fractal dimension error

On systems with high area distortion as measured by δA_{Tiss} we will slightly overestimate the value of the fractal dimension. We quantify this overestimation on the basis of area-length scalings and the definition of fractal dimension scalings: $L^{D_f} \sim A$, $D_f \sim \ln A / \ln L$, then:

$$\delta D_f \sim \sqrt{\left(\frac{\delta A_{Tiss}}{A \ln L}\right)^2 + \left(\frac{\ln A}{\ln L^2} \frac{\delta L}{L}\right)^2}. \quad (15)$$

4.2.5 Stream order error

We obtain the valley network stream order directly from previously published streamlines, where there is no error reported [18]. We therefore associate no

error with this metric.

4.2.6 Undulations error

The formal definition of the undulations metric is:

$$\Omega = \frac{\Delta H}{H} = \frac{z_{und} - z_{min}}{H}. \quad (16)$$

Any uncertainty can be written in terms of the differential expression:

$$\delta\Omega = \frac{\delta\Delta H}{H} - \frac{\Delta H \delta H}{H^2} \quad (17)$$

For the Martian valley networks, we can safely assume that:

$$\Delta H \ll H \text{ and } \delta H \sim \delta\Delta H, \quad (18)$$

so that the second term of the right hand side becomes second order, and therefore can be dropped. The error in the undulations metric is then approximated by:

$$\delta\Omega \sim \frac{\delta\Delta H}{H} \sim \frac{\delta z}{H} = \frac{1}{H} \sqrt{\delta z_{TH-MOL}^2 + \delta z_{MOLA}^2}, \quad (19)$$

which depends on the error associated with the elevation of each altimeter point, as reported before. Since the measurement is reported as a percentage, the associated error is multiplied by 100.

4.2.7 Bivariate plots and error bars

Figure 11 shows the most important metrics (by percentage variance captured) in bivariate plots against each other. Vertical and horizontal error bars show the error associated with each metric in each valley network. Left to right, the top row shows aspect ratio R , stream order S_n , minimum width λ , and undulations Ω . Top to bottom the left column shows the same metrics plus the junction angle γ . The horizontal axis correspond to the metric in the left column, whereas the vertical axis corresponds to the metric indicated in the top row.

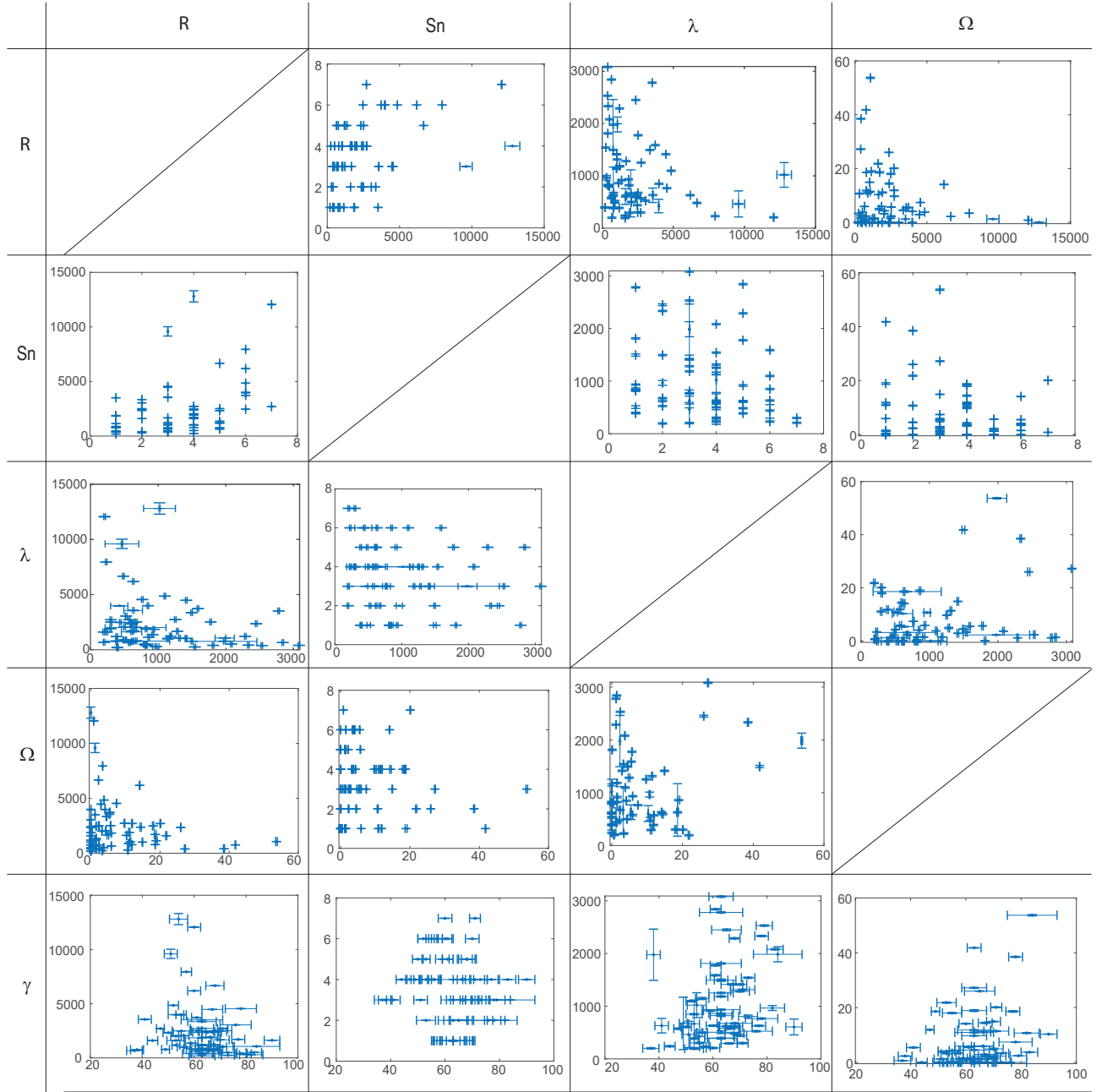


Figure 11: Bivariate plots of the most representative metrics showing errorbars

4.3 Error propagated to the principal components

The principal component analysis consists of a linear algebra transformation relating a set of six old variables (metrics) to a set of six new variables (the principal components). We can write each metric as its average value and its associated error, so that our metrics dataset is represented by a matrix such as this one:

$$VN_{ik} = \begin{pmatrix} \langle \lambda \rangle_1 \pm \delta \lambda_1 & \langle S_n \rangle_1 \pm \delta S_{n1} & \langle D_f \rangle_1 \pm \delta D_{f1} \\ \vdots & \ddots & \vdots \\ \langle \lambda \rangle_{66} \pm \delta \lambda_{66} & \langle S_n \rangle_{66} \pm \delta S_{n66} & \langle D_f \rangle_{66} \pm \delta D_{f66} \end{pmatrix}$$

$$\begin{pmatrix} \langle \Omega \rangle_1 \pm \delta \Omega_1 & \langle R \rangle_1 \pm \delta R_1 & \langle \gamma \rangle_1 \pm \delta \gamma_1 \\ \vdots & \ddots & \vdots \\ \langle \Omega \rangle_{66} \pm \delta \Omega_{66} & \langle R \rangle_{66} \pm \delta R_{66} & \langle \gamma \rangle_{66} \pm \delta \gamma_{66} \end{pmatrix} = \langle VN_{ik} \rangle \pm \delta VN_{ik}$$

Where the indexes i and k go from $i = 1 : 66$ and $k = 1 : 6$, with 66 being the number of valley networks we analyzed and 6 the number of metrics we measured. The projection of the metrics dataset into the Principal Component space of figure 2 (manuscript) is given by the transformation matrix. In essence, the transformation re-scales, rotates, and re-centers the metrics dataset VN_{ik} to display it using principal components PC_{jk} , that is, in the orthogonal basis representing the maximum variance of the data. There are 6 principal components ($j = 1 : 6$) and again, 66 valley networks ($k = 1 : 66$). In this new basis, our data matrix is given by principal components one to six instead of six metrics:

$$PC_{jk} = \begin{pmatrix} \langle PC \rangle_{11} \pm \delta PC_{11} & \langle PC \rangle_{12} \pm \delta PC_{12} & \langle PC \rangle_{13} \pm \delta PC_{13} \\ \vdots & \ddots & \vdots \\ \langle PC \rangle_{661} \pm \delta PC_{661} & \langle PC \rangle_{662} \pm \delta PC_{662} & \langle PC \rangle_{663} \pm \delta PC_{663} \end{pmatrix}$$

$$\begin{pmatrix} \langle PC \rangle_{14} \pm \delta PC_{14} & \langle PC \rangle_{15} \pm \delta PC_{15} & \langle PC \rangle_{16} \pm \delta PC_{16} \\ \vdots & \ddots & \vdots \\ \langle PC \rangle_{664} \pm \delta PC_{664} & \langle PC \rangle_{665} \pm \delta PC_{665} & \langle PC \rangle_{666} \pm \delta PC_{666} \end{pmatrix}$$

$$= \langle PC \rangle_{jk} \pm \delta PC_{jk}$$

The basis change is given by the matrix of basis change between metrics and principal components PCM_{ij} so that our initial dataset VN_{ik} and the principal components dataset PC_{jk} are related through a matrix multiplication:

$$PC_{jk} = PCM_{ij} [\langle VN \rangle_{ik} \pm \delta VN_{ik}] \quad (20)$$

Applying the distributive property we have:

$$PC_{jk} = PCM_{ij} \langle VN \rangle_{ik} \pm PCM_{ij} \delta VN_{ik} = \langle PC \rangle_{jk} \pm \delta PC_{jk}. \quad (21)$$

Identifying terms, the error of each of the principal components is $\delta PC_{jk} = PCM_{ij} \delta VN_{ik}$, that is, the PCA basis transformation matrix multiplied by the

matrix of metrics errors. Because we use the 1st, 2nd and 3rd principal components, we can use the matrix of the change of basis PCM_{ij} (visually displayed for PC 1, 2, 3, and 4 in figure 8) to find how the errors propagate from each metric measurement to the three principal components in each valley network. For example, for the first principal component and the valley network k we have:

$$\delta PC_{1k} = PCM_{11}\delta\lambda_k + PCM_{12}\delta S_{n_k} + PCM_{13}\delta D_{f_k} + PCM_{14}\delta\Omega_k + PCM_{15}\delta R_k + PCM_{16}\delta\gamma_k. \quad (22)$$

Figure 12 below shows the principal component analysis with added error bars. With the exception of three systems that show significant error bars (black dots), the classification of valley networks is not affected by the measurement error. The main source of uncertainty in the valley networks affected are the statistical variety in cross-sectional width measurements that propagates into the aspect ratio, and the irregularity and/or lack of data points in some valley network longitudinal profiles propagating into the undulations error. These are not sources of error we can diminish by repeating measurements or changing the projection.

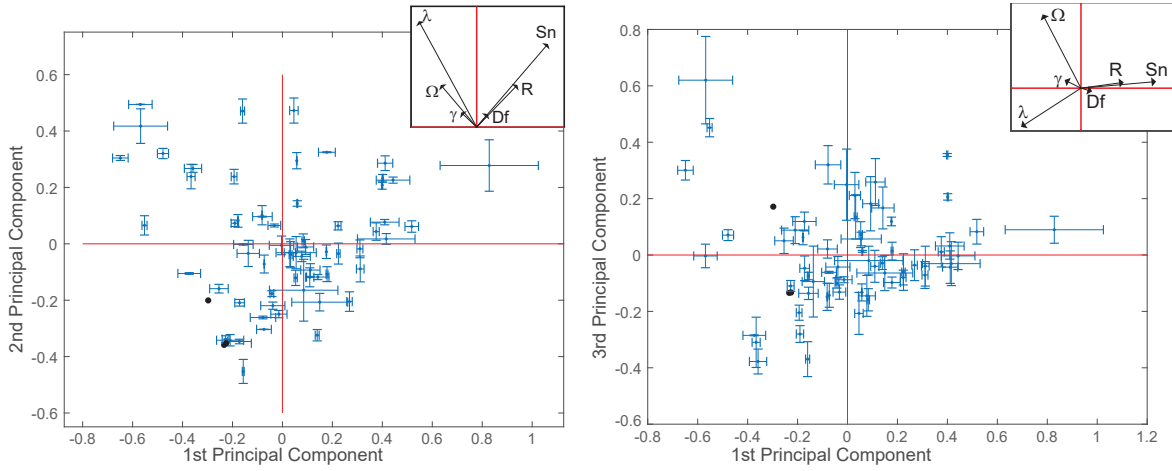


Figure 12: 1st and 2nd principal components (left panel), and 1st and 3rd principal components (right panel) with error bars calculated as stated. The upper right panels show arrows pointing towards the direction of metric increase, and with relative magnitude scaled with the variance captured respectively. Black dots are those valley networks with large error bars, and correspond to the same systems in both plots.

5 PCA end-member examples

Figure 13 shows characteristic examples of fluvial, glacial, sapping, and subglacial systems, chosen among the valley networks with highest confidence, as shown in figure 2 in the manuscript.

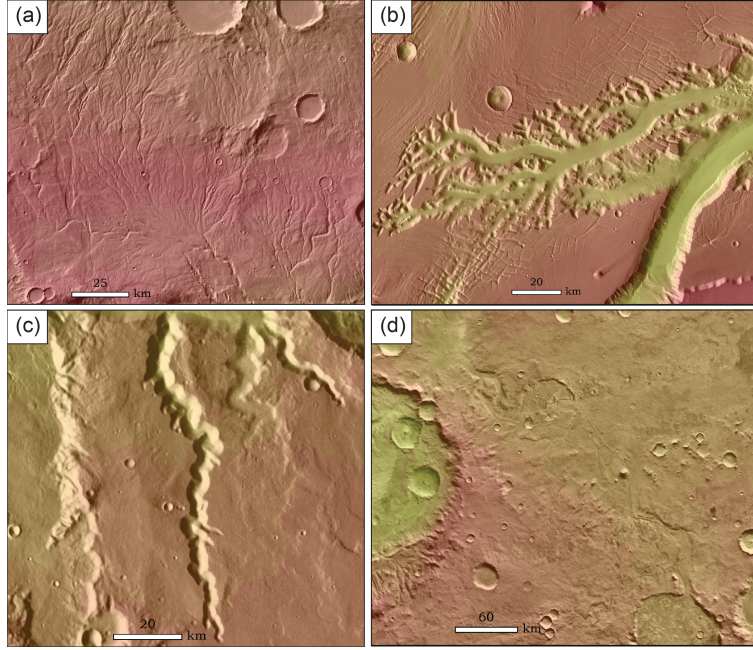


Figure 13: Representative examples of fluvial (upper left), glacial (upper right), sapping (lower left) and subglacial (lower right) valley networks. Warrego valles is the fluvial system ($\gamma = 55$, $D_f = 1.79$, $S_n = 6$, $\lambda = 420$ m, $R = 1233$, and $\Omega = 0\%$). The glacial example is M68, an unnamed valley within the Kasei system ($\gamma = 83$, $D_f = 1.5$, $S_n = 4$, $\lambda = 2080$ m, $R = 408$, $\Omega = 3.7\%$). The sapping example is Abus Vallis ($\gamma = 63$, $D_f = 1.31$, $S_n = 1$, $\lambda = 820$ m, $R = 341$, $\Omega = 1.5\%$). The subglacial example is Pallacopas Valles ($\gamma = 84$, $D_f = 1.5$, $S_n = 3$, $\lambda = 1985$ m, $R = 672$, $\Omega = 53.7\%$). Background uses THEMIS data overlain with MOLA/HRSC colorized elevation

6 Physical models of morphometry: metric predictions

To interpret the PCA we use synthetic valley networks, which are predictions derived from the physical models of each metric [12, 11]. Below we present the models used to generate the metric predictions for each erosional mechanism, in turn, fluvial, glacial, sapping, and subglacial, built on the following list of assumptions [12, 11]:

- Steady and well-developed incompressible and hydrostatic flow.
- Shallow slopes.
- Steady state landscape response to the flow regime of each system.
- Cross-sectional mechanical stability.

6.1 Fluvial regime

Open channel flow, linear rheology, and a mechanical equilibrium between lateral gradients of hydrostatic pressure and complex turbulent overturning motions, typically captured in terms of a net turbulent drag, are the main distinctive characteristics of fluvial systems. With the assumptions listed above, we derived in previous work [12, 11] the following models to physically describe each of the metrics. Supplementary table 2 (parameters) contains the list of parameters, units, references and definitions as they appear on the following equations.

6.1.1 Aspect ratio

$$R = L/W = \frac{R_\rho \Delta h_t}{d_s} \frac{2 \tan(\phi_r)}{\pi^2} \sqrt{\frac{1 + \tan(\phi)^2}{\theta_t^2 - \tan(\phi)^2}}, \quad (23)$$

where L is length along the longest flow path, W is channel width, scaled to valley network width using a linear conversion factor [37], h_t is the channel depth, θ_t is a non-dimensional Shields parameter, ϕ is the inclination angle of the channel walls, ϕ_r is the angle of repose of sediment in water, the effective density $R_\rho = \rho_w/(\rho_s - \rho_w) \sim 1.61$ compares the sediment and water density difference and $d_s \sim 0.45$ mm is taken to be the average grain size.

6.1.2 Width of order 1 tributaries

Order 1 fluvial valley spacing is a characteristic lengthscale of the landscape that emerges from the equilibrium between incision and mass wasting in the fluvial landscape evolution equation [38, 39]. If the ridges corresponding to first order tributaries are periodic, then valley width (distance from ridge to ridge) is equivalent to valley spacing (distance from minimum to minimum). In these conditions, the width of order 1 tributaries scales like:

$$\lambda \sim \frac{D^{\frac{1}{2m+1}}}{K}, \quad (24)$$

where D is the soil diffusivity, K is the erodability of the substrate, and m is the area exponent in the stream power equation [52, 38].

6.1.3 Junction angle

The junction angle in a steady-state valley network carved by a gravity driven flow on an open channel depends on the slopes of the main stem and tributary at the site of the junction [17, 12], which can be described by:

$$\gamma = \arctan(-\cot(\alpha) \sin(\beta)), \quad (25)$$

where α is the slope of the main stem and β the slope of the tributary before the junction.

6.1.4 Stream order

Fluvial networks (precipitation or snow-fed), and glacial networks, behave mathematically like tree-like systems [30, 12], with a Gaussian probability distribution that describes when two tributaries of a given order merge. The stream order of such systems increase logarithmically with the number of first order streams according to:

$$\Omega\{S_n\} = 1/2 \log_2 N_1 + O(1) \quad N_1 \gg 1, \quad (26)$$

Where N_1 is the number of order 1 tributaries in the network.

6.1.5 Fractal dimension

The fractal dimension of a system is a statistical measure of its scale invariant complexity [29, 42]. For a fluvial network, the following equation links D_f to both the channel order and Horton's laws [16]:

$$D_f = D_1 \frac{r_b}{r_l}, \quad (27)$$

where D_1 is the fractal dimension of 1st order tributaries and, again, r_b and r_l are the bifurcation and length ratios from Horton's laws.

6.1.6 Undulations

Undulations in the longitudinal profile arise from the presence of a pressure gradient that opposes gravity and drives water uphill opposing topography. In the absence of such force, such as in fluvial systems, undulations are not significant $\Omega \sim 0$.

6.2 Glacial regime

Open channel flow, rheology following a power-law relationship (Glenn's law) [35], and the mechanical balance between a driving force consisting of lateral differences in ice thickness and the friction arising in response to the contact with the bed and the walls [6] are the main distinctive characteristics of glacial systems. With the assumptions listed above, we derived in previous work [12] the following models to physically describe each of the metrics. Supplementary table 2 (parameters) contains the list of parameters, units and definitions as they appear on the following equations.

Stream order, fractal dimension and junction angle derive from the assumption of a gravity driven flow in an open channel, with a Gaussian probability of tributary merging, and thus describe both fluvial and glacial drainage systems [12]. Below, we give the equations for the remaining metrics.

6.2.1 Aspect ratio

$$R = \frac{\sqrt{R_b^4 u_s / C}}{\rho_i g h_i S_o \cos(\phi)}, \quad (28)$$

where u_s is basal sliding speed, R_b is the bed roughness, $C = 10^{-20} \text{ m}^2 \text{ s/kg}$ is a constant [35], ρ_i is the ice density, h_i is ice thickness, S_o is the topographic slope, and ϕ is the slope of the valley side [12].

6.2.2 Width of order 1 tributaries

The width of order 1 valleys is set by a competition between the thickening of ice in incipient valleys, which increases flow speed and erosion, and viscous stresses/ sidewall drag, which acts to impede glacier motion and erosion [36].

$$\lambda = \frac{2\tau_b}{S} \sqrt[3]{\pi^2 \eta_{eff} c / (\rho_i g)^2}, \quad (29)$$

where c is a bed-friction parameter such that $\eta_{eff} c \sim 10^{-2} \text{ m Pa}^{-1}$, and the basal shear stress is in the range $\tau_b = 0.5 \times 10^5 - 2.4 \times 10^5 \text{ Pa}$.

6.2.3 Undulations

Overdeepenings and undulations exist in glacial valley longitudinal profiles, particularly in those systems where subglacial hydrology strongly controls ice sliding speed [14]. We expect glacial valley undulations to exist, and in occasions be significantly larger than zero: $\Omega \geq 0$.

6.3 Sapping regime

Sapping valleys form when the height of the groundwater table locally intersects the ground level. The emergence of groundwater in springs can both incise channels, that grow forward in the direction in which groundwater enters the tip, and contribute to the overland flow [27, 8]. This has an important effect on network development: as erosion advances, tributaries bifurcate in the upstream direction [7]. Mass wasting represents an additional strong contribution in the areas of water seepage [28].

6.3.1 Aspect ratio

The sapping valley aspect ratio is given by the following equation[12]:

$$R = L/W = 4d_s/3R_\rho\pi \sqrt{\frac{2\sqrt{2}\kappa(1/2)g}{3\sin(2\pi/3)}} \sqrt{\frac{\theta_t^6}{C_f K_h S^2}}, \quad (30)$$

where d_s is the average grain size, C_f is the friction factor, κ is the complete elliptic integral of the first kind [7], S is the topographic slope, and K_h is the hydraulic conductivity.

6.3.2 Width of order 1 tributaries

To our knowledge, there are no models or scaling relationships that predict the width of first order sapping valleys. In our predictions, to compromise the identification of sapping valleys as little as possible, we adopted a value corresponding to the average width of order 1 tributaries in our dataset.

6.3.3 Junction angle

Observations for junction angles in terrestrial sapping valleys distribute around 72° [7], which has been explained analytically by solving the Poisson equation for the square of the water table height [7, 12] for negligible precipitation rates and imposing the symmetrical bifurcation of two tributary streams after a junction. The solution does not depend on the characteristics of the terrain other than assuming a basal impermeable layer and small precipitation rates, and therefore Martian sapping valleys should also distribute around 72° .

6.3.4 Stream order and fractal dimension

Tributary sapping valleys do not merge into main stems as tree-like systems, which include fluvial or glacial valleys, but rather bifurcate in the upstream direction as the network develops, parting from a main stem. The mechanisms that lead to this bifurcation are poorly understood, and therefore we cannot apply the model of [30] to predict stream order in a sapping network. The same is true for the fractal dimension metric, as the modelling [12] depends on the Horton laws [42], which are a consequence of the statistical merging pattern of tree-like networks [25].

6.3.5 Undulations

We do not expect undulations to appear in the steady state longitudinal profiles of sapping valleys.

6.4 Subglacial regime

Subglacial channels and the larger tunnel valleys (figure 3 c) originate where meltwater flows in pipe-like channels under large masses of ice, incised partly on the ice and partly on the bedrock. The turbulent flow regime characteristic of subglacial meltwater is similar to that of fluvial channels. However, subglacial flows are driven by variations in ice thickness in addition to topographic gradients, and are thus expected to be overpressured [43, 50, 45]. Consequently, subglacial channels can either follow or divert from topographic gradients, and present some remarkable morphologies [48, 13, 24, 11].

6.4.1 Aspect ratio

We use the model presented in [12] to describe the aspect ratio of subglacial channels:

$$R = L/W = \frac{(1 - f_m)R_\rho \Delta h_t}{d_s} \frac{2 \tan(\phi_r)}{\pi^2} \sqrt{\frac{1 + \tan(\phi)^2}{\theta_t^2 - \tan(\phi)^2}}, \quad (31)$$

which is similar to the fluvial aspect ratio, but with the introduction of a parameter f_m equivalent to the fraction of power lost to melt the icy roof. Upper and lower bounds for this parameter are in the supplementary table 2 (parameters).

6.4.2 Width of order 1 tributaries

The spacing between order 1 subglacial tributaries has been discussed in the literature, both for purely ice-incised channels (R-channels) and depositional features such as eskers [51, 45, 15]. However, order 1 subglacial channel spacing has not yet been successfully modelled even though it is observed in the field [11]. To quantify this metric, we take the same approach to sapping 1st order spacing and adopt the average minimum width value.

6.4.3 Junction angle

Junction angles observations or modeling descriptions in the subglacial literature are nonexistent to our knowledge. To quantify this metric, we take the same approach than in the previous metric and adopt the average junction angle value in our data.

6.4.4 Stream order and fractal dimension

Field observations and modelling results show that subglacial channels can develop into networks [48, 3, 4, 45]. There is no quantification, however, of their maximum stream order other than our own observations [12]. It is also unclear whether they follow Horton's laws or not [16], although their structure unlike tree-like systems suggest they do not [30, 12, 11], which makes the calculation of fractal dimension values difficult. We adopted the empirical range observed on Earth for stream orders, and the average value of this metric derived from this same dataset for both the mean stream order and the fractal dimension for the same reasons exposed above.

6.4.5 Undulations

Arguably the most distinctive characteristic of subglacial channels and tunnel valleys is the possible presence of undulations in their profiles and of deviations from topographic gradients, both characteristics noted extensively in the literature [48, 13, 24, 11]. Reliable predictions for the magnitude of undulations in the profiles of subglacial channels would require good constraints on both the

surface and the ice surface topographic gradients, which represents a significant challenge. Although we cannot quantify a number, we argue that undulations in the range between 0 and a significant fraction of the elevation drop are feasible for Martian subglacial channels.

7 Metric Prediction Results

Utilizing the models presented above to make predictions about the morphometry of Martian channels requires reasonable constraints on the Martian parameter space. Although the extensive exploration of the Martian surface has provided some reliable constraints on these parameters, particularly the rover missions Sojourner and Opportunity [49, 47], we acknowledge the fact that most of the Martian surface is unexplored and these parameters may not accurately represent all areas on Mars.

Supplementary table 2 (parameters) provides a list of parameters required for the models. The upper and lower bounds in that table give us the uncertainty space associated with each prediction, and with it the space within the PCA where the morphometry is best explained by rivers, glaciers, sapping valleys, or subglacial channels. The second sheet of the Supplementary table 2 (parameters) presents the average ($\langle \rangle$), upper (UB) and lower bound (LB) predicted for each metric. Table 1 in the manuscript summarizes the average predicted synthetic valley networks.

Some metrics, notably for the case of subglacial erosion, do not have physical equations that allow us to reliably predict their values for Mars. Specifying the 6 synthetic parameters for each erosional style is required for PCA display, so we used the average value of the dataset for that metric (i.e., for minimum width, we calculated the average of the minimum width $\langle \lambda \rangle$ of all valley networks, plus the standard deviation $\pm \sigma$). Our Montecarlo method then selects random values of the metric from the pool ($\langle \lambda \rangle - \sigma$, $\langle \lambda \rangle + \sigma$) for each subglacial synthetic valley network.

Choosing the average value of a metric when models do not (yet) exist is justified as follows. By definition of the transformation, the average of all metrics represents exactly the origin of coordinates (0, 0, 0, 0, 0, 0) in the principal component space. For those metrics and regimes without a physical predictive model, we choose the average value of the measured metric to introduce as little perturbation as possible in the predictions and let the more robust physical predictions dominate the characterization space of that erosional regime. This approach is by all means imperfect, but it avoids the introduction of any a priori values that would affect the positioning of the predicted cluster in the PCA space. A result of this choice is that the predicted cluster (i.e., subglacial, sapping, glacial clusters) will be shifted towards the origin of coordinates compared to what it would be if physical predictions existed. This is particularly important in metrics such as the minimum width, which explain a large portion of the total data variance. The effect is then minimized interpreting the results by combining the 1st-2nd PC plots (manuscript figure 2a) and 1st-3rd PC plots

(manuscript figure 2b), where the 2nd and 3rd PC have different dominant metrics. This effect is nearly negligible in metrics that have a small effect in the total data variance, such as the fractal dimension. Metrics that do not have predictive power and are purely empirical/statistical are detailed in section 6, and highlighted in red color in supplementary table 2 (parameters), sheet 2.

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Table 1: Sensitivity analysis

Sample size (VNs)	γ	D_f	S_n	λ	R	Ω
52	0.17	0.12	0.18	0.11	0.48	0.10
7	0.39	0.35	0.32	0.19	0.41	0.58
39	0.02	0.21	0.11	0.04	0.33	0.03
39	0.13	0.25	0.13	0.19	0.34	0.27
56	0.18	0.17	0.10	0.11	0.25	0.19
25	0.24	0.19	0.18	0.15	0.40	0.25
49	0.29	0.13	0.13	0.10	0.44	0.09
30	0.35	0.38	0.12	0.11	0.57	0.05
48	0.10	0.20	0.15	0.05	0.33	0.09
62	0.16	0.08	0.00	0.06	0.22	0.04
42	0.12	0.07	0.12	0.01	0.14	0.12
55	0.15	0.14	0.07	0.10	0.53	0.10
41	0.22	0.19	0.14	0.09	0.43	0.08
37	0.08	0.25	0.23	0.07	0.06	0.43
31	0.23	0.35	0.25	0.11	0.55	0.21
63	0.04	0.11	0.08	0.01	0.27	0.06
51	0.25	0.13	0.07	0.10	0.27	0.12
14	0.39	0.39	0.23	0.34	0.60	0.52
18	0.23	0.38	0.34	0.03	0.49	0.19
44	0.06	0.18	0.19	0.15	0.49	0.14
20	0.15	0.30	0.34	0.02	0.50	0.20
62	0.04	0.03	0.04	0.07	0.05	0.14
50	0.28	0.19	0.09	0.16	0.43	0.13
30	0.42	0.20	0.19	0.24	0.34	0.51
7	0.28	0.65	0.45	0.20	0.60	0.64
50	0.25	0.22	0.15	0.10	0.59	0.01
30	0.29	0.05	0.06	0.21	0.61	0.02
34	0.29	0.16	0.06	0.10	0.35	0.08
45	0.18	0.08	0.13	0.18	0.27	0.29
15	0.17	0.21	0.29	0.21	0.54	0.37
27	0.18	0.21	0.12	0.11	0.42	0.08
59	0.32	0.10	0.07	0.13	0.28	0.19
32	0.20	0.25	0.15	0.07	0.29	0.19
54	0.19	0.23	0.15	0.14	0.35	0.23
52	0.10	0.07	0.13	0.11	0.28	0.16
42	0.13	0.01	0.14	0.12	0.44	0.06
average	0.20	0.20	0.16	0.12	0.39	0.19
std	0.1	0.1	0.1	0.1	0.1	0.2