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The signal-burying game can explain why we obscure positive traits and good deeds

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Supplementary Information

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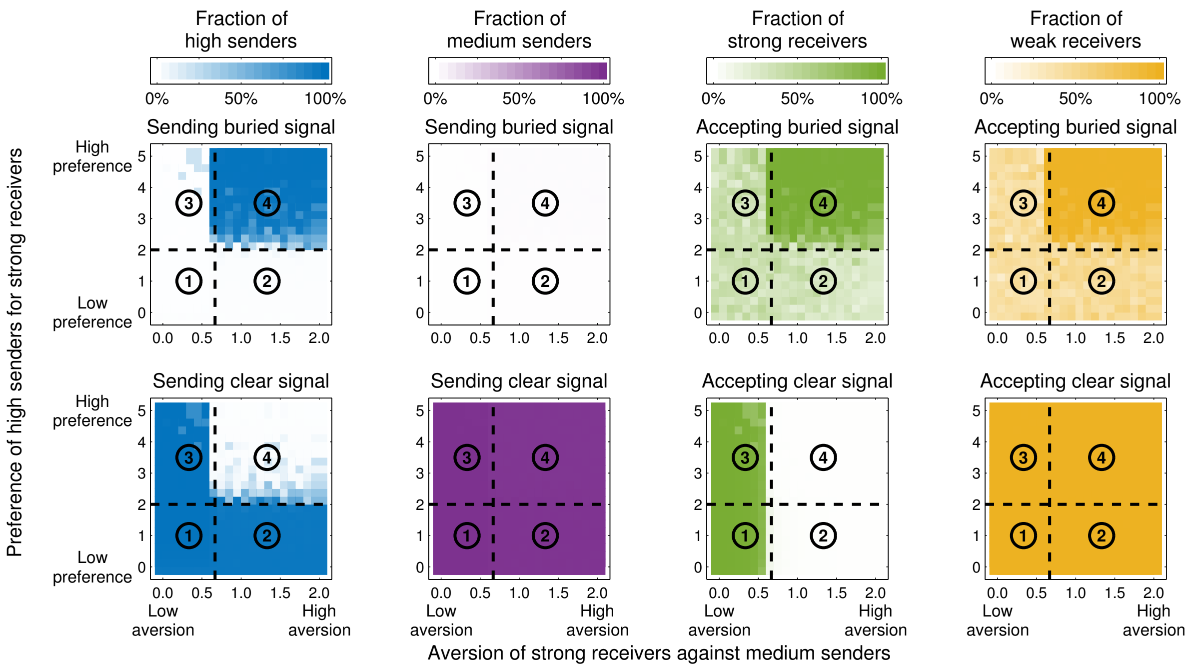
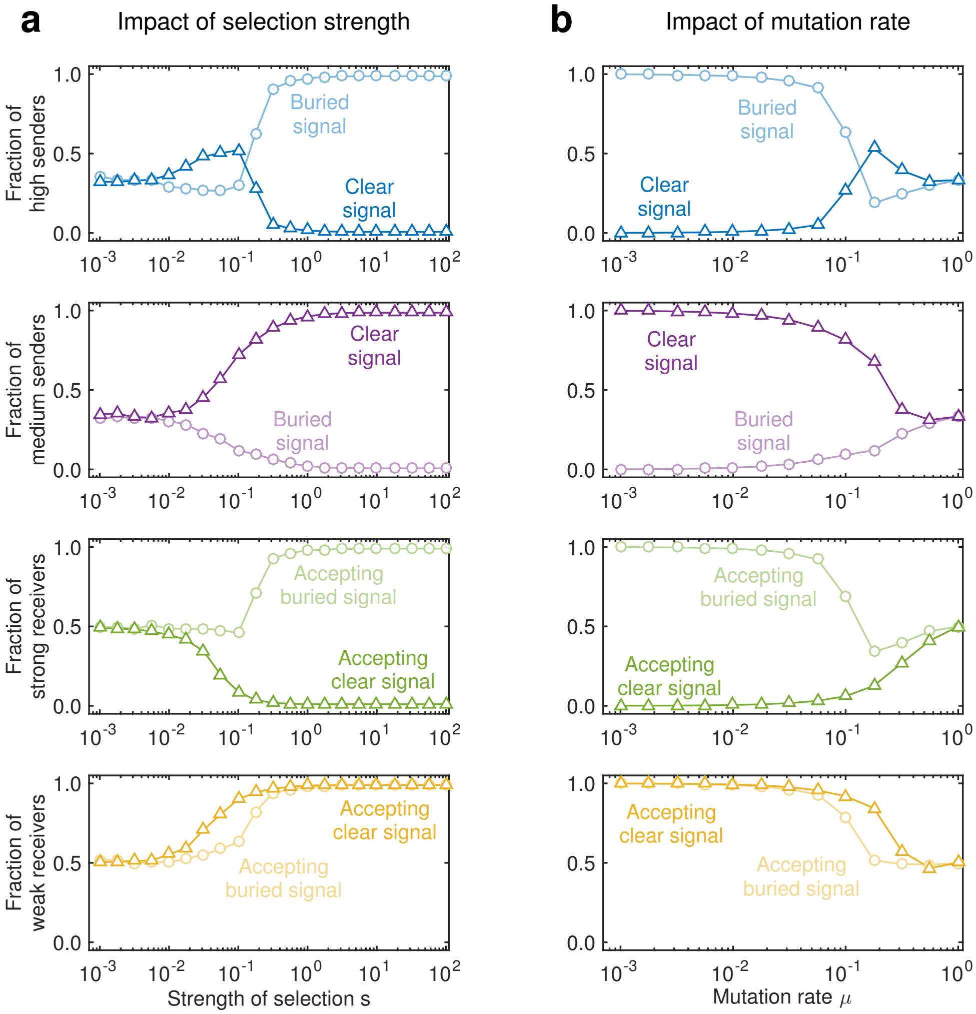
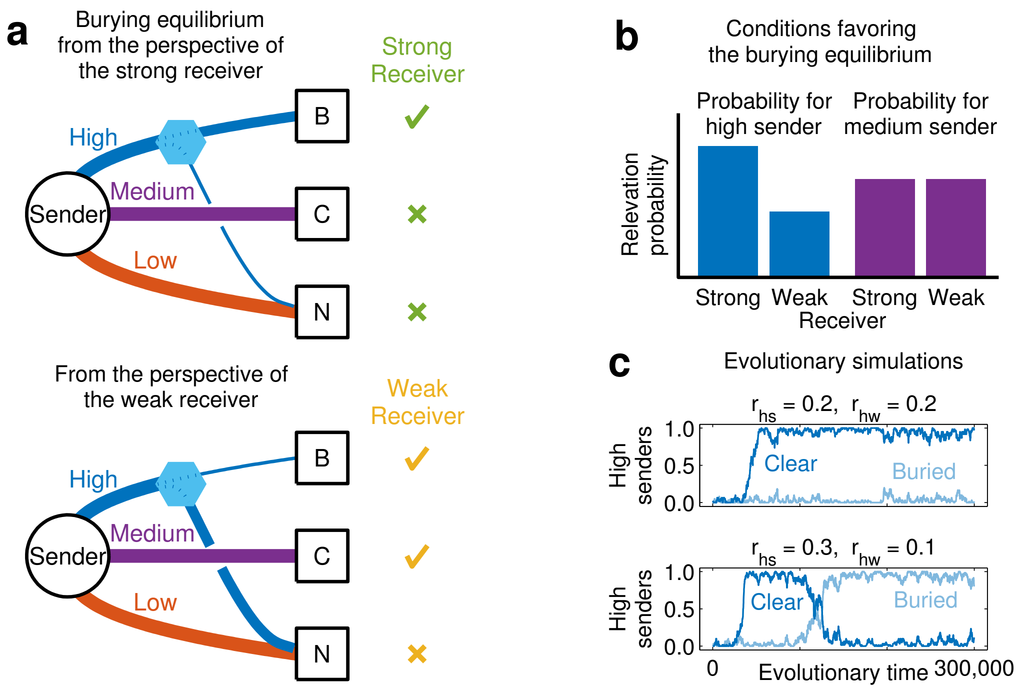
Supplementary Figures



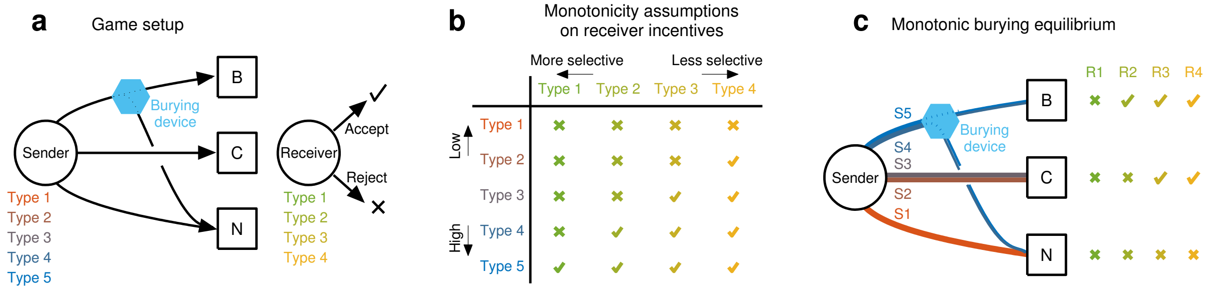
Supplementary Figure 1: Burying evolves if high senders especially value strong receivers and if strong receivers wish to avoid medium senders. The figure extends the evolutionary results presented in **Fig. 2** of the main text, by showing the eventual steady state frequencies for all player types. To explore when evolution leads to the emergence of burying, we have varied the relative preference of high senders for strong receivers, a_{hs}/a_{hw}, and the relative aversion of strong receivers against medium type senders, $-b_{ms}/b_{hs}$. According to static equilibrium considerations, there are four parameter regions. In the cases (1) and (2), we expect a pooling equilibrium where both high and medium senders use clear signals. In the parameter region (3), there are two possible cases, a pooling equilibrium or the burying equilibrium, whereas in parameter region (4) only the burying equilibrium can be sustained. Our evolutionary simulations fully support these predictions; eventually, each player adapts to act as expected from equilibrium considerations (the darker the color in each panel, the higher the respective frequency). In the only ambiguous case (3) where static considerations allow for two equilibria, we observe that the pooling equilibrium is favored (as it can be easier reached from an initial population where no one sends or accepts a signal).



Supplementary Figure 2: The stability of the burying equilibrium is robust with respect to changes in the evolutionary parameters. In the main text, simulations were performed using a fixed strength of selection $s = 1$, and a mutation rate $\mu = 0.02$. To explore the effect of these two critical evolutionary parameters, we have repeated the simulations for **Fig. 3** of the main text (where only the burying equilibrium satisfies the intuitive criterion) for **a,** different selection strengths and **b,** different mutation rates. The burying equilibrium is stable in all evolutionary scenarios, provided that selection is sufficiently strong for players to adapt according to the payoff consequences of their actions, and that mutations are sufficiently rare to allow populations to settle at an equilibrium. Each curve depicts the average fraction of each player type over $5 \cdot 10^6$ updating events, averaged over five simulations. Populations were initialized in the burying equilibrium.



Supplementary Figure 3: The evolution of burying when revelation probabilities also depend on the receiver's type. If the probability that a buried signal gets revealed also depends on the type of the receiver, the revelation probabilities take the form r_{ij} with $i \in \{h, m, l\}$ and $j \in \{s, w\}$. **a,** Strong and weak receivers can have different probabilities to identify high senders in a burying equilibrium. Here we depict a case in which buried signals are easy to observe for strong receivers, but hard to spot for weak receivers. **b,** An equilibrium analysis shows that inhomogeneous revelation probabilities favor the existence of a burying equilibrium if high type senders prefer strong receivers over weak receivers, and if strong receivers are more likely to spot the high senders' buried signals, see Corrolary 1 in the **SI** text for details. **c,** We have compared evolutionary scenarios in which revelation probabilities are independent of the receiver (upper panel) with scenarios in which they depend on the receiver type (lower panel), keeping the average revelation probability over the whole receiver population fixed. As predicted by the equilibrium analysis, high senders only learn to use buried signals in the second scenario. Except for the revelation probabilities, all other game parameters are the same as in **Fig. 3** of the main text.



Supplementary Figure 4: Burying equilibria in a model with many types. a We have extended our baseline model to allow for arbitrarily many types of senders and receivers. The depicted scenario shows a case with five different sender types and four receiver types. **b,** We derive qualitatively similar results as in the main text when we consider a type space such that senders can be ordered according to their quality (high sender types are preferred by more receivers), and that receivers can be ordered according to their selectivity (low receiver types are more selective). **c,** We explore all burying equilibria when high sender types particularly value strong receivers (for the exact conditions used, see **SI** text). In any such burying equilibrium, behaviors are monotonic: higher types of senders choose more elaborate signals than lower types, whereas less selective receivers accept a given signal more readily than more selective receivers.

Supplementary Notes

In the following, we first describe the setup of our baseline model (Section 1). Then we characterize all possible equilibria of our model, and we explore which of these equilibria satisfy the 'intuitive criterion', a standard equilibrium refinement for signaling games (Section 2). Thereafter, we discuss various model variations. First we consider a model variation in which sender types might differ in their probability to interact with certain receiver types (Section 3.1). Second, we study the case when the revelation probability of a buried signal may depend on the type of the receiver (Section 3.2). Third, we allow senders to choose the probability with which their buried signal becomes revealed (Section 3.3). Fourth, we contrast our model with a classical signaling model in which signals cannot be buried (Section 3.4). Fifth, we extend our baseline model such that it covers the case of arbitrarily many types of senders and receivers (Section 3.5). Finally, we provide an alternative model formulation in which there is only one type of receiver, and where the sender's payoff is a (non-linear) function of the receiver's ex-post belief (Section 3.6). The appendix contains the MATLAB code used to run our evolutionary simulations.

1 Description of the baseline model

The signal burying game is an asymmetric game between two players, a sender and a receiver. In the baseline model, we assume there are three different types of senders (low, medium, and high) and two different types of receivers (strong and weak). Players know their own type, but they cannot observe the type of their co-player directly. As a consequence, senders may wish to communicate their type by making use of a costly signal (e.g., by getting a degree from a prestigious university). In our model, senders can do so by choosing one of the following three options: they may either send no signal (and hence pay no cost), send a clear signal (pay a cost to obtain a signal, and make sure that the signal is observable), or they can bury a signal (pay a cost for the signal, but make sure that the signal is not directly observable). We assume that clear signals are noted by all receivers, whereas buried signals may only get revealed by chance (with a given probability that may depend on the sender's type). Receivers then have two possible actions. They decide whether or not to accept the sender as a partner, based on the signal they receive.
For the mathematical description of this model, we use the following parameters:

p_i ... Proportion of senders of type $i \in \{l, m, h\}$.

q_j ... Proportion of receivers of type $j \in \{s, w\}$.

c_i ... Cost to send a signal for a sender of type i.

r_i ... Probability that a buried signal of sender i is revealed.

a_{ij} ... Payoff to a sender of type i when accepted by a receiver of type j.

b_{ij} ... Payoff to a receiver of type j when accepting a sender of type i.

If a receiver rejects the sender, both players receive a partnering payoff of zero. We make the following assumptions on the players' incentives:

(A1) Senders (weakly) prefer to be accepted,

$$a_{ij} \geq 0 \text{ for all } i, j. \tag{1}$$

(A2) Receivers are selective:

$$\begin{array}{rl} b_{hj} > 0 \text{ for all } j & \text{All receivers get a positive payoff from accepting a high sender.} \\ b_{mw} > 0 > b_{ms} & \text{Only weak receivers get a positive payoff from medium senders.} \\ b_{lj} < 0 \text{ for all } j & \text{No receiver gets a positive payoff from accepting a low sender.} \end{array} \tag{2}$$

The assumptions (A1) and (A2) seem to be rather unproblematic. The first assumption states that senders have an incentive at all to persuade receivers to accept. Formally, this assumption rules out a few relevant scenarios. For example, in fashion senders of high status might sometimes want to avoid interactions with weakly selective receivers altogether (that is, $a_{hs} > 0$ but $a_{hw} < 0$). Most of our results on the burying equilibrium would equally apply to such a case. In particular, condition (1) in the main text (stating that high senders need to especially value partnering with strong receivers) is satisfied trivially. However, the interpretation of our results would change. When some of the a_{ij} values are allowed to be negative, it is no longer obvious whether senders bury their signals *despite* or *because* this makes them forego interactions with certain receiver types.

The second set of assumptions can be considered as a definition of what it means to be a sender of type $i \in \{l, m, h\}$, and what it means to be a receiver of type $j \in \{s, w\}$. For example, high senders are those that are universally appreciated, and strong receivers are those that are most selective. For the classification of all possible equilibria, it is useful to make the following additional assumptions:

(A3) Low senders do not have an incentive to pay the cost for a signal,

$$c_l > a_{lj} \text{ for all } j. \tag{3}$$

(A4) Receivers wish to avoid low senders under all circumstances,

$$\begin{aligned} p_l b_{ls} + p_h b_{hs} &< 0 \\ p_l b_{lw} + p_m b_{mw} + p_h b_{hw} &< 0. \end{aligned} \tag{4}$$

Taken together, these two assumptions imply that low senders never send a signal, and that receivers never accept senders who do not send a signal.

Finally, let us briefly justify why we have set up the model in this way. For burying to itself act as a costly signal, it needs to be the case that there is an opportunity cost to burying. This requires that there are receivers who would have accepted a clear signal, but who may not notice the buried signal and hence reject the sender. These receivers correspond to the weak types in our model. For weak receivers to reject senders who do not send a signal, it needs to be the case there is some low type of sender who does not send the costly signal and who receivers prefer not to interact with. In addition, our model also requires that burying leads to some benefit. This implies there must be another receiver type (which we call the strong type) who is only interested in a particular type of sender (the high type), but not in the type who would send clear signals (the medium type) or no signal (the low type). Other assumptions on the preferences that we have made merely simplified the equilibrium conditions but did not substantively change the analysis. Finally, to capture the applications we had in mind, burying had to reduce the probability of a desirable signal being observed, but also needed to have the property that receivers can sometimes tell that the signal was buried. Thus, our model was designed to be as simple as possible while still capturing the key costs and benefits of burying.

2 Bayesian Nash equilibria of the baseline model

2.1 Characterization of the Bayesian Nash equilibria

In the following, we characterize all perfect Bayesian Nash Equilibria (PBNE) of the signal-burying game. A PBNE requires players to act optimally (in expectation) given their beliefs, and that the players' beliefs in turn are determined by the players' strategies and by Bayes' rule. For signals that are not sent in equilibrium (such that Bayes' rule cannot be applied), beliefs are allowed to be arbitrary. For brevity, the following proofs typically omit specifying the receivers' beliefs for each case . All proofs are valid by simply positing that after observing a signal that is used in equilibrium, receivers form their beliefs according to Bayes' rule. If they observe a signal that is not used in equilibrium, we posit that receivers assume it was the low sender who has sent the signal.

Due to assumption (A3) we can assume that low senders do not send a costly signal. To characterize all equilibria, we therefore only need to analyze all possible strategy combinations for high and medium senders. Since there are three possible signals (no signal, clear signal, or buried signal), and assuming that senders use a pure strategy, this leads to $3^2 = 9$ different cases.

Claim 1 (Pooling equilibrium)**.** *There is always an equilibrium in which no one sends a signal. In any such equilibrium, receivers reject all senders.*

Proof. Consider the profile in which no type of sender pays the cost for a signal, and in which receivers reject everyone independent of the signal. It follows immediately that no sender can improve by paying the cost of a signal. The possible deviations of receivers are either neutral (e.g. if they accept signals), or they decrease the receivers' payoffs (if they accept senders with no signal; this follows from assumptions (A2) and (A4)). Therefore, this strategy profile is an equilibrium.
If there was another equilibrium of this sort in which receivers would accept some senders, then receivers would have to accept everyone (since senders are indistinguishable). This possibility is ruled out by (A4).

□

Claim 2. *There is an equilibrium in which medium senders send a clear signal and high type senders send no signal iff*

$$q_w a_{mw} \geq c_m \text{ and } q_w a_{hw} \leq c_h. \tag{5}$$

In any such equilibrium, weak receivers accept the signal whereas strong receivers do not.

Proof. Consider the profile in which medium senders send a clear signal, whereas low and high senders do not send a signal. Weak receivers only accept those with a clear signal, and strong receivers do not accept anyone. Low senders do not have an incentive to deviate because of (A3). Medium senders do not gain higher payoffs by not sending the signal because of the first inequality in (5); they do not gain higher payoffs by burying the signal (because then they would lose on some of the weak receivers without getting some of the strong receivers). High senders do not have an incentive to deviate by sending a signal because of the second inequality in (5); nor do they have an incentive to bury the signal. Given the senders' strategies, it follows from (A4) that the receivers' best response is to reject senders who do not send a signal, and it follows from (A2) that only weak receivers should accept senders with a signal. □

Claim 3. *There is an equilibrium in which medium senders send a buried signal, whereas high type senders send no signal iff*

$$r_m q_w a_{mw} \geq c_m \text{ and } r_h q_w a_{hw} \leq c_h. \tag{6}$$

In any such equilibrium, weak receivers accept a buried signal whereas strong receivers do not.

Proof. Follows by the same proof as for claim 2, using the following strategy profile: Medium senders use a buried signal, whereas low and high senders do not send a signal. Low receivers only accept buried signals, and strong receivers accept no one. □

Claim 4. *There is an equilibrium in which medium senders send no signal and high senders send a clear signal iff*

$$q_w a_{mw} + q_s a_{ms} \leq c_m \text{ and } q_w a_{hw} + q_s a_{hs} \geq c_h. \tag{7}$$

In any such equilibrium, both receivers accept clear signals.

Proof. Use the profile in which high senders send a clear signal, the other senders send no signal, and in which both receivers only accept clear signals. □

Claim 5. *There is an equilibrium in which medium and high type senders send a clear signal iff one of the following two cases holds*

$$\begin{aligned} &(a) \quad p_m b_{ms} + p_h b_{hs} \geq 0, \text{ and } q_w a_{mw} + q_s a_{ms} \geq c_m, \text{ and } q_w a_{hw} + q_s a_{hs} \geq c_h. \\ &(b) \quad p_m b_{ms} + p_h b_{hs} \leq 0, \text{ and } q_w a_{mw} \geq c_m, \text{ and } q_w a_{hw} \geq c_h. \end{aligned} \tag{8}$$

In the first case, both types of receivers accept the signal; in the second case, only weak receivers accept the signal.

Proof. For the senders, suppose low senders do not send a signal, whereas the other senders do. In case (a), use the strategy profile in which both receivers only accept players with a clear signal. In case (b), use the strategy profile in which weak receivers only accept players with a clear signal, and in which strong receivers accept no one. □

Claim 6. *There is no equilibrium in which medium senders send a buried signal, whereas high senders send a clear signal.*

Proof. In any such equilibrium it follows from (A2) that both types of receivers accept clear signals. In that case medium senders should deviate by not burying their signal. □

Claim 7. *There is an equilibrium in which medium senders send no signal, whereas high type senders send a buried signal iff*

$$r_m q_w a_{mw} + r_m q_s a_{ms} \leq c_m \text{ and } r_h q_w a_{hw} + r_h q_s a_{hs} \geq c_h. \tag{9}$$

In any such equilibrium, both receivers accept buried signals.

Proof. Suppose low and medium senders send no signal, and high senders send a buried signal. For the receivers, suppose they only accept buried signals. □

Claim 8 (Burying equilibrium)**.** *There is an equilibrium in which medium senders send a clear signal and high type senders send a buried signal iff the following four conditions hold*

$$\frac{r_m}{1-r_m} \leq \frac{q_w a_{mw}}{q_s a_{ms}}, \quad \frac{r_h}{1-r_h} \geq \frac{q_w a_{hw}}{q_s a_{hs}}, \quad q_w a_{mw} \geq c_m, \text{ and } r_h(q_w a_{hw} + q_s a_{hs}) \geq c_h. \tag{10}$$

In this equilibrium, weak receivers only accept clear and buried signals and strong receivers only accept buried signals.

Proof. The first condition ensures that medium senders do not wish to bury their signal, whereas the second condition ensures that high senders do not wish to send a clear signal. The last two conditions guarantee that it pays for medium and high senders to pay the cost of a signal. Receivers do not have an incentive to deviate because of the conditions in (A2). □

Claim 9. *There is an equilibrium in which medium and high type senders send a buried signal iff one of the following two cases holds*

$$\begin{aligned} &(a) \quad p_m b_{ms} + p_h b_{hs} \geq 0, \text{ and } r_m(q_w a_{mw} + q_s a_{ms}) \geq c_m, \text{ and } r_h(q_w a_{hw} + q_s a_{hs}) \geq c_h. \\ &(b) \quad p_m b_{ms} + p_h b_{hs} \leq 0, \text{ and } r_m q_w a_{mw} \geq c_m, \text{ and } r_h q_w a_{hw} \geq c_h. \end{aligned} \tag{11}$$

In the first case, both types of receivers accept the buried signal; in the second case, only weak receivers accept the buried signal.

Proof. Consider the strategy profile in which low senders do not send a signal, and the other types of senders send a buried signal. In case (a), suppose that both types of receivers only accept buried signals. In case (b), suppose that weak receivers only accept buried signals, whereas strong receivers accept no one. □

2.2 Equilibrium refinement using the Intuitive criterion

Some of the previously discussed equilibrium outcomes appear unreasonable: they require receivers to hold certain unrealistic beliefs about signals that are not sent along the equilibrium path. In the following, we use the intuitive criterion[1] to exclude such equilibria. The intuitive criterion posits that one should reject all equilibria that require receivers to believe that a given signal may be sent by a sender who should never send such signals in the first place (because that sender cannot possibly benefit from sending such a signal, no matter how the receiver would react). As we shall see, there are certain parameter regions in which the burying equilibrium is the only equilibrium that is in line with the intuitive criterion.

Claim 1′. *An equilibrium in which no one sends a signal fails the intuitive criterion (IC) if at least one of the following three conditions is satisfied:*

$$\begin{aligned} &(a) \quad q_w a_{mw} > c_m, \\ &(b) \quad q_w a_{mw} + q_s a_{ms} < c_m \ \textit{ and } \ q_w a_{hw} + q_s a_{hs} > c_h, \\ &(c) \quad r_m(q_w a_{mw} + q_s a_{ms}) < c_m \ \textit{ and } \ r_h(q_w a_{hw} + q_s a_{hs}) > c_h. \end{aligned} \tag{12}$$

Proof. Suppose there is an equilibrium in which no one sends a signal and condition (a) is satisfied. Then it must be the case that weak receivers reject clear signals (otherwise (a) implies that medium senders could deviate profitably by sending a clear signal). However, weak receivers have no incentive to do so: if they observe a clear signal, they should believe that the sender is either a medium or high type (because of (A3), low senders would never send a signal). Then (A2) suggests that weak receivers should accept the clear signal.

Now suppose condition (b) is satisfied. Due to the first inequality in (b), medium senders would not want to send a clear signal. Thus receivers need to believe that a clear signal is sent by a high sender. The second inequality in (b) then ensures that high senders have an incentive to send the clear signal.

Case (c) follows similarly, by replacing clear signals with buried signals. □

Claim 2′. *An equilibrium in which medium senders send a clear signal and high senders send no signal fails the IC if*

$$\frac{r_m}{1-r_m} < \frac{q_w a_{mw}}{q_s a_{ms}} \quad \textit{and} \quad r_h(q_w a_{hw} + q_s a_{hs}) > c_h. \tag{13}$$

Proof. Suppose we are in such an equilibrium (in particular, it follows by Claim 2 that only weak receivers accept clear signals). The conditions in (13) ensure that only high senders would ever send a buried signal. Because of (A2), both types of receivers should therefore accept buried signals; in turn, high senders should send a buried signal. □

Claim 3′. *There is no equilibrium that satisfies the IC in which medium senders send a buried signal and high senders send no signal.*

Proof. To achieve such an outcome in an equilibrium, it must be the case that weak receivers reject clear signals (otherwise medium senders could deviate by sending clear signals instead of buried signals). However, given that low types would never send a clear signal, it follows by (A2) that weak receivers should not reject clear signals. □

Claim 4′. *Any equilibrium in which medium senders send no signal and high senders send a clear signal satisfies the IC.*

Proof. The only signal that is not used along the equilibrium path is the buried signal. But given the conditions in (7), senders have no incentive to send buried signals, no matter how receivers would react. □

Claim 5′. *An equilibrium in which medium and high senders send a clear signal fails the IC if*

$$p_m b_{ms} + p_h b_{hs} < 0, \quad \frac{r_m}{1-r_m} < \frac{q_w a_{mw}}{q_s a_{ms}}, \ \textit{and} \ \frac{r_h}{1-r_h} > \frac{q_w a_{hw}}{q_s a_{hs}}. \tag{14}$$

Proof. Because of the first inequality in (14) and Claim 5, only weak receivers accept clear signals in such an equilibrium. The second inequality in (14) suggests that medium senders would never benefit from sending a buried signal, whereas the third inequality in (14) suggests that high senders may benefit from doing so. The IC thus suggests that both types of receivers should accept buried signals, and that high senders should send buried signals. □

Claim 7′. *An equilibrium in which medium senders send no signal and high senders send a buried signal fails the IC if at least one of the following three conditions is satisfied:*

$$(a)\ q_w a_{mw} > c_m, \quad (b)\ q_w a_{mw} + q_s a_{ms} < c_m, \quad (c)\ \frac{r_h}{1-r_h} < \frac{q_w a_{hw}}{q_s a_{hs}}. \tag{15}$$

Proof. In case (a), medium senders should send a clear signal (and weak receivers should accept such signals because of the IC). In case (b) or case (c), high senders should send a clear signal (in case (b), both types of receivers should accept the clear signal; in case (c), weak receivers should accept the clear signal). □

Claim 9′. *An equilibrium in which medium and high senders send a buried signal fails the IC if one of the following two conditions hold:*

$$\begin{aligned}
&(a) \quad p_m b_{ms} + p_h b_{hs} \geq 0 \ \textit{and either} \ \ \tfrac{r_m}{1-r_m} < \tfrac{q_w a_{mw}}{q_s a_{ms}}, \ \textit{or} \ \tfrac{r_h}{1-r_h} < \tfrac{q_w a_{hw}}{q_s a_{hs}}, \\
&(b) \quad p_m b_{ms} + p_h b_{hs} < 0
\end{aligned} \tag{16}$$

Proof. In case (a) the equilibrium fails the IC because either medium senders or high senders prefer to send the clear signal (which at least the weak receivers should accept). In case (b), both types of senders, medium and high, prefer to send the clear signal. □

Theorem 1. *Suppose parameters are such that there is a burying equilibrium, with all inequalities in (10) being strict.*

1. *If $p_m b_{ms} + p_h b_{hs} < 0$, the burying equilibrium is the only equilibrium that satisfies the IC.*
2. *Otherwise there are two possible equilibrium outcomes that are consistent with IC: the burying equilibrium, and a partial pooling equilibrium in which medium and high types send a clear signal (which is accepted by both types of receivers).*

Proof. Suppose parameters are such that all inequalities for the existence of a burying equilibrium in (10) are strict. Let us go through all cases to check whether there is any other equilibrium that is consistent with the IC.

1. Because of (10) and the assumption of strict inequality, we have $q_w a_{mw} > c_m$. Thus, the pooling equilibrium in Claim 1 is ruled out by the IC, see (12).

2. Conditions (10) and the assumption of strict inequality imply $\frac{r_m}{1-r_m} < \frac{q_w a_{mw}}{q_s a_{ms}}$ and $r_h(q_w a_{hw} + q_s a_{hs}) > c_h$. Thus, the equilibrium outcome described in Claim 2 is ruled out by the IC, see (13).

3. As shown in Claim 3', there is no equilibrium that satisfies the IC in which medium senders send buried signal and high senders send no signal.

4. Because of (7), the equilibrium in Claim 4 requires $q_w a_{mw} + q_s a_{ms} \leq c_m$. This is inconsistent with the requirement $q_w a_{mw} > c_m$, which follows from (10) and the assumption of strict inequality.

5. Because of Claim 5', the IC allows us to exclude the equilibrium outcomes described in (8b). However, the equilibrium outcomes described in (8a) are consistent with the conditions for a burying equilibrium (10), and they satisfy the IC. In this equilibrium, medium and high senders send a clear signal, which is accepted by both receivers.

6. As shown for Claim 6, there is no equilibrium in which medium senders send a buried signal and high senders send a clear signal.

7. The equilibrium outcome in Claim 7 requires $q_w a_{mw} + q_s a_{ms} \leq c_m$. This is ruled out by the assumption that $q_w a_{mw} > c_m$.

8. Case 8 corresponds to the burying equilibrium, which satisfies the IC trivially (because there are no unused signals).

9. Conditions (10) and the assumption of strict inequality imply $\frac{r_m}{1-r_m} < \frac{q_w a_{mw}}{q_s a_{ms}}$. As a consequence, one of the conditions in (16) is always met. Thus an equilibrium outcome in which medium senders and high senders send a buried signal violates the IC.

□

3 Model extensions

3.1 Extension 1: Non-uniform interaction probabilities

In the baseline model, we have assumed that every type of sender has the same chance to interact with a certain receiver type. In this subsection, we drop this assumption. We consider the same three sender types and two receiver types as in the baseline model. Again, the proportions of the three sender types are given by p_h, p_m and p_l, respectively, with $p_h + p_m + p_l = 1$. However, this time we assume that the probability that a sender of type i interacts with a receiver of type j is q_{ij}. These conditional probabilities are normalized such that for each sender type i, we have $q_{is} + q_{iw} = 1$. Moreover, we assume that $q_{ij} > 0$ for all senders i and all receivers j; that is, all pairings have a positive probability to occur.

The game proceeds similar to the baseline case. First, a sender type is randomly chosen according to the probability distribution $\mathbf{p} = (p_h, p_m, p_l)$. Then this sender of type i is randomly matched with a receiver, chosen with respect to the distribution $\mathbf{q_i} = (q_{is}, q_{iw})$. Players know their own type, and all relevant probability distributions $\mathbf{p}$ and $\mathbf{q_i}$, but they cannot tell the co-player's type directly. Then the sender either sends a buried signal, a clear signal, or no signal, and the receiver determines whether or not to accept the sender based on the observed signal. For simplicity, we assume here that all senders get the same payoff $a_{ij} := a$ from partnering with any given receiver, whereas the receiver's payoff b_{ij} still depends on both players' types. Moreover, we still suppose the assumptions (A1)–(A3) to hold (for what follows, Assumption (A4) will not be necessary). In this case, we get the following characterization for the existence of a burying equilibrium.

Theorem 2. *For the model extension with non-uniform interaction probabilities, there is a burying equilibrium in which high senders bury their signal, medium senders use a clear signal, and low senders do not signal if and only if*

$$r_h \geq q_{hw}, \quad r_m \leq q_{mw}, \quad r_h \cdot a \geq c_h, \quad \text{and} \quad q_{mw} \cdot a \geq c_m. \tag{17}$$

In this equilibrium, strong receivers only accept a sender when they observe a buried signal, whereas weak receivers additionally accept senders with clear signals.

The proof of this theorem is analogous to the corresponding proof of the baseline case, and is therefore omitted. But while the conditions (17) for a burying equilibrium are formally similar to the respective conditions (10) in the baseline case, the interpretation is now slightly different. Instead of requiring that high senders gain a sufficiently high payoff from interactions with strong receivers, the first condition in (17) now requires them to meet strong receivers sufficiently often.

This interpretation is closely related to the model by Carbajal *et al* ("Inconspicuous conspicuous consumption"[2]). Their model considers a population of senders who differ along two dimensions, wealth and social connectedness. To signal their type, senders can either buy a "subtle" status good, a "loud" status good, or no status good (which naturally map to our three signal types). After choosing consumption, the sender interacts with an observer who is either discerning or undiscerning. Only discerning observers take notice of subtle goods. Senders who are better connected are more likely to interact with discerning observers. Under these assumptions, it is shown that the equilibrium is monotonic with respect to a sender's connectedness: when two senders of equal wealth are compared, it can never occur that the less connected sender purchases a subtle good whereas the better connected sender does not.

Our model extension gives a similar conclusion: while it can never happen that a medium type sends a buried signal whereas the high type sends a clear signal (see Claim 6), the converse case can occur if high types are better connected with strong receivers (Theorem 2). However, we note that Carbajal *et al*[2] do not yield our original interpretation for a burying equilibrium: in their model, senders cannot use a subtle signal to communicate that they strongly prefer interactions with discerning observers to interactions with undiscerning observers.

3.2 Extension 2: Revelation probabilities that depend on the receiver's type

In the baseline model, the probability that a buried signal becomes revealed only depends on the type of the sender. Instead, we shall now consider the case in which a buried signal becomes revealed with probability r_{ij} with $i \in \{h, m, l\}$ being the type of the sender and $j \in \{s, w\}$ being the type of the receiver. Such a model may be realistic, for example, if strong receivers pay more attention to find buried signals, or if the sender choose the way to bury a signal exactly so as to exclude a particular group of receivers. When revelation probabilities depend on the receiver's type, strong and weak receivers have a different probability to identify high senders in a burying equilibrium, as depicted in **Fig. S3a**. Such a model extension leads to the following minor modifications for the conditions of a burying equilibrium.

Theorem 3. *In the model in which revelation probabilities also depend on the type of the sender, there exists a burying equilibrium if and only if*

$$\frac{r_{ms}}{1-r_{mw}} \leq \frac{q_w a_{mw}}{q_s a_{ms}}, \quad \frac{r_{hs}}{1-r_{hw}} \geq \frac{q_w a_{hw}}{q_s a_{hs}}, \quad q_w a_{mw} \geq c_m, \quad q_w r_{hw} a_{hw} + q_s r_{hs} a_{hs} \geq c_h. \tag{18}$$

The comparative statics of these conditions is similar to the baseline model – burying is most likely to arise if high senders particularly value interactions with strong receivers (i.e., if a_{hw}/a_{hs} is small), or if buried signals from high senders become more easily revealed (if r_{hs} and r_{hw} are close to one). However, as the following result shows, inhomogeneous revelation probabilities may further facilitate the emergence of a burying equilibrium (see **Fig. S3b,c** for a numerical example).

Corrolary 1. *Suppose that high senders prefer interactions with strong receivers over interactions with weak receivers, $a_{hs} > a_{hw}$, and suppose that buried signals of high senders are revealed with probabilities r_{hs} and r_{hw}, respectively. We define the associated homogeneous scenario by considering the case in which the buried signals of high senders become revealed with uniform probability*

$$r_h := r_{hs} \cdot q_s + r_{hw} \cdot q_w \tag{19}$$

to both receiver types. Then inhomogeneous revelation probabilities $r_{hs} \neq r_{hw}$ facilitate burying in the following sense:

1. *Whenever the conditions for a burying equilibrium are satisfied with respect to the homogeneous scenario, they are also satisfied with respect to any inhomogeneous scenario for which $r_{hs} > r_{hw}$.*
2. *Conversely, there are parameter regions in which inhomogeneous probabilities $r_{hs} > r_{hw}$ allow for a burying equilibrium, although the associated homogeneous scenario does not.*

Proof. Because $a_{hs} > a_{hw}$, the weighted sums satisfy the inequality

$$\lambda_1 a_{hs} + (1 - \lambda_1) a_{hw} > \lambda_2 a_{hs} + (1 - \lambda_2) a_{hw} \tag{20}$$

if and only if $\lambda_1 > \lambda_2$. By defining $\lambda_1 := r_{hs} q_s / r_h$ and $\lambda_2 := q_s$, we may thus conclude that

$$\frac{r_{hs} q_s}{r_h} a_{hs} + \frac{r_{hw} q_w}{r_h} a_{hw} > q_s a_{hs} + q_w a_{hw} \tag{21}$$

if and only if $r_{hs} > r_h$, or equivalently, if and only if $r_{hs} > r_{hw}$.

For the first statement of the corollary, we note that if the conditions in (18) pertaining to the high type are satisfied in the homogeneous scenario, and hence

$$\begin{aligned} r_h(q_s a_{hs} + q_w a_{hw}) &\geq q_w a_{hw} \\ r_h(q_s a_{hs} + q_w a_{hw}) &\geq c_h, \end{aligned} \tag{22}$$

then (21) guarantees that also the analogous conditions for the inhomogeneous scenario hold,

$$\begin{aligned} r_{hs} q_s a_{hs} + r_{hw} q_w a_{hw} &\geq q_w a_{hw} \\ r_{hs} q_s a_{hs} + r_{hw} q_w a_{hw} &\geq c_h, \end{aligned} \tag{23}$$

provided that $r_{hs} > r_{hw}$. The second statement of the corollary follows from the fact that the inequality in (21) is strict. □

Thus, if high senders particularly value strong receivers, a burying equilibrium emerges more easily if strong receivers are in turn more likely to observe the buried signals of high senders.

Given the conditions in (18) are satisfied, we note again that the burying equilibrium is the only equilibrium that is consistent with the intuitive criterion if $p_m b_{ms} + p_h b_{hs} < 0$. Otherwise, if $p_m b_{ms} + p_h b_{hs} \geq 0$, the intuitive criterion also supports a pooling equilibrium in which both high and medium senders use a clear signal.

3.3 Extension 3: Sender determines revelation probability

In the baseline model, we have taken the revelation probability of a buried signal as an exogenous parameter. However, in many applications the sender may have some discretion over the likelihood that a hidden signal will eventually be revealed. In the following, let us incorporate this scenario as follows. As before, we assume that the sender can choose whether or not to pay the cost of the signal. If he does, he additionally determines a probability $r \in \mathcal{R}$ with which the signal will be revealed, where $\mathcal{R} \subseteq [0, 1]$ is the set of feasible revelation probabilities, which we posit to include 1. Choosing $r = 1$ then corresponds to the previous case of a clear signal, whereas any choice with $r < 1$ entails some burying. If a signal gets revealed, the receiver observes the signal as well as the chosen revelation probability r. The receiver then decides whether to accept or reject the sender, given his information. Overall, the sender's strategy is thus a function of the sender's type to the set of possible signals, $\{h, m, l\} \to \{\emptyset, \mathcal{R}\}$, where $\emptyset$ refers to not sending a signal. A receiver's strategy, on the other hand, takes the form of a function $\{\emptyset, \mathcal{R}\} \to \{\text{Accept}, \text{Reject}\}$.

The following result shows that the previously discussed burying equilibrium has a natural analogue in the extended game.

Theorem 4. *In the extended game, there is an equilibrium in which high types choose a buried signal ($r_h < 1$) whereas medium types choose a clear signal ($r_m = 1$) if and only if*

$$q_w a_{mw} \geq c_m, \tag{24}$$

and there is an $\hat{r} \in \mathcal{R}$ such that

$$\frac{q_w a_{hw}}{q_s a_{hs}} \leq \frac{\hat{r}}{1 - \hat{r}} \leq \frac{q_w a_{mw}}{q_s a_{ms}} \;\; \text{and} \;\; \hat{r} \geq \frac{c_h}{q_w a_{hw} + q_s a_{hs}}. \tag{25}$$

In the only such equilibrium compatible with the intuitive criterion, high senders choose the maximum revelation probability,

$$\hat{r}_h = \max\left\{ \hat{r} \in \mathcal{R} \;\middle|\; \hat{r} \text{ satisfies the inequalities } (25) \right\}. \tag{26}$$

Proof. Consider the following strategy profile: high senders choose a $\hat{r}_h$ that satisfies the inequalities (25); medium senders choose $\hat{r}_m = 1$; low senders decide not to pay the cost for a signal; strong receivers accept senders for which they know that $r \leq \hat{r}_h$; and weak receivers accept everyone who sends a signal. We first show that this is an equilibrium.

High senders have no incentive to deviate: by reducing their r_h they are only less likely to be identified as high senders without having any additional advantage; by increasing r_h they lose all strong receivers, which is unprofitable by the first inequality in (25); finally, sending no signal is unprofitable by the last inequality in (25). Medium senders have no incentive to deviate either: by reducing their r_m, they may get accepted by strong receivers, but they also risk losing some of the weak receivers; the net effect is zero or negative, due to the second inequality in (25). They also have no incentive not to send a signal, because of (24). Low senders do not benefit from deviating because of assumption (A3). Given the senders' actions played in equilibrium, receivers react optimally according to assumption (A2). Hence the above specified strategy profile constitutes an equilibrium.

Conversely, if either (24) or one of the inequalities in (25) does not hold, there can be no equilibrium with the desired properties. If (24) was violated, medium senders would prefer not to send the clear signal. If the first inequality in (25) was violated, high senders would prefer to send a clear signal instead of a buried signal. If the second inequality in (25) was violated, medium senders would prefer to bury their signal too. And if the last inequality was violated, high senders would prefer not to pay the costs for a signal.

Finally, suppose high senders choose a revelation probability compatible with inequalities (25), but not the maximally possible value given by (26). By the intuitive criterion, both receiver types would also accept higher values of r_h, provided that the inequalities (25) are still satisfied. Hence high senders can deviate by choosing a higher revelation probability. □

The above result thus ensures that burying equilibria can also emerge when revelation probabilities are not exogenously given, but endogenously determined by the sender. Moreover, the result suggests that populations would naturally settle at an equilibrium where revelation probabilities are chosen optimally – they are chosen as high as possible, subject to the constraint that they still allow high type senders to differentiate themselves from medium types.

In **Fig. 5**, we discuss an explicit example. Senders can choose between two different burying devices. The first burying device yields a slightly buried signal (with revelation probability $2/3$), whereas the second burying device leads to a considerably buried signal (with revelation probability $1/3$). Thus the set of feasible revelation probabilities is $\mathcal{R} = \{1/3, 2/3, 1\}$. In that case, Theorem 4 implies that there can only be a burying equilibrium in which high senders use a considerably buried signal if

$$\frac{q_s a_{hs}}{q_w a_{hw}} \geq 2 \geq \frac{q_s a_{ms}}{q_w a_{mw}}. \tag{27}$$

Similarly, there can only be a burying equilibrium in which high senders use a slightly buried signal if

$$\frac{q_s a_{hs}}{q_w a_{hw}} \geq 1/2 \geq \frac{q_s a_{ms}}{q_w a_{mw}}. \tag{28}$$

If the latter equilibrium exists, it is also the equilibrium that is selected by the intuitive criterion. **Fig. 5b** summarizes these findings in a graph where the x-axis depicts the medium sender's relative preference for strong receivers, a_{ms}/a_{mw}, whereas they y-axis gives the high sender's relative preference for strong receivers, a_{hs}/a_{hw} (assuming that the other conditions required by Theorem 4 are satisfied). **Fig. 5c** confirms these predictions using evolutionary simulations. If the medium sender shows no particular interest in strong receivers, high senders only need to slightly bury their signal to achieve separation. However, as medium senders become increasingly interested in strong receivers, high senders are either forced to considerably bury their signal, or full separation becomes infeasible.

3.4 Extension 4: Comparison with a classical signaling model

In the previous models, high senders could only distinguish themselves from medium senders by burying their signal. It may thus appear that burying a signal is just a mechanism to make a given signal more expensive, and that a similar effect could be achieved by considering a classical signaling model in which players can choose among various signals that come with different costs. In the following, we thus extend our model by allowing for alternative signals. We show that while a classical signaling model may also lead to full separation, the comparative statics is different.

To see this, let us consider a model in which there are two types of signals, signal 1 and signal 2, with costs c_i^1 and c_i^2 for $i \in \{l, m, h\}$, respectively (see **Fig. 4a** of the main text). We shall assume that signal 2 is more costly for each type, $c_i^1 < c_i^2$ for all i. Senders may choose whether they want to pay the cost for one of the signals, and if they choose signal 1, whether they want to bury it. Receivers again need to decide whether or not to accept the respective sender. We say that players are in a burying equilibrium if they are in an equilibrium in which high and medium types choose the first signal, but only high types bury their signal. In contrast, we say that players are in a classical separating equilibrium, if high type senders use signal 2 and medium senders use signal 1. In the following, we provide conditions for these two equilibrium scenarios to occur.

Theorem 5 (Burying equilibria versus classical signaling)**.**

1. *There is a strategic burying equilibrium if and only if*

$$\frac{r_m}{1-r_m} \leq \frac{q_w a_{mw}}{q_s a_{ms}}, \quad \frac{r_h}{1-r_h} \geq \frac{q_w a_{hw}}{q_s a_{hs}}, \quad q_w a_{mw} \geq c_m^1, \quad q_w a_{hw} + q_s a_{hs} \geq \frac{c_h^1}{r_h}. \tag{29}$$

2. *There is a classical separating equilibrium if and only if*

$$c_m^2 - c_m^1 \geq q_s a_{ms}, \quad c_h^2 - c_h^1 \leq q_s a_{hs}, \quad q_w a_{mw} \geq c_m^1, \quad q_w a_{hw} + q_s a_{hs} \geq c_h^2. \tag{30}$$

Proof.
1. The conditions for the burying equilibrium are the same as before (see Eqs. (10)). To make sure this is an equilibrium also in the extended model, we only need to specify that both types of receivers would reject any sender using the second signal.
2. Consider the strategy profile according to which high senders use signal 2, medium senders use signal 1, low senders do not send a signal, strong receivers only accept signal 2, and weak receivers accept any costly signal. Then the first condition in (30) ensures that medium senders prefer

sending signal 1 over signal 2; the second condition guarantees that high senders prefer signal 2 over signal 1. The last two conditions again ensure that both sender types have an incentive to send some signal. Given these behaviors of the senders, the receivers' strategies in the strategy profile correspond to a best response.

□

While the two conditions (29) and (30) are formally similar, there is one important difference. The first two conditions in (30) involve *absolute* payoff terms, a_{ms} and a_{hs}, whereas the first two conditions in (29) involve *relative* payoff terms, a_{mw}/a_{ms} and a_{hw}/a_{hs}. For classical signaling, high senders need to be compensated for paying the higher cost of the signal; for the burying equilibrium, high senders need to be compensated for losing out on the weak receivers. If weak receivers are of little interest to high senders, the latter requirement is easy to satisfy.

To further facilitate a comparison between burying a signal and classical signaling, let us consider the special case in which the costs of the second signal correspond to the effective cost of burying the first signal, $c_i^2 = c_i^1/r_i$. In that case, conditions (29) and (30) can be rewritten as

$$\text{Burying equilibrium:} \quad \frac{c_m^2 - c_m^1}{c_m^1} \geq \frac{q_s a_{ms}}{q_w a_{mw}}, \quad \frac{c_h^2 - c_h^1}{c_h^1} \leq \frac{q_s a_{hs}}{q_w a_{hw}}, \quad q_w a_{mw} \geq c_m^1, \quad q_w a_{hw} + q_s a_{hs} \geq c_h^2.$$

$$\text{Classical separation:} \quad c_m^2 - c_m^1 \geq q_s a_{ms}, \quad c_h^2 - c_h^1 \leq q_s a_{hs}, \quad q_w a_{mw} \geq c_m^1, \quad q_w a_{hw} + q_s a_{hs} \geq c_h^2.$$

The latter two inequalities in these two sets of conditions coincide. But in the first two inequalities for the classical separating equilibrium, only absolute quantities matter (e.g., the absolute value of strong receivers). For the burying equilibrium, on the other hand, only relative quantities matter (e.g., the value of strong receivers compared to the value of weak receivers).

To illustrate these differences, let us consider an example in which all sender types have the same revelation probabilities, $r_h = r_m = r_l = r = 1/3$, and where medium and high senders have the same costs for sending a signal, $c_h^1 = c_m^1 = c^1 = 1/3$ and $c_h^2 = c_m^2 = 3$ (the costs for the low sender are assumed to be prohibitively high to prevent low types from sending any signal). Furthermore, we assume that the players derive the following payoffs from an interaction (the first value gives the payoff of the sender, the second value is the payoff of the receiver),

	s	w
h	a_{hs} , 1	a_{hw} , 1
m	$3, -2$	$3, 1$
l	$3, -10$	$3, -10$

Given these parameter values, the conditions (29) imply that there is a burying equilibrium if high senders derive a sufficient payoff to be willing to pay the cost of a signal, $r(q_s a_{hs} + q_w a_{hw}) \geq c^1$, and if they prefer to bury the signal rather than sending a clear signal, $a_{hs} \geq (1-r_h)/r_h \cdot q_w a_{hw}/q_s$. Similarly, it follows from (30) that we can get a classical separating equilibrium if high senders find it worthwhile to pay the higher cost, $a_{hs} \geq (c^2 - c^1)/q_s$. **Fig. 4** in the main text illustrates these two possible outcomes in a graph in which the x–axis shows the payoff of a high sender against a strong receiver, and in which the y–axis gives his payoff against a weak receiver. In particular, the graph shows that when high senders derive an intermediate value from interacting with strong receivers, but a low payoff from weak receivers, then only burying allows for full separation.

Classical signaling and the burying of a signal do not only differ in their comparative statics – they also differ in their respective welfare consequences. In a classical separating equilibrium, the expected payoff over the whole population sums up to

$$\pi_{CS} = p_h \Big[q_s \big(a_{hs} + b_{hs} \big) + q_w \big(a_{hw} + b_{hw} \big) - c_h^2 \Big] + p_m \Big[q_w (a_{mw} + b_{mw}) - c_m^1 \Big] \tag{31}$$

In contrast, the overall payoff in the burying equilibrium amounts to

$$\pi_{BU} = p_h \Big[r_h \Big(q_s \big(a_{hs} + b_{hs} \big) + q_w \big(a_{hw} + b_{hw} \big) \Big) - c_h^1 \Big] + p_m \Big[q_w (a_{mw} + b_{mw}) - c_m^1 \Big] \tag{32}$$

In particular, in a classical signaling model, the price for full separation is entirely paid by the sender population (in form of the cost terms c_h^2 and c_m^1). In contrast, in a burying equilibrium, the senders' way to achieve full separation also affects the receivers' payoffs – in form of opportunity costs $(1-r_h)b_{hj}$ when the hidden signal does not become revealed. Overall, burying a signal will be more efficient if

$$\pi_{BU} - \pi_{CS} = p_h \Big[\big(c_h^2 - c_h^1 \big) - (1-r_h) \big(q_s (a_{hs} + b_{hs}) + q_w (a_{hw} + b_{hw}) \Big] > 0; \tag{33}$$

that is, if the cost of the more elaborate signal is large compared to the negative consequences of sending a buried signal.

3.5 Extension 5: Burying in a model with many types

The previous models were meant to be as simple as possible – we introduced the minimum number of sender and receiver types necessary to allow for burying equilibria. In this model extension, we aim to show how our framework can be extended to the case of many sender and receiver types (see **Fig. S4** for a graphical representation). We consider a model in which there are m different types of senders, such that the type space for senders takes the form $\mathcal{S} = \{S_1, \ldots, S_m\}$. The proportion of senders of type S_i is given by $p_i > 0$. Similarly, we consider n different receivers and we denote the set of all possible receiver types as $\mathcal{R} = \{R_1, \ldots, R_n\}$, with $q_j > 0$ denoting the respective proportion of each receiver type. As in the baseline model, a sender of type S_i can choose to pay a cost $c_i > 0$ for a signal, and he can decide whether or not he wants to bury the signal. If buried, his signal is revealed with probability r_i, in which case it becomes known to the receiver that the signal was hidden. If a sender of type S_i interacts with a receiver of type R_j, the sender's payoff is a_{ij}, whereas the receiver's payoff is b_{ij}. As in the baseline model, we assume here that senders always prefer to be accepted,

(A1′) $a_{ij} \geq 0$ for all i, j.

Later we will formulate additional assumptions. These assumptions will allow us to derive conditions for a burying equilibrium in which the quality of a sender can be inferred from his chosen signal. Under the given assumptions, we will see that senders who choose to bury the signal are exactly those senders who receivers prefer the most, and receivers who condition their behavior on buried signals are exactly those receivers who are most selective.

We first introduce some further notation. For a given pure strategy profile, let $\mathcal{S}_B$, $\mathcal{S}_C$, and $\mathcal{S}_N$ denote those sender types who send a buried signal, a clear signal, and no signal, respectively (in particular, $\mathcal{S}_B$, $\mathcal{S}_C$, and $\mathcal{S}_N$ are disjoint sets such that $\mathcal{S}_B \cup \mathcal{S}_C \cup \mathcal{S}_N = \mathcal{S}$). Similarly, let $\mathcal{R}_B$, $\mathcal{R}_C$, and $\mathcal{R}_N$ refer to the subset of receiver types who accept the respective signal; since receivers may accept more than one signal type, these sets do not need to be disjoint. Using this notation, we get the following characterization of pure equilibria for the extended model.

Theorem 6 (Equilibria of the model with arbitrarily many types)**.** *A pure strategy profile for the extended game with multiple types is an equilibrium if and only if*

$$\textit{For all } S_i \in \mathcal{S}_B: \quad \frac{r_i}{1-r_i}\left(\sum_{R_j \in \mathcal{R}_B} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_C} q_j a_{ij}\right) \geq \sum_{R_j \in \mathcal{R}_C} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \quad \textit{and} \quad \sum_{R_j \in \mathcal{R}_B} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \geq \frac{c_i}{r_i} \tag{34a}$$

$$\textit{For all } S_i \in \mathcal{S}_C: \quad \frac{r_i}{1-r_i}\left(\sum_{R_j \in \mathcal{R}_B} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_C} q_j a_{ij}\right) \leq \sum_{R_j \in \mathcal{R}_C} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \quad \textit{and} \quad \sum_{R_j \in \mathcal{R}_C} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \geq c_i \tag{34b}$$

$$\textit{For all } S_i \in \mathcal{S}_N: \quad \sum_{R_j \in \mathcal{R}_C} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \leq c_i \quad \textit{and} \quad \sum_{R_j \in \mathcal{R}_B} q_j a_{ij} - \sum_{R_j \in \mathcal{R}_N} q_j a_{ij} \leq \frac{c_i}{r_i} \tag{34c}$$

$$\textit{For all } R_j \in \mathcal{R}_B: \quad \sum_{S_i \in \mathcal{S}_B} p_i b_{ij} \geq 0; \qquad \textit{For all } R_j \notin \mathcal{R}_B: \quad \sum_{S_i \in \mathcal{S}_B} p_i b_{ij} \leq 0; \tag{34d}$$

$$\textit{For all } R_j \in \mathcal{R}_C: \quad \sum_{S_i \in \mathcal{S}_C} p_i b_{ij} \geq 0; \qquad \textit{For all } R_j \notin \mathcal{R}_C: \quad \sum_{S_i \in \mathcal{S}_C} p_i b_{ij} \leq 0; \tag{34e}$$

$$\textit{For all } R_j \in \mathcal{R}_N: \quad \sum_{S_i \in \mathcal{S}_N} p_i b_{ij} \geq 0; \qquad \textit{For all } R_j \notin \mathcal{R}_N: \quad \sum_{S_i \in \mathcal{S}_N} p_i b_{ij} \leq 0; \tag{34f}$$

Proof. The inequalities in (34a) guarantee that senders who make use of buried signals do not have an incentive to deviate; the first inequality ensures that they do not want to send a clear signal, and the second inequality makes sure that they want to pay the cost c_i for a signal. Similarly, the first inequality in (34b) ensures that senders with a clear signal do not wish to bury their signal, while the second inequality ensures that they want to send a signal, as opposed to sending no signal. According to the first inequality in (34c), senders who do not send a signal cannot benefit from sending a clear signal, whereas the second inequality implies that they do not benefit from sending a buried signal. The conditions (34d) – (34f) state that receivers accept exactly those signals for which they can expect to end up with a partner that yields a non-negative expected payoff. □

As an immediate consequence of this Theorem, we note that it only pays to pay the cost of a signal, or to bury a signal, if this is required to get access to certain receivers.

Corrolary 2. *For any pure equilibrium in which some senders use buried signals, $\mathcal{S}_B \neq \emptyset$, it follows that there are receivers who require exactly such signals, $\mathcal{R}_B \nsubseteq \mathcal{R}_C$ and $\mathcal{R}_B \nsubseteq \mathcal{R}_N$. Similarly, in any pure equilibrium in which senders use clear signals $\mathcal{S}_C \neq \emptyset$, there are receivers who accept clear signals but who wouldn't accept no signal, $\mathcal{R}_C \nsubseteq \mathcal{R}_N$.*

Proof. If $\mathcal{R}_B \subseteq \mathcal{R}_N$ then the second inequality in (34a) cannot be satisfied; if $\mathcal{R}_B \subseteq \mathcal{R}_C$, the first inequality in (34a) cannot be satisfied. An analogous argument shows that $\mathcal{R}_C \subseteq \mathcal{R}_N$ is incompatible with the assumption that the strategy profile is an equilibrium. □

Formally, the conditions in Theorem 6 resemble the previous inequalities needed for a burying equilibrium, see conditions (10). However, in contrast to the results of the baseline model, the conditions in Theorem 6 do not yet allow us to give an intuitive explanation for who will bury the signal, and for who will require buried signals. To derive such an intuitive explanation, we will make use of some further assumptions in the following, which make burying most likely to occur. First, as in the baseline model, let us assume that receivers can be ordered according to their selectivity, and senders can be ordered according to their quality as an interaction partner.

(A2′) If $i < j$ and $k < l$, then $b_{ik} < b_{jk}$ and $b_{ik} < b_{il}$.

The first inequality for receiver payoffs $b_{ik} < b_{jk}$ incorporates our assumption that senders S_j with a higher index j are more preferred interaction partners. The second inequality $b_{ik} < b_{il}$ reflects the assumption that receivers with a lower index k have an incentive to be more selective, as they derive a lower benefit from interactions with any given sender. Senders S_i with a low index i thus resemble the senders of low quality of our baseline model, whereas receivers R_k with a low index k resemble the strong receivers. The monotonicity assumption (A2′) is illustrated in **Fig. S4b**. When this assumption holds, the behavior of receivers in equilibrium is in fact monotonic, as formalized in the following Proposition.

Proposition 1. *Consider a pure equilibrium that involves some burying, $\mathcal{S}_B \neq \emptyset$. Then $k < l$ and $R_k \in \mathcal{R}_B$ imply $R_l \in \mathcal{R}_B$. Similarly, provided that $\mathcal{S}_C \neq \emptyset$ and $\mathcal{S}_N \neq \emptyset$, we can conclude that $R_k \in \mathcal{R}_C$ implies $R_l \in \mathcal{R}_C$, and that $R_k \in \mathcal{R}_N$ implies $R_l \in \mathcal{R}_N$, respectively.*

Proof. By Theorem 6, the assumption $R_k \in \mathcal{R}_B$ requires $\sum_{S_i \in \mathcal{S}_B} p_i b_{ik} \geq 0$. Assumption (A2') then implies that $\sum_{S_i \in \mathcal{S}_B} p_i b_{il} > 0$, and thus Theorem 6 again guarantees $R_l \in \mathcal{R}_B$. The proof for $\mathcal{R}_C$ and for $\mathcal{R}_N$ are analogous. □

Proposition 1 allows us to rank two different receivers R_k and R_l according to their tolerance level in equilibrium. It is always the receiver with the higher index who is more tolerant – when $k < l$ and receiver R_k accepts a given sender, then so will receiver R_l. Together with Corrolary 2, this allows us to derive the following conclusion:

Corrolary 3. *In any pure equilibrium in which $\mathcal{S}_B$, $\mathcal{S}_C$ and $\mathcal{S}_N$ are non-empty, there are numbers $n_O < n_C < n_N$ such that the sets of receivers who accept a given signal type take the form*

$$\begin{aligned} \mathcal{R}_N &= \{R_{n_N}, R_{n_N+1}, \dots, R_n\} \\ \mathcal{R}_C &= \{R_{n_C}, R_{n_C+1}, \dots, R_n\} \\ \mathcal{R}_B &= \{R_{n_O}, R_{n_O+1}, \dots, R_n\}, \end{aligned} \tag{35}$$

where the case $n_X > n$ is allowed to produce the empty set, $\mathcal{R}_X = \emptyset$. In particular, the behaviors of receivers are ordered, $\mathcal{R}_N \subsetneqq \mathcal{R}_C \subsetneqq \mathcal{R}_B$.

This corrolary states that any receiver who accepts no signals would also accept clear signals, and any receiver who accepts clear signals would also accept buried signals. That is, more elaborate signals are universally appreciated; it is never perceived as negative if a sender has been observed to bury his signal.

If we aim to establish a similar ranking among sender types, it is useful to introduce a few further assumptions. First, to facilitate comparisons, let us restrict ourselves to senders who are comparable in their revelation probabilities, and in their absolute costs to send the signal, $r_i = r$ and $c_i = c$ for all $1 \leq i \leq m$. Moreover, let us assume that the senders' payoffs are subject to the following condition.

(A3′) If $i < j$ and $k < l$, then $a_{ik} < a_{jk}$ and $\dfrac{a_{ik}}{a_{il}} < \dfrac{a_{jk}}{a_{jl}}$.

The first inequality on the senders' payoffs, $a_{ik} < a_{jk}$, states that high type senders (who are more preferred by receivers) also have a higher incentive to engage in interactions with receivers. The second inequality in addition requires that the higher the type of the sender, the more the sender prefers interactions with strong receivers (with small index k) to interactions with weak receivers (with higher index l). The latter inequality in assumption (A3′) is motivated by the conditions required for the burying equilibrium in (10). Also there we have seen that burying is more likely to arise in equilibrium if high types are preferentially interested in strong receivers. To proceed further, we need the following technical result.

Lemma 1. *Consider two positive n-tuples $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ such that for all $1 \leq k < l \leq n$ we have $x_k/x_l < y_k/y_l$. Then*

$$\frac{\sum_{k\in K} x_k}{\sum_{l\in L} x_l} < \frac{\sum_{k\in K} y_k}{\sum_{l\in L} y_l} \tag{36}$$

for all non-empty index subsets $K, L \subseteq \{1, \dots, n\}$ such that $\max(K) < \min(L)$.

Proof. Due to the stated assumptions, we have $x_k y_l < x_l y_k$ for all $k \in K$ and $l \in L$. Thus we get the same relationship when we sum up over all $k \in K$ and $l \in L$, yielding

$$\sum_{k \in K,\, l \in L} x_k y_l < \sum_{k \in K,\, l \in L} x_l y_k, \tag{37}$$

which is equivalent to inequality (36). $\square$

Using Lemma 1 and assumption (A3′), we can establish the following monotonicity relationship among senders.

Proposition 2. *Consider a pure equilibrium in which all signals are used, $\mathcal{S}_B, \mathcal{S}_C, \mathcal{S}_N \neq \emptyset$, and let $i < j$. Then $S_i \in \mathcal{S}_B$ implies $S_j \in \mathcal{S}_B$ – if the lower type buries in equilibrium, then so does the higher type. Similarly, if $i < j < i'$ and $S_i, S_{i'} \in \mathcal{S}_C$ then $S_j \in \mathcal{S}_C$, and if $S_{i'} \in \mathcal{S}_N$ then $S_j \in \mathcal{S}_N$.*

Proof. When $S_i \in \mathcal{S}_B$, Theorem 6 and Corrolary 3 imply $\sum_{R_k \in \mathcal{R}_B \setminus \mathcal{R}_N} q_k a_{ik} \geq \frac{c}{r}$. Due to assumption (A3′), it follows that $\sum_{R_k \in \mathcal{R}_B \setminus \mathcal{R}_N} q_k a_{jk} > \frac{c}{r}$, and hence $S_j \notin \mathcal{S}_N$. Similarly, because of Theorem 6 and Corrolary 3 we have

$$\frac{\sum_{R_k \in \mathcal{R}_B \setminus \mathcal{R}_\mathcal{C}} q_k a_{ik}}{\sum_{R_l \in \mathcal{R}_C \setminus \mathcal{R}_\mathcal{N}} q_k a_{il}} \geq \frac{1-r}{r}. \tag{38}$$

As of assumption (A3′), we have $a_{ik}/a_{il} < a_{jk}/a_{jl}$ for all $k < l$. Moreover, by Corrolary 3, the indices k used in the numerator on the left hand's side of (38) are strictly smaller than the indices l used in the denominator. Hence, by defining $\mathbf{x} = (a_{i1}, \ldots, a_{in})$ and $\mathbf{y} = (a_{j1}, \ldots, a_{jn})$, we can use Lemma 1 to conclude

$$\frac{\sum_{R_k \in \mathcal{R}_B \setminus \mathcal{R}_\mathcal{C}} q_k a_{jk}}{\sum_{R_l \in \mathcal{R}_C \setminus \mathcal{R}_N} q_k a_{jl}} > \frac{1-r}{r}, \tag{39}$$

such that Theorem 6 implies $S_j \notin \mathcal{S}_C$. Hence, $S_j \in \mathcal{S}_B$.

To show that $i < j < i'$ and $S_i, S_{i'} \in \mathcal{S}_C$ implies $S_j \in \mathcal{S}_C$, we can proceed similarly. Again, the conditions $i < j$, $S_i \in \mathcal{S}_C$ imply $S_j \notin \mathcal{S}_N$, whereas the conditions $j < i'$, $S_{i'} \in \mathcal{S}_C$ imply $S_j \notin \mathcal{S}_B$. We conclude $S_j \in \mathcal{S}_C$. Analogously one can show that $j < i'$ and $S_{i'} \in \mathcal{S}_N$ imply $S_j \in \mathcal{S}_N$. $\square$

As in the case of different receivers, Proposition 2 allows us to rank different senders S_i and S_j according to their behavior in a burying equilibrium.

Corrolary 4. *In any pure equilibrium in which $\mathcal{S}_B$, $\mathcal{S}_C$ and $\mathcal{S}_N$ are non-empty, there are numbers $1 < m_C < m_O < m$ such that the sets of senders who use a given signal take the form*

$$\begin{aligned} \mathcal{S}_N &= \{S_1, S_2, \ldots, S_{m_C - 1}\} \\ \mathcal{S}_C &= \{S_{m_C}, S_{m_C+1}, \ldots, S_{m_O - 1}\} \\ \mathcal{S}_B &= \{S_{m_O}, S_{m_O+1}, \ldots, S_m\}. \end{aligned} \tag{40}$$

Thus, the higher the type of the sender, the more elaborate the type of the signal that he will use in equilibrium.

We can summarize these results by rephrasing Theorem 6 as follows.

Theorem 7 (Burying equilibria in a model with arbitrarily many types)**.** *Under the assumptions (A1′) – (A3′), and given that $r_i = r$ and $c_i = c$ for all senders S_i, there is a pure equilibrium for which $\mathcal{S}_B, \mathcal{S}_C, \mathcal{S}_N \neq \emptyset$ if and only if there are numbers $1 < m_C < m_O < m$ and $n_O < n_C < n_N$ such that*

$$\text{For all } S_i \text{ with } i \geq m_O: \quad \frac{r}{1-r} \geq \frac{\sum\limits_{n_C \leq j < n_N} q_j a_{ij}}{\sum\limits_{n_O \leq j < n_C} q_j a_{ij}} \quad \text{and} \quad \sum_{n_O \leq j < n_N} q_j a_{ij} \geq \frac{c}{r} \tag{41a}$$

$$\text{For all } S_i \text{ with } m_C \leq i < m_O: \quad \frac{r}{1-r} \leq \frac{\sum\limits_{n_C \leq j < n_N} q_j a_{ij}}{\sum\limits_{n_O \leq j < n_C} q_j a_{ij}} \quad \text{and} \sum_{n_C \leq j < n_N} q_j a_{ij} \geq c \tag{41b}$$

$$\text{For all } S_i \text{ with } i < m_C: \quad \sum_{n_C \leq j < n_N} q_j a_{ij} \leq c \quad \text{and} \quad \sum_{n_O \leq j < n_N} q_j a_{ij} \leq \frac{c}{r} \tag{41c}$$

$$\text{For all } R_j \text{ with } j \geq n_O: \quad \sum_{i \geq m_O} p_i b_{ij} \geq 0; \qquad \text{For all } R_j \text{ with } j < n_O: \quad \sum_{i \geq m_O} p_i b_{ij} \leq 0; \tag{41d}$$

$$\text{For all } R_j \text{ with } j \geq n_C: \quad \sum_{m_C \leq i < m_O} p_i b_{ij} \geq 0; \qquad \text{For all } R_j \text{ with } j < n_C: \quad \sum_{m_C \leq i < m_O} p_i b_{ij} \leq 0; \tag{41e}$$

$$\text{For all } R_j \text{ with } j \geq n_N: \quad \sum_{i \geq m_O} p_i b_{ij} \geq 0; \qquad \text{For all } R_j \text{ with } j < n_N: \quad \sum_{i \geq m_O} p_i b_{ij} \leq 0; \tag{41f}$$

The senders S_i with $i \geq m_O$ are exactly those senders who bury their signal in equilibrium, whereas senders with $m_C \leq i < m_O$ use clear signals and senders with $i < m_C$ use no signal. Similarly, receivers R_j with $j \geq n_O$ accept buried signals, and receivers with $j \geq n_C$ and $j \geq n_N$ accept clear signals and no signal, respectively.

We note that conditions (41a) and (41b) exactly reproduce conditions (10) for the existence of a burying equilibrium in the baseline model. There, condition (41c) is trivially satisfied because of Assumption (A3), and the conditions (41d) – (41f) are met due to the assumption (A2).

In this subsection, we have considered a model that allows for more than three different types of senders and for more than two different receiver types. With Theorem 6 we provide a general characterization of all pure equilibria that can occur. As one may expect, these conditions are more elaborate than the conditions in the baseline model, potentially leading to somewhat irregular behaviors in equilibrium. To give more intuitive insights, we have added further assumptions, stating that receivers can be ordered according to their selectivity, senders can be ordered according to their quality, and high senders prefer to interact with strong receivers. Given these assumptions, we have derived simple monotonicity relationships. In a burying equilibrium, the quality of a sender can be inferred from his chosen signal. Senders who choose to bury the signal are exactly those senders who receivers prefer the most. Conversely, receivers who condition their behavior on buried signals are exactly those receivers who are most selective, as depicted in **Fig. S4c**.

3.6 An alternative model for burying

In the following, let us consider an alternative model that yields a complementary interpretation for why players may bury their positive signals. Again, there are three types of senders (low, medium, and high). As before, a sender's type is private information: senders know their own type, but receivers only know the respective distribution $\mathbf{p} = (p_l, p_m, p_h)$. Senders can signal their type by choosing an element in $\sigma \in \{\mathbf{b}, \mathbf{c}, \mathbf{n}\}$. The element $\mathbf{n}$ incurs no cost, and corresponds to the option not to send any signal. The element $\mathbf{c}$ incurs a cost c_i that depends on the type of the sender $i \in \{l, m, h\}$, and is interpreted as a clear signal. When choosing the element $\mathbf{b}$, which incurs the same cost c_i as a clear signal, the signal is buried. Buried signals either become revealed and tagged as being buried with probability r_i and $i \in \{l, m, h\}$; otherwise buried signals go unnoticed. In the latter case, the receiver does not know whether the sender has used a buried signal (which the receiver failed to notice), or whether the sender has not sent a signal at all.

In contrast to our previous model, we now only consider one type of receiver. After observing the sender's signal σ, the receiver forms updated beliefs $\mathbf{p}' = (p'_l, p'_m, p'_h)$ about the distribution of the sender's type according to Bayes' rule, given the strategies used by the senders. In this alternative model, receivers are not required to make a decision whether to accept the sender. Instead, the payoff of a sender of type i is proportional to some non-negative benefit function $f_i(\mathbf{p}')$ of the receiver's updated belief. We assume that the functions f_i satisfy the following monotonicity conditions for any updated beliefs $\mathbf{p}' = (p'_l, p'_m, p'_h)$ and $\tilde{\mathbf{p}}' = (\tilde{p}'_l, \tilde{p}'_m, \tilde{p}'_h)$:

1. If $p'_l = \tilde{p}'_l$, $p'_m < \tilde{p}'_m$, and $p'_h > \tilde{p}'_h$ then $f_i(\mathbf{p}') > f_i(\tilde{\mathbf{p}}')$.
2. If $p'_l < \tilde{p}'_l$, $p'_m = \tilde{p}'_m$, and $p'_h > \tilde{p}'_h$ then $f_i(\mathbf{p}') > f_i(\tilde{\mathbf{p}}')$.
3. If $p'_l < \tilde{p}'_l$, $p'_m > \tilde{p}'_m$, and $p'_h = \tilde{p}'_h$ then $f_i(\mathbf{p}') > f_i(\tilde{\mathbf{p}}')$.

These conditions guarantee that senders prefer to be seen as of higher quality. For example, if two posteriors $\mathbf{p}'$ and $\tilde{\mathbf{p}}'$ are the same, except for the fact that according to $\mathbf{p}'$ it is more likely that the sender is of high type as opposed to a medium type, then senders get a higher benefit under $\mathbf{p}'$ than under $\tilde{\mathbf{p}}'$.

If a sender's estimated quality is $\mathbf{p}'$, then the payoff of a sender of type i is $u_i = f_i(\mathbf{p}') - c_i$ if the sender paid a cost for a signal, and it is $u_i = f_i(\mathbf{p}')$ otherwise. We assume that $f_l(\mathbf{p}') < c_l$ for all posteriors $\mathbf{p}'$, such that low sender types never have an incentive to use a signal.

Again, we refer to a burying equilibrium as an equilibrium in which high senders bury their signal, medium senders send a clear signal, and low senders use no signal. In such an equilibrium, the receivers' updated beliefs $\mathbf{p}'_\mathbf{b}$, $\mathbf{p}'_\mathbf{c}$ and $\mathbf{p}'_\mathbf{n}$ after observing the signals $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{n}$ are

$$\mathbf{p}'_\mathbf{b} = (0,0,1), \quad \mathbf{p}'_\mathbf{c} = (0,1,0), \quad \mathbf{p}'_\mathbf{n} = \left(\frac{p_l}{p_h(1-r_h)+p_l}, 0, \frac{p_h(1-r_h)}{p_h(1-r_h)+p_l}\right). \tag{42}$$

We note that our three assumptions above imply $f_i(\mathbf{p}'_\mathbf{b}) > f_i(\mathbf{p}'_\mathbf{n})$ and $f_i(\mathbf{p}'_\mathbf{b}) > f_i(\mathbf{p}'_\mathbf{c})$ for all player types i. That is, when a player's buried signal becomes revealed, the player's benefit from the buried signal exceeds the benefit from a clear signal and the benefit from no signal. The equilibrium payoffs of the three sender types are given by

$$\begin{aligned} u_h &= r_h f_h(\mathbf{p}'_\mathbf{b}) + (1 - r_h) f_h(\mathbf{p}'_\mathbf{n}) - c_h, \\ u_m &= f_m(\mathbf{p}'_\mathbf{c}) - c_m, \\ u_l &= f_l(\mathbf{p}'_\mathbf{n}). \end{aligned} \tag{43}$$

Using this, we get the following characterization for the existence of a burying equilibrium (the proof is analogous to the respective proof in the baseline model):

Theorem 8 (Burying equilibria in the alternative model)**.** *There is a strategic burying equilibrium if and only if the following four conditions hold,*

$$\begin{aligned} &r_h \geq \frac{f_h(\mathbf{p}'_\mathbf{c}) - f_h(\mathbf{p}'_\mathbf{n})}{f_h(\mathbf{p}'_\mathbf{b}) - f_h(\mathbf{p}'_\mathbf{n})}, \quad r_m \leq \frac{f_m(\mathbf{p}'_\mathbf{c}) - f_m(\mathbf{p}'_\mathbf{n})}{f_m(\mathbf{p}'_\mathbf{b}) - f_m(\mathbf{p}'_\mathbf{n})}, \\ &\frac{c_h}{r_h} \leq f_h(\mathbf{p}'_\mathbf{b}) - f_h(\mathbf{p}'_\mathbf{n}), \quad c_m \leq f_m(\mathbf{p}'_\mathbf{c}) - f_m(\mathbf{p}'_\mathbf{n}). \end{aligned} \tag{44}$$

Two consequences follow immediately from the first two conditions in Eq. (44). First, in case that high and medium senders derive the same benefit from their perceived quality, $f_h = f_m$, strategic burying can only arise if a high type's signal is more likely to be revealed, $r_h \geq r_m$. Second, in case that both types have the same revelation probability $r_h = r_m$, it follows that strategic burying can only occur if

$$\frac{\Delta_h^{\mathbf{bc}}}{\Delta_h^{\mathbf{bc}} + \Delta_h^{\mathbf{cn}}} \geq \frac{\Delta_m^{\mathbf{bc}}}{\Delta_m^{\mathbf{bc}} + \Delta_m^{\mathbf{cn}}}, \tag{45}$$

where $\Delta_i^{\sigma\tilde{\sigma}} := f_i(\mathbf{p}'_\sigma) - f_i(\mathbf{p}'_{\tilde{\sigma}})$ is the relative advantage of type $i \in \{l, m, h\}$ from sending signal σ instead of $\tilde{\sigma}$ in the burying equilibrium. This condition is more easily satisfied, for example, when high types care a lot about being perceived as high and not as medium (such that $\Delta_h^{\mathbf{bc}}$ is large), whereas they would not really care whether they are perceived as medium or low (such that $\Delta_h^{\mathbf{cn}}$ is small).

The conditions (44) can be interpreted in terms of our baseline model. To this end, let us consider the special case that the relative advantage of signal σ compared to signal $\tilde{\sigma}$ for the various types is given by

$$\begin{aligned} &\Delta_h^{\mathbf{bc}} := q_s a_{hs} \quad \Delta_h^{\mathbf{cn}} := q_w a_{hw} \\ &\Delta_m^{\mathbf{bc}} := q_s a_{ms} \quad \Delta_m^{\mathbf{cn}} := q_w a_{mw}. \end{aligned} \tag{46}$$

In that case, the conditions for the existence of a burying equilibrium in (44) become

$$
\begin{aligned}
&r_h \geq \frac{q_w a_{hw}}{q_w a_{hw} + q_s a_{hs}}, \quad r_m \leq \frac{q_w a_{mw}}{q_w a_{mw} + q_s a_{ms}}, \\
&\frac{c_h}{r_h} \leq q_w a_{hw} + q_s a_{hs}, \qquad c_m \leq q_w a_{mw}.
\end{aligned} \tag{47}
$$

These four conditions coincide with the conditions for the existence of a burying equilibrium in our baseline model, see Eqs. (1)–(4) in the main text.

Instead of explicitly distinguishing between various kinds of receivers, this alternative model incorporates the benefit of signaling in a more abstract way, by introducing the benefit function $f(\mathbf{p}')$. This more abstract formulation has the added advantage that it is amenable to alternative interpretations. For example, let us consider a scenario in which there is only one type of receiver, but this receiver may choose among three different actions $\{a_1, a_2, a_3\}$ after observing the sender's signal. Moreover, let us assume the receiver has an incentive to choose action a_1 if she believes that the sender is certainly a high type (i.e., if $p'_h = 1$), that she wants to choose action a_2 if she believes that the sender may be either a high or a medium type (i.e., if $p'_l = 0$ and $p'_h < 1$), and that she wants to choose action a_3 otherwise (i.e., if $p'_l > 0$). In such a model, the conditions for a burying equilibrium are again given by the inequalities in (44), where $f_i(\mathbf{p}'_\mathbf{b})$, $f_i(\mathbf{p}'_\mathbf{c})$, $f_i(\mathbf{p}'_\mathbf{n})$ need to be interpreted as the benefit that an i-type sender derives if the receiver chooses action a_1, a_2, or a_3, respectively.

Appendix: Simulation code used for the evolutionary analysis

In the following, we reproduce our MATLAB code for the baseline model that we have used to run the evolutionary simulations.

```
function [xhvec,xmvec,xlvec,ysvec,ywvec]=evolProc();
%% function [xhvec,xmvec,xlvec,ysvec,ywvec]=evolProc();
% The output matrices xhvec, xmvec, xlvec ysvec,ywvec give the population
% composition in each of the 3 sender-subpopulations and 2
% receiver-subpopulations over the course of an evolutionary simulation.

global distrS distrR aMat bMat cvec rvec beta

%% Specifying the parameters
nUp=2*10^6; % number of updating events for the simulation
ahs=12; ahw=3; ams=4; amw=4; als=1; alw=1; aMat=[ahs,ahw; ams,amw; als,alw];
% Defining the payoffs of the sender population in a 3x2 matrix
bhs=6; bhw=6; bms=-10; bmw=4; bls=-10; blw=-10; bMat=[bhs,bhw; bms,bmw; bls,blw];
% Defining the payoffs of the receiver population in a matrix
ch=1; cm=1; cl=100; cvec=[ch,cm,cl]; % Defining a vector containing the costs
```

```
rh=1/3; rm=1/3; rl=1/3; rvec=[rh,rm,rl]; % Defining the revelation probabilities
distrS=[0.2 0.3 0.5]; % distribution among senders (p_high / p_medium / p_low)
distrR=[0.5 0.5]; % distribution among receivers (q_strong / q_weak)
NS=300; NR=300; % size of the respective populations
mu=0.02; beta=1; % mutation rate and strength of selection

%% Setting up an initial population
% For senders:  3-dim vector with number of respective senders who bury / send
clear signal / send no signal
xh=[0,0,NS*distrS(1)]; % initially, all high senders send no signal
xm=[0,0,NS*distrS(2)]; % same for medium senders
xl=[0,0,NS*distrS(3)]; % same for low senders
% For receivers:  8-dim vector encoding the 2^3 possibilities how receivers can
react to the sender's signal
ys=[NR*distrR(1),zeros(1,7)]; % initially, strong receivers reject everyone
yw=[NR*distrR(2),zeros(1,7)]; % same for weak receivers
% Initializing a matrix for recording the population composition in each evolutionary
timestep
xhvec=[xh; zeros(nUp-1,3)]; xmvec=[xm; zeros(nUp-1,3)]; xlvec=[xl; zeros(nUp-1,3)];
ysvec=[ys; zeros(nUp-1,8)]; ywvec=[yw; zeros(nUp-1,8)];

%% Actual Simulation
for t=2:nUp
   rn=rand(1);
   if rn < mu % Chance event determines whether there is a mutation event or
a selection event
      [z,nr]=Mutate(xh,xm,xl,ys,yw); % Starting the mutation subroutine
   else
      [z,nr]=Select(xh,xm,xl,ys,yw); % Starting the selection subroutine
   end
   if nr==1, xh=z; elseif nr==2, xm=z; elseif nr==3, xl=z; elseif nr==4, ys=z;
   else yw=z; end % Updating of the respective sender or receiver population
   xhvec(t,:)=xh; xmvec(t,:)=xm; xlvec(t,:)=xl; ysvec(t,:)=ys; ywvec(t,:)=yw;
   % Recording the population composition in timestep t
end
end


function [z,nr]=PickPop(xh,xm,xl,ys,yw);
%% function [z,nr]=PickPop(xh,xm,xl,ys,yw);
% Subroutine that randomly picks one of the sender/receiver subpopulations, proportional
to their respective size
```

```
global distrS distrR

rn=rand(1,2);
if rn(1)<1/2 % With probability 1/2, a sender population is chosen
   if rn(2)<distrS(1)
      z=xh; nr=1;
   elseif rn(2)<1-distrS(3)
      z=xm; nr=2;
   else
      z=xl; nr=3;
   end
else % Otherwise, a receiver subpopulation is chosen
   if rn(2)<distrR(1)
      z=ys; nr=4;
   else
      z=yw; nr=5;
   end
end
end


function [z,nr]=Mutate(xh,xm,xl,ys,yw);
%% function [z,nr]=Mutate(xh,xm,xl,ys,yw);
% Subroutine that randomly selects one of the subpopulations and changes the
strategy of one of the players

[z,nr]=PickPop(xh,xm,xl,ys,yw); % Selecting a random subpopulation
n=length(z); rn=rand(1); zcs=cumsum(z)/sum(z);
jOld=1; % Starting a routine to select a player to change her strategy
for i=2:n
   if rn>zcs(i-1) & rn<=zcs(i)
      jOld=i;
   end
end
jNew=randi(n); % Taking a random new strategy
z(jNew)=z(jNew)+1; z(jOld)=z(jOld)-1;
end


function [z,nr]=Select(xh,xm,xl,ys,yw);
%% function [z,nr]=Select(xh,xm,xl,ys,yw);
% Subroutine that incorporates the selection process.  A learner and a role model
are randomly chosen; the learner adopts the role model's strategy with some probability
```

```
depending on the players' payoffs

global aMat bMat cvec rvec beta

[z,nr]=PickPop(xh,xm,xl,ys,yw); % Selecting a random subpopulation
if max(z)~= sum(z) % Selection can only occur if the population is heterogeneous
   n=length(z); rn=rand(1,3); zcs=cumsum(z)/sum(z);
   jLearn=1; jRole=1; % Starting a routine to select a learner and a role model
   for i=2:n
      if rn(1)>zcs(i-1) & rn(1)<=zcs(i)
         jLearn=i;
      end
      if rn(2)>zcs(i-1) & rn(2)<=zcs(i)
         jRole=i;
      end
   end
   if jLearn ~= jRole % Selection can only occur if the strategy of the learner
is different from the strategy of the role model
      [payh,paym,payl,pays,payw]=CalcPay(xh,xm,xl,ys,yw,aMat,bMat,cvec,rvec);
% Calculating the payoffs given the current population
      if nr==1, pay=payh; elseif nr==2, pay=paym; elseif nr==3, pay=payl;
      elseif nr==4, pay=pays; else, pay=payw; end
      piRole=pay(jRole); piLearn=pay(jLearn); % Deriving the payoffs of the learner
and of the role model
      rho=1/(1+exp(-beta*(piRole-piLearn))); % Calculationg the imitation probability
      if rn(3)<rho
         z(jLearn)=z(jLearn)-1; z(jRole)=z(jRole)+1; % Updating the population
if imitation occurs
      end
   end
end
end


function [payh,paym,payl,pays,payw]=CalcPay(xh,xm,xl,ys,yw,a,b,c,r);
%% function [payh,paym,payl,pays,payw]=CalcPay(xh,xm,xl,ys,yw,a,b,c,r);
% Calculates the payoffs of h,m,l senders and s,w receivers
% xh=(xhB,xhC,xhN) gives the absolute numbers of high senders that bury, send
a clear signal, or send no signal; similarly for xm, xl.
% ys=(ys000,ys001,ys010,ys011,ys100,ys101,ys110,ys111) gives the absolute number
of strong receivers who accept(1) or reject(0) the respective signal type (bury/clear/no
signal)
% a = (ahs,ahw; ams, amw; als, alw); b=(bhs,bhw; bms, bmw; bls; blw);
```

```
% c=(ch,cm,cl) cost vector for different sender types
% r=(rh,rm,rl) revelation probability vector

% Calculation of important quantities
ns=sum(ys); % Absolute number of strong individuals
ysB=sum(ys(5:8)); ysC=sum(ys([3,4,7,8])); ysN=sum(ys([2:2:8])); % Number of strong
players who accept buried signals, clear signals, and no signal, respectively
nw=sum(yw); ywB=sum(yw(5:8)); ywC=sum(yw([3,4,7,8])); ywN=sum(yw([2:2:8])); %
Same for weak players
nSen=sum(xh)+sum(xm)+sum(xl); nRec=ns+nw; % Absolute size of the sender and receiver
population
ysB=ysB/nRec; ysC=ysC/nRec; ysN=ysN/nRec; % Calculating the fraction of strong
players who accept buried / clear / no signals.
ywB=ywB/nRec; ywC=ywC/nRec; ywN=ywN/nRec; % Same for weak receivers

% Calculation of the senders' payoffs
payh=[r(1)*(a(1,1)*ysB+a(1,2)*ywB)+(1-r(1))*(a(1,1)*ysN+a(1,2)*ywN)-c(1),...
   a(1,1)*ysC+a(1,2)*ywC-c(1),...
   a(1,1)*ysN+a(1,2)*ywN];
paym=[r(2)*(a(2,1)*ysB+a(2,2)*ywB)+(1-r(2))*(a(2,1)*ysN+a(2,2)*ywN)-c(2),...
   a(2,1)*ysC+a(2,2)*ywC-c(2),...
   a(2,1)*ysN+a(2,2)*ywN];
payl=[r(3)*(a(3,1)*ysB+a(3,2)*ywB)+(1-r(3))*(a(3,1)*ysN+a(3,2)*ywN)-c(3),...
   a(3,1)*ysC+a(3,2)*ywC-c(3),...
   a(3,1)*ysN+a(3,2)*ywN];

% Calculation of the receivers' payoffs
PaysB=b(1,1)*xh(1)*r(1)+b(2,1)*xm(1)*r(2)+b(3,1)*xl(1)*r(3); % Payoff that a
strong receiver gets from accepting senders who bury their signal
PaywB=b(1,2)*xh(1)*r(1)+b(2,2)*xm(1)*r(2)+b(3,2)*xl(1)*r(3); % Same for weak
receivers
PaysC=b(1,1)*xh(2)+b(2,1)*xm(2)+b(3,1)*xl(2); % Payoff of a strong receiver for
accepting a clear signal
PaywC=b(1,2)*xh(2)+b(2,2)*xm(2)+b(3,2)*xl(2); % Same for weak receiver
PaysN=b(1,1)*xh(3)+b(2,1)*xm(3)+b(3,1)*xl(3) + ...
   + b(1,1)*xh(1)*(1-r(1))+b(2,1)*xm(1)*(1-r(2))+b(3,1)*xl(1)*(1-r(3)); % Payoff
of strong receiver for accepting no signal
PaywN=b(1,2)*xh(3)+b(2,2)*xm(3)+b(3,2)*xl(3) + ...
   + b(1,2)*xh(1)*(1-r(1))+b(2,2)*xm(1)*(1-r(2))+b(3,2)*xl(1)*(1-r(3)); % Same
for weak receivers

pays=(PaysN*[0,1,0,1,0,1,0,1]+PaysC*[0,0,1,1,0,0,1,1]+PaysB*[0,0,0,0,1,1,1,1])/nSen;
% Calculating an 8-dim vector with the payoffs for each strategy of strong receivers
```

```
payw=(PaywN*[0,1,0,1,0,1,0,1]+PaywC*[0,0,1,1,0,0,1,1]+PaywB*[0,0,0,0,1,1,1,1])/nSen;
% Same for weak receivers
end
```

Supplementary References

[1] Cho, I.-K. & Kreps, D. M. Signaling games and stable equilibria. *Quarterly Journal of Economics* **102**, 179–221 (1987).

[2] Carbajal, J. C., Hall, J. & Li, H. Inconspicuous conspicuous consumption. *Peruvian Economic Association,* Working paper No. 38 (2015).