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SUPPLEMENTARY INFORMATION

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Value-based attention but not divisive normalization influences decisions with multiple alternatives

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Supplementary Results

Additional regression analyses of distractor value on relative choice accuracy.

Supplementary Table 1 shows the results of different implementations of the logistic regressions analysis that regressed relative choice accuracy between target options onto target values (V1, V2) and distractor value (V3) or normalized distractor value (normV3) for both experiments. In principle, the analysis can be performed using two different approaches. In a random-effects ("two-stage") approach, the regression is performed independently for each participant and the individual regression coefficients are then tested against 0 with a one-sample *t*-test at the group level. In a fixed-effects approach, all data is collapsed across participants and a single regression analysis is performed (mixed-effects approaches are also possible, but we do not provide them here; instead we provide a hierarchical Bayesian analysis approach; see main text, Fig. 2d, and Supplementary Figure 4). The fixed-effects analyses for the direct replication experiment are performed once with participant ID 3 included and once with ID 3 excluded. As stated in the Methods, this participant assigned a value of 0 to all but two of the distractors. Furthermore, this participant had extremely high choice accuracy (i.e., best option chosen in 95% of all trials), apparently because s/he had a few favorite snacks and many snacks that s/he did not like at all. Thus, a random-effects analysis (as well as a median-split of distractors into high and low value) could not be performed for ID 3, and this participant was excluded from any further analysis. However, for the sake of completeness but also for pointing out a problem of fixed-effects analyses in the current context, we report and discuss the results for the fixed effects analyses with ID 3 being included here.

Supplementary Table 1 *Tests of the distractor effect on relative choice accuracy*

Test	ID 3 included	Effect of V1	Effect of V2	Effect of V3 / normV3
Direct replication experiment				
RFX, V3	No	**1.11 (1.17)*** **	**-0.81 (0.84)*** **	-0.01 (0.16)
RFX, normV3	No	**1.10 (1.17)*** **	**-0.81 (0.84)*** **	-0.01 (0.17)
FFX, V3	No	**1.69 {0.04}*** **	**-1.61 {0.04}*** **	-0.01 {0.02}
FFX, normV3	No	**1.68 {0.04}*** **	**-1.60 {0.04}*** **	-0.08 {0.06}
RFX, V3	Yes	-	-	-
RFX, normV3	Yes	-	-	-
FFX, V3	Yes	**1.71 {0.04}*** **	**-1.63 {0.04}*** **	-0.02 {0.02}
FFX, normV3	Yes	**1.70 {0.04}*** **	**-1.62 {0.04}*** **	**-0.11 {0.06}* **
Eye-tracking experiment				
RFX, V3	-	**1.10 (0.64)*** **	**-0.92 (0.55)*** **	-0.02 (0.15)
RFX, normV3	-	**1.09 (0.63)*** **	**-0.92 (0.55)*** **	-0.01 (0.16)
FFX, V3	-	**1.23 {0.05}*** **	**-1.29 {0.05}*** **	**0.07 {0.02}** **
FFX, normV3	-	**1.23 {0.05}*** **	**-1.29 {0.05}*** **	**0.07 {0.02}** **

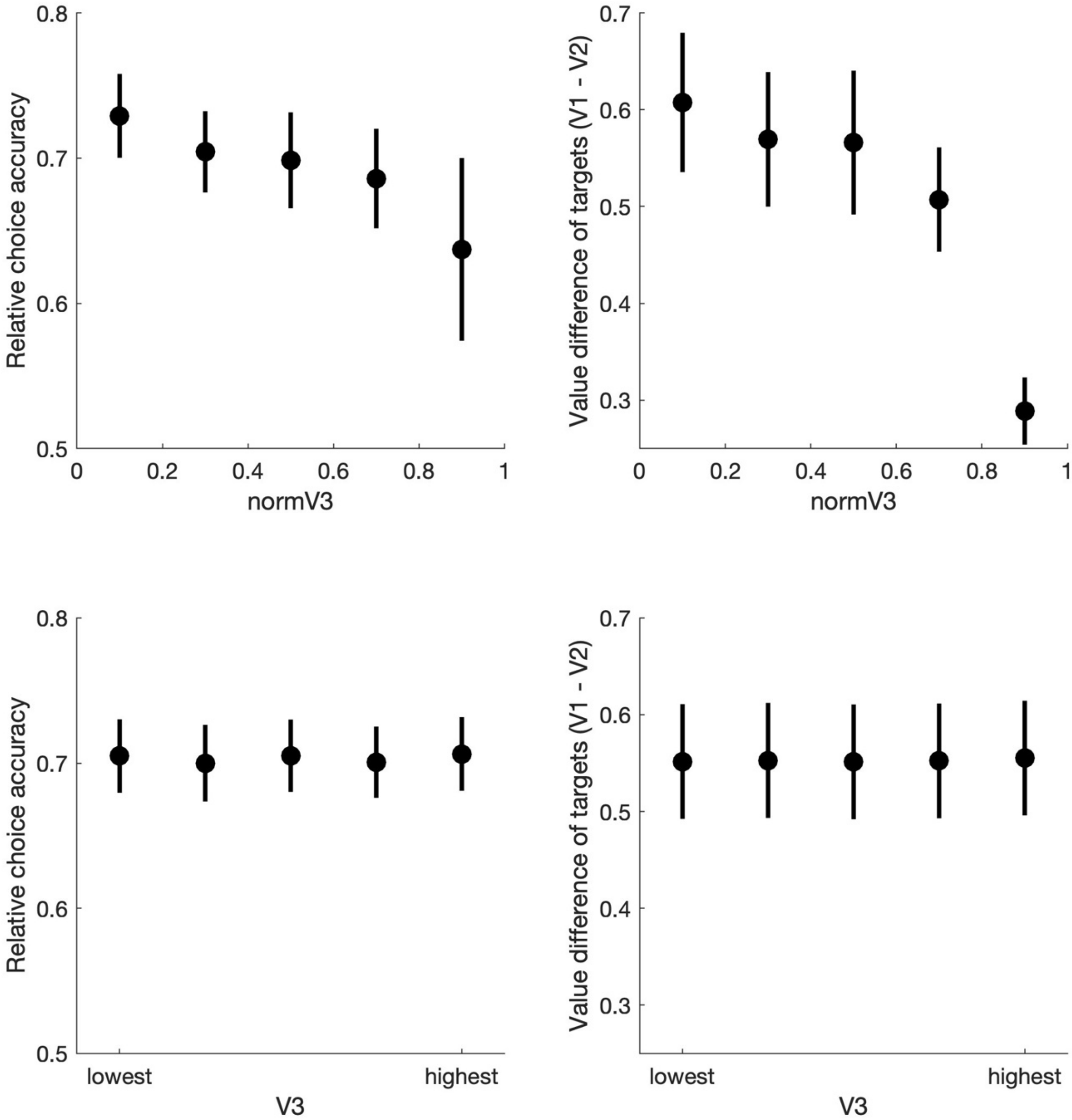
Note. Regression coefficients for different implementations of the regression analysis that regressed relative choice accuracy onto target (V1, V2) and distractor (V3) values. For the random-effects (RFX) analyses, group means are shown together with standard deviations in parentheses. For the fixed-effects analyses (FFX), coefficients are shown together with standard errors in curly brackets. The FFX analyses of the direct replication experiment are repeated with participant ID 3 included, who had only two distractors with a non-zero subjective value and very high choice accuracy. RFX analyses with ID 3 included are not reported as conducting the analysis within ID 3 was impossible. All analyses were conducted once with V3 and once with normV3 as distractor value. As can be seen, the FFX analysis with normV3 and ID 3 included yielded a significantly negative effect of normV3 (p = .040), which would be in line with divisive normalization. However, a more plausible explanation for this result is that the FFX analysis counts all the (very accurate) decisions of ID 3 to the lower end of the distractor-value range. Oddly, the FFX analyses for the eye-tracking experiment yielded significantly positive effects, suggesting the opposite of a normalization effect. In general, we see these results as evidence for the inappropriateness of FFX analyses in the current context (i.e., with strong variations of value ranges and choice accuracy across participants). Regression analyses were performed with standardized predictor variables (standardization for the FFX analyses was performed after merging the individual data). *p < .05, **p < .01, ***p < .001.

As can be seen from Supplementary Table 1, all random-effects analyses suggest a non-significant effect of distractor value on relative choice accuracy. The results of the fixed-effects analyses are much less consistent: The two analyses of the direct replication with ID 3 excluded and the analysis of the direct replication with ID 3 included and V3 as predictor suggest a null effect; the analysis of the direct replication with ID 3 included and normV3 as predictor suggests a significantly negative effect; the two analyses of the eye-tracking experiment suggest a significantly positive effect. The fact that including ID 3 leads to a significantly negative effect in the fixed-effects analysis (only when using normV3 as predictor) derives from the above-mentioned facts that this participant assigned very low values to distractors and had a very high choice accuracy. Thus, when collapsing the data across participants in the fixed-effects analysis, all the decisions of this participant are associated with low-value distractors (because her/his distractor ratings were always much lower than the group average), and her/his very high choice accuracy produces the spurious effect that low normV3 appear to be associated with higher accuracy. We assume that the significantly positive effects in fixed-effects analyses of the eye-tracking experiment derive from similar issues of data aggregation. In general, we take these results as evidence for the inappropriateness of fixed-effects analyses in the current context, because interindividual differences in the use of the willingness-to-pay and rating scales as well as interindividual differences in choice accuracy necessitate the modeling of variance on the individual level[1]. Our hierarchical Bayesian analysis supports this conclusion (here, the effect of ID 3 is mitigated, because the extremely high accuracy of this participant makes her/his data uninformative). It is conceivable that similar artificial effects are present in the original data of Louie and colleagues[2].

The variable normV3 but not V3 is confounded by choice difficulty.

As outlined in the preregistration protocol, we decided to deviate from the original analysis by replacing the normalized distractor value normV3 (i.e., normV3 = V3 / 0.5*[V1 + V2]) by the "raw" distractor value V3 when performing our statistical analyses. The reason for doing so is that normV3 is confounded with target choice difficulty (i.e., V1 – V2) as well as with the summed target values (i.e., V1 + V2), because high values of normV3 are only possible when both V1 and V2 are not far away from V3. For example, assume V3 = 1. To achieve normV3 = 0.8 (i.e., a high normV3 value), the sum of V1 and V2 must be 2.5 (since 1/(0.5*2.5) = 0.8). Because V2 > V3 by definition, V2 > 1. Therefore, V1 < 2.5 – 1 = 1.5. Thus, V1 – V2 < 0.5. In words, the target value sum must be 2.5 and the difference must be smaller than 0.5. In contrast, to achieve only normV3 = 0.4, the target value sum must be 5, and the target value difference must be smaller than 3 (i.e., there are more possibilities for larger target-value differences).

It follows that normV3 (but not V3) should be negatively correlated with V1 – V2 and with V1 + V2. To test the presence of these confounds in our data (direct replication experiment), we regressed normV3 onto V1 – V2 and V1 + V2 in a random-effects linear regression. Indeed, we found that both regression coefficients were significantly negative (V1 – V2: $t(101) = -3.04$, $p = .003$, $d = -0.30$; V1 + V2: $t(101) = -16.81$, $p < .001$, $d = -1.66$). In contrast, the variable V3 is not confounded (V1 – V2: $t(101) = -1.20$, $p = .233$, $d = -0.12$; V1 + V2: $t(101) = -0.46$, $p = .644$, $d = -0.05$). Supplementary Figure 1 shows the relationship between normV3 and V3 with relative choice accuracy and choice difficulty (following the layout of Figure 5A in Louie et al.[2]). It becomes obvious that the seemingly negative relationship between normV3 and relative choice accuracy is an artefact of the (expectably) negative relationship between choice difficulty and relative choice accuracy.



Supplementary Figure 1 The variable normV3 but not V3 is confounded with target choice difficulty. The upper left panel shows the relative choice accuracy between targets as a function of the normalized distractor value normV3 (compare with Figure 5A in Louie et al.[2]). It appears as if there is a substantial negative relationship. However, the upper right panel shows that this relationship is likely to be an artefact because normV3 is also negatively related to the difference between V1 and V2, which essentially reflects the difficulty to choose between targets. The lower panels show that when replacing normV3 by V3 there is neither a relationship with relative choice accuracy nor with target-value difference.

To lend further support for our deviation from the original analysis but also to provide evidence for the suitability of using V3 as a proxy for testing choice effects that are consistent with divisive normalization, we performed simulations of the divisive normalization model to test the sensitivity and specificity of our statistical tests. Thereto, we performed 1’000 simulations of our direct replication experiment, using the empirical values from the BDM-auction (i.e., V1, V2, V3/normV3) as inputs and generating choices under two models, one with and one without divisive normalization. Then, we performed our main statistical tests (i.e., comparing relative choice accuracy for trials with high- vs. low-value distractors and the random-effects logistic regression analysis with V1, V2, and V3/normV3) to see how often a true divisive normalization effect is detected (i.e., sensitivity) and how often the absence of a divisive normalization effect is suggested (i.e., specificity). The parameter values for the simulated models were as follows (see Methods): K = 100 and S = 0 for all (simulated) participants and models; $\sigma_H = 1$ and w = 1 for all participants of the model with divisive normalization; $\sigma_H = 11$ and w = 0 for all participants of the model without divisive normalization (σ_H was adjusted to compensate for the smaller denominator value of Equation 1 [see below] when w is set to 0, so that the two models featured similar overall choice accuracies); σ^2_{fixed} was determined for each participant and each model independently by drawing a value from a normal distribution with $M = 25$ and $SD = 20$ truncated to values ≥ 1 to achieve levels and variations of choice accuracy that were comparable to our empirical results. Supplementary Table 2 lists the results of these simulations. While the V3-based analyses achieve desirable levels of sensitivity and specificity (i.e., α-error below 5%, β-error below 20%), the normV3-based paired *t*-test between high- and low-value distractors has an unacceptably high false-positive rate of more than 27% (because of the above-mentioned confound of normV3 and choice difficulty).

Supplementary Table 2 *Sensitivity and specificity of distractor effect analyses on relative choice accuracy*

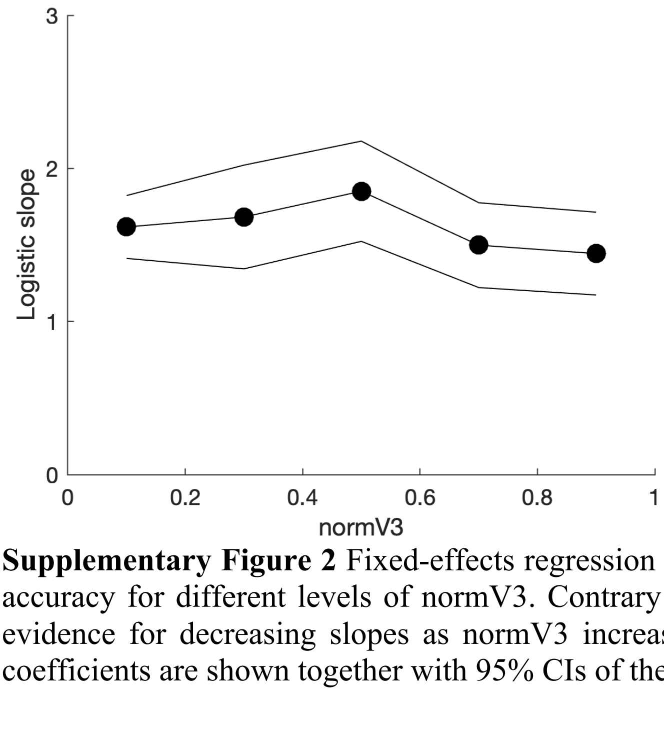
Test	Sensitivity	Specificity
Paired *t*-test: high vs. low V3	.852	.976
Paired *t*-test: high vs. low normV3	.982	.726
RFX regression: V1, V2, V3	.869	.967
RFX regression: V1, V2, normV3	.849	.966

Note. Sensitivity refers to the proportion of simulations that were generated under the assumption of divisive normalization being present for which the respective statistical test yielded a statistically significant effect (i.e., a true positive effect). Specificity refers to the proportion of simulations that were generated under the assumption of divisive normalization being not present for which the respective statistical test did not yield a statistically significant effect (i.e., a true negative effect). The group comparison between high and low normV3 provides unacceptably low specificity (i.e., more than 27% false positives). Details about the simulations can be found in the Supplementary Text. RFX = random effects analysis.

Further analyses requested by the first author of the original study.

The first author of the original study[2], Kenway Louie, asked us to perform a series of additional analyses. Most of these analyses are fixed-effects analyses and use normV3 rather than V3 to test for a normalization effect. For the reasons outlined above (i.e., normV3 is confounded by choice difficulty, which affects fixed-effects analyses in particular; V3 is more appropriate and provides good sensitivity and specificity), we consider these analyses problematic. But for the sake of completeness, we report them here. We restricted these analyses to the direct replication experiment (ID3 excluded). In short, none of these analyses provided evidence that would support the normalization model.

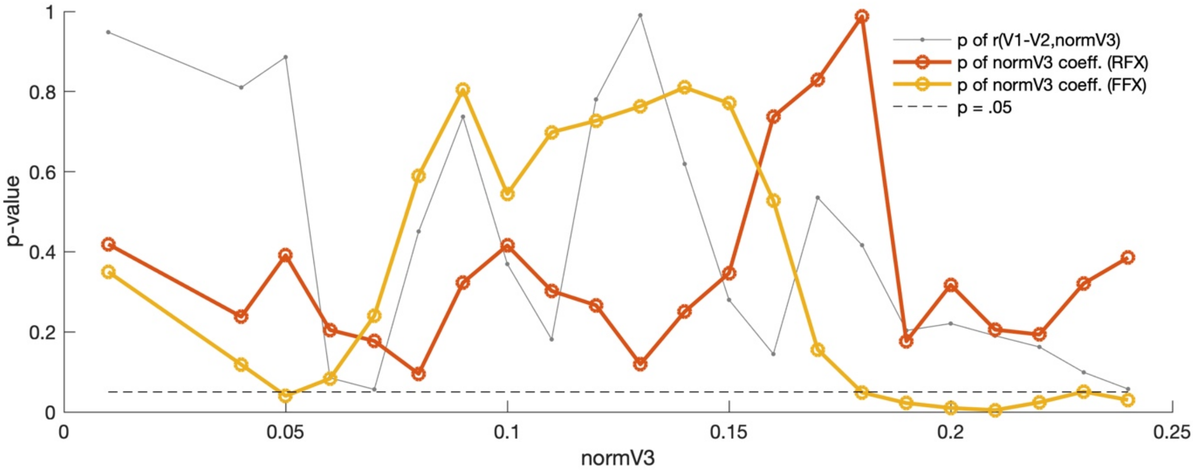
Logistic regressions for different ranges of normV3. This analysis is supposed to reproduce Figure 5C from Louie et al.[2]. Trials are sorted into 5 bins of normV3 values (i.e., from 0 to 1 in steps of .2), and in each bin relative choice accuracy is regressed onto the target value difference V1 – V2 (fixed-effects analyses). Louie and colleagues[2] showed that the regression coefficients (i.e., the "logistic slopes") decreased for increasing normV3 values (except for the last bin) in their dataset. In our data, there is no evidence for a dependency of the regression coefficients on normV3 (Supplementary Figure 2).



Supplementary Figure 2 Fixed-effects regression coefficients of V1 – V2 on relative choice accuracy for different levels of normV3. Contrary to Figure 5C in Louie et al.[2], there is no evidence for decreasing slopes as normV3 increases in our data. As in the original study, coefficients are shown together with 95% CIs of the parameter estimates.

Regression analysis with V1 – V2. As an alternative to the original regression analysis, the two separate predictors for V1 and V2 could be replaced by a single predictor for the difference V1 – V2. We performed this regression trying both V3 and normV3 and both random- and fixed effects analyses. None of them yielded a statistically significant effect that would be in line with the normalization account (V3, random-effects: $t(101) = -0.42$; $p = .672$; $d = -0.04$; normV3, random-effects: $t(101) = -0.89$; $p = .377$; $d = -0.09$; V3, fixed-effects: $\beta_{V3} = 0.01$; $p = .441$; normV3, fixed-effects: $\beta_{normV3} = -0.10$; $p = .071$).

Restricting analyses to values of normV3 that are not correlated with V1 – V2. One could try to restrict the analyses to ranges of normV3 for which there is no significantly negative correlation between normV3 and V1 – V2. This approach is problematic because even a non-significant negative correlation could bias the results. Furthermore, in our dataset, we had to restrict the analysis by including only trials with normV3 < 0.24. This led to the exclusion of at least 33 of 102 participants for the random-effects analysis and at least 68.5% of all trials for the fixed-effects analysis. We report the results of these analyses here nonetheless. Specifically, we conducted the random-effects and fixed-effects analyses (logistic regressions with V1, V2, and normV3) for all possible ranges from normV3 < .01 to normV3 < 1 for which V1 – V2 and normV3 were not correlated in the fixed-effects dataset (as stated above, the highest possible cutoff was normV3 < 0.24). With respect to the random-effects analysis, all of these analyses did not yield a statistically significant effect of normV3 (Supplementary Figure 3). With respect to the fixed-effects analysis, the majority of these analyses (i.e., 15 of 22) did not yield a statistically significant normV3 effect (Supplementary Figure 3).



Supplementary Figure 3 Random-effects (RFX) and fixed-effects (FFX) regression analyses for ranges of normV3 without a significantly negative correlation with V1 – V2. The gray line shows the p-value of the correlation between the restricted normV3 range and V1 – V2. The orange and yellow lines show the p-value of the normV3 effect for the RFX and FFX analyses, respectively.

Alternative specification of normV3. One could redefine the variable normV3 as V3 divided by the mean of all options (of all trials of a participant). This avoids the confound with choice difficulty (which arises from the trial-specific division of V3 by 0.5*(V1+V2) in the original specification of normV3). Performing the fixed-effects analysis with this new specification of normV3, however, did not yield a statistically significant distractor effect (β_{normV3} = -0.05; p = .249). Note that the random-effects analysis is unaffected by this modification, because dividing V3 by a participant-wise constant only changes the absolute values of the regression coefficient but not the inferential statistics.

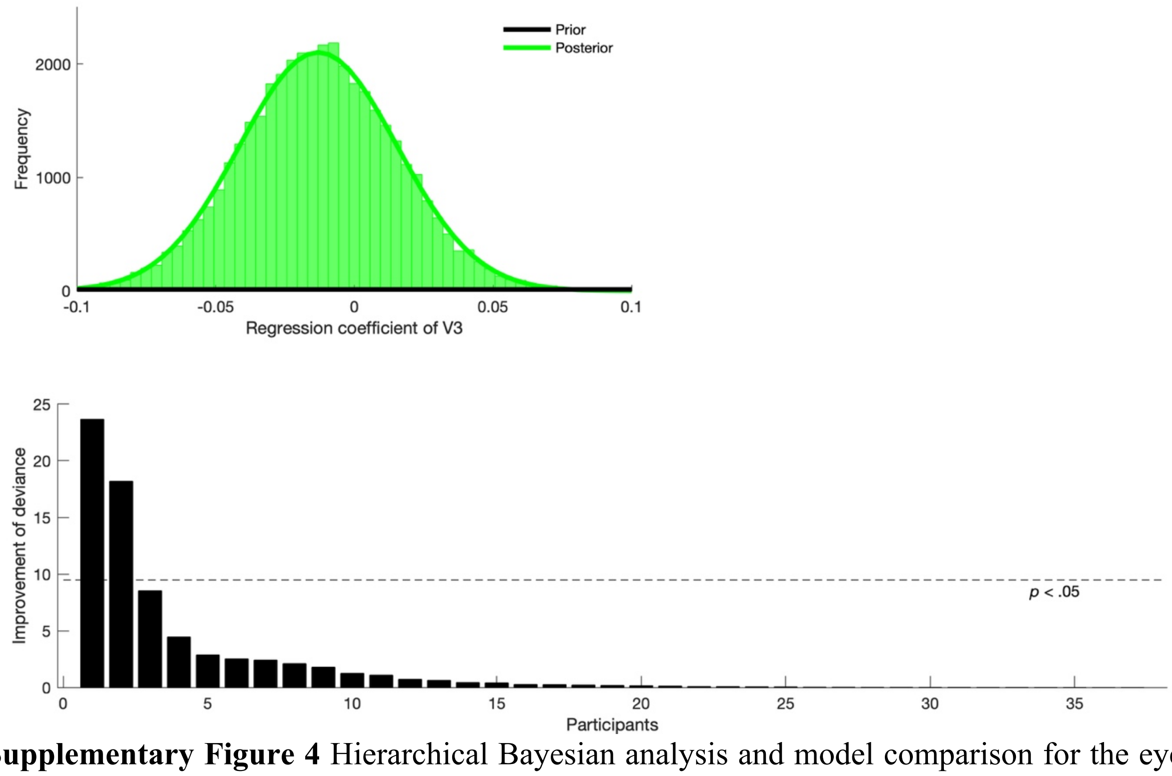
Definition of high- and low-value distractors based on group medians. As explained in the Methods (and the pre-registration protocol), we separated high- and low-value distractors by a participant-wise median-split based on V3, as this ensures equal trial numbers with high- and low-value distractors in every participant. In Louie et al.[2], the median-split was based on the group median of normV3. This approach is problematic for the reasons outlined above (i.e., confound with difficulty), but also because it results in an unequal number of trials of low- and high-value distractors in every participant. In fact, some participants must even be excluded from the analysis, because all of their normV3 values lie either entirely above or entirely below the group median (in our case, 17 of 102 participants had to be excluded). We performed the comparison of relative choice accuracy between high- and low-value distractors based on this median-split nonetheless. There was no statistically significant distractor effect (high V3: M = 70.8%, SD = 11.9; low V3: M = 70.2%, SD = 11.6; $t(84)$ = 1.07; p = .289; d = 0.12). Kenway Louie also suggested to use a trial-independent definition of normV3 for this analysis, namely by dividing V3 by the minimum target value (i.e., lowest V2) per participant. Using this alternative definition of normV3 to specify the group median and to split the data accordingly let to the exclusion of only 5 of 102 participants. However, there was again no statistically significant distractor effect (high V3: M = 70.3%, SD = 12.5; low V3: M = 70.4%, SD = 12.3; $t(96)$ = -0.29; p = .769; d = -0.03).

Analyses of the dataset of Krajbich & Rangel (2011).

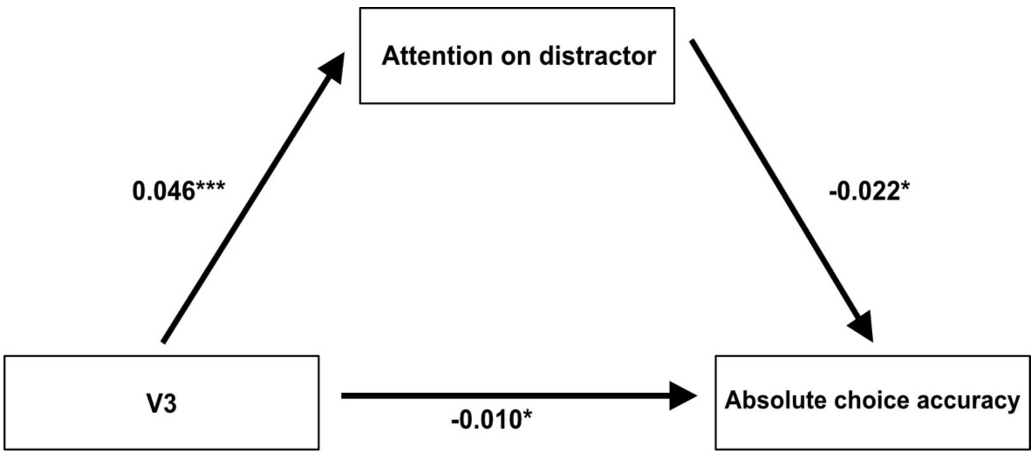
This dataset was examined for effects of distractor value on relative choice accuracy and on RT, and for early and late distractor effects. Most of these analyses were equivalent to those conducted with our own datasets. As an additional test for late effects of value-based attention, a random-effects logistic regression analysis was conducted regressing whether a fixation was on an option onto the option's value, the within-trial fixation number, and the interaction between value and fixation number. Instead of excluding the last fixation of a trial, we added a fourth predictor variable that was set to 1 if the current fixation was the last fixation and made on the chosen option, and to 0 otherwise. Because the dataset contained fewer trials per participant (100 instead of 300), the criterion for including fixation numbers was lowered from ≥ 30 to ≥ 10 recorded fixations per fixation number per participant.

Additional analyses of the eye-tracking experiment.

Here, we provide additional analyses of the eye-tracking experiment, including the hierarchical Bayesian regression analyses (Supplementary Figure 4, upper panel), the model comparison between the probit and the divisive normalization model (Supplementary Figure 4, lower panel), and the path analysis of a mediation the influence of distractor value (V3) on absolute (rather than relative) choice accuracy on via attention (Supplementary Figure 5). Absolute choice accuracy refers to the probability of choosing the best option when including all decisions (i.e., also choices of the distractor). A dependency of absolute choice accuracy on V3 does not constitute a violation of IIA and does not provide evidence for divisive normalization. In the legend of Supplementary Figure 4, we also mention the Bayes Factors in favor of the null hypothesis when performing the hierarchical Bayesian regression analysis with normV3 rather than with V3.



Supplementary Figure 4 Hierarchical Bayesian analysis and model comparison for the eye-tracking experiment. The upper panel shows the prior and posterior distributions of the V3 coefficient according to a hierarchical Bayesian logistic regression (analogous to Fig. 2d). Similar to the direct replication, the posterior density at 0 is very high. The Bayes Factor in favor of the null hypothesis is 158, which is seen as *very strong* evidence[3]. Note that we also conducted the hierarchical Bayesian logistic regression with normV3 instead of V3 as predictor variable. The Bayes Factors in favor of the null hypothesis were 69 / 67 (with / without ID3 included) for the direct replication and 40 for the eye-tracking experiment, all of which are seen as *strong* evidence[3]. The lower panel shows the individual improvements of model fit for the more complex divisive normalization model compared to the probit model in the eye-tracking experiment. Similar to the direct replication, there is only a minority of participants (i.e., 2 out of 37) whose decisions are explained significantly better by the divisive normalization model, and the increased complexity is not justified by the small improvement of fit at the group level ($\chi^2(148) = 73$; $p > .999$).



Supplementary Figure 5 A mediating role of attention on absolute (but not relative) choice accuracy. Here, we adapted the path analysis that tested a mediating role of attention (on distractors) on decision making (see Fig. 4a in the main text) by including also those trials in which the distractor was chosen. In this analysis, relative choice accuracy is replaced by absolute choice accuracy, which is the probability to choose the best option in all trials[4]. In this analysis, the negative influence of V3 on choice accuracy is indeed partially mediated by attention. More specifically, there is a positive effect of V3 on distractor attention ($t(36) = 3.66$, $p < .001$, $d = 0.60$), indicating that participants looked more on high- compared to low-value distractors, and there is a negative effect of distractor attention on absolute choice accuracy ($t(36) = -2.58$, $p = .014$, $d = -0.42$), indicating that distractors were chosen more often if they were fixated. In addition, there was also a direct effect of V3 on absolute choice accuracy ($t(36) = -2.42$, $p = .021$, $d = -0.40$). These results are consistent with findings from our previous work (see Fig. 4B in Gluth et al.[4]).

Computational modeling I: comparison of probit and divisive normalization.

We conducted a nested model comparison between the divisive normalization model, the probit model[5], and a baseline model that assumed random choices. Essentially, the normalization model is a generalization of the probit model, because both models assume normally distributed errors. According to the normalization model, decisions are based on the normalized neural firing rate μ_i of value V_i:

$$\mu_i = \mathrm{K} * \frac{V_i}{\sigma_\mathrm{H} + \sum_j^n \mathrm{w} * V_j}, \quad (1)$$

where n refers to the number of options in the choice set and K, σ_H, and w are free parameters of the model representing gain, semisaturation, and a weight term, respectively[2,6,7]. Firing rates are also affected by two sources of (Gaussian) noise, the fixed noise term with mean 0 and variance $\sigma^2_{\mathrm{fixed}}$ and a firing-rate dependent noise term with mean 0 and variance $\mathrm{S} * \mu_i$. Thus, σ_{fixed} and S are the fourth and fifth free parameters of the model. The probability p_i to choose option i is thus given by the probability that the firing rate μ_i is higher than the firing rates μ_j of the alternatives j. For decisions between more than two options, no closed-form solution for the normalization model (or the probit model) exists, but p_i can be written as the integral:

$$p_i = \int_{-\infty}^{\infty} p(\mu_i = x) * \prod_{j \neq i}^{n-1} p(\mu_j < x) \; dx , \quad (2)$$

where $p(\mu_i = x)$ refers to the probability that the firing rate of option i has value x, n refers to the number of choice options, and $p(\mu_j < x)$ refers to the probability that the firing rate of alternative option j is smaller than x. The two p-terms in Equation 2 are given by:

$$p(\mu_i = x) = \phi\left(\frac{x - \mu_i}{\sqrt{\mathrm{S}\mu_i + \sigma^2_{fixed}}}\right), \quad (3)$$

and

$$p(\mu_j < x) = \Phi\left(\frac{x-\mu_j}{\sqrt{S\mu_j+\sigma_{fixed}^2}}\right), \tag{4}$$

where ϕ and Φ refer to the standard normal and cumulative standard normal distribution, respectively. The divisive normalization model is reduced to the probit model by fixing w and S to 0, and fixing K and σ_H to any positive value (following Louie et al.[2], we set K = 100 and $\sigma_H = 50$), so that σ_{fixed} remains as a free parameter. The probit model is reduced to the baseline model by setting $\sigma_{fixed} = \infty$.

For parameter estimation, we used a maximum likelihood estimation approach. For each participant, the probit model was estimated first, starting with a grid search with 10 steps for σ_{fixed} (equally spaced between 0.1 and 18.1), followed by 10 runs of a constrained simplex minimization as implement in the Matlab function *fminsearchcon.m* (1 run used the best parameter value from grid search as initial value, 3 runs used the second-to-fourth best grid search values, 6 runs picked initial values from all grid search values randomly). Parameter σ_{fixed} was constrained to values between $2.22*10^{-16}$ and 30. The best fit and the associated parameter value were saved. Then, the divisive normalization model was estimated in a similar way (i.e., grid search with 10 steps per parameter [K and σ_H: 1 to 181; w and S: 0 to 1.8; σ_{fixed} as for probit]; 10 runs of simplex minimization, with the last run using the parameter set of the estimated probit model as initial values; constraints: $0 \leq K \leq 200$, $2.22*10^{-16} \leq \sigma_H \leq 200$, $0 \leq w, S \leq 10$, σ_{fixed} as for probit). The deviance for each model and participant was calculated as two times the negative log-likelihood of the model, summed over participants, and subjected to a likelihood ratio test for nested model comparison[8]. The integral of Equation 2 was approximated using numerical integration as implemented in the Matlab function *integrate.m*. Choice probabilities were cut at $.01 \leq p \leq .99$. Estimated parameter values of the two models are provided in Supplementary Table 3.

Supplementary Table 3 *Parameter values for probit and divisive normalization models*

Model	Parameters				
	K	σ_H	w	S	σ^2_{fixed}
Direct replication experiment					
Probit	100 (fixed)	50 (fixed)	0 (fixed)	0 (fixed)	1.53 (2.47)
Normalization	82.4 (60.9)	88.1 (83.9)	1.91 (3.08)	0.13 (0.32)	0.36 (0.80)
Eye-tracking experiment					
Probit	100 (fixed)	50 (fixed)	0 (fixed)	0 (fixed)	1.27 (0.85)
Normalization	71.4 (60.3)	92.4 (72.4)	1.67 (3.12)	0.05 (0.09)	0.31 (0.42)

Note. Average parameter estimates are shown together with standard deviations in parentheses.

Computational modeling II: simulations of the extended aDDM.

The multi-alternative version of the aDDM is introduced and explained in Krajbich and Rangel[9]. Here, we reiterate only its core assumptions and specifications. The model assumes that at every time step t (Δt = 1 ms) evidence for an option i is accumulated according to the following equation:

$$E_{i,t} = E_{i,t-1} + d * R_i * \left(\theta + [1 - \theta] * F_{i,t}\right) + \varepsilon_{i,t} \ , \tag{5}$$

where d is a free parameter controlling the speed of accumulation, R_i is the input value (here, the average BDM-auction bid or rating from the first task in each experiment, which we multiplied by 2 to match the range used in previous studies[9,10]), θ is a free parameter controlling the reduction of evidence accumulation for non-fixated options, $F_{i,t}$ is an index variable indicating whether option i is currently fixated ($F_{i,t}$ = 1) or not ($F_{i,t}$ = 0), and $\varepsilon_{i,t}$ is Gaussian noise with mean 0 and standard deviation σ. Decisions are not determined directly by $E_{i,t}$ but by a comparison of each accumulator with the best alternative accumulator:

$$V_{i,t} = E_{i,t} - \max\left(E_{j,t}\right) , \qquad i \neq i. \tag{6}$$

As soon as any relative decision value $V_{i,t}$ reaches a fixed threshold with value 1, the respective option is chosen at time point t. The durations of fixations were drawn from a log-normal distribution with mean 5.1835 and standard deviation 0.53293 (these parameter values provided an acceptable fit to our empirical fixation durations for first and middle fixations). Before the first and between each fixation, we modeled a fixation gap of 60 ms (which is roughly the median delay between consecutive fixations in our data). During this gap, the accumulation process starts / proceeds without a fixation bias by setting $F_{i,t}$ for every option to 0 and replacing θ by:

$$\theta_{gap} = \frac{(1+[n-1]*\theta)}{n} \ . \tag{7}$$

Note that this specification ensures the same sum of attention weights across options during gaps and fixations, so that the overall rate of accumulation does not change (i.e., during gap:

$3*\theta_{gap}$; during fixations: $2*\theta+1$; e.g., if $\theta = 0.4$, then $\theta_{gap} = 0.6$, so that $3*\theta_{gap} = 1.8$ and $2*\theta+1 = 1.8$). In the extended aDDM, the probability to fixate an option is a function of the option's accumulated value. Following our previous work[4], we used a logistic function to describe this relationship:

$$p(fixate\ i) = \frac{\exp\left(\gamma * [E_{i,t} - \min(E_{j,t})]\right)}{\sum_j \exp\left(\gamma * [E_{j,t} - \min(E_{j,t})]\right)}\ , \qquad (8)$$

where γ is a free parameter controlling the influence of value-based attention on fixation probabilities. Finally, the model includes a non-decision time parameter to model sensory encoding and motor execution time.

Our goal was to show that this extended aDDM is able to capture the qualitative patterns seen in our choice, RT and eye-tracking data. Therefore, we simulated the model using the average ratings and choice sets of our eye-tracking sample as input values and trials, respectively, and searched for an "optimal" set of the parameters d, θ, σ, and γ with a grid search of 9 steps per parameter (equally spaced steps for d from 0.00005 to 0.00045, for θ from 0.1 to 0.9, for σ from 0.01 to 0.018, for γ from 0 to 0.8; the non-decision time was always assumed to be 300 ms). We then checked how many empirical data patterns could be predicted by each model simulation and identified the best parameter set by taking the simulation with the highest score. More specifically, the score depended on whether the model prediction fell into the 95% CIs of the following features of the group data: choice proportions of the best, second-best and distractor option, mean and median RT, mean and median number of fixations per trial, the means of the first ten within-trial fixations for best, second-best and distractor option (see Fig. 5e in the main text and Supplementary Figures 6–10). Matches of choice proportions and RT patterns were counted twice to achieve a more balanced weighting between scoring behavioral and eye-tracking predictions. We note that this scoring system and the chosen set of summary statistics is somewhat arbitrary. Future research should attempt to provide a more principled method of estimating parameters based on combined behavioral and eye-tracking data on the single-trial level, for instance, by adopting the joint modeling approach[11]. For the purpose of the present work, which is concerned with the model's ability to predict the qualitative patterns seen in our data, we consider this scoring system being sufficient. After identifying the best parameter set for our model, the model was simulated 100 times with the best parameter set to obtain average effect sizes of the model's predictions (see Results and Fig. 5 in the main text).

We compared our proposed extension of the aDDM with five other variants. The first of these variants was the aDDM with random fixations. The best parameter set for this model was obtained by searching within the above-described grid search for the best match among those simulations that assumed $\gamma = 0$. The second variant was an aDDM with random fixations and (a simplified form of) divisive normalization of input values. For this model, the grid search was performed with parameter ν instead of γ (which was set to 0) ranging from 0 to 40 in 9 equally spaced steps (parameter ν represents the semisaturation parameter in the divisive normalization model[2]). The input values R_i where then normalized as follows:

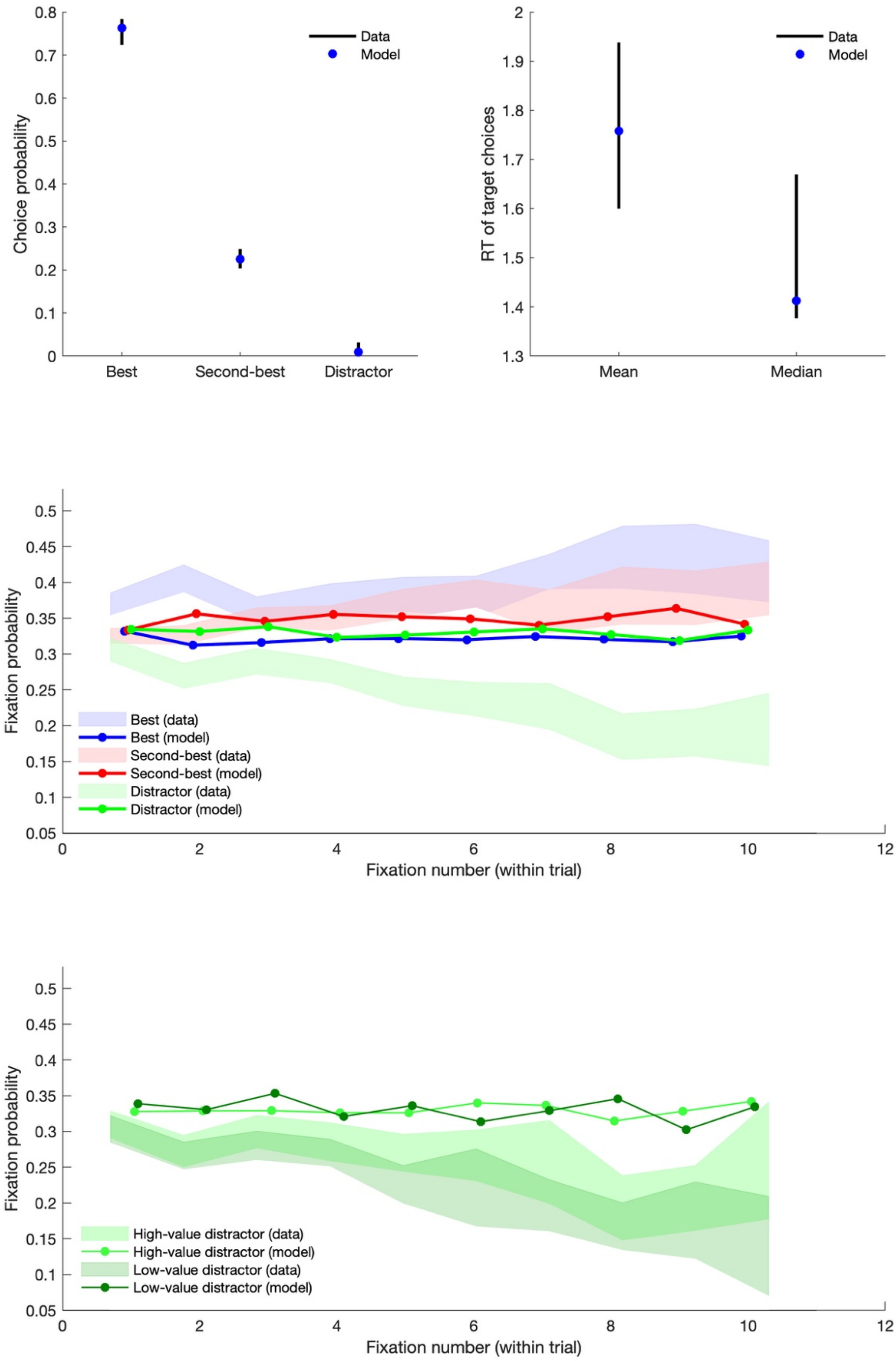
$$normR_i = \frac{R_i}{\nu + \sum_j R_j} \quad . \tag{9}$$

The variable $normR_i$ was then rescaled to values between 0 and 10 to match the range of previous studies[9,10] and replaced R_i in Equation 5. The third variant was the aDDM with the empirical transition patterns reported in Krajbich and Rangel[9]. Notably, these patterns included a dependency of late fixations on whether an option was best, second-best or worst, but no mechanistic explanation of these effects were provided and no further (value-based) distinction within option types was made. The grid search for this variant included only the parameters d, θ, and σ. The fourth variant assumed an alternative implementation of value-based attention, which was not based on accumulated values but on input values by replacing the term $E_{i,t} - \min(E_{j,t})$ by R_i in Equation 8. Notably, such a form of value-based attention has been proposed in earlier work, including our own[4,12]. The grid search for this model was conducted with γ ranging from 0 to 0.4 in 9 equally spaced steps. The last variant, which was briefly outlined in the preregistration protocol, assumed the presence of a lower "reject" threshold (and random fixations). According to this model, if the relative decision value V_i of an option i decreases so much that it reaches this reject threshold (before any other option reaches the upper "choice" threshold), the option is excluded from the choice set and the decision is made between the two remaining options. The grid search for this model replaced parameter γ (which was set to 0) by a reject threshold parameter ρ with values ranging from -1.75 to -0.25 in 9 equally spaced steps. The predictions of the five alternative aDDM variants with their best parameter sets are provided in Supplementary Figures 6–10. Best parameter values of all models are provided in Supplementary Table 4.

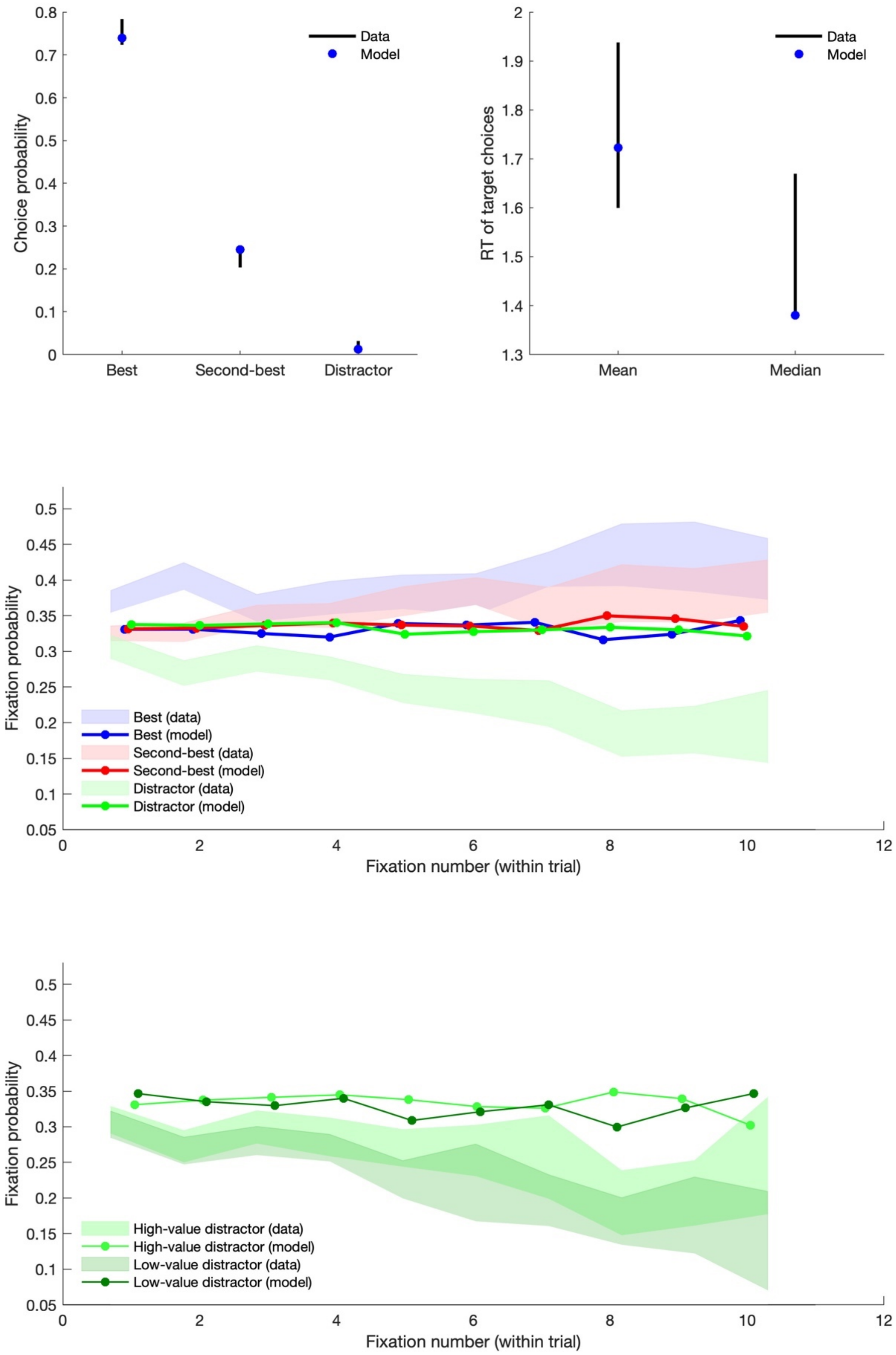
Supplementary Table 4 *Parameter values for aDDM variants*

aDDM variant	Parameters					
	d	θ	σ	γ	ν	ρ
Value-based attention I	0.00035	0.9	0.017	0.3	-	-
Random fixations	0.0004	0.8	0.017	-	-	-
Divisive normalization	0.00045	0.7	0.016	-	40	-
Empirical fixation transitions	0.00035	0.9	0.018	-	-	-
Value-based attention II	0.00035	0.8	0.017	0.1	-	-
Reject threshold	0.00025	0.3	0.018	-	-	-1.1875

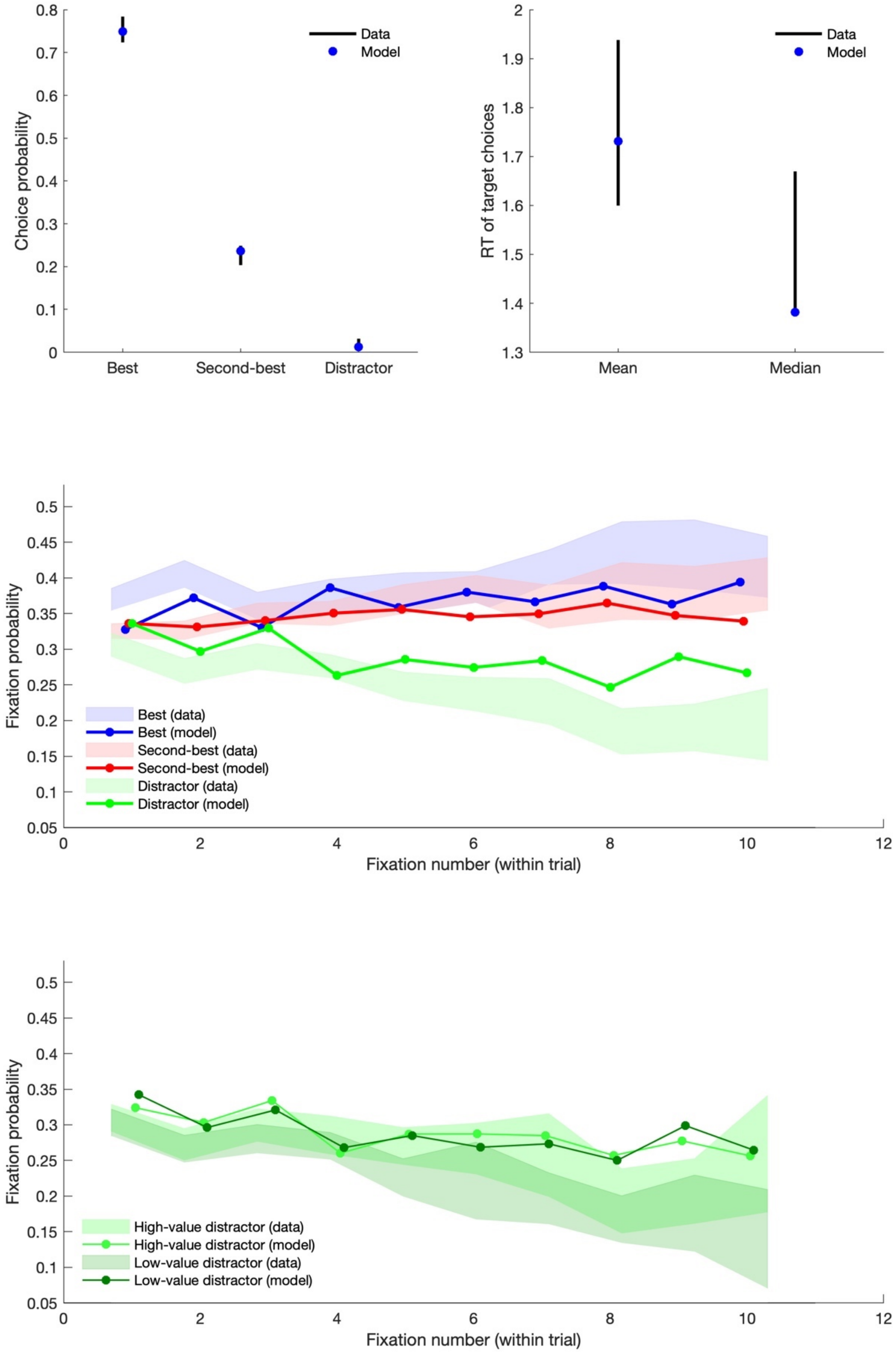
Note. For each aDDM variant, the set of parameter values are reported that provided the best match with the qualitative choice, RT, and eye-tracking patterns seen in our data. Value-based attention I refers to our proposed mechanism that relies on accumulated values, value-based attention II refers to a mechanism that relies on input values.



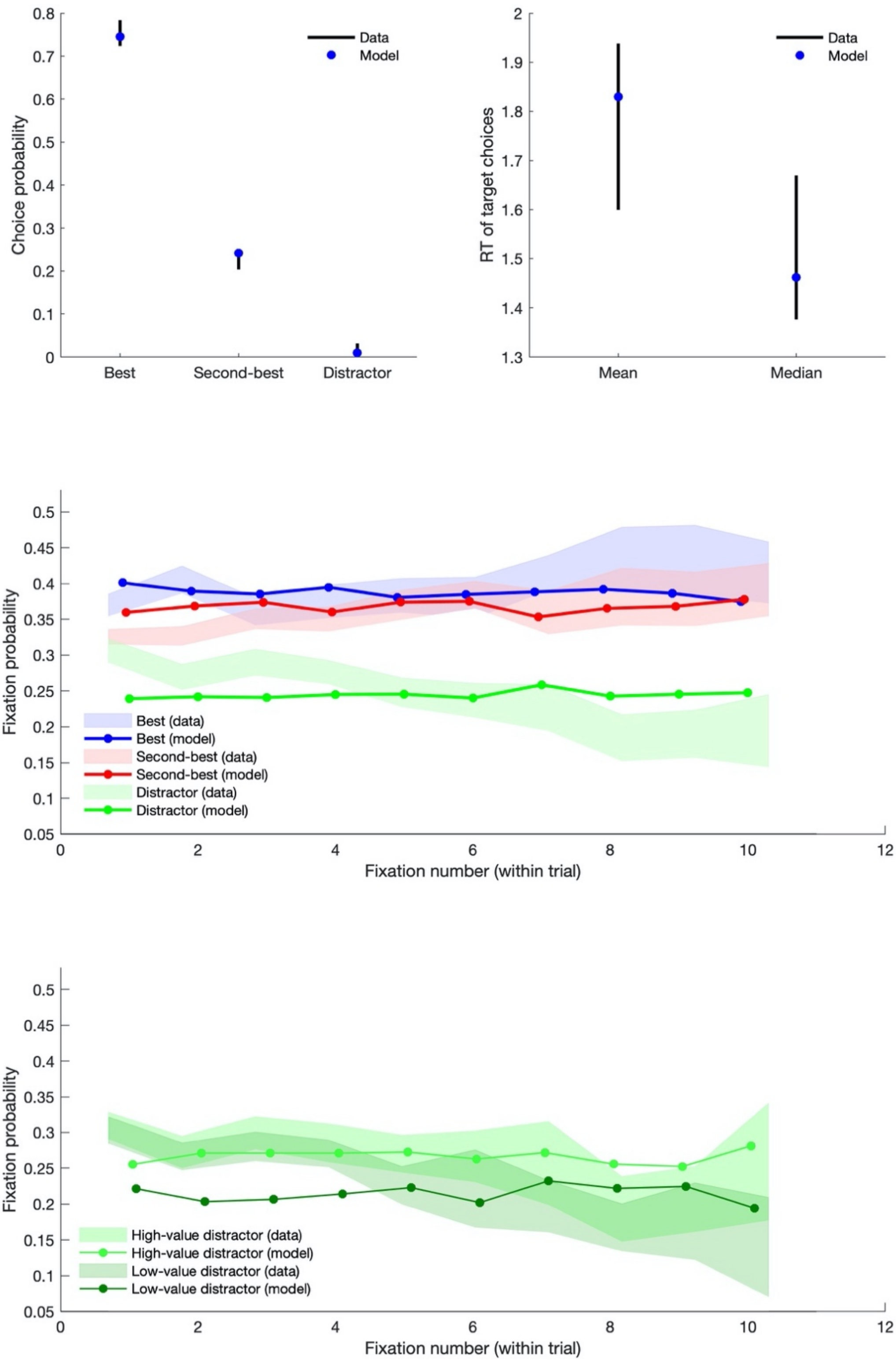
Supplementary Figure 6 Predictions of the aDDM with random fixations. Model predictions refer to simulations with the parameter set that provided the closest fit to the data according to our "qualitative scoring" (see Methods). The model is compared to the data (for which the 95% CIs are shown) with respect to choice probabilities of all three options (upper left panel), mean and median RT of target choices (upper right panel), within-trial fixation patterns of all three options (middle panel) and of high- and low-value distractors (lower panel). The aDDM with random fixations captures the choice and RT data but not the fixation patterns.



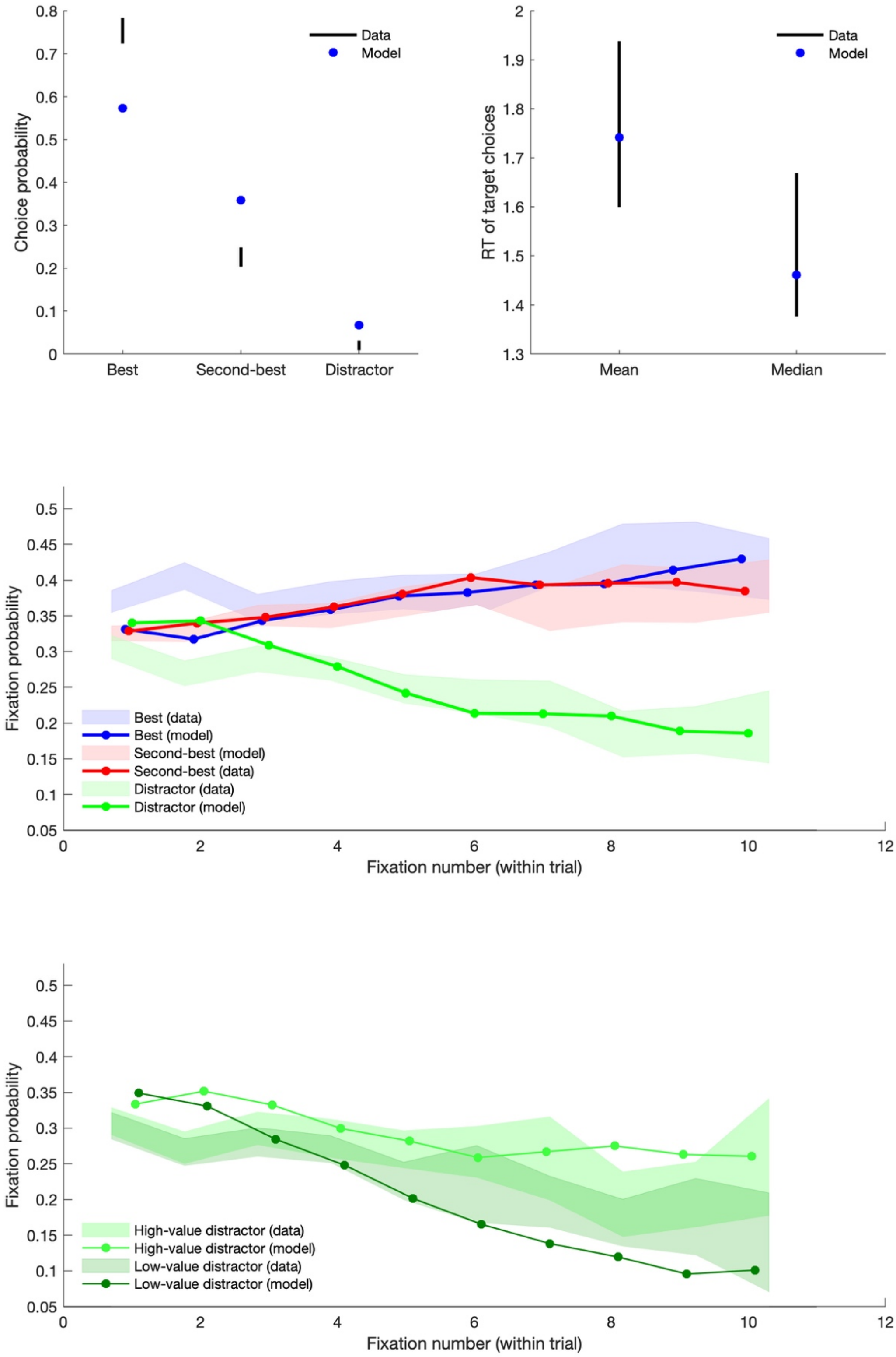
Supplementary Figure 7 Predictions of the aDDM with divisive normalization. This model assumes random fixations but divisive normalization of input values to the accumulator. The model predictions are equivalent to the aDDM with random fixations but without normalization (i.e., choice and RT data are captured well, fixation patterns are not captured).



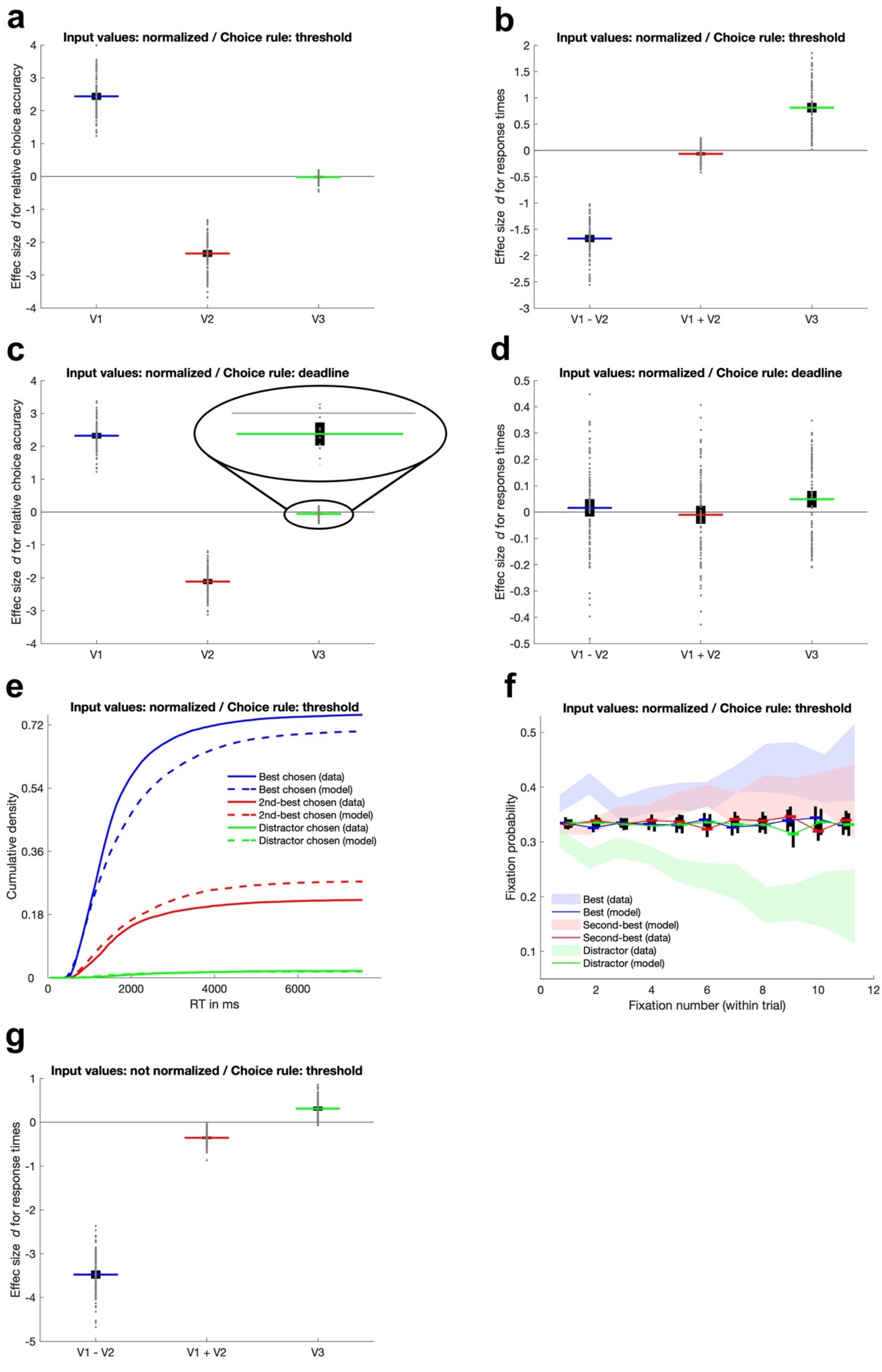
Supplementary Figure 8 Predictions of the aDDM with empirical fixation transitions. This aDDM variant samples fixations according to the empirical transitions patterns reported in Krajbich and Rangel[9]. For late fixations, these transition probabilities differed between best, second-best and worst options. Thus, the model captures the development of fixations in our data to some extent. However, it does not predict a different development for high- compared to low-value distractors, because it lacks a mechanistic link between value and fixation probability.



Supplementary Figure 9 Predictions of the aDDM with modulation of attention by input values. Inspired by previous work including our own MIVAC model[4,12], this aDDM variant assumes that the probability to fixate options is a function of the options' input values (i.e., the average values taken from the rating task). This model (over-)predicts a dependency of the first fixation on value. Most importantly, the model is unable to predict the decline of distractor fixations over time, because its mechanism is not dynamic (i.e., does not depend on accumulated value).



Supplementary Figure 10 Predictions of the aDDM with a reject threshold. This variant of the aDDM assumes random fixations and a reject threshold. If the relative decision value (see Fig. 5a) of an option decreases so much that it reaches a lower reject threshold, then the option is excluded and the decision is made between the two remaining options. This model is able to predict the decline of distractor fixations over time and the steeper decline of low- compared to high-value distractors. However, the model fails to capture the choice proportions, and the predicted relationship between value and fixations emerges too late (i.e., after the first two fixations).



Supplementary Figure 11 Predictions of a DDM with normalized input values and two different choice-rule strategies. Similar to the model presented in Supplementary Figure 7, the results shown here refer to an aDDM variant that uses normalized values as input to the accumulators. However, the influence of attention on evidence accumulation is taken out (by setting $\theta = 1$; see Equation 5), thus reducing the aDDM to a DDM. Furthermore, the influence of divisive normalization is maximized (by setting $\nu = 0$; see Equation 9). The model is then simulated under two different choice-rule strategies, once under the assumption that a decision is made as soon as one accumulator reaches a threshold (of 1), and once under the assumption that a decision is made after a pre-determined RT deadline (at which the option with the highest accumulator value is chosen). The deadline in each trial is drawn from the actual RT values of our eye-tracking experiment. **a-b** Under the assumption of a "threshold" choice rule, the model predicts a distractor effect on RT (**b**) but not on choices (**a**). **c-d** Under the assumption of a "deadline" choice rule, the model predicts a distractor effect on choices (**c**) but not on RT (**d**). Note that the predicted distractor effect on choices is very small (see the zoomed-in insertion; average effect size d = -0.06, 95% CI = [-0.09, -0.03]). Hence, this model allows to reconcile the different findings of the original study (which reported a distractor effect on choices) and the current study (which reported no distractor effect on choices but on RT) under the assumption that participants of the two different studies used two different choice-rule strategies. However, the model makes several other predictions that are not in line with the data. Under the assumption of a "deadline" choice rule, the model does not predict an influence of target-value difference (i.e., difficulty) on RT (**d**), which is at odds with the robustness of this effect in the current study and many other studies on decision making[13] (but we do not know whether this effect was present in the original study[2], which did not report RT effects). Under the assumption of a "threshold" choice rule, the model does not provide a sufficient account of the empirical cumulative RT distributions of the three choice options (**e**) and of the eye-tracking results (**f**). Finally, the assumption of normalization of input values is not necessary to account for the distractor effect on RT seen in the current study. This becomes evident when simulating the model with input values that are not subjected to divisive normalization (**g**). In this case, the model still predicts a positive effect of distractor value on RT. In other words, the prediction that high-value distractors slow down responses emerges naturally from the sequential sampling framework (at least when implemented as a DDM or an aDDM) and does not provide evidence for divisive normalization. In all panels, gray dots show the effect sizes for each simulation (we performed 100 simulations using the dataset of the eye-tracking experiment), colored lines and black error bars show the overall means and their 95% CIs, respectively. Shaded areas in **f** represent the 95% CIs of the data.

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