
A quantum dot in germanium proximitized by a superconductor

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Introduction

In the Supplementary Information we present additional data as well as a minimal model for our hybrid system. In section I, we present details on the experimental setup, describing both DC and RF measurements. In section II, we present data in a low regime of coupling between the superconductor (SC) and quantum dot (QD) that showcases the tunability of our system and complements the results presented in the main article. We also present similar results measured using radiofrequency (RF) reflectometry, a technique ubiquitous for spin-based qubits, crucial to develop readout for hybrid superconductor-semiconductor devices incorporating quantum dots, such as Kitaev chains. Section III describes critical field measurements for different orientations of the applied magnetic field. In Section IV we present data on PtSiGe Hall bars and wires providing an insight into the SC properties of PtSiGe films. Finally, Section V presents a qualitative model of the observed spectrum of our proximitized quantum dot.

Section I. Details of Experimental Setup

In this section we describe the experimental setup used for both the low-frequency (lock-in) measurements in the main text, and the RF-reflectometry setup used for the additional data presented in the supplement (Section II).

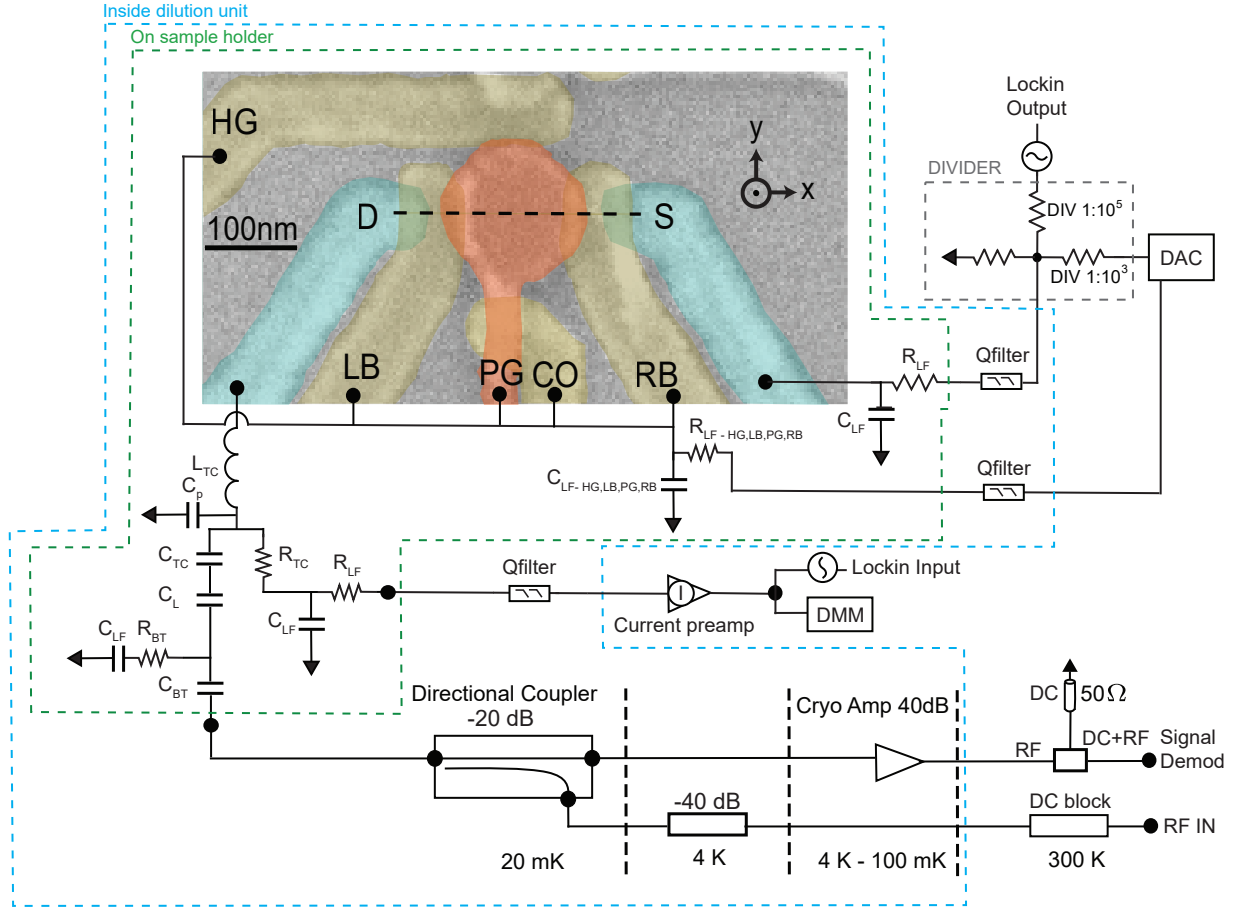


FIG. S1. False coloured SEM of a nominally identical device, including the experimental setup used for DC, differential conductance and RF measurements.

All measurements in Figures 1-4 in the main text are performed using a two-terminal measurement setup, measuring a DC current and/or the differential conductance G . For two-terminal measurements in which a DC current is measured, we apply a voltage on the source (S) via a QDevil QDAC-II with 20-25 bit resolution, and a ± 10 V output voltage. This voltage passes through a 1:1000 voltage divider on the DAC output. The voltage is applied to a DC-loop via a QFilter-II RC low-pass filter ($f_{co}=65$ kHz) and LC low-pass filter ($f_{co}=225$ MHz), followed by a QDevil

sampleboard incorporating further RC low-pass filtering ($f_{co} = 130$ kHz, $R_{LF} = 1.2$ k Ω , $C_{LF} = 1$ nF). The wirebond connected to the drain contact connects to the PCB and branches into an RF-reflectometry setup, which is described below, and a DC readout line. The DC readout line on the drain (D) then passes through the same filtering setup as described above. Outside the cryostat, the current is converted to a voltage via a Basel SP983c current pre-amplifier set to a gain of 1×10^{-7} V/A before being digitized by a Keysight 34461A digital multimeter. The total line resistance of the sample board, QFilter-II, and DC looms amount to a line resistance of $R_{line} \sim 5.80 \pm 0.03$ k Ω .

The differential conductance measurements are performed by applying an AC excitation via a Stanford Research Systems SR860 lock-in amplifier. This excitation is combined with the DC signal via a 1:10000 voltage divider which otherwise undergoes the same filtering and amplification as the DC current. We furthermore subtract the line resistance of ~ 5.8 k Ω from each point on the V_{SD} axis, $V_{SD} = V_{DAC} - (I_{DC} \cdot R_{line})$, where V_{DAC} is the voltage being output from the DAC, I_{DC} is the measured current, and R_{line} the line resistance. We optimize and measure the in-phase component and convert the signal to units of quantum of conductance by $G = G_0^{-1} \cdot (\frac{V_{AC}}{X \cdot \text{Gain}} - R_{Line})^{-1}$, with $V_{AC} = 2.5$ μ V for all figures, except Figure 2a ($V_{AC} = 10$ μ V), and Figure 4 ($V_{AC} = 3.2$ μ V), X being the measured in-phase signal in units of volts, the Gain being 1×10^{-7} V/A, and $G_0 = 7.748091719 \times 10^{-5}$ S.

Furthermore, we incorporate high-frequency measurement via standard RF-reflectometry methods [1] using a cryogenic tank circuit with a frequency of 192.3 MHz (as seen in the measurements in the following section, in Figure S3). A typical RF excitation voltage at the sample is 3 μ V, which is applied and demodulated using a Quantum Machines OPX+. The RF-reflectometry setup has a bias-tee with resistance $R_{BT} = 50$ k Ω , capacitance $C_{BT} = 22$ nF, and an inductance $L_{TC} = 560$ nH. C_P is a parasitic capacitance that together with L_{TC} forms the resonant (tank) circuit, estimated to have a value of about 1.2 pF. The capacitances $C_{TC} = 100$ pF and $C_L = 100$ pF are coupling capacitances.

Section II. Low Γ_S B-field Studies in Transport and RF-Reflectometry

We study the magnetic field dependence of the subgap states in a regime of low coupling between SC and QD, complementary to the strong-coupling regime in the main text.

Setting the tunnel barriers to $V_{RB} = -1.39$ V and $V_{LB} = -0.8350$ V, we tune the QD into a many-hole regime using the plunger gate PG. We perform bias spectroscopy at 0 mT, observing filling of the QD with eye-like features characteristic of YSR states as described in the main text, and shown in Figure S2a. In Figure S2b the out-of-plane magnetic field is set to 100 mT, revealing lifting of what we believe is subgap spin degeneracy of the $|D\rangle$ groundstate, as indicated by the black arrows.

In Figure S2c the splitting is increased as shown by the black arrows, when we apply a field of 200 mT. We then configure the PG to $V_{PG} = -2.2365$ V at the subgap splitting in Figure S2c (orange notch), and vary the out-of-plane magnetic field as seen in Figure S2d. A Zeeman splitting is measured in bias spectroscopy starting from 0 mT. Extracting the g-factor using the standard Zeeman formula yields a $g_{\perp} = 4.5 \pm 0.6$, consistent with previous experimental findings of large g-factors in planar Ge/SiGe, and ~ 3 times larger than the g-factor we found for the strongly hybridized system described in the main text in this study. As the magnetic field is rotated along the principal axis at a perpendicular field of 200 mT a strong anisotropy is seen, in agreement with previous studies carried out in planar Ge/SiGe.

We proceed to tune the QD to a few-hole regime and perform bias spectroscopy using standard RF-reflectometry methods as described above. Note that the Figures in S3 show the normalized demodulated signal V_{RF} .

Figures S3c-e depict Coulomb diamonds measured at magnetic field strengths of 0 mT, 750 mT and 990 mT respectively, in the few-hole, lower-coupling regime described in this Supplementary Information section. Suppression of transport exists in an energy window of gap of $\pm 2\Delta_0$, as well as Coulomb diamonds with charging energies exceeding 2 mV. The x-axis is labeled with a virtual parameter δV_{PG} , where $\delta V_{PG} = 0$ corresponds to $V_{PG} = -2$ V sweeping 15 mV above and below. Figures S3c-e depict bias spectroscopy from which we extract $g_{\perp}$ outside the superconducting gap. Figures S3f-h correspond to zoomed-in plots of figures S3c-e, revealing a magnetic field dependent splitting. The extracted g-factor is $g = 5.3 \pm 0.8$, in agreement within error with the subgap spin splitting that we measured using low-frequency transport techniques, in Figure S2.

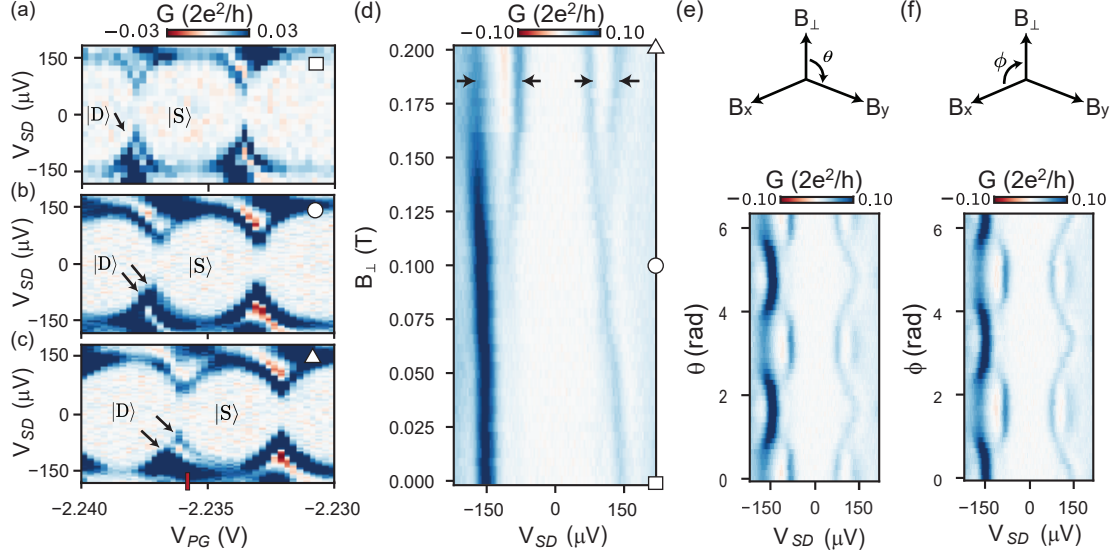


FIG. S2. **(a-c)** Bias spectroscopy with tunnel barriers set to $V_{RB} = -1.39$ V, $V_{LB} = -0.8350$ V, and $V_{HG} = -1$ V at 0 mT (square in (a)), 100 mT (circle in (b)) and 200 mT (triangle in (c)). Black arrows indicate spin splitting at the transition between a $|D\rangle$ groundstate and a $|S\rangle$ groundstate. **(d)** Magnetic field sweep at $V_{PG} = -2.2365$ V (red notch in (c)), showing spin-splitting of the odd-occupied state. We extract an out of plane g -factor of $g_{\perp} = 4.5 \pm 0.6$. **(e-f)** Bias spectroscopy as a function of magnetic field angle at total magnetic field strength $|B| = 200$ mT and $V_{PG} = -2.2365$ V. Above each panel, we state the rotational angle with respect to the sample direction. A strong g -tensor asymmetry is observed for the QD between out-of-plane and in-plane magnetic field orientations.

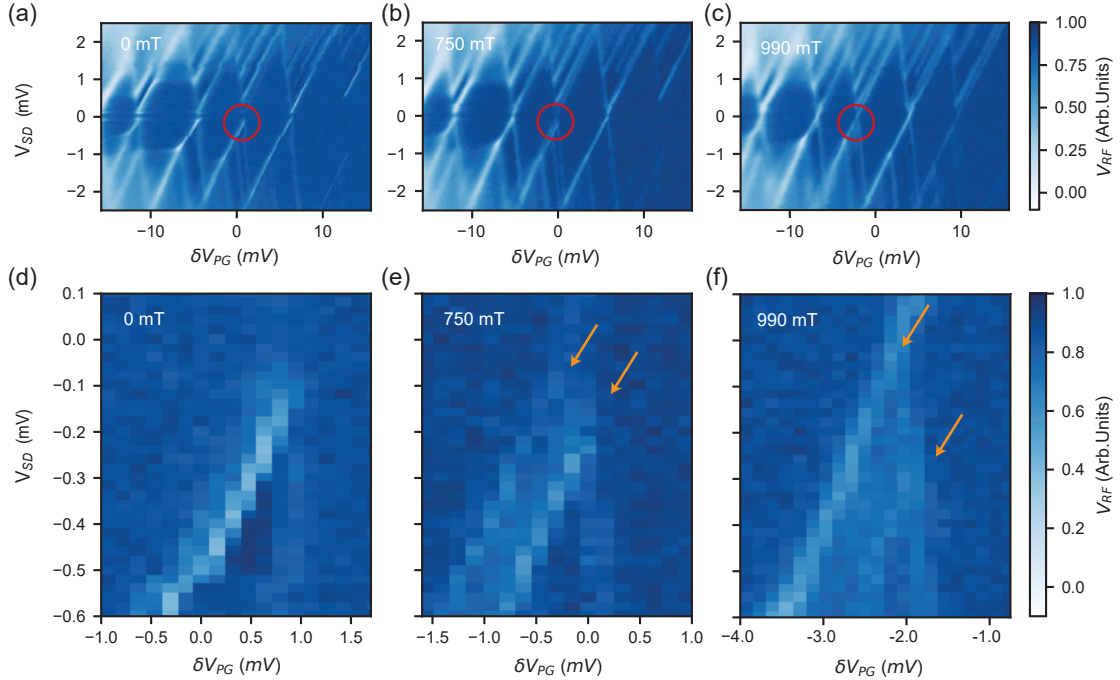


FIG. S3. **(a-c)** Bias spectroscopy, measured using RF reflectometry, at 0 mT (a) with electrostatic gates set to $V_{RB} = -1.48$ V, $V_{LB} = -1.11$ V, and $V_{HG} = 0$ V, 750 mT (b) and 990 mT (c). **(d-f)** Zoomed-in scan within the region marked by red circles in panels c-e respectively. Orange arrows indicate spin splitting, with a g -factor of $g = 5.3 \pm 0.8$.

Section III. Measurements of Critical Field

We perform bias spectroscopy as a function of magnetic field in Figure S4a-c, with the device configured in the few-hole regime. Figure S4b shows the critical in-plane field parallel to the direction of transport (see Figure S1a for a definition of the x direction). This configuration is expected to have a higher critical field than the perpendicular magnetic direction due to a smaller surface area, thus making it harder for the magnetic field flux to pin and destroy superconductivity. Finally, Figure S4c depicts the critical in-plane field perpendicular to the direction of transport (see Figure S1a for a definition of the y direction). The critical field in this direction is marginally lower than the out-of-plane magnetic field.

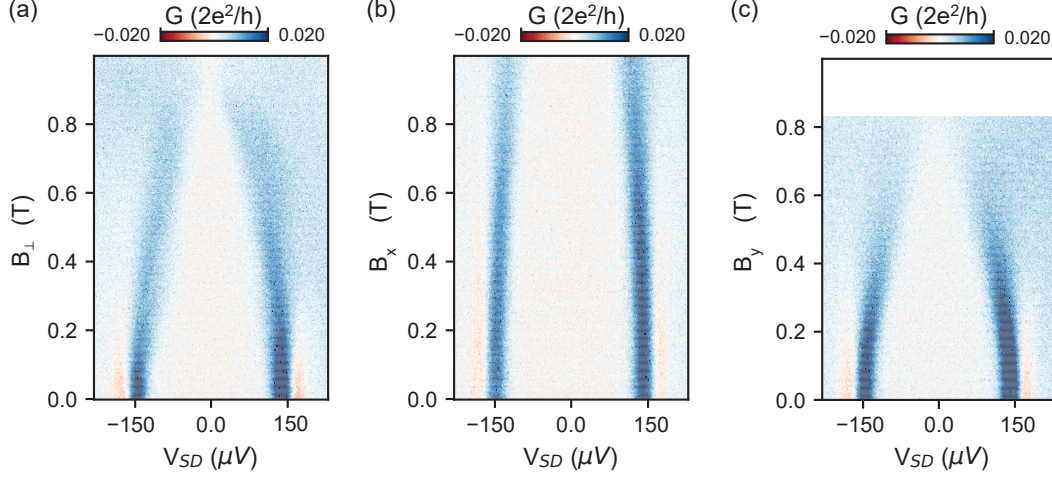


FIG. S4. Magnetic fields sweeps at same electrostatic gate settings as in main Figure 3a. **(a)** Magnetic field sweep from 0 to 1 T out of plane with respect to the device. **(b)** Magnetic field sweep from 0 to 1 T for in-plane field, parallel to the direction of transport **(c)** Magnetic field sweep from 0 to 0.82 T for in-plane field, perpendicular to the direction of transport.

	Device Type	B_x^* (T)	B_y^* (T)	$B_\perp^*$ (T)
HB1	Hallbar	1.54(1)	-	-
HB2	Hallbar	>1	1.34(1)	0.474(1)
HB3	Hallbar	>1	>1	0.123(6)
SL1	Wire	0.831(6)	> 1	0.643(1)
SL2	Wire	0.827(1)	0.633(1)	0.695(1)*

TABLE I. Summary of measured critical magnetic fields for different Hall bar and wire devices under test. *We note that for wire SL2, two critical magnetic fields emerge, which we attribute to different geometries of the wire and the fanout. The number reported is the higher of the two critical fields. For completeness, the lower critical field occurs at $B_\perp = 0.500 \pm 0.001$ T.

Section IV. Electrical Characterization of PtSiGe Films

To study superconducting properties, we also perform measurements of the critical magnetic field strengths, for PtSiGe Hall bars and wires fabricated using identical methods and on the same wafer as the device reported in the manuscript. Figure S5a shows a schematic of the Hall bar dimensions and magnetic field orientation. We perform current-biased measurements using a lock-in amplifier at 33 Hz excitation frequency and 0.5 V amplitude, applied through a 1 M Ω resistor. Figures S5b-d show measurements of differential resistance as a function of applied magnetic field strength for two separate Hall bars HB1 and HB2. Differential resistance is calculated by dividing the in-phase lock-in signals of the longitudinal voltage drop V_{xx} by the measured current at the drain electrode I . We fit a sigmoid function to the data of the form $R(B) = R_N/(1 + f(B))$ where $f(B) = \exp[-(B - B_c)/c]$. Here, R_N is the normal state resistance, B^c is the critical magnetic field, and c represents the broadening of the normal-superconducting transition. Critical magnetic fields for each Hall bar are reported in Table 1. As expected, the out-of-plane critical field $B_\perp^c$ is smaller compared to the in-plane fields for a Hall Bar geometry, as observed in ref [2].

We also perform measurements of critical magnetic fields on wires, whose geometries are shown in figure S5e. Two identical devices S1 and S2 are studied. Critical magnetic fields are reported in Table 1. We note that in figure S5h, there appear to be two transitions as shown using light and dark green. This is likely due to the design of the device, where the voltage probes will detect drops across part of the fanout, which have a much larger surface area than the wires themselves, resulting in a much smaller critical field. This is further evidenced when comparing the critical fields of the Hall bar devices and the wires, whereby the wires typically exhibit much higher out-of-plane critical magnetic fields.

We further study the electrical properties of the PtGeSi material, by measuring longitudinal and Hall resistances as a function of out of plane magnetic field, in Hall bar HB3 (Figure S6a). Sweeping magnetic field strength from -6 T to 6 T, we observe a linear dependence of the Hall resistance R_{xy} in the normal phase of the material. Fitting this, we extract a sheet carrier density of $n_{2D} = 5.2 \times 10^{16} \text{ cm}^{-2}$. We use this value combined with the longitudinal (normal) resistance $R_N = 37.9 \Omega$, to extract a carrier mobility of $\mu = 6 \text{ cm}^2/\text{Vs}$.

Figure S6b shows current-biased measurements through a wire device SL1, from which we extract a critical current of 4 μA . Assuming a cross section of 100 nm x 15 nm for the wire, we calculate a critical current density of $J = 2.7 \text{ A}/\text{m}^2$. Due to the similarity in feature size, we expect this current density to reflect that of the superconducting leads in the device reported in the main manuscript. We note that measurements in the main text are taken at currents restricted to around 1 nA or lower (Main text Figure 1d) significantly lower than the expected critical current of 2.4 μA , and are therefore unlikely to exhibit features due to depairing [3].

Finally, we study the critical temperature of the PtSiGe material in a wire device (SL2). We use a PID feedback loop on a heater placed at the mixing chamber temperature stage of the cryostat, to control the temperature of the mixing chamber plate. This temperature is stable within a range of ± 10 mK of the setpoint. We note that the temperature of the mixing chamber plate is not necessarily reflective of the temperature of the sample, as the sample is loaded inside a puck via a fast sample exchange unit, and is likely to underestimate the actual temperature by some tens of mK. We perform out-of-plane magnetic field sweeps between -0.2 T and 0.2 T, while stepping the mixing chamber temperature setpoint of the heater in steps of 20 mK, from 20 mK to 600 mK. After each temperature step, a settle time of 5 minutes is allowed before beginning the magnetic field sweep. Figure 6c shows the results of this measurement, indicating a clear switching between superconducting and normal phases. We fit the critical magnetic fields and critical temperatures using a sigmoid function, finding $T_c = 408 \text{ mK}$, and $B_\perp^c = 0.116 \text{ T}$. We plot the scaled phase diagram in figure S6d, and fit the standard BCS relation $T/T_c = 1.74\sqrt{1 - B_\perp/B_\perp^c}$ with good agreement.

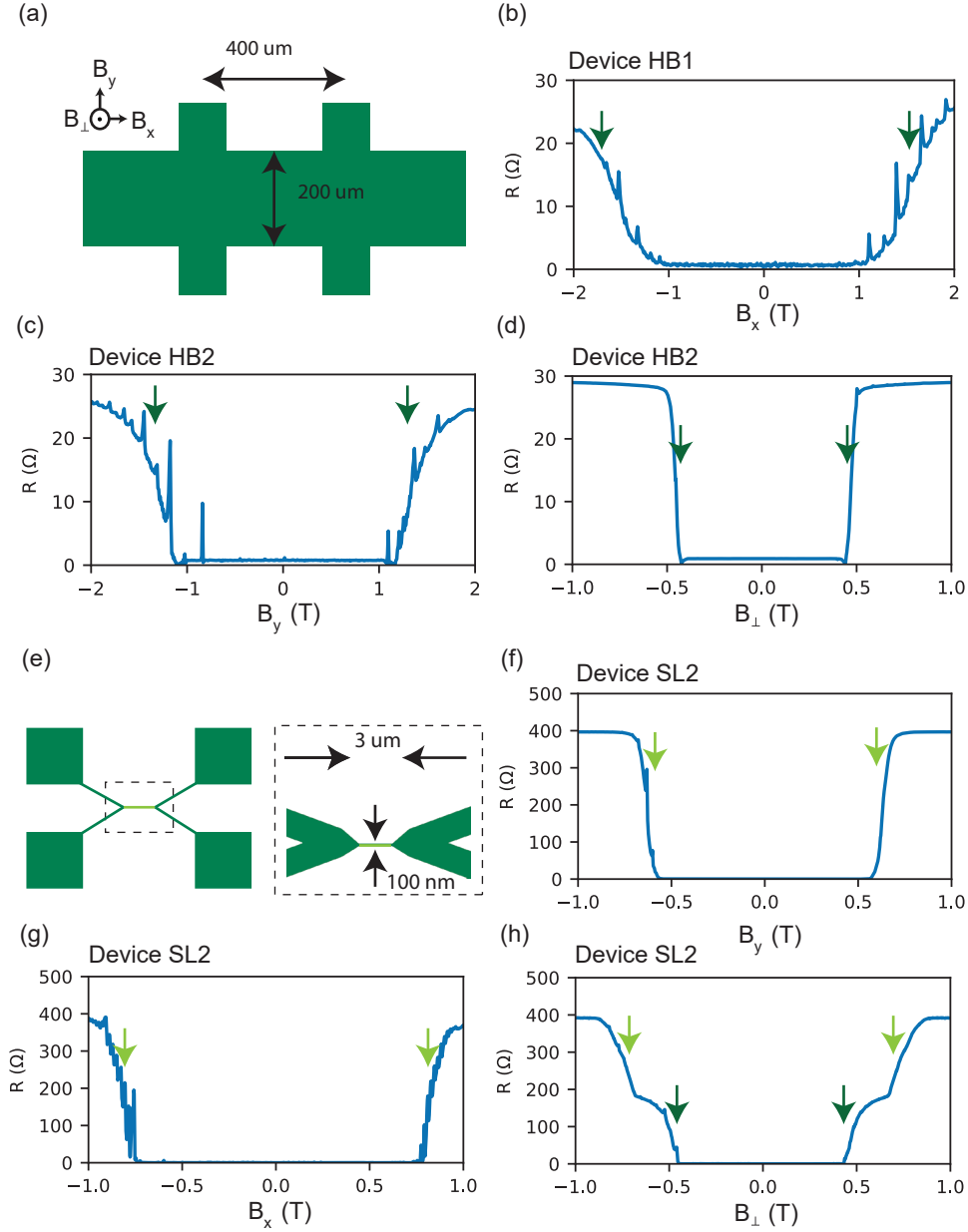


FIG. S5. **(a-d)** Critical magnetic field measurements for PtSiGe Hall bars. (a) Schematic of Hall bar geometry and alignment of magnetic field directions. Here, B_x indicates a magnetic field applied in the plane of the heterostructure, aligned with the direction of transport, B_y indicates a magnetic field applied in the plane of the heterostructure, perpendicular with the direction of transport, and $B_\perp$ indicates a magnetic field applied out-of-plane with respect to the heterostructure. Dark green arrows represent the magnetic field at which a superconducting-normal state transition occurs according to the fit. **(e-g)** Critical magnetic field measurements for PtSiGe wires. (e) Schematic of wire geometry. The wires are $3\ \mu\text{m}$ long, $100\ \text{nm}$ wide, with a Pt deposition thickness of $15\ \text{nm}$. In (h), two critical magnetic field onsets occur due to the voltage probes being sensitive to voltage drops in part of the fanout (inset of (e)). The dark green arrows represent the superconducting-normal state transition of the larger ‘fanout’ section, while the light green arrows represent the superconducting-normal state transition of the wires.

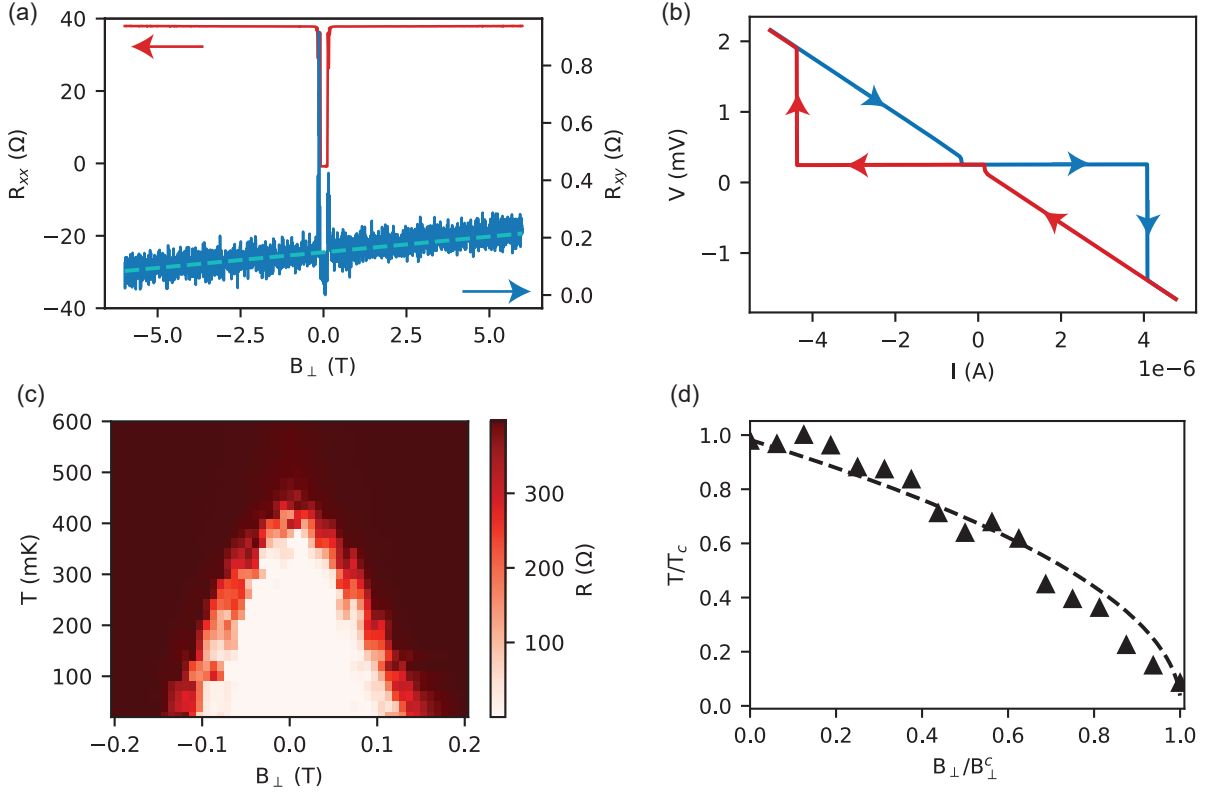


FIG. S6. **(a)** Longitudinal (red) and Hall (blue) resistance of Hall bar HB3 for a symmetric out of plane magnetic field range of -6 T to 6 T. A linear fit of the Hall resistance of the device in the normal state is shown in the cyan dashed line, resulting in an estimated carrier density of $5.2 \times 10^{16} \text{ cm}^{-2}$. We estimate the mobility of the carriers in the PtSiGe Hall bar (aspect ratio $L/W = 2$) to be $6 \text{ cm}^2/\text{Vs}$. **(b)** Critical current measurements on a 100 nm wide PtSiGe wire at zero magnetic field and at base cryostat temperature. Arrows indicate the direction of the sweep. A critical current density of 2.7 A/m^2 is extracted assuming a PtSiGe thickness of 15 nm. **(c)** Critical out-of-plane magnetic field as a function of temperature recorded at the mixing chamber plate of the cryostat. Instabilities in PID loop impart approximately a $\pm 10 \text{ mK}$ error bar on the temperature axis. A critical temperature of $T_c = 408 \text{ mK}$ and critical out-of-plane magnetic field of 0.116 T are extracted from sigmoid function fits to the data. **(d)** Fitted critical temperature and magnetic field from c) scaled by their $B_{\perp}^c(T=0)$ and $T^c(B_{\perp}=0)$ values respectively. The dashed line is a BCS theory fit to the data.

Section V. Modeling of Subgap States

The device in the main text consists of a QD weakly coupled to one superconducting lead, and strongly coupled to another. We therefore write the Hamiltonian of the total system as:

$$H = H_{QD} + H_T + H_{SC}. \quad (1)$$

Where H_{QD} is defined as

$$H_{QD} = \sum_{\sigma} \epsilon d_{\sigma}^{\dagger} d_{\sigma} + U d_{\downarrow}^{\dagger} d_{\downarrow} d_{\uparrow}^{\dagger} d_{\uparrow} + \frac{E_z}{2} (d_{\uparrow}^{\dagger} d_{\uparrow} - d_{\downarrow}^{\dagger} d_{\downarrow}) \quad (2)$$

Here, ϵ is the electrochemical potential, U is the charging energy and E_z is the Zeeman energy. As one superconducting lead is weakly coupled, we consider only one bulk superconductor for H_{SC} , and ignore possible phase differences between their order parameters. In the superconducting Anderson impurity model, H_{SC} consists of a sum over multiple orbitals in the superconductor. Following Refs. [4, 5], we replace this sum by a single orbital and write:

$$H_{SC} = \xi \sum_{\sigma} c_{\sigma}^{\dagger} c_{\sigma} - \Delta (c_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} + h.c.) + \frac{E_z^{SC}}{2} (c_{\uparrow}^{\dagger} c_{\uparrow} - c_{\downarrow}^{\dagger} c_{\downarrow}) \quad (3)$$

Where ξ is the single-electron energy, Δ is the parent gap and E_z^{SC} is the Zeeman energy. We set $\xi = 0$, as the superconductor is grounded throughout the experiment. The Zeeman splitting of the superconductor is commonly excluded, as metallic superconductors usually have significantly lower g factors than the semiconductors they are coupled to. We however choose not to exclude it, in view of the low in-plane g-factor of Ge/SiGe heterostructure QDs, as well as the little-known magnetic field behavior of the novel PtGeSi superconductor. Finally, we can write the coupling of the superconductor and QD as:

$$H_T = \Gamma_S \sum_{\sigma} (c_{\sigma}^{\dagger} d_{\sigma} + h.c.) \quad (4)$$

where Γ_S is the spin-conserving hybridization energy. The full basis consists of 16 states, which we write as the tensor product of the number states of the superconductor and QD $|SC, QD\rangle$:

$$\{|0, 0\rangle, |\uparrow, 0\rangle, |\downarrow, 0\rangle, |2, 0\rangle, |0, \uparrow\rangle, |\uparrow, \uparrow\rangle, |\downarrow, \uparrow\rangle, |2, \uparrow\rangle, |2, \downarrow\rangle, |\uparrow, \downarrow\rangle, |\downarrow, \downarrow\rangle, |2, \downarrow\rangle, |0, 2\rangle, |\uparrow, 2\rangle, |\downarrow, 2\rangle, |2, 2\rangle\} \quad (5)$$

To model the spectrum, we consider excitations between the even and odd parity states. As the tunnel probe is a superconductor, we offset all excitation energies by Δ .

In Main Text Fig. 4b,c a Zeeman splitting of the superconductor was required to match the model to the experiment. For the model overlays seen in main text Figure 4a-c, the parameters used were a superconducting gap of $\Delta = 71 \mu\text{eV}$, a charging energy of $U = 1.6 \text{ meV}$, a hybridization energy of $\Gamma_S = 110 \mu\text{eV}$ along with an energy splitting on both SC and QD. We phenomenologically include the magnetic field dependence of the parent gap as: $\Delta(B) = 2\Delta_0 \sqrt{1 - (B/B_c)^2}$, where the factor of 2 is due to both leads having a superconducting density of states, resulting in quasiparticle transport between the bulk superconductors at $E = 2\Delta$ and $B_c = 0.9 \text{ T}$ as the critical field. This is required for agreement between the experimental data and the model, as states are observed at $E_{\text{state}} + \Delta$ due to the superconducting tunnel probe.

Using the described ZBW model we can compute the energy spectrum of the QD hybridizing with the SC. In S7d and g, we adding a Zeeman energy of $E_z = 56 \mu\text{eV}$ to the QD energy and nothing on the superconductor, which results in a spin-splitting at the charge degeneracy points $\epsilon(\Delta) = 0$ and $\epsilon(\Delta) = -1.6$, but no splitting in the even parity regions ($|S\rangle$), contradicting the data in Figure S7g. In supplement Figure S7e and h, we add a Zeeman energy of $E_z = 56 \mu\text{eV}$ only to the SC, resulting in a splitting in the even parity states ($|S\rangle$) resembling the energy separation seen in main text Figure 4b-c, however we see no spin-splitting at the charge degeneracy points $\epsilon(\Delta) = 0$ and $\epsilon(\Delta) = -1.6$; furthermore, the characteristic eye-like curvature of the $|D\rangle$ state is significantly reduced, and does not match the data in Figure S7h. In Figure S7f and i, a Zeeman energy $E_z = 56 \mu\text{eV}$ was added to both the QD and the SC, resulting in qualitative agreement to the data in Figure S7i. We stress that our model is quite simple and is not meant to fit the data quantitatively, and as such the magnitude and sign of the respective Zeeman terms have not been varied extensively. For example, future theoretical work could include the spin-orbit coupling, as well as a more detailed description of the superconductor beyond zero band width.

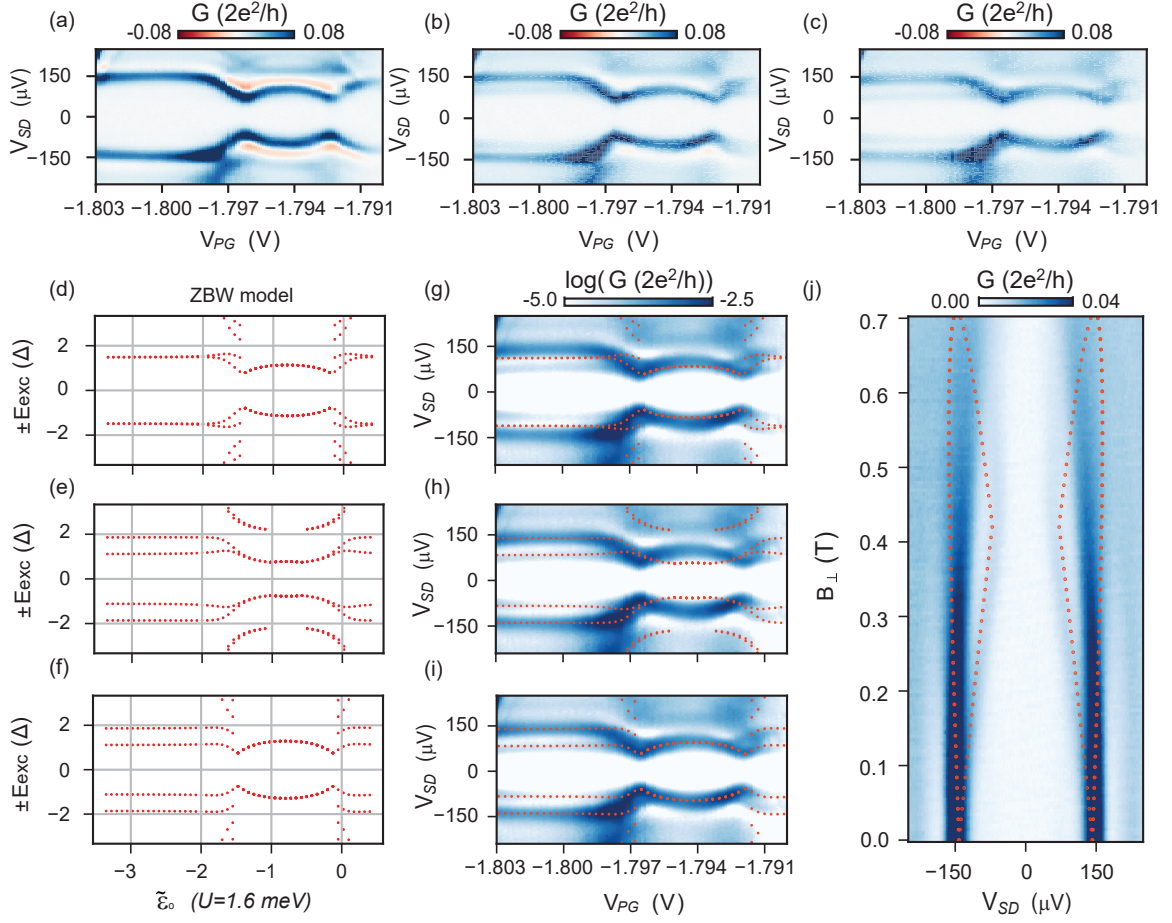


FIG. S7. **(a-c)** Bias spectroscopy data from main text Figures 4(a-c) without the logarithmic scale. **(d)** Excitation energy in units of $\Delta = 71\mu eV$ as a function of chemical potential in units of charging energy $U = 1.6 meV$, with a Zeeman energy only on the QD of $E_z=56 \mu eV$. **(e)** Excitation energy in units of $\Delta = 71\mu eV$ as a function of chemical potential in units of charging energy $U = 1.6 meV$, with a Zeeman energy of $E_z=56 \mu eV$ on SC only. **(f)** Excitation energy in units of $\Delta = 71 \mu eV$ as a function of chemical potential in units of charging energy $U = 1.6 meV$, Zeeman energy on both SC and QD of $E_z=56 \mu eV$. **(g-i)** Bias spectroscopy data from main Figure 4c, overlaid with the computed model (red) from supplement Figure 4d-f respectively. **(j)** Bias spectroscopy data from main Figure 4d overlaid with the computed ZBW model (orange) correcting for the gap closing as the magnetic field increases using a g-factor of 1.5 on both SC and QD.

Finally, in Fig S7j we show the bias spectroscopy data from main text Figure 4d, overlaid with the computed ZBW spectrum. There is a good agreement between the model and the data until about 400 mT, when the splitting becomes larger than the SC pairing energy. We attribute this discrepancy between model and data to the ZBW model, which cannot model the Zeeman effect on a bulk superconducting density of states.

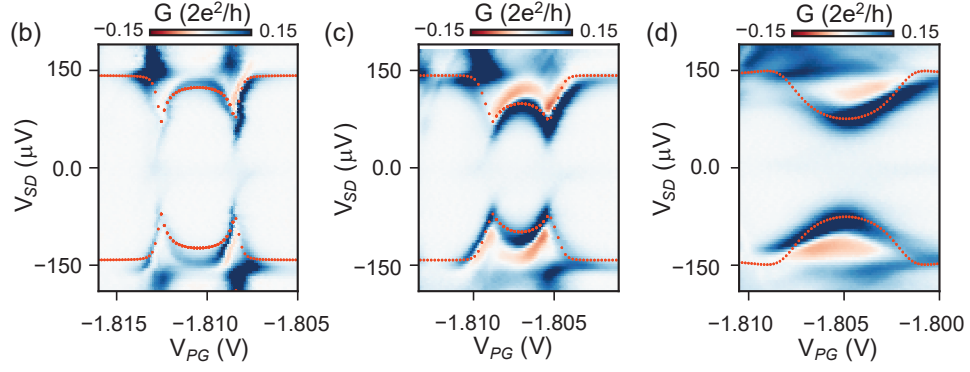


FIG. S8. **(a)** Bias spectroscopy data from main text Figure 2b overlaid with the computed ZBW model, setting the gap energy to $\Delta = 71 \mu\text{eV}$, a charging energy of $U = 1.6 \text{ meV}$ and a $\Gamma_S = 70 \mu\text{eV}$. **(b)** Bias spectroscopy data from main text Figure 2c overlaid with the computed ZBW model, setting the gap energy to $\Delta = 71 \mu\text{eV}$, a charging energy of $U = 1.6 \text{ meV}$ and a $\Gamma_S = 110 \mu\text{eV}$. **(c)** Bias spectroscopy data from main text Figure 2d overlaid with the computed ZBW model, setting the gap energy to $\Delta = 71 \mu\text{eV}$, a charging energy of $U = 1.6 \text{ meV}$ and a $\Gamma_S = 150 \mu\text{eV}$.

In order to quantify the hybridization energy of SC and QD Γ_S , we model the subgap states in main text Figure 2b-d in the ZBW model, using the superconducting gap and charging energy extracted in the main text. We manually tune the parameter Γ_S until a qualitative match between data and model is found. For Figures S8a-c we set the SC gap to $\Delta = 71 \mu\text{eV}$ and the charging energy to $U = 1.6 \text{ meV}$, for which we extract an estimate of $\Gamma_S = 70 \mu\text{eV}$ for Figures S8a, $\Gamma_S = 110 \mu\text{eV}$ for Figure S8b, and $\Gamma_S = 150 \mu\text{eV}$ for the fully coupled groundstate in Figure S8c.

[†] These authors contributed equally.

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