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Vanishing skyrmion Hall effect at the angular momentum compensation temperature of a ferrimagnet

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Supplementary Text

Note S1. Measurement of Dzyaloshinskii-Moriya interaction (DMI) induced effective magnetic field $\mu_0 H_{\text{DMI}}$ based on the magnetic droplet nucleation model

In order to measure the DMI, we use the magnetic droplet nucleation model proposed by Kim *et al.*^{S1}, which is a simple scheme to quantify the DMI-induced effective magnetic field $\mu_0 H_{\text{DMI}}$. We measure the hysteresis loop by sweeping the magnetic field $\mu_0 H$ with an angle ϕ , and obtain the magnetization switching field $\mu_0 H_{\text{SW}}$. The tilted angle ϕ of the magnetic field is defined from the +z direction to the +x direction. The coercivity field $\mu_0 H_c$ and the in-plane magnetic field $\mu_0 H_x$ are then given as $\mu_0 H_c \equiv \mu_0 H_{\text{SW}} \cos \phi$ and $\mu_0 H_x \equiv \mu_0 H_{\text{SW}} \sin \phi$, respectively. Figure S1 shows the measured $(\mu_0 H_c)^{\frac{1}{2}}$ as a function of $\mu_0 H_x$ for the GdFeCo/Pt sample. The tendency of $(\mu_0 H_c)^{\frac{1}{2}}$ with respect to $\mu_0 H_x$ shows a threshold, above which $\mu_0 H_c$ decreases with increasing $\mu_0 H_x$. This threshold corresponds to $\mu_0 H_{\text{DMI}}$ ^{S1} (indicated by an arrow). The red solid lines in Fig. S1 show the best fits based on the magnetic droplet nucleation model. The good conformity supports the validity of this model.

On the other hand, the domain wall (DW) hard-axis anisotropy field $\mu_0 H_S$ is estimated using the relation $\mu_0 H_S \cong \left(\frac{2 \ln 2}{\pi^2}\right) \mu_0 M_S \lambda / t_f$ ^{S2,S3}, where M_S is the saturation magnetization, t_f is the thickness of the magnetic layer, and λ is the DW width. The estimated $\mu_0 H_S$ is 0.9 mT by using $M_S = 10^4$ A/m, $\lambda = 15$ nm^{S4}, and $t_f = 30$ nm. Since $\mu_0 H_S$ is much smaller than $\mu_0 H_{\text{DMI}}$ (= 63.3 mT), homochiral magnetic bubbles with a well-defined Q can be stabilized in the structure.

To quantify D ($\equiv \lambda M_S \mu_0 H_{\text{DMI}}$), we measured the magnetic properties $K_{\text{eff}} = 9$ kJ/m³ and $M_S = 3 \times 10^4$ A/m using superconducting quantum interference device (SQUID)

magnetometry, where K_{eff} is the effective magnetic anisotropy energy density. λ is defined as $\sqrt{A/K_{\text{eff}}}$, where A is the exchange stiffness constant, and thus, λ can be estimated from the measured K_{eff} and $A = 10$ pJ/m. From this approach, we obtained $D = 0.06$ mJ/m².

Note S2. Estimation of the temperature rise due to the Joule heating effect

To estimate the temperature rise due to the current-induced Joule heating, we investigate the temperature-dependent electrical resistivity of wires^{S5}. Figure S2a shows the resistivity ρ of a wire as a function of the current I . Since there exists the Joule heating effect, ρ exhibits a quadratic dependence on the current I , i.e., $\rho/\rho_0 = 1 + \sigma_I I^2$, where ρ_0 is the resistivity at the ambient temperature (300 K) and σ_I is a proportionality constant. Figure S2b shows the resistivity ρ of a wire as a function of the temperature T , which is described by $\rho/\rho_0 = 1 + \sigma_T \Delta T$ with the temperature rise ΔT and another proportionality constant σ_T . Combining these two measurements, we estimate the temperature rise as $\Delta T = (\sigma_I/\sigma_T)I^2$. In experiments, σ_I and σ_T are estimated to be 1.46 A^{-2} and $3.45 \times 10^{-4} \text{ K}^{-1}$, respectively. For the current $I = 0.14 \text{ A}$, ΔT reaches about 83 K.

Note S3. Experimental reason causing the pinning problem near half-ring shaped writing line

Franken *et al.* reported that the domain wall energy landscape can be directly engineered by modulating the magnetic anisotropy via ion irradiation using a focused ion beam (FIB)^{S6}. Irradiation leads to changing magnetic properties and therefore giving the desired variation of the potential landscape for the domain walls. Similarly, the pinning can be commonly generated at the edges of wires due to the photolithography and ion-milling

process^{S7}. For this reason, we believe that lithography-induced damage could induce a change in the domain wall energy landscape that serves as a strong pinning potential near half-ring shaped writing line. This explanation is supported by experimental observations that the bubbles are created only near the writing line.

Note S4. Additional measurement of the elongation angle θ as a function of T by using the short pulse generator

To minimize the Joule heating effect, we also measure the current-driven elongation of bubble domains by using a pulse generator (Picosecond 10, 300B), where we inject the voltage pulse (amplitude $V = 15.4$ V and duration $\Delta t = 100$ ns). Due to the short pulse length, we expect that the current-induced Joule heating is ignorable. As the temperature of our pulse set-up varies only in the range from 300 K to 350 K, we measure the elongation angle in this temperature range. Figure S3 summarizes the elongation angle θ as a function of T with the magnetization-DOWN states. Similar to Fig. 3d, θ linearly increases with T . The linear extrapolation (red dotted line) shows that θ becomes zero at the temperature close to T_A . Together with the results shown in Fig. 3d in the main text, this result confirms that the skyrmion Hall effect vanishes at T_A .

Note S5. Analytical description of the elongation dynamics of a pinned magnetic bubble driven by SOT

I. The SOT-induced pressure on the Neel-type DW. According to the spin Hall phenomenology^{S8}, the SOT can be written as,

$$\boldsymbol{\tau}_{\text{SOT}} = \eta(\hat{\mathbf{z}} \times \mathbf{j}) \times \mathbf{n} + \vartheta \mathbf{n} \times (\hat{\mathbf{z}} \times \mathbf{j}) \times \mathbf{n}, \quad (\text{S1})$$

where η and ϑ are the reactive and the dissipative coefficients characterizing spin-orbit interactions present in the system. Here, $\mathbf{n}$ is the spin order parameter, $\mathbf{j}$ is the charge current density, and $\hat{\mathbf{z}}$ is the normal to the film plane. The first and the second terms on the right-hand side are the so-called field-like and the damping-like components of the SOT, respectively.

Let us derive the pressure (per unit length) on a Néel-type DW along the x direction, whose profile is given by

$$\mathbf{n}(x, y - Y, z) = \left(0, q_1 \operatorname{sech}\left(\frac{y - Y}{\lambda}\right), q_2 \tanh\left(\frac{y - Y}{\lambda}\right) \right) \quad (\text{S2})$$

where λ parametrizes the width of the DW and $q_1, q_2 = \pm 1$ specify the type of the DW. The pressure (the density of the force $\mathbf{F}_Y$ per unit length) on the DW is given by (The direction of the pressure is normal to the wall, i.e., along the y direction)

$$P = \int dy \mathbf{n} \times \boldsymbol{\tau}_{\text{SOT}} \cdot \frac{\partial \mathbf{n}}{\partial Y} = -\pi q_1 q_2 \vartheta j_y, \quad (\text{S3})$$

within the collective-coordinate approach^{S9}. In the vectorial notation, $\mathbf{P} = -\pi q_1 q_2 \vartheta \mathbf{j}_\perp$, where $\mathbf{j}_\perp$ is the component of the current density perpendicular to the DW string. The product $q_1 q_2$ is determined by the sign of the DMI of the material. Let us fix it as $q_1 q_2 = -1$ in this note. Then, $\mathbf{P} = \pi \vartheta \mathbf{j}_\perp$.

II. The elongation dynamics of a magnetic bubble. A magnetic bubble can be considered as a loop of a magnetic DW. Here, we consider an elongated bubble with one pinned end [see Fig. 4a] as a composite object that consists of a small rigid ball at the free end (representing a half-skyrmion) and a stretchable straight rod with the fixed tension $\mathbf{F}_T$ (representing an energy density per unit length of a DW along the circumference of a bubble) [see Fig. 4a]. This model

is similar to that used in Ref. S10, where small skyrmions were created from big magnetic bubbles. There are two dynamic variables: the length of the line $l(t)$ (or, equivalently, the distance between the origin and the ball) and the angle $\theta(t)$ from the current direction $\mathbf{j} = j\hat{x}$ to the elongation direction.

A. The simple model

We are interested in the steady-state solution with a constant angle $\theta(t) = \theta_0$ and the constant growing rate of the line $l(t) = l_0 + vt$. There are several types of forces acting on the rigid ball (a half skyrmion): $\mathbf{F}_j = \int_{\text{half-skyrmion circumference}} d\mathbf{l} \mathbf{P} = 2\pi r \vartheta j (\cos \theta \hat{\mathbf{l}} + \sin \theta \hat{\mathbf{l}} \times \hat{\mathbf{z}})$, $\mathbf{F}_g = 4\pi s_{\text{net}} Q (\hat{\mathbf{z}} \times \hat{\mathbf{v}})$, and $\mathbf{F}_d = -\alpha s_{\text{total}} D \hat{\mathbf{v}}$, where r is the radius of a half skyrmion, $\hat{\mathbf{l}}$ is the unit vector along the elongation, $Q = \pm 1/2$ is the topological charge of a half-skyrmion, s_{net} is the net spin density that changes its sign across the angular momentum compensation temperature T_A , s_{total} is the sum of the magnitudes of the sublattice spin density, D is the dimensionless number of order 1 (see, e.g., Ref. S11). In this model for explaining the constant-angle elongation dynamics, we neglect the SOT-induced pressure on the straight rod since it can be easily cancelled by the disorder-induced pinning potential in an analogous way that the dynamics of a long DW is easily pinned by disorders. There is also a tension acting on the rigid ball, $\mathbf{F}_T$, exerted by the DW attached to it, which is twice the DW energy density per unit length. We remark here that the tension $\mathbf{F}_T$ does not depend on the velocity or the angle; it is constant over time. To have the constant angle θ_0 , the net torque should vanish:

$$4\pi s_{\text{net}} Q v - 2\pi r \vartheta j \sin \theta_0 = 0. \quad (\text{S4})$$

To have the constant growth rate v , the forces should balance:

$$2\pi r \vartheta j \cos \theta_0 - \alpha s_{\text{total}} D v = F_T. \quad (\text{S5})$$

We can remove the angle variable θ_0 , which results in

$$2\pi r \vartheta j \cos \theta_0 = \frac{\alpha s_{\text{total}} D}{2s_{\text{net}} Q} \vartheta j \sin \theta_0 + F_T. \quad (\text{S6})$$

In the large-bubble limit (large r) or in the high-current limit (large j), where we can neglect the tension on the right-hand side over the left-hand side, we obtain

$$\tan \theta_0 \approx \frac{4\pi s_{\text{net}} Q}{\alpha s_{\text{total}} D}, \quad \text{for } r \vartheta j \gg F_T, \quad (\text{S7})$$

which is identical to the expression for the skyrmion Hall angle obtained for an isolated skyrmion^{S12}. This result explains the experimental results reasonably well. For a circular ansatz for a half magnetic bubble, $D = 2\pi r/\lambda$, where λ is the DW width. In this case, we obtain the equation of the skyrmion Hall angle θ_{SKH} ($\theta_{\text{SKH}} = \theta_0$),

$$\theta_{\text{SKH}} \approx \tan^{-1} \left(\frac{2s_{\text{net}} Q \lambda}{\alpha s_{\text{total}} r} \right). \quad (\text{S8})$$

Note S6. Numerical simulations based on the atomistic Landau-Lifshitz-Gilbert equation with both SOTs

The spin Hall effect in the Pt substrate generates not only damping-like SOT but also field-like SOT. To clarify the effect of field-like SOT, we perform numerical simulations based on the atomistic Landau-Lifshitz-Gilbert equation including both SOTs : $\partial \mathbf{S}_i / \partial t = -\gamma_i \mathbf{S}_i \times \mathbf{H}_{\text{eff},i} + \alpha \mathbf{S}_i \times \partial \mathbf{S}_i / \partial t - \gamma_i c_j \mathbf{S}_i \times (\mathbf{S}_i \times \hat{\mathbf{y}}) - \chi \gamma_i c_j \mathbf{S}_i \times \hat{\mathbf{y}}$, where χ is the ratio of field-like SOT to damping-like SOT. Previous report^{S13} shows that for the Pt layer, χ is smaller than 1.0. For this reason, we carry out simulations with χ of 0, ± 0.1 , ± 0.2 , ± 0.5 , and ± 1.0 . As shown in the Fig. S4, the elongation angle does not change much with χ .

Note S7. Quantitative comparison among experiment, theory, and modeling

As it is rather difficult to directly compare the experimental results with the modeling ones because of very different dimensionality of bubbles, we perform two separate comparisons; i.e., between experiment and theory [Eq. (2)], and between modeling and theory.

For the former, material parameters such as M_{FeCo} , M_{Gd} , s_{FeCo} , and s_{Gd} are estimated by measuring the net magnetic moment of GdFeCo film, M_{net} , for various temperatures. The Gd and FeCo sub-lattice magnetic moments, M_{FeCo} and M_{Gd} , are obtained based on the power law criticality [see details in Ref. S14]. The spin densities, s_{FeCo} and s_{Gd} , are calculated using the known Landé g-factor of FeCo and Gd (the Landé g-factor of FeCo is 2.2 and that of Gd is 2.0)^{S14-S17}. Figure S5a shows the temperature-dependent M_{net} . Symbols indicate the experimental data and solid line is the best fit from the power law criticality. The good conformity supports the validity of this model. Figures S5b-d show the temperature-dependent sub-lattice magnetic moment M_i , sub-lattice angular momentum s_i , and the ratio $s_{\text{net}}/s_{\text{total}}$, respectively, where $s_{\text{net}} = s_{\text{FeCo}} - s_{\text{Gd}}$ and $s_{\text{total}} = |s_{\text{FeCo}} + s_{\text{Gd}}|$. Here, we use the relative temperature defined as $T - T_A$ to investigate $s_{\text{net}}/s_{\text{total}}$.

Figure S6 shows the experimentally obtained elongation angle as a function of $s_{\text{net}}/s_{\text{total}}$. At $s_{\text{net}}/s_{\text{total}} = 0$, the elongation angle is nearly zero, consistent with both experiment and theory. The solid lines are the best linear fits. The slope for the UP magnetization state ($Q = +1/2$) is 10.17 and the slope for the DOWN magnetization state ($Q = -1/2$) is -8.81. From Eq. (2), the slope is given by $(2Q\lambda)/(\alpha r)$. By using $\lambda = 20 \text{ nm}^{\text{S4}}$ and $\alpha = 0.003^{\text{S4}}$, the estimated $(2Q\lambda)/(\alpha r)$ is $6.67/r$, where the unit of r is μm . From the MOKE images, we find that r is about $5 \mu\text{m}$. It gives the slope of 6.67, which shows a similar

order of magnitude with experimental slopes (i.e., 10.17 or -8.81).

We also quantitatively compare the numerical results to the theory [Eq. (2)]. Figure S7 shows the numerically obtained elongation angle as a function of s_{net} . The bubble radius r estimated from numerical results is $10 \text{ nm} \pm 2 \text{ nm}$. Other parameters such as λ , α , s_{TM} , and s_{RE} are shown in Methods and Table S1. Symbols are numerical results and lines correspond to Eq. (2) with different r . Figure S7 shows that the numerical results are in reasonable agreement with the theory.

Table S1. Parameters used in the numerical simulation. M_{TM} (M_{RE}) is the magnetic moment of transition-metal (rare-earth) element.

Index	1	2	3	4	5	6	7	8	9
M_{TM} [kAm ⁻¹]	570	565	560	555	550	545	540	535	530
M_{RE} [kAm ⁻¹]	540	530	520	510	500	490	480	470	460
$s_{\text{net}} [\equiv -\frac{\hbar}{\mu_B} \left(\frac{M_{\text{TM}}}{g_{\text{TM}}} - \frac{M_{\text{RE}}}{g_{\text{RE}}} \right)]$ [10 ⁻⁷ Jsm ⁻³]	1.24	0.93	0.62	0.31	0	-0.31	-0.62	-0.93	-1.24

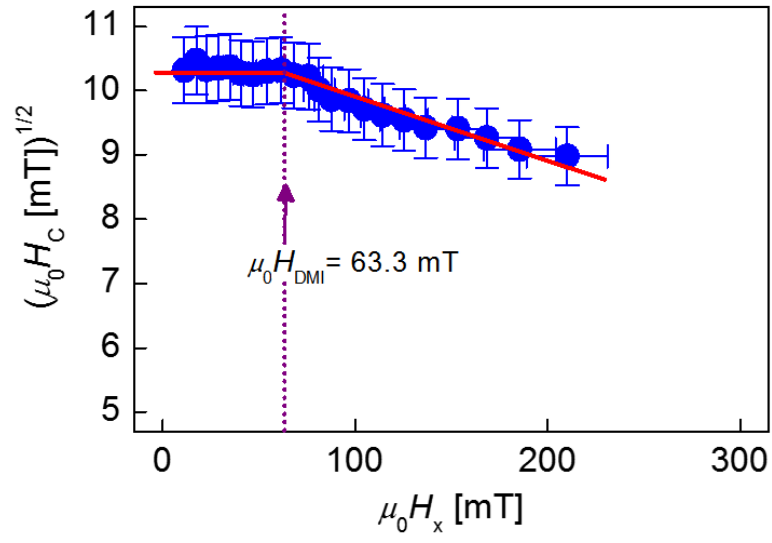


Figure S1. DMI measurement in GdFeCo/Pt sample using the magnetic droplet model. Coercivity field $\mu_0 H_c$ and in-plane magnetic field $\mu_0 H_x$. A red solid line is the best fit with the magnetic droplet model. A vertical arrow indicates the effective DMI field.

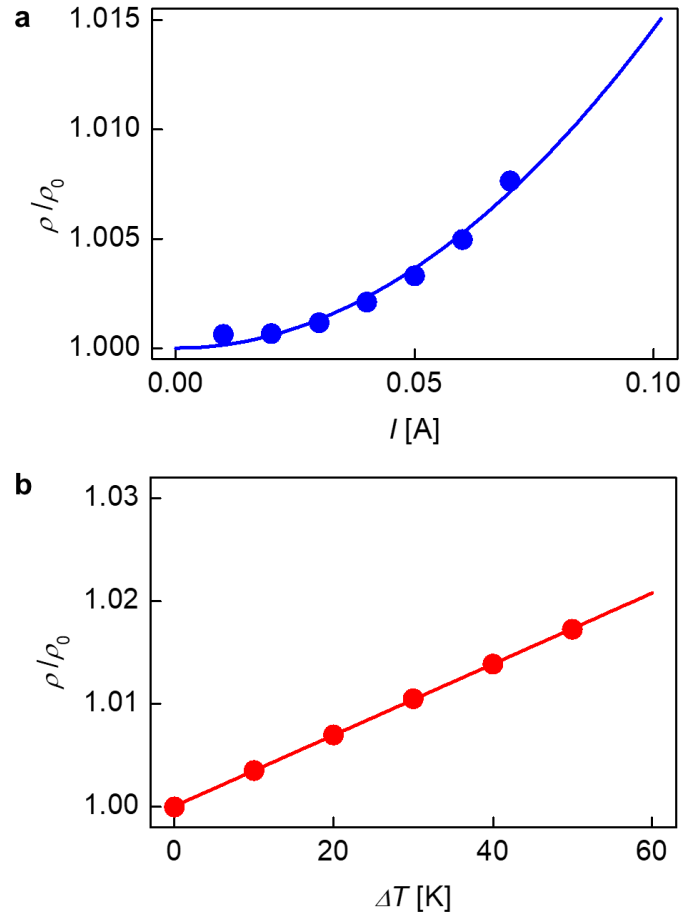


Figure S2. Estimation of the temperature rise due to the Joule heating effect. **a**, The variation in resistivity ρ/ρ_0 as a function of current I . The solid line is the best fit with $\rho/\rho_0 = 1 + \sigma_I I^2$. **b**, The variation in resistivity ρ/ρ_0 as a function of ΔT . The solid line is the best fit with $\rho/\rho_0 = 1 + \sigma_T \Delta T$.

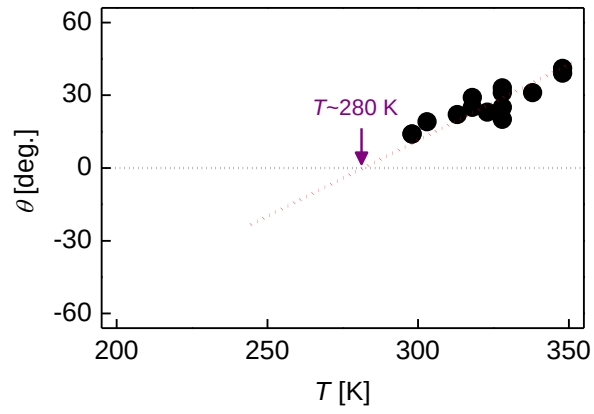


Figure S3. The elongation angle θ as a function of T for magnetization-DOWN state. A short current pulse (100 ns) is used for the measurement. A red dotted line is the linear extrapolation.

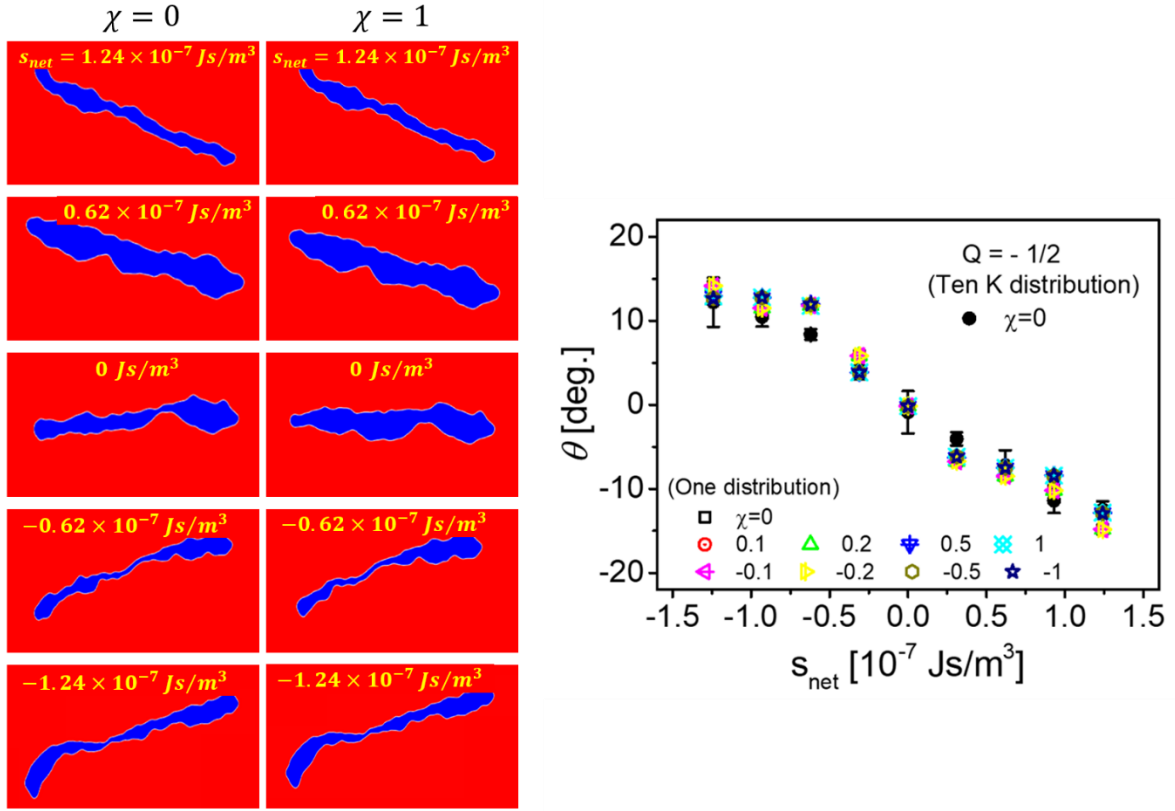


Figure S4. Effect of field-like SOT on the elongation angle. Images on the left-hand side are for $\chi = 0$ and 1.0, where χ is the ratio of field-like SOT to damping-like SOT. A plot on the right-hand side summarizes the results. No meaningful change due to the field-like torque is observed for the tested ranges of χ .

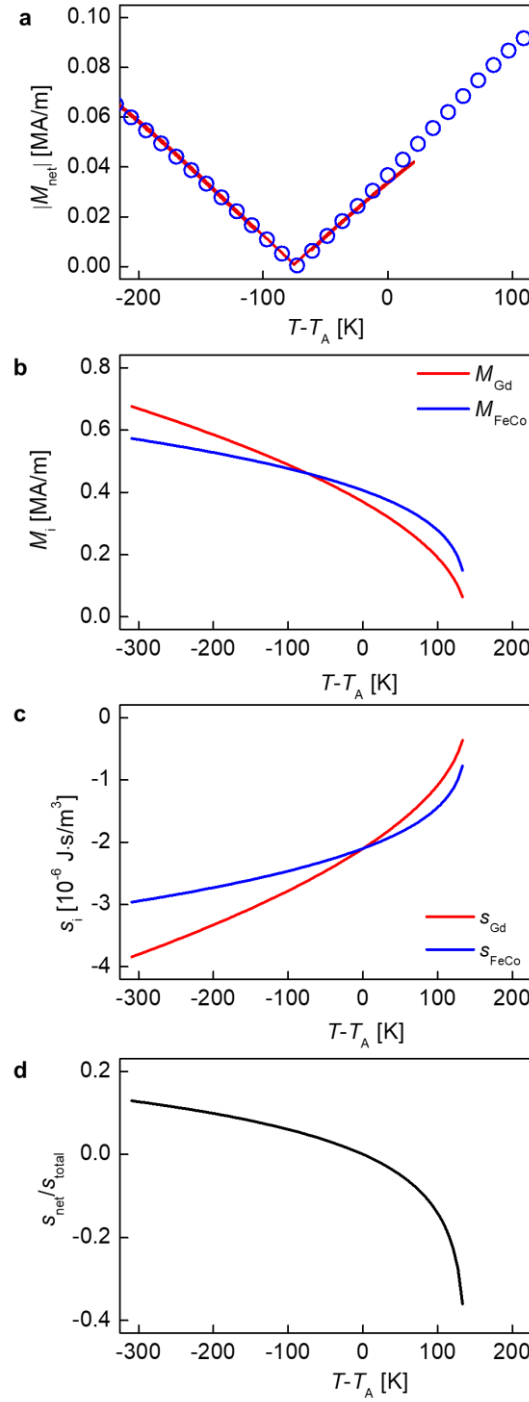


Figure S5. Temperature dependence of magnetic and spin moments. The material parameters such as M_{FeCo} , M_{Gd} , s_{FeCo} , and s_{Gd} are estimated by measuring the net magnetic moment of GdFeCo film. **a**, temperature-dependent $|M_{\text{net}}|$. **b**, the temperature-dependent sub-magnetic moment M_i . **c**, the temperature-dependent sub-angular momentum s_i . **d**, the temperature-dependent the ratio between $s_{\text{net}}/s_{\text{total}}$.

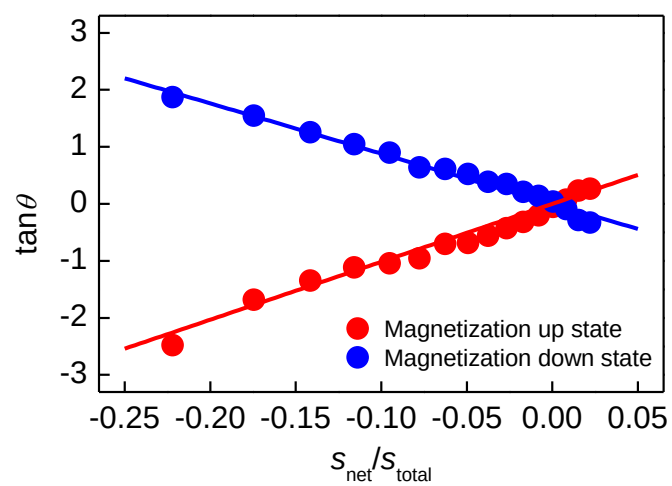


Figure S6. Experimentally obtained elongation angle as a function of $s_{\text{net}}/s_{\text{total}}$.

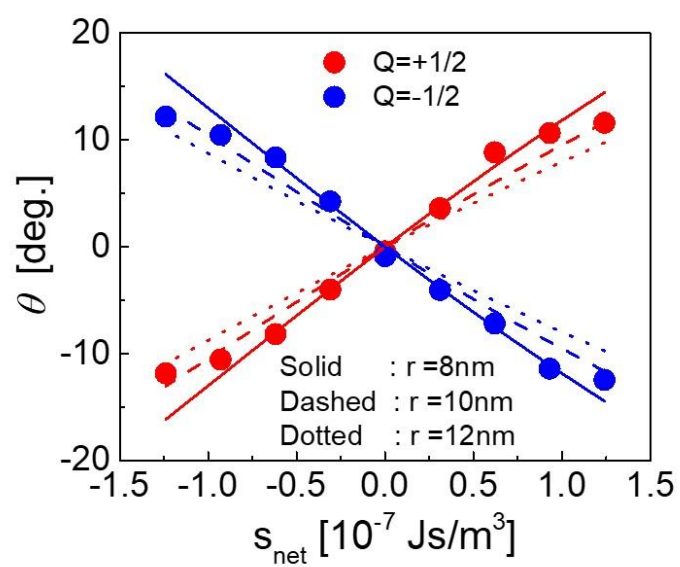


Figure S7. Numerically obtained elongation angle as a function of s_{net} .

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