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A high-performance topological bulk laser based on band-inversion-induced reflection

Zeng-Kai Shao^{1,2,3,6}, Hua-Zhou Chen^{1,2,3,6}, Suo Wang^{1,2,3,6}, Xin-Rui Mao^{1,2,3}, Zhen-Qian Yang^{1,2,3}, Shao-Lei Wang¹, Xing-Xiang Wang^{4,5}, Xiao Hu^{4,5} and Ren-Min Ma^{ 1,2,3*}

¹State Key Lab for Mesoscopic Physics and School of Physics, Peking University, Beijing, China. ²Collaborative Innovation Center of Quantum Matter, Beijing, China. ³Frontiers Science Center for Nano-optoelectronics, Peking University, Beijing, China. ⁴International Center for Materials Nanarchitectonics, National Institute for Materials Science, Tsukuba, Japan. ⁵Graduate school of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Japan. ⁶These authors contributed equally: Zeng-Kai Shao, Hua-Zhou Chen, Suo Wang. *e-mail: renminma@pku.edu.cn

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[†]These authors contributed equally to this work.

*Correspondence should be addressed to renminma@pku.edu.cn

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1. Comparison of topological bulk laser with other representative micro-/nano-scale lasers

Key merits of topological bulk lasers include: (1) Topological bulk cavity provides a novel lasing-mode selection mechanism. The band-inversion induced reflection occurs for a small range of wavevectors around the Γ point, which not only limits the number of cavity modes with efficient confinement but also renders the mode closer to the band edge a higher quality factor. This novel mode selection mechanism gives our laser devices a side mode suppression ratio over 36 dB. For the same cavity size, there are a number of competing modes in a dielectric cavity laser, which is unfavorable for single-mode lasing. We have fabricated a semiconductor disk laser with the same shape and size as adopted in the topological bulk laser, where we see more than 10 lasing modes simultaneously in a sharp contrast to our topological bulk laser (Supplementary Fig. 1, middle row). The outputs from multimode lasers are subject to random fluctuations and instabilities due to mode competition for limited gain. (2) The band-inversion induced reflection enables topological bulk lasers with emission directionality, because it only occurs around the gamma point. One of the essential characteristics of laser device is the ability to produce directional lasing emission. As is well known, a common limitation of microscale laser devices is their poor directionality of the emitted light, which is overcome in our device. (3) Our topological band-inversion induced confinement provides a more efficient feedback than the photonic crystal bandgap induced confinement at a given cladding size, since an efficient feedback from photonic crystal bandgap needs fine fabrication and numbers of periodic structures. This effect has been verified by both experimental and simulation results (Supplementary Figs. 1 and 2).

In order to make a fair comparison of our system with conventional current methods, we have taken two approaches: (1) We compared the performance of laser based on the band-inversion induced confinement to other representative small lasers and commercial laser diodes reported in literatures which is presented in Table 1 below. This comparison is intended to differentiate the performance of our lasers from state-of-the-arts small lasers based on conventional current methods. (2) We also compared the performance of lasers with our band inversion induced confinement to photonic crystal band gap induced confinement and index contrast induced confinement. This comparison is intended to contrast lasing performances based on different confinement mechanisms but with the same gain material, cavity size and shape. These results are presented in Figs. 1 and 2 of Supplementary Materials.

1. Table S1: Comparison of topological bulk laser with other representative small lasers

Reference number	Type	Size	Wavelength	Temperature	Threshold	Linewidth	SMSR	Directionality	Single mode
This work	2D topological bulk laser	Hexagonal side length: ~7.4 μm	~1596 nm	RT	0.00027 kW/cm ² (average) 4.5 kW/cm ² (peak)	0.25 nm	~36 dB	Yes	Yes
[1]	Commercial VCSEL	Diameter: ~ 15 μm	~1550 nm	RT	1.13 kA/cm ²	0.15 nm	35 dB	Yes	Yes
[2]		Diameter: ~ 20 μm	~840 nm		0.32 kA/cm ²	0.4 nm	-		No
[3]	Coupled VCSEL	6 μm×8 μm	~880 nm	RT	-	-	~30 dB	Yes	Yes
[4]		20 μm×24 μm	~862 nm		~ 1.46 kA/cm ²	< 0.5 nm	~20 dB		
[5]	2D topological edge laser	47 μm×43 μm	~1532 nm	RT	0.075 kW/cm ² (average) 22.7 kW/cm ² (peak)	4 nm	11.3 dB	No	Yes
[6]		150 μm×150 μm	~1555.5 nm	—	16 kW/cm ² *	0.64 nm	~10 dB		
[7]	1D topological edge laser	Length: 83 μm	~1523 nm	—	0.032 kW/cm ² (average) 81 kW/cm ² (peak)	~2 nm	~20 dB	No	Yes
[8]		Length: 44 μm	~1412 nm	T=4 K	32.2 kW/cm ² (peak)	0.050 nm	~15 dB		
[9]		Length: 160 μm	~1590 nm	—	26 kW/cm ² *	~4 nm	~10 dB		
[10]	BIC laser	19 μm×19 μm	1551.4 nm	RT	0.014 kW/cm ² (average)	~0.33 nm	—	Yes	Yes
[11]		~55 μm×90 μm	825 nm	T=77 K	0.0014 kW/cm ² (average)	~0.3 nm	~30 dB		
[12]	Photonic crystal nanolaser array	15 μm×15 μm	1534 nm	RT	0.0135 kW/cm ² (average)	0.23 nm	—	No	Yes
[13]		120 μm×120 μm	1580-1590 nm		(0.0013-0.0041) kW/cm ² (average)	0.2 nm	—		No
[14]	Plasmonic nanolaser array	18 μm×18 μm	860 nm	RT	3.33 GW/cm ² (peak)	< 0.5 nm	~0.3 dB	No	No
[15]		~14 mm × 14 mm	882 nm		0.27 GW/cm ² (peak)	~2 nm	—		Yes
[16]		4 mm × 4 mm	877 nm	—	~3.33 GW/cm ² (peak)	~1.6 nm	—		
[17]		~40μm × 40 μm	1478 nm	T=5 K	5.5 kW/cm ²	~2 nm	~ 30 dB		
[18]	Microdisk laser	Diameter: 10 μm	1560 nm	RT	1.9 kA/cm ²	~1 nm	~25 dB	Waveguide coupled	Yes
[19]		Diameter: 16 μm	1557 nm		2.2 kA/cm ²	0.19nm	40 dB		
[20]		Diameter: 27 μm	~ 1082 nm		1.7 kA/cm ²	0.0026nm	25 dB	No	
[21]		Diameter: 10 μm	~ 1080 nm		4.1 kA/cm ²	0.02nm	29.9 dB		
[22]	Photonic crystal nanolaser	Length: ~5 μm	1566 nm	RT	51 kW/cm ² (peak)	~ 0.5 nm	40 dB	No	Yes
[23]		Length: ~1 μm	~1520 nm		0.0054 kW/cm ² (peak)	< 0.4 nm	—		
[24]		Length: 1μm	1556 nm		0.098 kW/cm ² (average)	—	—		
[25]		Length: ~1.5μm	~1535 nm		0.084 kW/cm ² (average)	0.3 nm	< 10 dB		
[26]	2D photonic crystal laser	Diameter: 500 μm	~939 nm	RT	~ 1.0 kA/cm ²	0.007 nm	—	Yes	Yes
[27]	Plasmonic nanolaser	98.9 nm×1.37 μm×2.0 μm	706.4 nm	RT	0.0027 kW/cm ² (average)	0.28 nm	12 dB	No	Yes
[28]		Diameter: 22 nm	528 nm		1.43 kW/cm ² (average)	1-2 nm	< 10 dB		
[29]		1.7 μm×1.15 μm×1.39 μm	1591 nm		~ 100 kW/cm ² (peak)	0.5 nm	—		
[30]		Height: 230 nm Diameter: 400-620 nm	1400 nm	T=78 K	~ 80 kW/cm ² (peak)	~0.5 nm	20 dB		
Control experiments	Photonic bandgap induced confinement (Supplementary Fig. 1)	Hexagonal side length: ~7.4 μm	—	RT	No lasing	—	—	No	No
	Index contrast induced confinement (Supplementary Fig. 1)	Hexagonal side length: ~7.4 μm	Multi-mode (1300-1600 nm)		—	9.3 nm	~0.5 dB		

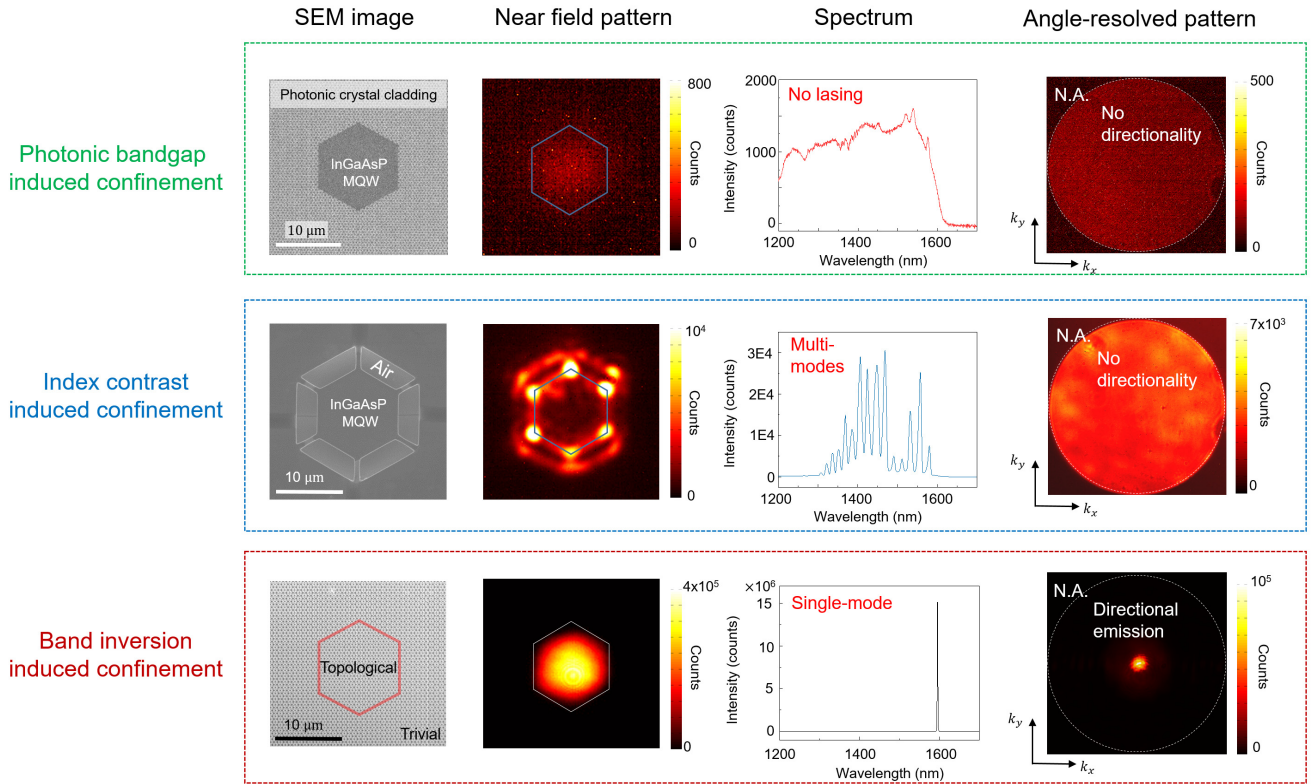
IEEE Standard: 30 dB SMSR is used for 10 Gb/s PMD; 35dB SMSR is used for 40 Gb/s PMD (IEEE Std 802.3TM-2015).
SMSR: Side-mode suppression Ratio; RT: Room temperature; VCSEL: Vertical cavity surface emitting laser

Note 1. The threshold marked with * indicates that the article does not explicitly mention whether it is the peak power density or the average power density.

- [1] Commercial 4 Gbps 1550 nm single mode VCSEL, RayCan Co., Ltd. in Korea, <http://www.raycan.com/>
- [2] Commercial 10 Gbps 850 nm multimode VCSEL, Global Communication Semiconductors, LLC. in California, <http://www.gcsincorp.com/>
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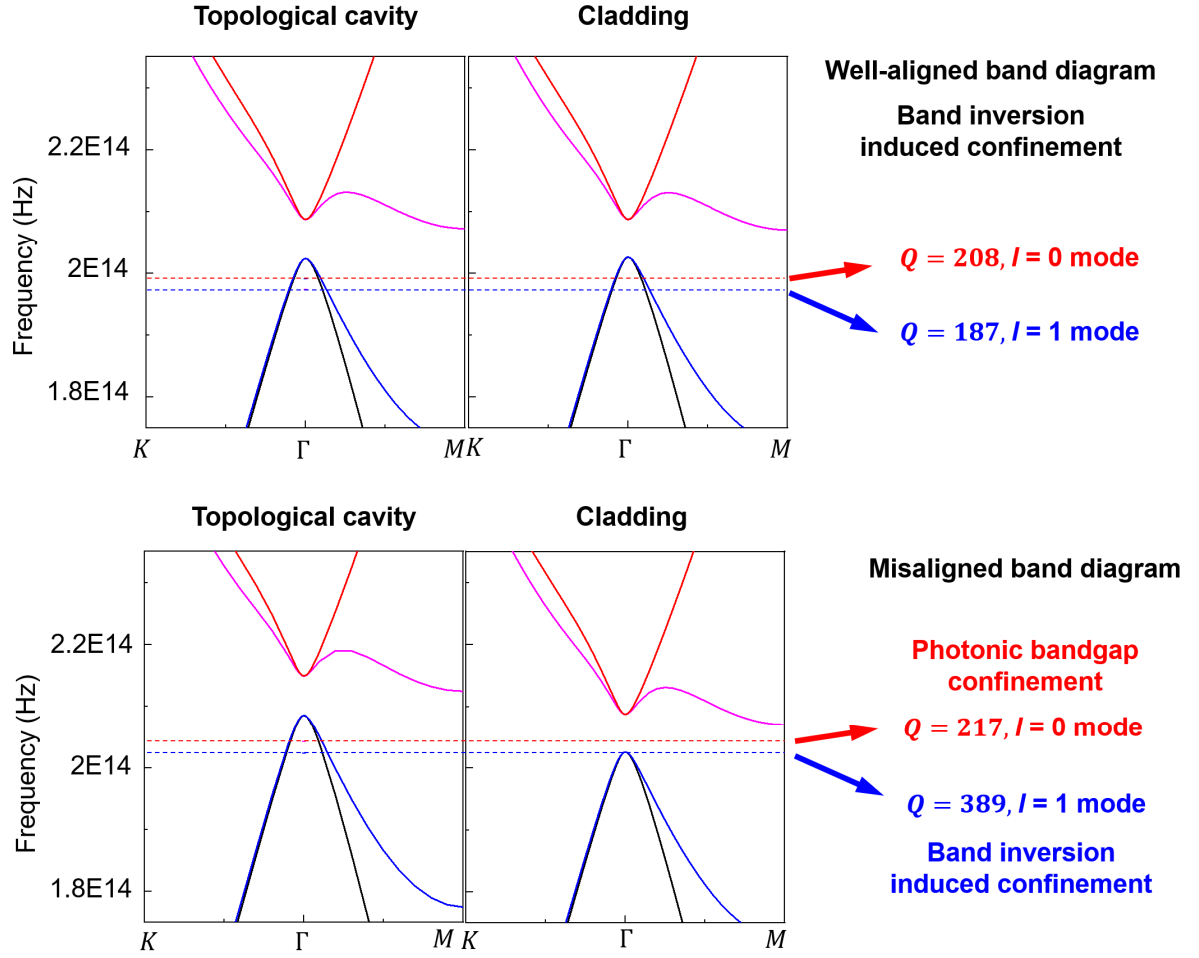
Efficient feedback of topological bulk lasers

Our topological band-inversion induced confinement provides a more efficient feedback than the photonic crystal bandgap induced confinement at a given cladding size, since an efficient feedback from photonic crystal bandgap needs fine fabrication and numbers of periodic structures. We confirm this by the following experiment and simulation. First, we replaced the topological photonic crystal at the center by a uniform hexagonal MQWs disk while keeping the cladding photonic crystal unchanged (1st row, Supplementary Fig. 1). No lasing behavior is observed at any pump power. Second, we have conducted numerical simulations to compare the two confinement mechanisms (Supplementary Fig. 2). The simulation results support our experimental observations. As shown in Supplementary Fig. 2, when the band diagrams of the trivial and topological photonic crystals are well aligned, the $l = 0$ mode exhibits a higher Q factor than that of the $l = 1$ mode. When the band diagrams of the trivial and topological photonic crystals are misaligned, the $l = 0$ mode is confined by the photonic crystal bandgap, while the $l = 1$ mode is still confined by the band-inversion induced reflection, where the calculated quality factor for the $l = 1$ mode is higher than the $l = 0$ mode.



Supplementary Fig. 1 | Comparison among the photonic bandgap induced confinement, index contrast induced confinement and band-inversion induced confinement. With the same cavity size, the photonic crystal cladded laser shows no lasing emission, the index contrast confined disk laser shows multi-modes lasing without emission directionality, whereas the topological bulk laser shows single-mode lasing with nice emission directionality. 1st row: hexagonal MQWs disk cavity cladded by a topological photonic crystal. No lasing behavior is observed for any

pump power. Spontaneous emission spectrum is obtained at a pump power 5 times higher than the threshold of topological bulk laser. **2nd row:** hexagonal MQWs disk laser cladded by air. **3rd row:** topological bulk laser based on the band-inversion induced confinement. Hexagons in SEM image and near field patterns indicate the cavity boundaries. Dashed circles in angle resolved patterns indicate numerical aperture (N.A., 0.42) of the collection objective.



Supplementary Fig. 2 | Comparison between band-inversion induced confinement and photonic crystal bandgap induced confinement. Numerically simulated quality factors of cavity modes in a well-aligned band diagram and a misaligned band diagram. In the well-aligned structure, both $l = 0$ and $l = 1$ modes are confined by the band-inversion induced reflection, and the $l = 0$ mode has a higher quality factor than the $l = 1$ mode, since the $l = 0$ mode is closer to Γ point. In the misaligned structure, $l = 0$ mode experiences the photonic crystal bandgap induced confinement and has a lower quality factor than $l = 1$ mode which is still confined by the band-inversion induced reflection. In these two simulations, the trivial cladding consists of four periods in the radial direction. In the calculation of cavity quality factor, the imaginary part of material refractive index is not considered and the quality factor is calculated via $Q = \text{Re}[\omega] / (2\text{Im}[\omega])$, where $\text{Re}[\omega]$ and $\text{Im}[\omega]$ are the real and imaginary parts of the eigenfrequency of the desired cavity mode.

2. p - d band-inversion and nontrivial photonic topology

A $k \cdot p$ theory can be formulated for Hamiltonian $H_\Gamma(\mathbf{k})$ describing the band structures near Γ point presented in Fig. 2b (also see METHODS):

$$H_\Gamma(\mathbf{k}) = \begin{pmatrix} H_+(\mathbf{k}) & 0 \\ 0 & H_-(\mathbf{k}) \end{pmatrix}, \quad H_\pm(\mathbf{k}) = \begin{pmatrix} M + Bk^2 & Ak_\pm \\ A^*k_\mp & -M - Bk^2 \end{pmatrix} \quad (1.1)$$

where the basis is taken as $[p_+ \ d_+ \ p_- \ d_-]^t$, $k_\pm = k_x \pm ik_y$, $k^2 = k_x^2 + k_y^2$, with the momenta measured from Γ point, $B < 0$, $M = \omega_p^2 - \omega_d^2$, and A is purely imaginary. ω_p and ω_d is the eigenfrequency for p - and d - modes at Γ point. The nontrivial photonic topology can be seen clearly if one rewrites the $k \cdot p$ Hamiltonian (1.1) as

$$H_\pm(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau} \quad (1.2)$$

where $\boldsymbol{\tau}$ are the three Pauli matrices with the fictitious field given by

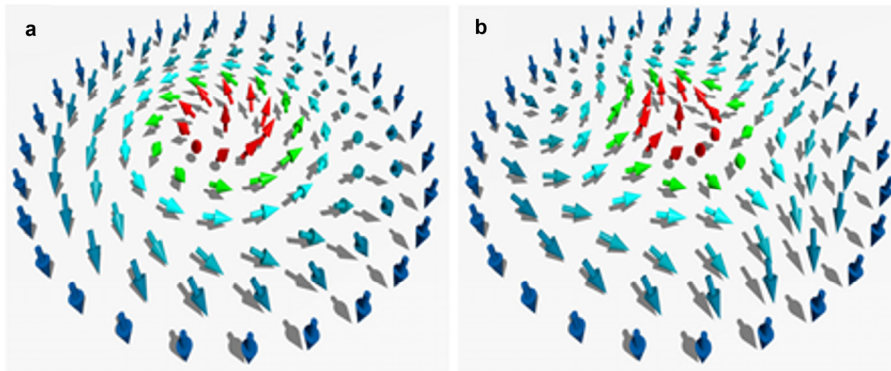
$$d_x = \mp |A|k_y, \quad d_y = -|A|k_x, \quad d_z = M + Bk^2 \quad (1.3)$$

For the case of $BM < 0$, the fictitious field forms two skyrmions in the reciprocal space (Supplementary Fig. 3). The skyrmions carry two topological charges with opposite signs

$$c_\pm = \frac{1}{2\pi} \iint_{\text{BZ}} dk_x dk_y \mathbf{d} \cdot (\partial_{k_x} \mathbf{d} \times \partial_{k_y} \mathbf{d}) = \pm 1 \quad (1.4)$$

Therefore, it is clear that the band inversion between the p and d orbitals in the honeycomb-type lattices gives rise to the nontrivial photonic topology. One can also obtain the same topological charges by evaluating directly the Berry curvature and the Berry phase (*Phys. Rev. Lett.* **114**, 223901 (2015)).

It is noticed that the photonic $k \cdot p$ Hamiltonian (1.1) takes the same form as that given by BHZ model for the quantum spin Hall effect (*Science* **314**, 1757-1761 (2006)), the first known electronic topological state with time-reversal symmetry. As a counterpart of the spin degree of freedom of electron, a pseudospin degree of freedom for the present photonic system can be defined by the direction of the orbital angular momentum.



Supplementary Fig. 3 | Skyrmions in reciprocal space. a-b, for the case of $BM < 0$, two skyrmions are formed in reciprocal space in the pseudospin-up and -down sectors.

3. Tight-binding model on honeycomb lattice

Basing on the tight-binding model (*Phys. Rev. Lett.* **114**, 223901 (2015); *New J. Phys.* **18**, 113013 (2016); *Sci. Rep.* **6**, 24347 (2016)), we can obtain the Hamiltonian H of the honeycomb lattice includes contributions from couplings inside one unit cell H_0 and those between two unit cells H_1 (also see METHODS):

$$H = H_0 + H_1 \quad (2.1)$$

$$H_0 = -t_0 \Psi^\dagger \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \Psi \quad (2.2)$$

$$H_1 = -t_1 \Psi^\dagger \begin{pmatrix} 0 & 0 & 0 & e^{ika_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{ika_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{ik(a_2-a_1)} \\ e^{-ika_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-ika_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-ik(a_2-a_1)} & 0 & 0 & 0 \end{pmatrix} \Psi \quad (2.3)$$

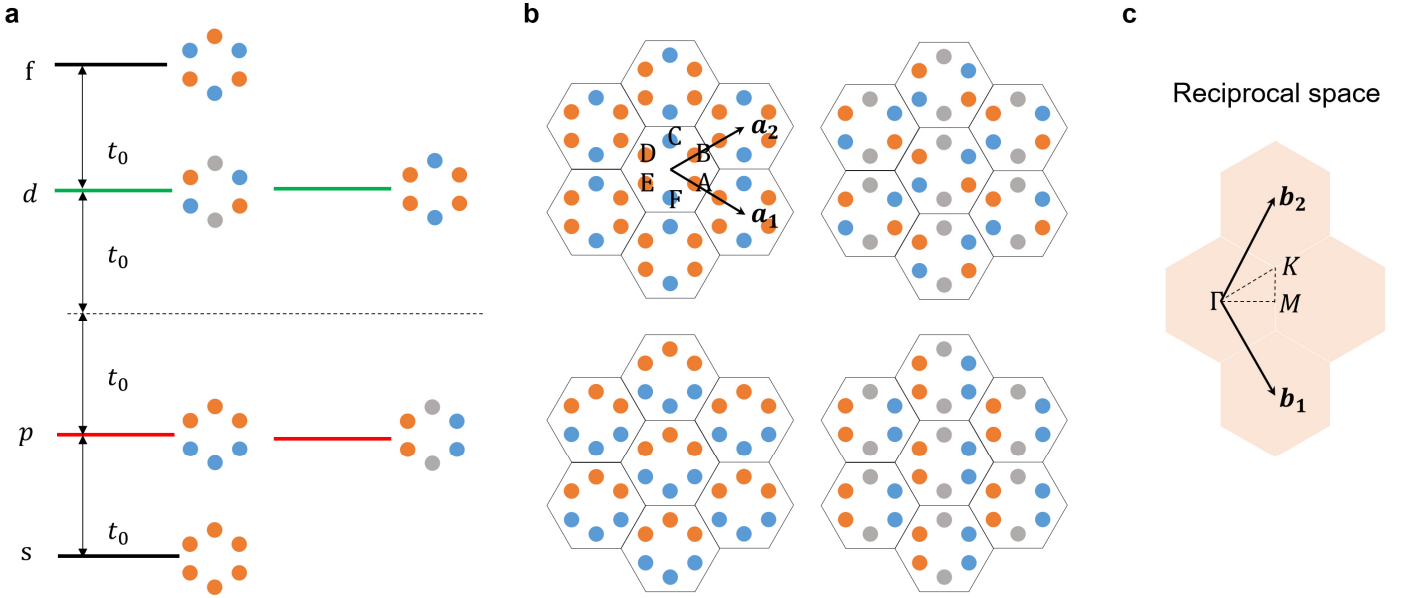
where t_0 and t_1 describe the intra-unit cell and inter-unit cell coupling strength respectively. $\mathbf{a}_1$ and $\mathbf{a}_2$ are the primitive lattice vectors (Fig. S4b). $\mathbf{k}$ is the momenta measured from Γ point. Ψ includes basic states describing the Wannier functions located at the six different sites A, B...F for our system, which are denoted by:

$$|A\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |B\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \dots |F\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (2.4)$$

Using the eigenstates of H_0 as the new basis, the Hamiltonian describing the interactions between p - and d -modes at Γ point can be written as:

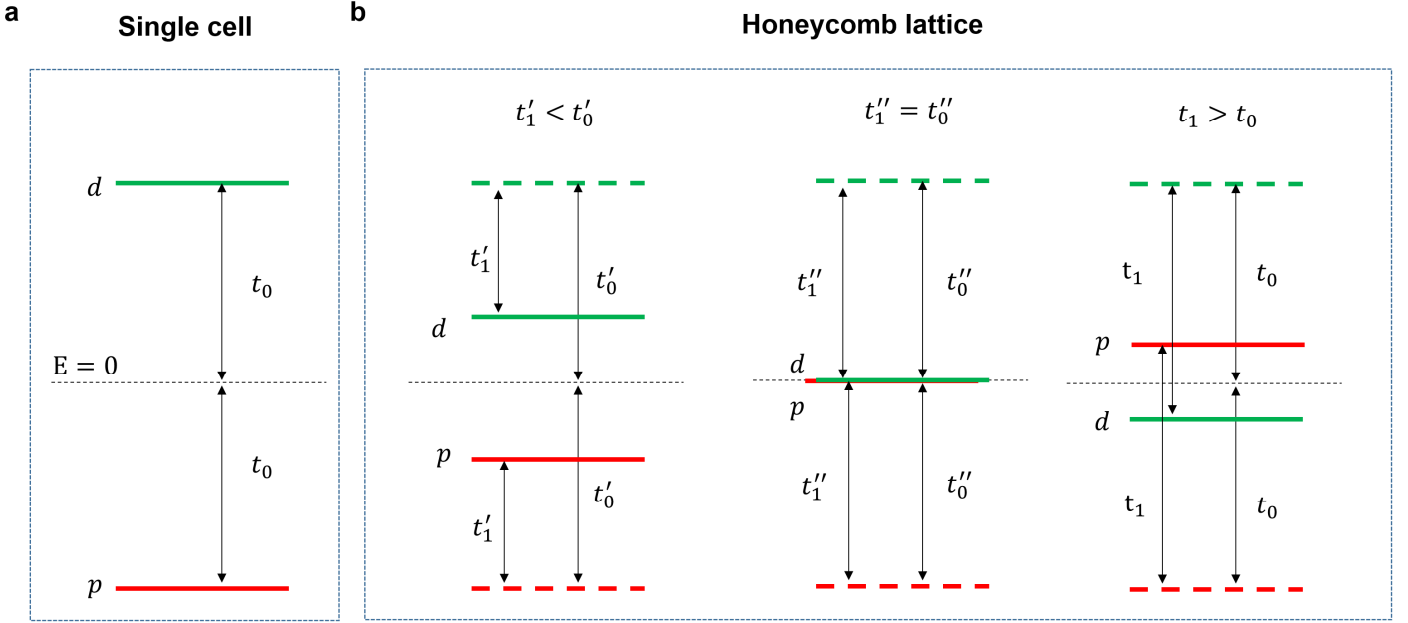
$$H = \begin{pmatrix} -t_0 + t_1 & 0 & 0 & 0 \\ 0 & -t_0 + t_1 & 0 & 0 \\ 0 & 0 & t_0 - t_1 & 0 \\ 0 & 0 & 0 & t_0 - t_1 \end{pmatrix}, \quad (2.5)$$

In the presence of inter-cell couplings, at Γ point the eigen energy of the two p -modes shifts to $-t_0 + t_1$, while the eigen energy of the two d -modes shifts to $t_0 - t_1$ (Supplementary Fig. 4).



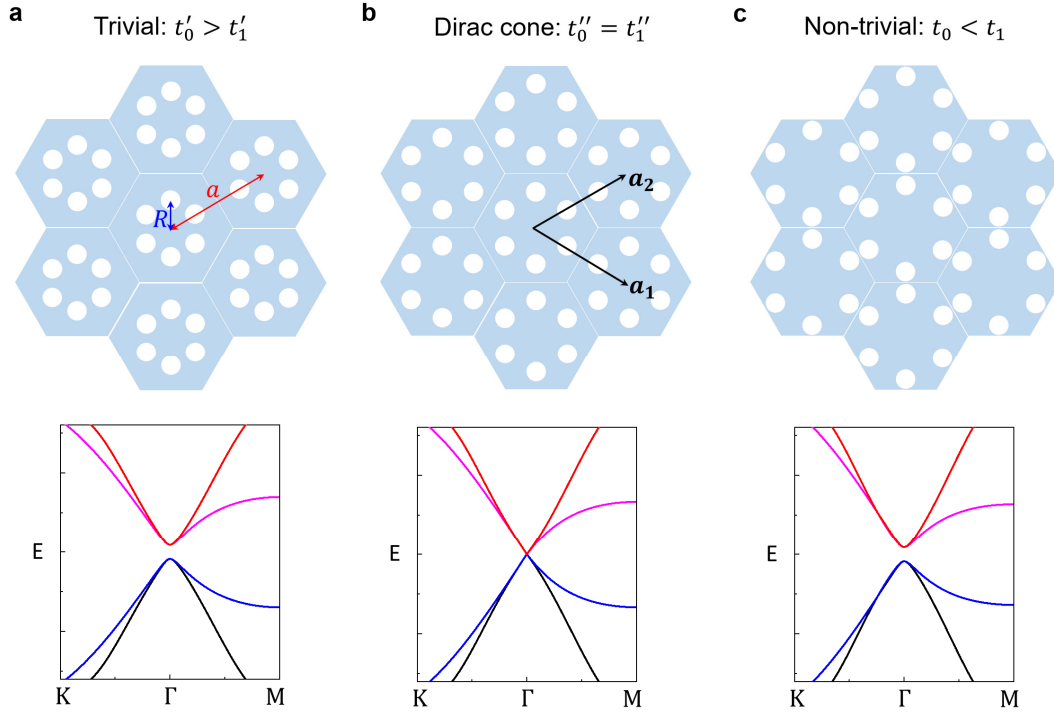
Supplementary Fig. 4 | Eigenmodes accommodated in honeycomb lattice. **a**, Schematic of the energy levels of a single unit cell which is composed by six sites. The eigenvector profiles are also shown, where the blue and orange colors indicate positive and negative values of the wavefunction. **b**, Schematics highlighting the inter-unit cell coupling at Γ point of d -modes (upper panel) and p -modes (lower panel) for honeycomb lattice. $\mathbf{a}_1$ and $\mathbf{a}_2$ indicate the primitive lattice vectors. The labels A, B, C, D, E, F denote each lattice site within a unit cell. **c**, Reciprocal space of the honeycomb lattice with $\mathbf{b}_1$ and $\mathbf{b}_2$ indicating the two reciprocal lattice vectors.

Supplementary Fig. 5 shows the schematics of energy spectra of the p - and d -modes in a single hexagonal unit cell and honeycomb lattice at Γ point, respectively. In a single cell, d -modes take a higher energy than p -modes. In normal honeycomb lattice, the inter- and intra-unit cell couplings are equal, and d - and p -modes become degenerate (middle one in Supplementary Fig. 5b). When the inter-unit cell coupling is weaker than the intra-unit cell coupling (i.e. $t_0' > t_1'$), there is no band inversion and the lattice possesses a trivial band structure (left one in Supplementary Fig. 5b). When the inter-unit cell coupling is stronger than the intra-unit cell coupling (i.e. $t_0 < t_1$), a band inversion takes place at Γ point, where p -modes take a higher energy (right one in Supplementary Fig. 5b), resulting in a topological band structure.



Supplementary Fig. 5 | Band-inversion in honeycomb lattice at Γ point. **a**, Schematic of the energy levels of the p - and d -modes supported in a single hexagonal unit cell. **b**, Schematics of the variations of energy levels of p - and d -modes in the presence of the inter-unit cell coupling in honeycomb lattice. Because of inter-unit cell couplings as shown in Supplementary Fig. 4, the energy of p - (d -) modes increases (decreases) by the amount of t_1 . When the inter-unit cell coupling t'_1 is smaller than the intra-unit cell coupling t'_0 , d -modes take a higher energy than p -modes, forming the trivial band structure (left). When the inter- and intra-unit cell couplings are equal (i.e. $t''_1 = t''_0$), d - and p -modes become degenerate in energy (middle). When the inter-unit cell coupling t_1 is larger than the intra-unit cell coupling t_0 , p -modes take a higher energy than d -modes, leading to the topological band structure (right).

The intra- and inter-unit cell couplings in honeycomb lattice can be tuned by changing the distance R between the centroid of each hole and the center of unit cell. As shown in Supplementary Fig. 6b, in normal honeycomb lattice, $R = a/3$, the distances between all sites are the same, leading to degenerate d - and p -modes and a Dirac-cone like dispersion. Reducing the distance R below $a/3$ results in a larger intra-unit cell coupling (i.e. $t'_0 > t'_1$), which opens a trivial bandgap around Γ point (Supplementary Fig. 6a). A topological bandgap can be obtained when R is larger than $a/3$ (Supplementary Fig. 6c) where a band inversion takes place.



Supplementary Fig. 6 | Deformed honeycomb lattice with trivial and topological band structures. **a**, To realize a trivial bandgap with $t'_0 > t'_1$, the cluster of the six holes is shrunk ($R < a/3$), where a is the lattice constant of the triangular lattice of hexagonal unit cells, and R is the distance between the center of unit cell to the center of individual holes. **b**, Doubly degenerate Dirac cones formed by d - and p -modes appear when $R = a/3$ with $t''_0 = t''_1$. $\mathbf{a}_1$ and $\mathbf{a}_2$ indicate the primitive lattice vectors. **c**, The cluster is expanded ($R > a/3$) to realize a topological bandgap with $t_0 < t_1$.

4. Cavity modes in a topological bulk laser

Starting from the Hamiltonian given in the main text, we can derive the dispersion relation where two p modes and two d modes appear around the bandgap around Γ point. Using the $k \cdot p$ approach and taking the basis $\{|p_+\rangle, |d_+\rangle, |p_-\rangle, |d_-\rangle\}$, we can derive a 4×4 effective Hamiltonian, which is block diagonalized into a pair of 2×2 Hamiltonians

$$H_{\pm} = \begin{bmatrix} -(t_0 - t_1) - \frac{1}{2}a^2t_1k^2 & -\frac{i}{2}at_1k_{\pm} \\ \frac{i}{2}at_1k_{\mp} & t_0 - t_1 + \frac{1}{2}a^2t_1k^2 \end{bmatrix} \quad (3.1)$$

with $k_{\pm} = k_x \pm ik_y$, $k^2 = k_x^2 + k_y^2$, where the lowest-order terms of momentum relevant to the topological property are taken into account. k_x and k_y are eigen wavenumbers in the direction of $\mathbf{a}_1 + \mathbf{a}_2$, $\mathbf{a}_2 - \mathbf{a}_1$ respectively, which form the two orthogonal coordinates (Supplementary Fig. S4b). a is the common lattice constant of the two photonic crystals. For the case that a trivial photonic crystal is at the center of the device, one has $t_0 > t_1$ and at Γ point the lower and higher energy state is $|p_+\rangle$ and $|d_+\rangle$, respectively. Let us first concentrate on the pseudospin-up sector.

In order to perform perturbation analysis, we define

$$H_+ = H_+^0 + H_+' \quad (3.2)$$

with the energies at Γ point

$$H_+^0 = \begin{bmatrix} -(t_0 - t_1) & 0 \\ 0 & t_0 - t_1 \end{bmatrix}, \quad (3.3)$$

and the perturbation due to finite momenta

$$H_+' = \begin{bmatrix} -\frac{1}{2}a^2t_1k^2 & -\frac{i}{2}at_1k_{\pm} \\ \frac{i}{2}at_1k_{\mp} & \frac{1}{2}a^2t_1k^2 \end{bmatrix}. \quad (3.4)$$

The two perturbed energies are given as

$$\begin{aligned} E_1 &= -(t_0 - t_1) + \langle p_+ | H_+' | p_+ \rangle + \frac{|\langle d_+ | H_+' | p_+ \rangle|^2}{-2(t_0 - t_1)} \\ &= -(t_0 - t_1) - \frac{1}{2}a^2t_1k^2 - \frac{a^2t_1^2}{8(t_0 - t_1)}k^2, \end{aligned} \quad (3.5)$$

and

$$E_2 = t_0 - t_1 + \frac{1}{2}a^2t_1k^2 + \frac{a^2t_1^2}{8(t_0 - t_1)}k^2 \quad (3.6)$$

up to the order of k^2 . The perturbed wavefunctions are

$$\begin{aligned} |1, +\rangle &= |p_+\rangle + \frac{\langle d_+ | H'_+ | p_+ \rangle}{-2(t_0 - t_1)} |d_+\rangle \\ &= |p_+\rangle - \frac{iat_1k_-}{4(t_0 - t_1)} |d_+\rangle, \end{aligned} \quad (3.7)$$

and

$$|2, +\rangle = |d_+\rangle - \frac{iat_1k_+}{4(t_0 - t_1)} |p_+\rangle, \quad (3.8)$$

up to the order of k .

In terms of the basis $\{|1, +\rangle, |2, +\rangle\}$, we have the following eigenvalue equation

$$\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \begin{bmatrix} \psi_{1,+} \\ \psi_{2,+} \end{bmatrix} = \lambda \begin{bmatrix} \psi_{1,+} \\ \psi_{2,+} \end{bmatrix}, \quad (3.9)$$

Now we consider a circular cavity of radius r_c formed by the trivial photonic crystal surrounded by a topological one, and compute the real-space wavefunctions inside the cavity. For this purpose, we replace k^2 in the perturbed energies E_1 and E_2 by the operator $k^2 \rightarrow -\nabla^2$. Because we are concentrating on the band edge below the bandgap, the first component in Eq. (3.9) should be picked up and one obtains

$$\nabla^2\psi_{1,+}(r, \phi) + k^2\psi_{1,+}(r, \phi) = 0, \quad (3.10)$$

where $\psi_{1,+}(r, \phi)$ is the profile wavefunction of the state $|1, +\rangle$ for the unit cell located at r and the azimuthal angle ϕ measured from the center of the circular cavity, and the eigen wavenumber k is given by the eigen energy λ via (see Eq. (3.5))

$$k^2 \equiv \frac{8[-(t_0 - t_1) - \lambda](t_0 - t_1)}{a^2t_1(4t_0 - 3t_1)}. \quad (3.11)$$

We are looking for solutions of Eq. (3.10) with $k^2 \geq 0$, which should exist since the dispersion curve below (above) the bandgap is convex upward (downward) as seen in Fig. 2B in the main text.

Eq. (3.10) is the standard Helmholtz equation, and the solution regular at the center of the system should be

$$\psi_{1,+}(r, \phi) = J_l(kr)e^{il\phi}, \quad (3.12)$$

where $J_l(kr)$ is the Bessel function of the first kind.

For the pseudospin-down sector, we have similarly

$$|1, -\rangle = |p_-\rangle - \frac{iat_1 k_+}{4(t_0 - t_1)} |d_-\rangle, \quad (3.13)$$

$$|2, -\rangle = |d_-\rangle - \frac{iat_1 k_-}{4(t_0 - t_1)} |p_-\rangle. \quad (3.14)$$

The wavefunction for the pseudospin-down state $|1, -\rangle$ is

$$\psi_{1,-}(r, \phi) = J_l(kr)e^{il\phi}, \quad (3.15)$$

which is the same as that for the pseudospin-up state $|1, +\rangle$.

Due to the mismatch between the symmetries of wavefunctions in the trivial and topological photonic crystals, the wavefunction inside the cavity should be suppressed to zero at the interface, namely $J_l(kr_c) = 0$. Therefore, the momentum k can only take discrete values given by the zeros of the Bessel functions

$$\begin{aligned} k_0 &= 2.4048/r_c \text{ for } l = 0, \\ k_1 &= 3.8317/r_c \text{ for } l = 1, \\ k_2 &= 5.1356/r_c \text{ for } l = 2, \end{aligned} \quad (3.16)$$

and so on, which gives discrete eigen frequencies.

For the case that the topological photonic crystal is at the center of the device, which corresponds to $t_0 - t_1 < 0$ in Eq. (3.1), the cavity modes are described by the second component in Eq. (3.9)

$$\nabla^2 \psi_{2,+}(r, \phi) + k^2 \psi_{2,+}(r, \phi) = 0, \quad (3.17)$$

with

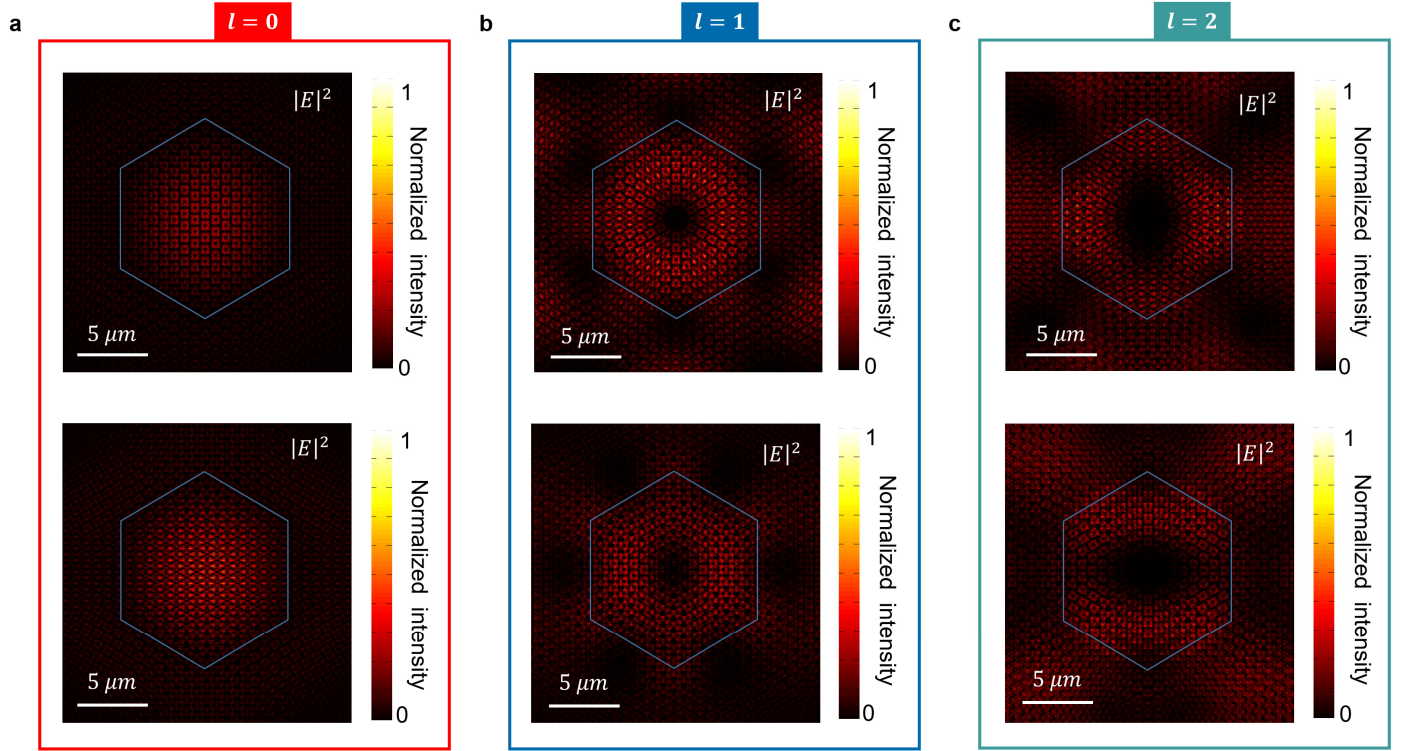
$$k^2 \equiv \frac{8[\lambda - (t_0 - t_1)](t_0 - t_1)}{a^2 t_1 (4t_0 - 3t_1)}. \quad (3.18)$$

The wavefunctions for the up and down pseudospin sectors are $|2, +\rangle$ in Eq. (3. 8) and $|2, -\rangle$ in Eq. (3. 14) with the profiles of cavity modes

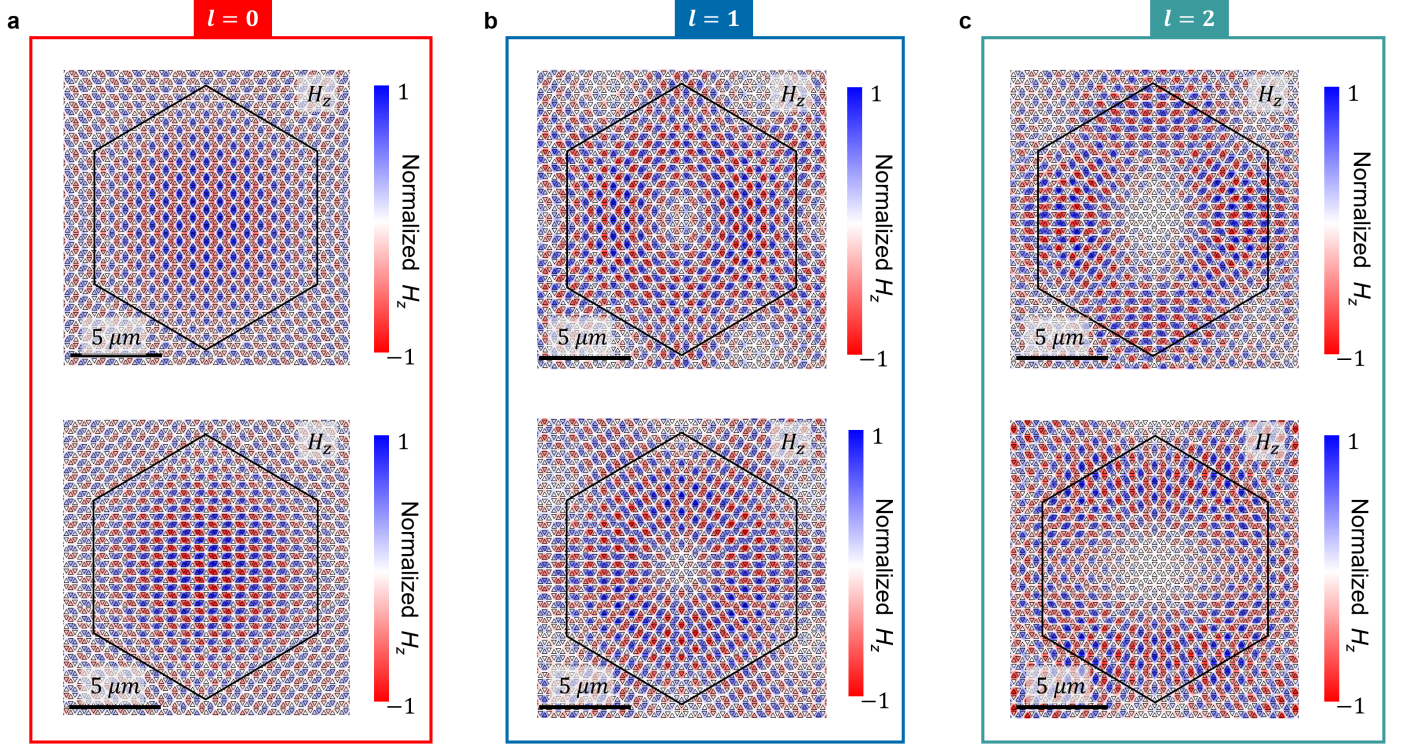
$$\psi_{2,+}(r, \phi) = \psi_{2,-}(r, \phi) = J_l(kr)e^{il\phi}. \quad (3.19)$$

It should be noticed that the position and the azimuthal angle ϕ in the profile wavefunction given by the Bessel function and the phase winding are measured with respect to the center of the system and refer to centers of individual hexagonal unit cells. The total wavefunction is also contributed from the pseudospin state which indicates the local phase winding in individual unit cells, in a stark contrast to conventional disc cavity.

The intensity and phase patterns of the three modes ($l = 0, 1, 2$) with high quality factor are shown in Supplementary Figs. 7 and 8.

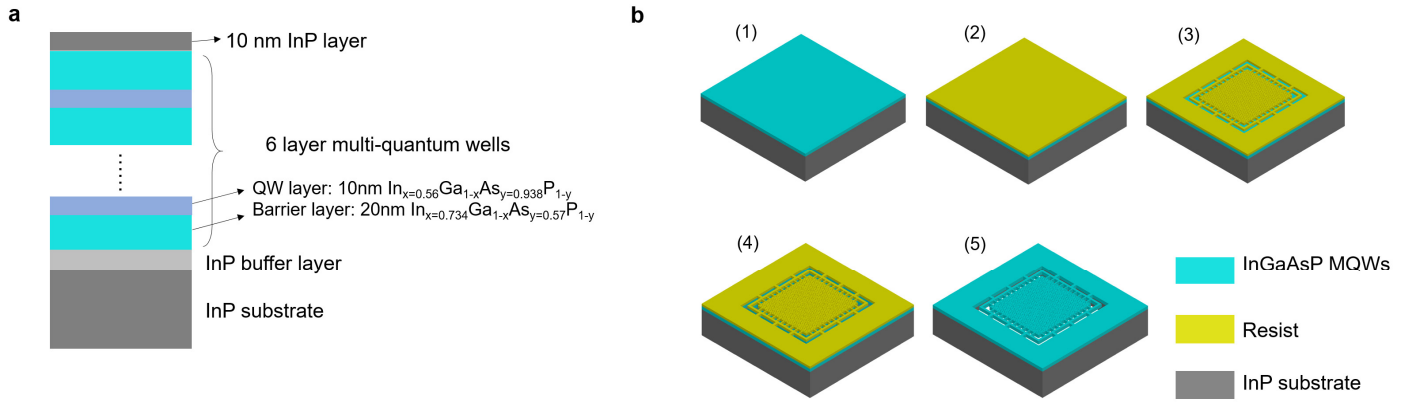


Supplementary Fig. 7 | Simulated intensity patterns for topological bulk laser modes of $l = 0$, 1 , and 2 for the in-plane electric field. **a, Field patterns of $l = 0$ modes. **b**, Field patterns of two $l = 1$ cavity modes. **c**, Field patterns of two $l = 2$ cavity modes. The blue hexagons indicate the topological cavity areas.**



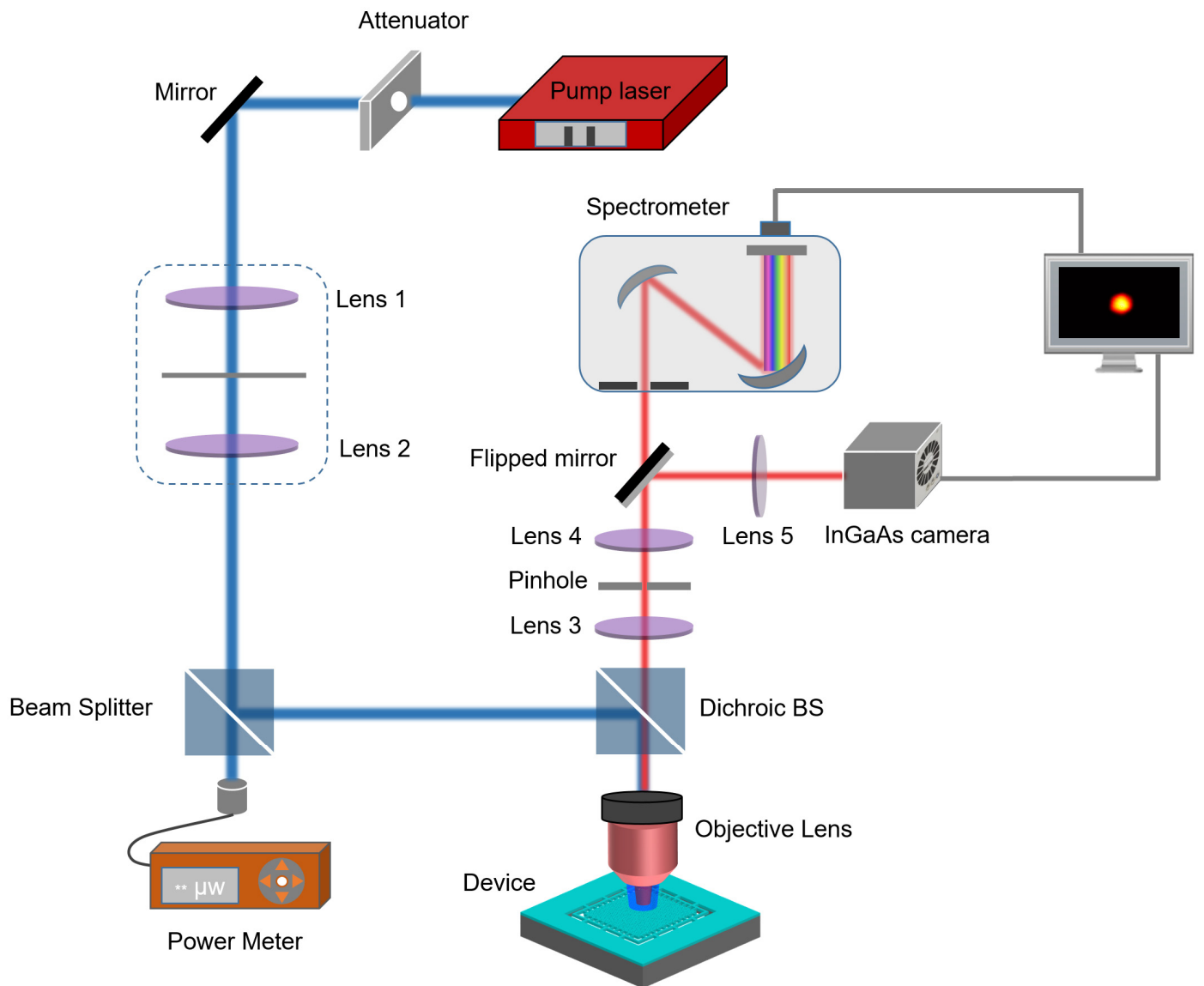
Supplementary Fig. 8 | H_z -field distributions for modes of $l = 0, 1$, and 2 depicted in Supplementary Fig. 7. **a, H_z -field distributions for two $l = 0$ modes. All unit nanocavity cells resonate in phase. **b**, H_z -field distributions for two $l = 1$ cavity modes. The d -modes within a unit cell rotate 2π along a closed circle. **c**, H_z -field distributions for two $l = 2$ cavity modes. The d -modes within a unit cell rotate 4π along a closed circle. The black hexagons indicate the topological cavity areas.**

5. Device Fabrication



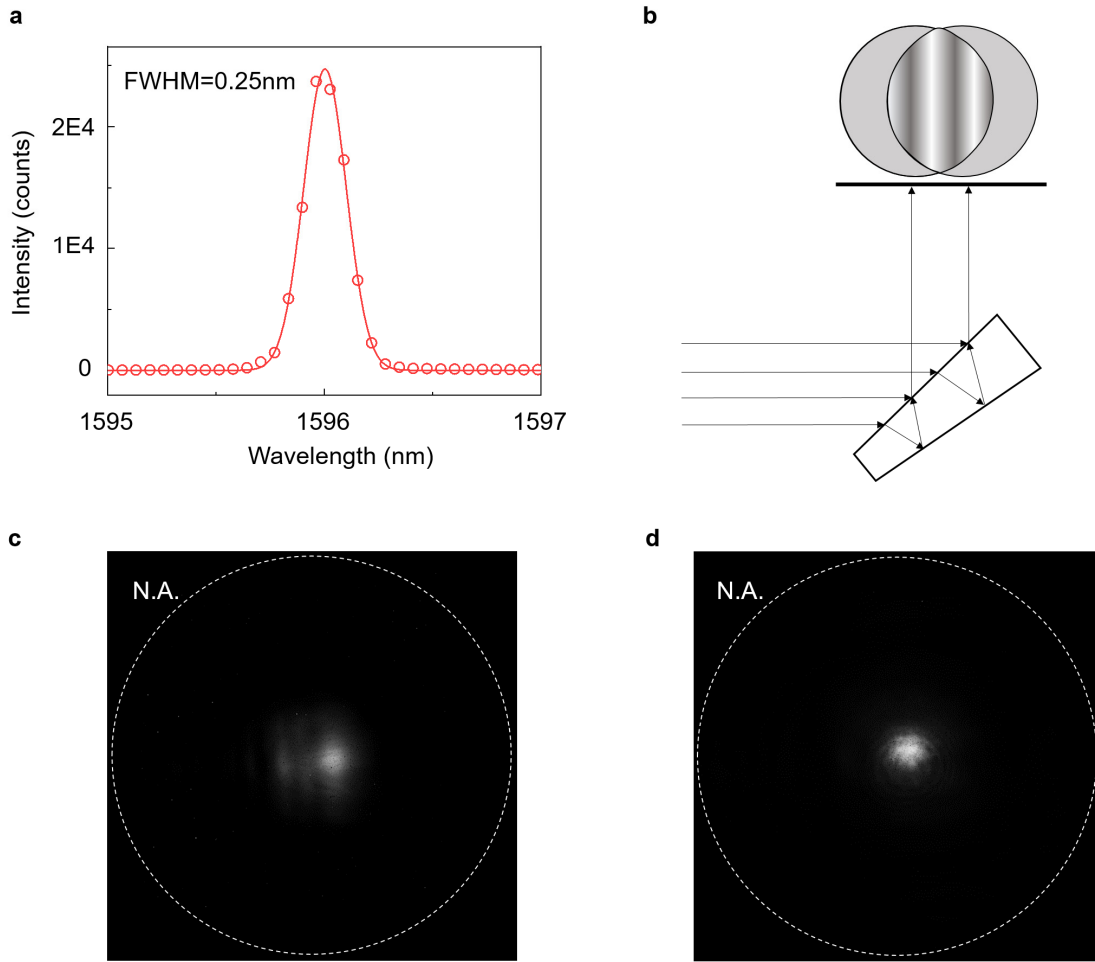
Supplementary Fig. 9 | Device fabrication. **a**, Cross-section of the InGaAsP MQWs wafer. **b**, Fabrication process of topological bulk laser: (1) pre-cleaning the wafer, (2) spin-coating e-beam resist, (3) writing pattern on the resist by e-beam lithography, (4) etching through the 200 nm MQWs by inductively coupled plasma etching, (5) wet etching InP substrate to form a membrane structure.

6. Optical characterization



Supplementary Fig. 10 | Experimental setup for lasing characterization. The setup consists three systems: the pump system on top, the collection system with mounted device on the lower right, and the analysis system with the real-space imaging and spectrum analysis on the top right.

7. Lasing linewidth and coherence of the topological bulk laser



Supplementary Fig. 11 | Coherence of the laser emission. **a**, Lasing linewidth characterized by a spectrometer with high spectral resolution, which gives a full width at half maximum (FWHM) of 0.25 nm. Dots: data, line: fitting. **b**, The emission beam of the topological bulk laser is guided to a piece of glass to form equal thickness interference. **c**, The interference pattern of the far field emission beam. **d**, The far field emission beam without interference of the same laser device depicted in panel b. The dashed circles in (c) and (d) indicate numerical aperture (N.A., 0.42) of the collection objective.

8. Novel lasing-mode selection mechanism in topological bulk lasers

There is a novel lasing-mode selection mechanism in our topological bulk laser. In the present honeycomb-type photonic crystals, the band inversion between p -orbital and d -orbital occurs in a small range of wavevectors around Γ point, which limits the number of cavity modes supported by efficient confinement, and renders a higher quality factor for the mode closer to the band edge at Γ point. This phenomenon has been verified by simulations as shown in Supplementary Fig. 12.

Couplings between the bulk modes and the topological interface modes provides further lasing-mode selection, as can be understood from the detailed wavefunctions. As addressed in Supplementary Part 3, associated with the quantum number of orbital angular momentum $l = 1$, there are four degenerate states $J_1(k_1 r)e^{\pm i\phi}|d_{\pm}\rangle$, where for clarity the small contributions from $|p_{\pm}\rangle$ modes are not shown explicitly since we are interested in the vicinity of Γ point below the bandgap. On the other hand, due to the pseudospin-momentum locking in the quantum pseudospin Hall effect, the topological interface states are given exclusively by $e^{i\phi}|d_{+}\rangle$ and $e^{-i\phi}|d_{-}\rangle$, where small contributions from $|p_{\pm}\rangle$ modes are not shown for the same reason as for the bulk modes. The two interface states couple with two bulk states $J_1(k_1 r)e^{i\phi}|d_{+}\rangle$ and $J_1(k_1 r)e^{-i\phi}|d_{-}\rangle$, which degrades their confinements and excludes them from cavity modes formed by the band-inversion induced reflection. When time-reversal symmetry preserves, the two remaining cavity modes $J_1(k_1 r)e^{i\phi}|d_{+}\rangle$ and $J_1(k_1 r)e^{-i\phi}|d_{-}\rangle$ form standing waves,

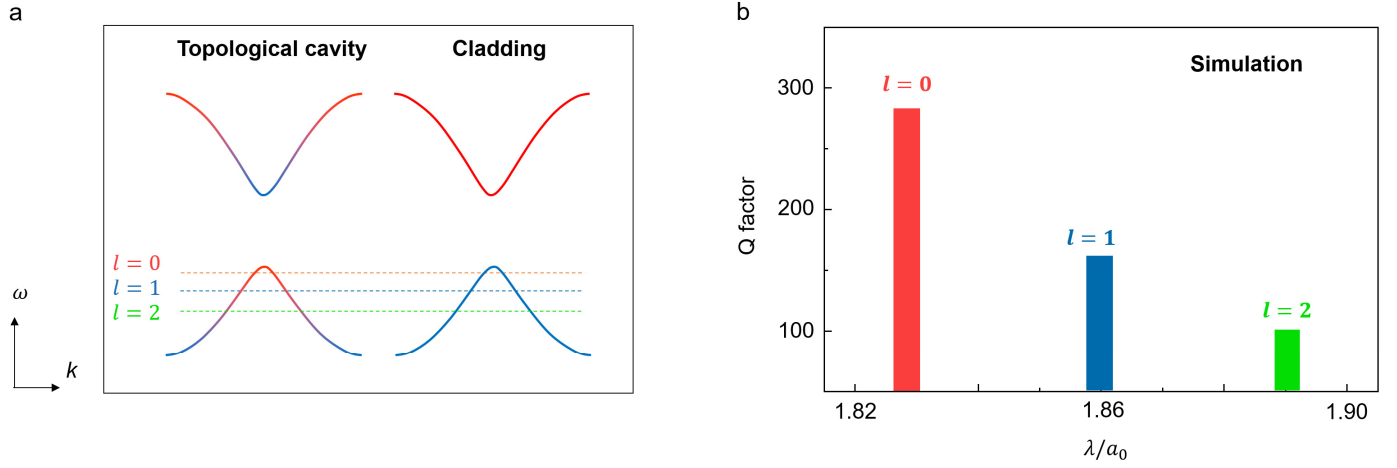
$$\psi_{l=1}^{(1)}(r, \phi) = J_1(k_1 r)(e^{-i\phi}|d_{+}\rangle + e^{+i\phi}|d_{-}\rangle) \quad (4.1)$$

$$\psi_{l=1}^{(2)}(r, \phi) = -iJ_1(k_1 r)(e^{-i\phi}|d_{+}\rangle - e^{+i\phi}|d_{-}\rangle) \quad (4.2)$$

It is clear that the two bulk cavity modes $J_0(k_0 r)|d_{xy}\rangle$ and $J_0(k_0 r)|d_{x^2-y^2}\rangle$ associated with $l = 0$ do not couple with the topological interface modes.

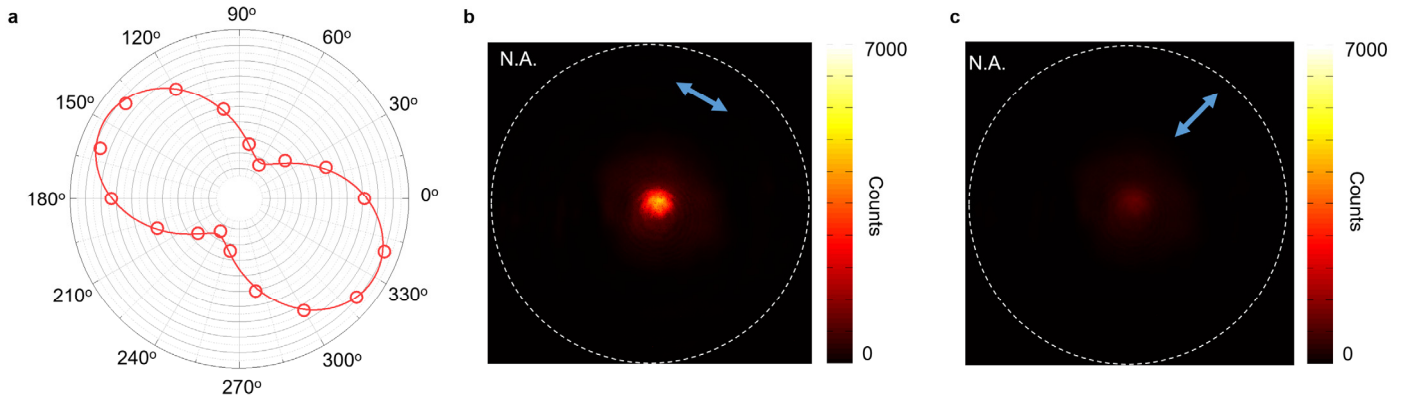
As for the laser performance, this novel lasing-mode selection mechanism with topology origin gives our laser devices a side mode suppression ratio over 36 dB, which is about 16 dB higher than all the topological lasers reported so far. As a further comparison, we have fabricated a dielectric disk laser with the same cavity size of the topological bulk laser, which shows more than 10 modes lasing simultaneously (2nd row Supplementary

Fig. 1). The outputs from multimode lasers are subject to random fluctuations and instabilities because of mode competition for limited gain.



Supplementary Fig. 12 | Lasing-mode selection mechanism in topological bulk laser. **a**, Schematic of band structures in topological and trivial photonic crystals. A mode with resonant frequency closer to the band edge experiences more efficient band-inversion induced confinement, since the band inversion only occurs in a small range of wavevectors around Γ point. **b**, Averaged quality factors for the degenerate $l = 0, 1$ and 2 topological bulk laser modes obtained by simulations, where the trivial cladding photonic crystal consists of eleven periods in the radial direction. In the calculation of cavity quality factor, the imaginary part of material refractive index is not considered.

9. Emission polarization of the topological bulk laser



Supplementary Fig. 13 | Polarization of the emission beam. **a**, Normalized emission intensity as a function of the polarization angle, using a linear polarizer placed in front of the CCD. Circles: data, line: fitting. **b-c**, Two representative polarization resolved emission patterns in momentum space. The arrows in (b) and (c) indicate the polarization directions of the linear polarizer placed in front of the CCD. The dashed circles in (b) and (c) indicate numerical aperture (N.A., 0.42) of the collection objective.