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Ultrasensitive torque detection with an optically levitated nanorotor

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Supplementary Information for "Ultrasensitive torque detection with an optically levitated nanorotor"

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1 Rotational vacuum friction.

Quantum vacuum friction due to electromagnetic fluctuations in vacuum has been theoretically studied by Pendry [1] and other researchers [2, 3, 4]. Considering two parallel surfaces with relative motion, the electromagnetic fluctuation will induce instantaneous charges on the surfaces which try to slow down the moving object. The same mechanism works for a rotating sphere near a planar surface. The electromagnetic fluctuations generate a frictional torque that tries to slow down the rotation of the sphere. Because the vacuum friction is extremely small, no experiment has been carried out to measure it yet. Our levitated optomechanical system provides a great platform to measure the vacuum friction. The calculation of the vacuum frictional torque experienced by a rotating sphere is shown below.

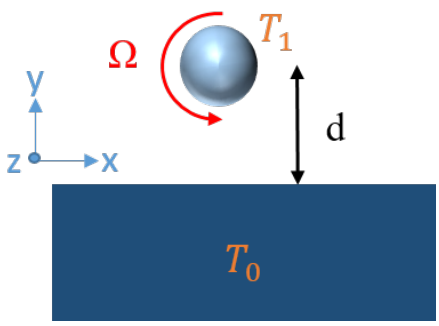
Assume that there is a silica sphere of radius R rotating at a frequency Ω along z axis near a planar surface as shown in Supplementary Fig. 1, the sphere will experience a vacuum friction which tries to slow down the rotation of the sphere. The distance between the center of the sphere and the surface is d. The temperatures of the sphere and the surface are T_1 and T_0, respectively. Adapting the theory presented in [2, 3], the vacuum frictional torque acting on the sphere along the z-axis is obtained as

$$M_{vacuum} = -\frac{2\hbar}{\pi}\int_{-\infty}^{\infty}[n_1(\omega-\Omega)-n_0(\omega)]\mathrm{Im}\{\alpha(\omega-\Omega)\}\mathrm{Im}\bar{G}(\omega)d\omega \quad (1)$$

where $n_j(\omega) = [\exp(\hbar\omega/k_B T_j)-1]^{-1}$ is the Bose-Einstein distribution function at a temperature T_j. $j = 0, 1$ stands for the substrate and the sphere, respectively. $\alpha(\omega)$ is the polarizability of the sphere and $\bar{G}(\omega)$ is the green function to connect the dipole moment fluctuation of the sphere and the induced electromagnetic field on the substrate surface. The relation between the fluctuated dipole moment $\vec{p}^{fl}$ and the induced electromagnetic field $\vec{E}^{ind}$ or the opposite way can be given explicitly as

$$\vec{E}^{ind}(\omega) = \vec{G}(\omega)\vec{p}^{fl}(\omega), \quad (2)$$

$$\vec{p}^{ind}(\omega) = \alpha(\omega)\vec{E}^{fl}(\omega). \quad (3)$$



Supplementary Fig. 1: Sketch of a sphere rotating near a surface. A sphere at temperature T_1 with radius R rotates along the z-axis with angular velocity Ω. Its center is positioned at a distance d from the semi-infinite homogeneous isotropic medium at temperature T_0. The z-axis is parallel to the surface.

Supplementary Table 1: Optical phonon frequencies of three common phonon polaritonic materials.

Material	ϵ_∞	ω_{LO} [cm^{-1}]	ω_{TO} [cm^{-1}]	γ [cm^{-1}]	Reference
SiO_2	2.09	1186	1167	4.43	[5]
		1112	1064	15.53	
		1106	1058	0.42	
		814	799	12.94	
		525	434	54.14	
Si_3N_4	3.90	948	826	6	[5]
		962	925	10	
		1084	1060	6	
		1710	1189	3253	
		21162	17873	0	
SiC	6.70	969	793	4.76	[4]

The Green function used in the calculation is $\bar{G}(\omega) = [G_{xx}(\omega) + G_{yy}(\omega)]/2$, where G_{ij} is the electromagnetic Green tensor. When R and d are much smaller than the wavelength of the electromagnetic fluctuation, the Green tensor components for this near planar surface geometry are obtained as

$$G_{xx}(\omega) = G_{zz}(\omega) = G_{yy}(\omega)/2 = \frac{1}{4\pi\epsilon_0(2d)^3}(\frac{\epsilon_{sub}(\omega)-1}{\epsilon_{sub}(\omega)+1}), \tag{4}$$

where d is the distance between the sphere and the surface and $\epsilon_{sub}(\omega)$ is the dielectric function of the substrate material. The polarizability of the sphere is given as

$$\alpha = 4\pi\epsilon_0 R^3(\frac{\epsilon_{sp}(\omega)-1}{\epsilon_{sp}(\omega)+2}). \tag{5}$$

$\epsilon_{sp}(\omega)$ is the dielectric function of the sphere.

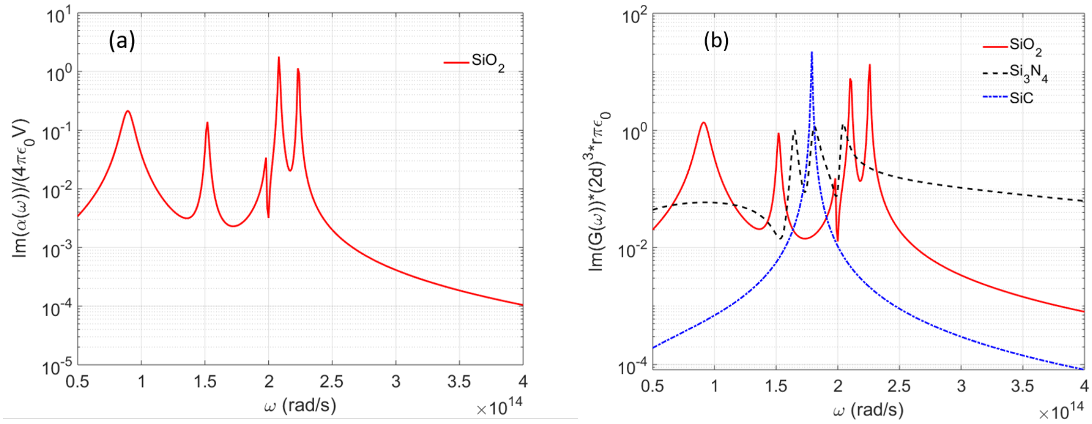
Here the dielectric functions of different materials are modeled as Lorentz oscillators, which treat each discrete vibrational mode as a classical damped harmonic oscillator:

$$\epsilon(\omega) = \epsilon_\infty(1 + \sum \frac{\omega_{LO}^2 - \omega_{TO}^2}{\omega_{TO}^2 - \omega^2 - j\omega\gamma}). \tag{6}$$

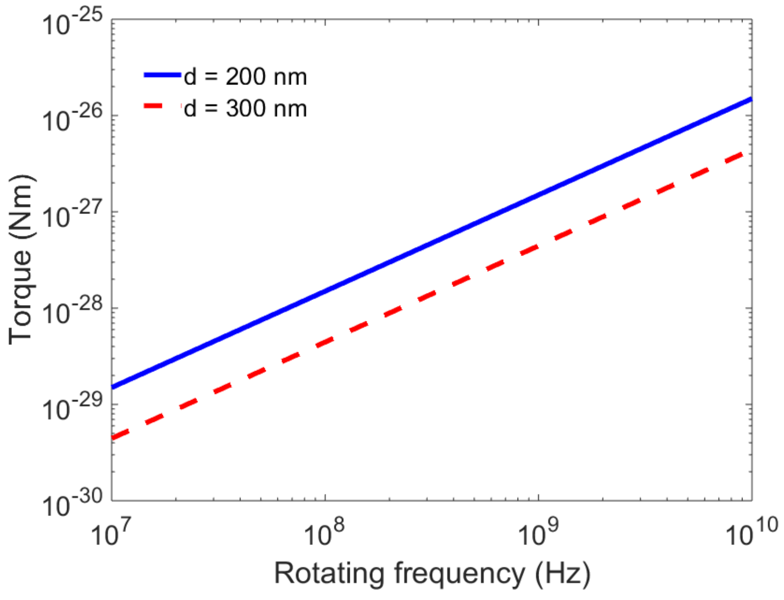
where ω_{LO} is the longitudinal polar-optic phonon frequency, ω_{TO} is the transverse polar-optic phonon frequency, γ is the damping coefficient and ϵ_∞ is the permittivity in the high-frequency limit. The optical phonon parameters for SiO_2, Si_3N_4 and SiC can be obtained from [4, 5] and they are shown in Supplementary Table 1. Only the imaginary parts of the polarizability and the green function contribute to the vacuum friction. Here we assume that the sphere is made of silica which is the case for our experiment, while the substrate can be made of various materials. Three common phonon polaritonic materials are used in the calculation. The imaginary part of the polarizability of SiO_2 nanosphere with a radius of 75 nm is shown in Supplementary Fig.2(a). Similarly, the imaginary part of the Green function $\bar{G}(\omega)$ connecting between the sphere and the SiO_2, Si_3N_4 and SiC surfaces are shown in Supplementary Fig.2(b). Note that the resonance peak frequencies are around 10^{14} rad/s for three materials.

Based on the polarizability and the Green function of different materials,the vacuum frictional torque can be calculated by using Eq.(1). The calculated torque between a SiO_2 sphere and three substrate materials are shown in Fig. 4 in the main text. It is noticeable that the torque between a SiO_2 sphere and a SiO_2 substrate has the largest value.

Rotating speed is also a significant factor in the vacuum friction torque. The dependence of the friction torque on the rotating speed is shown in Supplementary Fig.3. The temperature is set to be 1000 K for both the sphere and the substrate and they are both made of SiO_2. The blue solid line indicates friction torque at a separation of 200 nm and the red dashed line is for $d = 300$ nm. As expected, the torque increases linearly with the rotating frequency $\Omega/2\pi$ under the limit that $\Omega \ll \omega_{TO}$.



Supplementary Fig. 2: (a). The imaginary part of the polarizability of SiO_2 nanosphere with a radius of 75 nm. (b). The imaginary part of the Green function $\bar{G}(\omega)$ of the connection between the SiO_2 sphere and three different surface materials. The red solid curve is for the SiO_2 surface, while the black dashed line and the blue dotted line are for the Si_3N_4 SiC, respectively.



Supplementary Fig. 3: Calculated vacuum friction torque as a function of the rotating frequency. The sphere and the substrate are assumed to have the temperature of 1000 K. The blue solid line is the case when the separation d is 200 nm while the red dashed line is for the 300 nm separation case.

2 Air damping.

The air drag torque on a single sphere with diameter D rotating at the angular speed Ω is given as [6]

$$M_{air} = \frac{\pi p D^4 \Omega}{11.976} \sqrt{\frac{2m_{gas}}{\pi k_B T}}, \tag{7}$$

where p is the air pressure, $m_{gas} = 4.6 \times 10^{-26}$kg is the mass of the air molecule and T is the temperature of the surrounding air molecules.

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