

Widely tunable and narrow-linewidth chip-scale lasers from near-ultraviolet to near-infrared wavelengths

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1. Photonic chip design and characteristics

1.1. Microring resonator and optical losses

The estimated free spectral range (FSR) of the microring resonator with 10 μm radius is 1.07 nm in near-ultraviolet (near-UV) (404 nm), 1.38 nm in deep blue (450 nm), 1.66 nm in blue (488 nm), 1.90 nm in green (520 nm), 3.22 nm in red (660 nm) and 4.74 nm in near-infrared (near-IR) (785 nm). We calculate the FSR via the expression $FSR = \lambda^2 / (\bar{n}_g L)$, where λ is the wavelength, L is the circumference of the ring and $\bar{n}_g = \frac{1}{L} \int_0^L n_g(z) dz$ is the average group index of the optical mode throughout the resonator at that wavelength^{1,2}. We obtain n_g by simulating the fundamental TE optical mode for the different cross-sections of the resonator.

Fig. S1 shows the microring resonances used for the self-injection locking from deep blue to near-IR. We measure the resonances using commercial tunable lasers systems (section 3). We split the light from the lasers between a wavemeter (HighFinesse WSU-10) to monitor the frequency detuning, and the photonic chip, whose output we monitor with a photodetector (Thorlabs APD120A2).

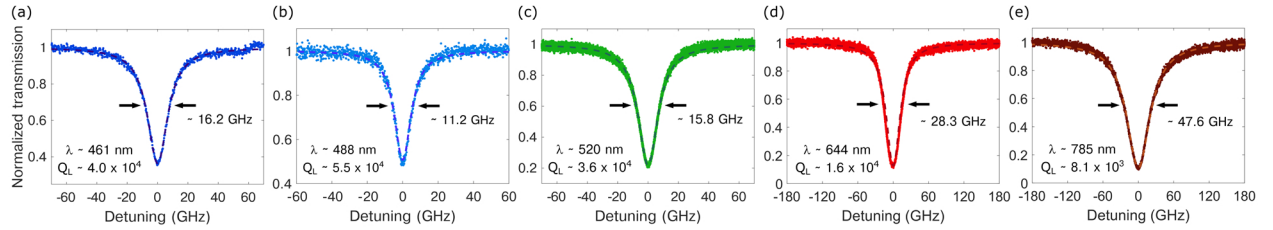


Figure S1: Ring resonances used for the self-injection locking from deep blue to near-infrared. We

fit the measured ring transmission with a Lorentzian shape (dashed lines) to extract the loaded quality factor Q_L . (a) Resonance in deep blue (461 nm), with $Q_L \sim 4.0 \times 10^4$. (b) Resonance in blue (488 nm), with $Q_L \sim 5.5 \times 10^4$. (c) Resonance in green (520 nm), with $Q_L \sim 3.6 \times 10^4$. (d) Resonance in red (644 nm), with $Q_L \sim 1.6 \times 10^4$. (e) Resonance in near-infrared (785 nm), with $Q_L \sim 8.1 \times 10^3$.

Our tapered resonator design allows for simultaneous micrometer-scale footprint, broadband low-loss single-mode operation, and tunability, which are characteristics needed to enable high-performance tunable and narrow linewidth integrated visible lasers. Compact resonators typically suffer from increased optical losses, originating from the increased overlap of the optical mode with the sidewalls³ and the stronger extra losses from the coupling region(s)⁴. While low optical losses at visible wavelengths have been previously demonstrated in high-confinement platforms, most results were obtained either from straight waveguides^{5–11} or resonators with very large radii ($\geq 100\text{ }\mu\text{m}$)^{12–18}. A few resonators with tighter bends ($\geq 22\text{ }\mu\text{m}$ radius) have been demonstrated, but either without tunability due to the lack of integrated phase shifters^{11,19–27} or with multimode operation^{28,29}. In low-confinement silicon nitride platform, even lower losses have been demonstrated, but at the expense of millimeter-to-centimeter-scale bending radii^{30–35}. Since none of these platforms simultaneously achieve broadband and tunable low-loss, single-mode operation with compact footprint ($\lesssim 10\text{ }\mu\text{m}$ bending radius), they are not suitable for a high-performance chip-scale visible lasers platform. With our tapered microresonator design, we achieve all these properties simultaneously and enable such a platform.

Our microrings operate in the undercoupled regime for any coupling strength between the ring and the waveguides, since we choose the two coupling regions (Fig. 1a inset, main text and Fig. S2) to always have the same gap. With this choice we ensure that the microring presents the same behavior when the light goes from the input port to the drop port (i.e., from the Fabry-Perot laser diode to the feedback loop) and the other way around.

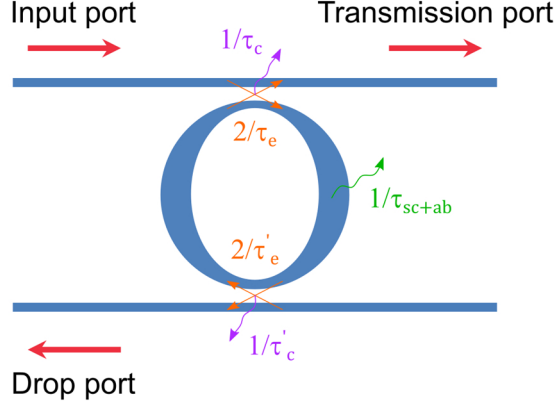


Figure S2: Schematic of the microring resonator with all the relevant ports and decay rates. $1/\tau_e$ and $1/\tau'_e$ are the coupling rates into the bus waveguide and the drop waveguide, respectively. $1/\tau_{sc+ab}$ is the decay rate associated with the material absorption and the scattering losses inside of the resonator. $1/\tau_c$ and $1/\tau'_c$ are the decay rates associated with the additional loss incurred by each coupling region. The intrinsic decay rate $1/\tau_i$ associated with the total loss is $1/\tau_i = 1/\tau_{sc+ab} + 1/\tau_c + 1/\tau'_c$. Since we choose the coupling gaps to be equal, $1/\tau'_e = 1/\tau_e$ and $1/\tau'_c = 1/\tau_c$.

In order to estimate the optical losses of the microrings at the different spectral ranges, we measure the rings transmissions for several coupling gaps at each wavelength (Figs. S3a to S7a) and model the results as follows. For a microring resonator with transmission and drop ports (Fig. S2), the transmission is given by Eq. 1³⁶:

$$T = \left| 1 - \frac{\frac{2}{\tau_e}}{i(\omega - \omega_0) + \frac{1}{\tau_i} + \frac{1}{\tau_e} + \frac{1}{\tau'_e}} \right|^2 \quad (1)$$

where ω is the angular frequency of the input light, ω_0 is the resonant angular frequency, $1/\tau_i$ is the decay rate due to loss, and $1/\tau_e$ and $1/\tau'_e$ are the coupling rates into the bus waveguide and the drop waveguide, respectively (Fig. S2). The quality factor associated with each rate $1/\tau$ is calculated via $Q = \omega\tau/2$. For our design, $1/\tau'_e = 1/\tau_e$ since the gaps are equal. On resonance ($\omega = \omega_0$), the transmission is the extinction ratio ER and is given by Eq. 2:

$$ER = \left(\frac{\frac{1}{\tau_i}}{\frac{1}{\tau_i} + \frac{2}{\tau_e}} \right)^2 = \left(\frac{\frac{1}{Q_i}}{\frac{1}{Q_i} + \frac{2}{Q_e}} \right)^2 = \left(\frac{\frac{1}{Q_i}}{\frac{1}{Q_L}} \right)^2 \quad (2)$$

where we replace the rates by their corresponding quality factors and define the loaded quality factor Q_L associated with the loaded (total) decay rate $1/\tau_L$ via $1/Q_L = 1/Q_i + 2/Q_e$. Q_L is related to the linewidth $\Delta\omega$ of the resonance via $Q_L = \omega_0/\Delta\omega$. Consequently, Q_L and ER are the parameters that we extract directly from the measured resonance shapes. From Eq. 2, we calculate the intrinsic quality factor Q_i associated with the loss decay rate $1/\tau_i$ via

$$Q_i = \frac{Q_L}{\sqrt{ER}} \quad (3)$$

and we obtain the extrinsic quality factor Q_e associated with the waveguide-to-ring coupling rate $1/\tau_e$ from

$$Q_e = \frac{2Q_L}{1 - \sqrt{ER}} \quad (4)$$

The intrinsic quality factor Q_i is associated with the total optical loss of the resonator, which is typically characterized by material absorption and scattering. However, due to the small footprint and strong coupling between the microring and the waveguides, the coupling regions introduce an additional source of loss for the resonator, as has been previously observed in silicon microring filters⁴. This extra loss is likely due to a stronger scattering of light at those regions, since the optical modes interact more intensively with the waveguide sidewalls. Therefore, in the most general case, the total loss has two terms: a fixed contribution from absorption and scattering determined by the geometry of the resonator, and a gap-dependent contribution from additional loss at the coupling regions. Mathematically, this means that

$$\frac{1}{Q_i} = \frac{1}{Q_{sc+ab}} + \frac{1}{Q_c} + \frac{1}{Q'_c} = \frac{1}{Q_{sc+ab}} + \frac{2}{Q_c} \quad (5)$$

where Q_{sc+ab} is the quality factor related to absorption and scattering losses and Q_c is the quality factor related to the additional loss in each coupling region ($1/Q'_c = 1/Q_c$ since the gaps are equal). It is possible

to experimentally estimate Q_{sc+ab} as the Q_i of microrings with large gaps, since in this case $1/Q_c \rightarrow 0$, $ER \rightarrow 1$ and $1/Q_e \rightarrow 0$.

Figs. S3b-d to S7b-d show the quality factors Q_{sc+ab} , Q_i , Q_c and Q_e calculated from Q_L and ER of the measured resonances (Figs. S3a to S7a). We approximate Q_{sc+ab} as the Q_i for the resonance with largest gap measured, which is supported by the fact that Q_i saturates for large gaps (Figs. S3c to S7c). For all spectral ranges, increasing the gap causes an increase in all quality factors, since the coupling and loss rates associated with the gap decrease. We observe an exponential dependence of Q_e with the microring gap across all the wavelengths (Figs. S3d to S7d), which is expected since the coupling strength scales with the overlap between the microring and the exponential tail of the optical mode of the waveguide³⁶. We also observe a mostly exponential relationship between Q_c and the gap across all the wavelengths (Figs. S3d to S7d). This behavior suggests that the additional loss incurred at the coupling regions is likely dominated by the extra surface scattering caused by the overlap between the exponential tail of the optical mode of the microring and the sidewall roughness of the adjacent waveguide. In terms of the estimated Q_{sc+ab} , we observe a monotonic increase from near-IR to green and a monotonic decrease from green to deep blue (Figs. S3c to S7c). This behavior is caused by the interplay between material absorption and scattering losses, which have different geometrical and spectral dependences. Material absorption increases linearly with optical mode confinement and exponentially with photon energy as it approaches the optical bandgap of the material³⁷, which for LPCVD silicon nitride is in the deep ultraviolet spectral range. Surface scattering losses, on the other hand, increase linearly with the optical mode overlap with the waveguide walls and scales as $\sim 1/\lambda^2$. At near-IR wavelengths, there is little material absorption, but the optical mode is delocalized since the silicon nitride film is thin, causing increased scattering losses due to strong overlap with sidewalls and potentially radiative losses due to the small bending radius of the resonator. As the wavelength shortens towards red and green, the optical confinement increases, which reduces those sources of loss and increases the quality factor. While the absorption and the surface scattering strength also increase

as the wavelength shortens, they only become dominant around the blue spectral range, beyond which the total loss starts to increase.

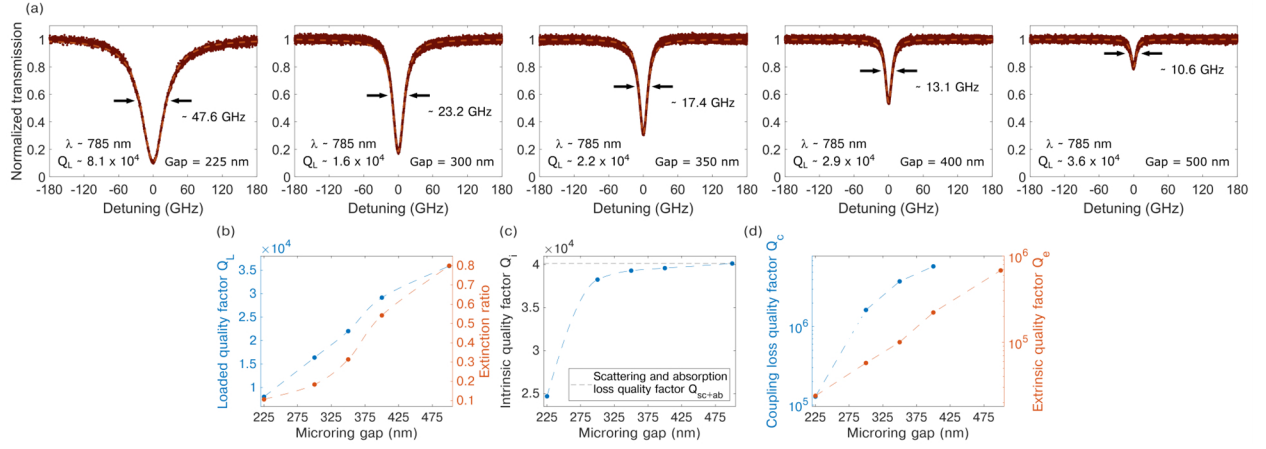


Figure S3: Microring resonances for varying gaps in the near-infrared (785 nm) spectral range and corresponding quality factors. (a) Measured resonances for different coupling gaps and their corresponding loaded quality factors (Q_L). The dashed lines are Lorentzian fittings of the experimental points. (b) Loaded quality factors (Q_L) and extinction ratios (ER) extracted from the resonances in (a). (c) Intrinsic quality factors Q_i calculated from the data in (b). We estimate the scattering and absorption losses quality factor Q_{sc+ab} to be 4.0×10^4 . (d) Coupling loss quality factors Q_c and extrinsic quality factors Q_e calculated from the data in (b). In (b)-(d), the dashed lines are guides to the eye.

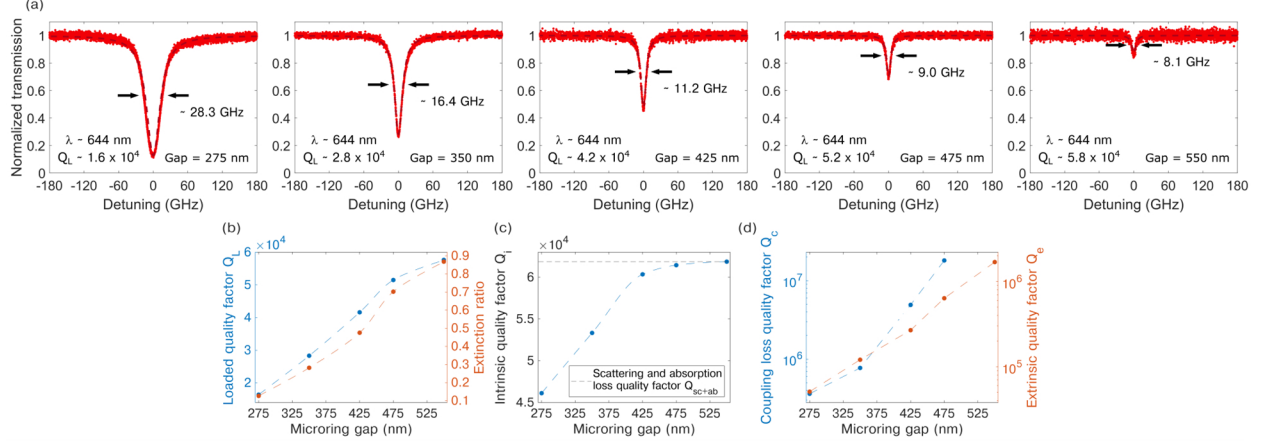


Figure S4: Microring resonances for varying gaps in the red (644 nm) spectral range and corresponding quality factors. (a) Measured resonances for different coupling gaps and their corresponding loaded quality factors (Q_L). The dashed lines are Lorentzian fittings of the experimental points. (b) Loaded quality factors (Q_L) and extinction ratios (ER) extracted from the resonances in (a). (c) Intrinsic quality factors Q_i calculated from the data in (b). We estimate the scattering and absorption losses quality factor Q_{sc+ab} to be 6.2×10^4 . (d) Coupling loss quality factors Q_c and extrinsic quality factors Q_e calculated from the data in (b). In (b)-(d), the dashed lines are guides to the eye.

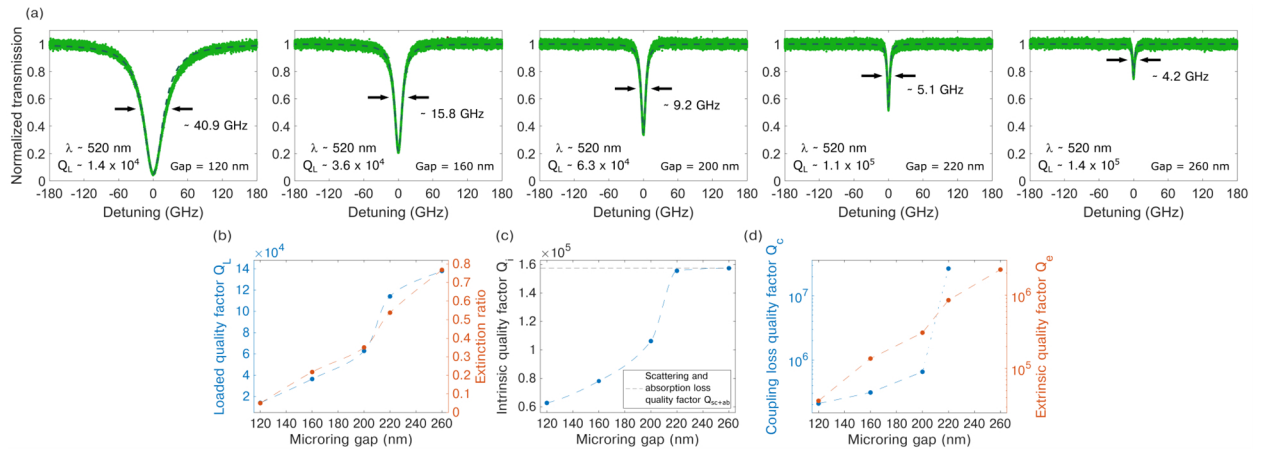


Figure S5: Microring resonances for varying gaps in the green (520 nm) spectral range and corresponding quality factors. (a) Measured resonances for different coupling gaps and their corresponding loaded quality factors (Q_L). The dashed lines are Lorentzian fittings of the experimental

points. (b) Loaded quality factors (Q_L) and extinction ratios (ER) extracted from the resonances in (a). (c) Intrinsic quality factors Q_i calculated from the data in (b). We estimate the scattering and absorption losses quality factor Q_{sc+ab} to be 1.6×10^5 . (d) Coupling loss quality factors Q_c and extrinsic quality factors Q_e calculated from the data in (b). In (b)-(d), the dashed lines are guides to the eye.

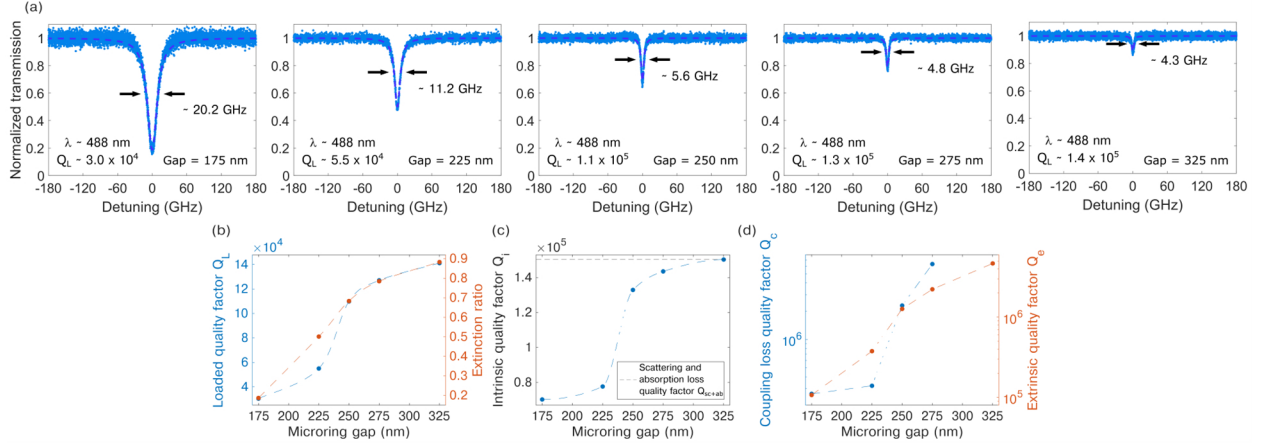


Figure S6: Microring resonances for varying gaps in the blue (488 nm) spectral range and corresponding quality factors. (a) Measured resonances for different coupling gaps and their corresponding loaded quality factors (Q_L). The dashed lines are Lorentzian fittings of the experimental points. (b) Loaded quality factors (Q_L) and extinction ratios (ER) extracted from the resonances in (a). (c) Intrinsic quality factors Q_i calculated from the data in (b). We estimate the scattering and absorption losses quality factor Q_{sc+ab} to be 1.5×10^5 . (d) Coupling loss quality factors Q_c and extrinsic quality factors Q_e calculated from the data in (b). In (b)-(d), the dashed lines are guides to the eye.

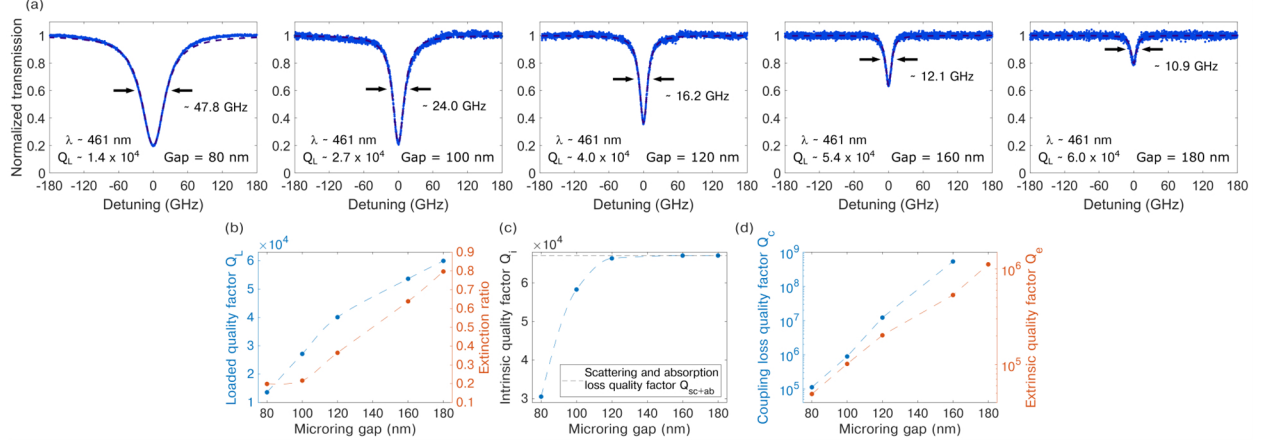


Figure S7: Microring resonances for varying gaps in the deep blue (461 nm) spectral range and corresponding quality factors. (a) Measured resonances for different coupling gaps and their corresponding loaded quality factors (Q_L). The dashed lines are Lorentzian fittings of the experimental points. (b) Loaded quality factors (Q_L) and extinction ratios (ER) extracted from the resonances in (a). (c) Intrinsic quality factors Q_i calculated from the data in (b). We estimate the scattering and absorption losses quality factor Q_{sc+ab} to be 6.7×10^4 . (d) Coupling loss quality factors Q_c and extrinsic quality factors Q_e calculated from the data in (b). In (b)-(d), the dashed lines are guides to the eye.

1.2. Feedback loop

We design the feedback loop at the drop port of the microring resonators to efficiently reflect the light back to the resonator, which then redirects it to the Fabry-Perot laser diode (FPLD) to cause self-injection locking. The feedback loop consists of a symmetric 1 x 2 splitter with interconnected output ports (Fig. S8), such that it works as a mirror by reversing the propagation direction of the input light. Following the nomenclature of Fig. S8, the scattering matrix of the symmetric 1 x 2 splitter reads

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \sqrt{S} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (6)$$

where S is the fraction of the input power that goes to each output port of the splitter. For a lossless splitter, $S = 0.5$. We design the waveguides that connect the output ports to have gradual bends to not incur

additional optical losses, and to be symmetric around the center of the splitter such that light accumulates the same phase ϕ in either direction. Consequently, we have

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = e^{i\phi} \begin{bmatrix} b_3 \\ b_2 \end{bmatrix} \quad (7)$$

By combining Eqs. 6 and 7, we conclude that $b_1 = \sqrt{S}(a_2 + a_3) = e^{i\phi} 2Sa_1$. The amplitude reflection coefficient $\Gamma_{loop} = |b_1/a_1|$ of the optical feedback loop is then

$$\Gamma_{loop} = 2S \quad (8)$$

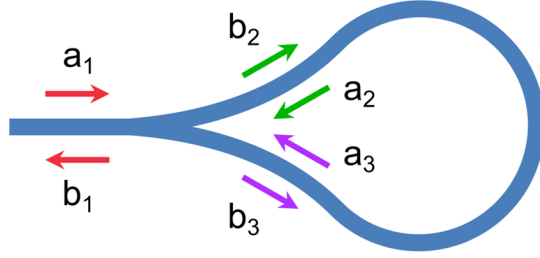


Figure S8: Schematic of the optical feedback loop and its ports. We use this nomenclature of the port signals in Eqs. 6 and 7 to derive the amplitude reflection coefficient $\Gamma_{loop} = |b_1/a_1|$ of the loop.

For the symmetric 1 x 2 splitters, we design multimode interference (MMI) couplers for each spectral range. Fig. S9 shows the simulated power transmission per port (S) and dimensions of each MMI. We perform the numerical simulations in Lumerical MODE using the EigenMode Expansion (EME) solver. Alternatively, one could use a broadband Y-splitter instead of the MMIs as the symmetric 1x2 splitters, thus allowing for the exact same feedback design for all visible wavelengths.

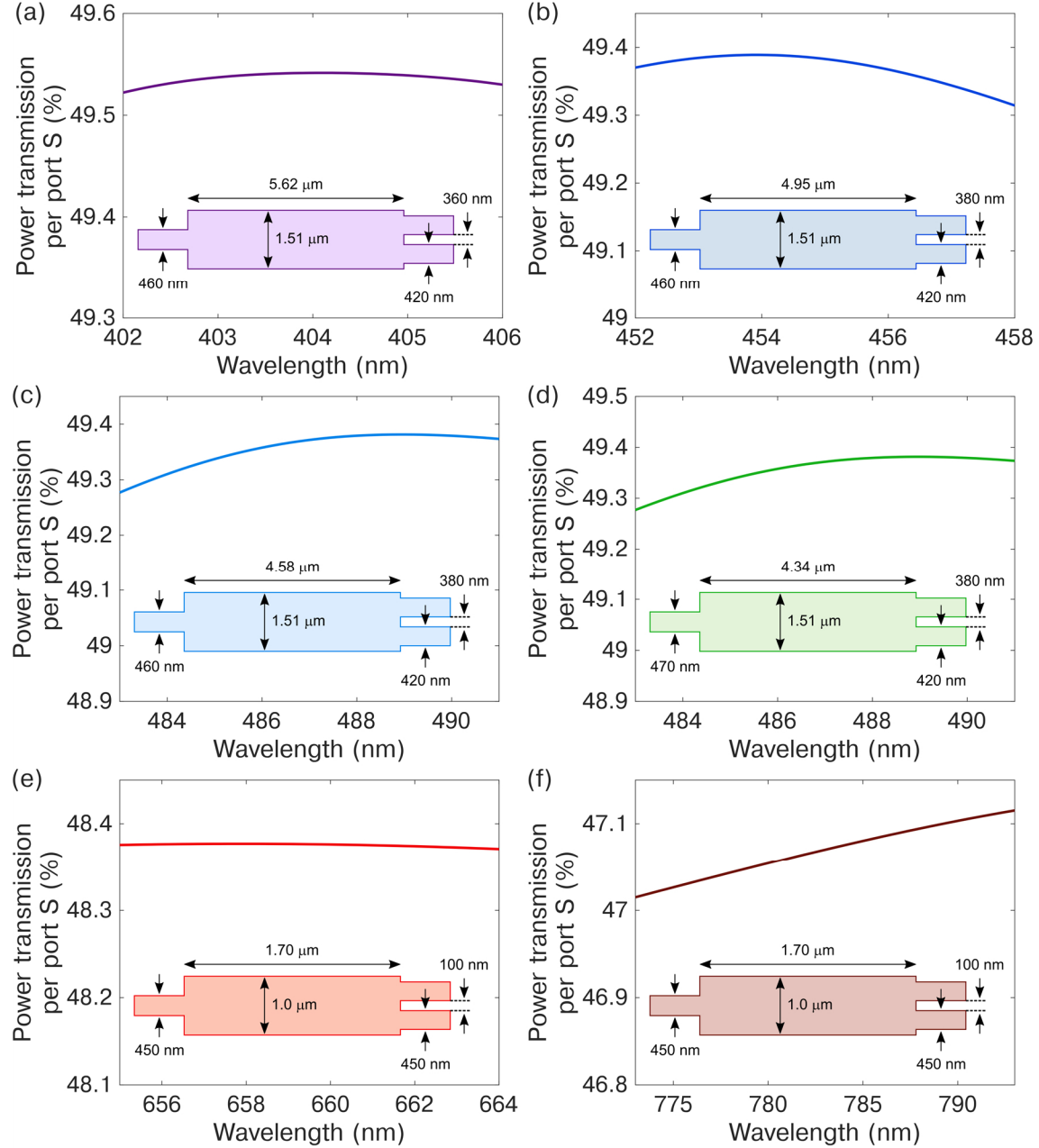


Figure S9: Power transmission per port (S) and dimensions of the MMIs designed for the different spectral ranges. (a) Near-UV. (b) Deep blue. (c) Blue. (d) Green. (e) Red. (f) Near-IR.

1.3. Edge couplers

For the input edge couplers, we use inverse tapers for near-IR and red and horn tapers for green, blue, deep blue, and near-UV in order to maximize the coupling between the FPLDs and the photonic chip.

For the output edge couplers, we use inverse tapers for all wavelengths to increase the overlap with the optical mode of the cleaved fibers and to avoid undesired facet reflections. We estimate the FPLD-to-chip coupling C from the overlap between the measured FPLD mode profiles (Fig. S18, section 2.4) and the simulated modes of the input edge couplers. Similarly, we estimate the chip-to-fiber coupling C_f from the overlap between the simulated modes of the output inverse tapers and the fundamental mode of the optical fibers. We simulate the photonic chip modes using Lumerical MODE and use the typical mode field diameters (MFDs) for the cleaved fibers (780HP for near-IR, SM600 for red, and SM450 for green, blue, deep blue and near-UV) across the different wavelengths³⁸. For the near-UV laser, we add an extra 4.6 dB loss to the simulated FPLD-to-chip and chip-to-fiber couplings to effectively account for the lower-than-expected coupling efficiency observed experimentally at this wavelength. We suspect that the non-optimal coupling is due to linear and angular misalignments between the FPLD/fiber and the photonic chip. Table S1 summarizes all the couplings and relevant dimensions.

We observe that the coupling losses tend to be larger at shorter wavelengths due to the larger mode mismatch between the on-chip optical modes and the modes of the FPLDs and fibers. While the chip-to-fiber coupling C_f only affects the fiber-coupled output power of the chip-scale lasers, the FPLD-to-chip coupling C affects their overall performance, since it determines the amount of effective optical feedback to the FPLDs (see section 2.13 for a quantitative analysis). The main factor causing larger FPLD-to-chip coupling losses from near-UV to green is the significant mismatch of the optical modes in the vertical direction, since the MFDs are around 175 nm for the horn taper and around $1.57\ \mu\text{m} - 1.73\ \mu\text{m}$ for the FPLDs (section 2.4). We can improve the mode matchings in a variety of ways³⁹, including bi-layer couplers^{40,41}, multi-tip couplers^{42,43}, spot-size converters^{44–52} and photonic wirebonding^{53–55}.

Table S1: Summary of the estimated couplings and the relevant dimensions of the Fabry-Perot laser diodes, photonic chip and optical fibers.

Wavelength (nm)	FPLD vertical/horizontal MFD (μm)	Input/output taper width (nm)	Fiber type	Fiber MFD (μm)	FPLD-to-chip coupling C	Chip-to-fiber coupling C_f
785	2.2/3.17	110/110	780HP	5.18	0.83	0.89
660	1.58/2.62	110/110	SM600	4.26	0.85	0.62
520	1.73/3.15	4000/50	SM450	3.56	0.32	0.92
488	1.64/3.03	4000/90	SM450	3.43	0.36	0.21
450	1.71/2.86	4000/70	SM450	3.31	0.29	0.31
404	1.57/2.22	4000/50	SM450	3.14	0.09*	0.15*

* We added an extra 4.6 dB loss to the simulated coupling to effectively account for the lower-than-expected coupling efficiency observed experimentally.

2. Fabry-Perot and chip-scale lasers characteristics and measurements

2.1. Threshold behavior, linewidth and longitudinal modes evolution

Fig. S10 depicts the typical spectral evolution of the light emitted by a FPLD as we increase the pump current from below to above threshold. Below threshold (Fig. S10, red curve), the optical gain is not enough to cause lasing and the emitted incoherent light comes from spontaneous emission. The light is spectrally broad and does not present any prominent peaks, reproducing the shape of the gain profile of the gain medium. At threshold (Fig. S10, yellow curve), the optical gain for one or more longitudinal modes of the FP cavity is enough to match the intrinsic losses and cause the onset of lasing. The lasing modes stand out from the broad spontaneous emission background as narrow peaks. As we increase the pump current beyond threshold (Fig. S10, violet curve), the transition from spontaneous to stimulated emission

intensifies, as demonstrated by the presence of a few powerful narrower peaks and the suppression of the non-lasing modes (the “ripples” on the emission background disappear). For pump currents not too much above threshold (Fig. S10, violet curve), only one or a few longitudinal modes lase, as they experience enough optical gain difference to the other longitudinal modes to win the mode competition^{56,57}. While the linewidths of the peaks in Fig. S10 are limited by the resolution of the optical spectrum analyzer, the linewidth narrowing from below to above threshold is evident, clearly demonstrating the transition from spontaneous (incoherent) to stimulated (coherent) emission that is signature of lasing (Fig. S10b, inset). For FPLDs operating close to single-mode regime, it is well known that increasing the pump current leads to narrowing of the linewidth⁵⁸, which can typically range from below MHz to tens of MHz^{59–62}. As we further increase the pump current (Fig. S10, green and solid blue curves), more longitudinal modes of the FP cavity experience enough optical gain to lase. The presence of multiple lasing modes causes an effective broadening of their linewidths due to nonlinear gain dynamics and mode partition noise⁵⁸. Typical linewidths in this regime can range from tens of MHz to several GHz⁶³.

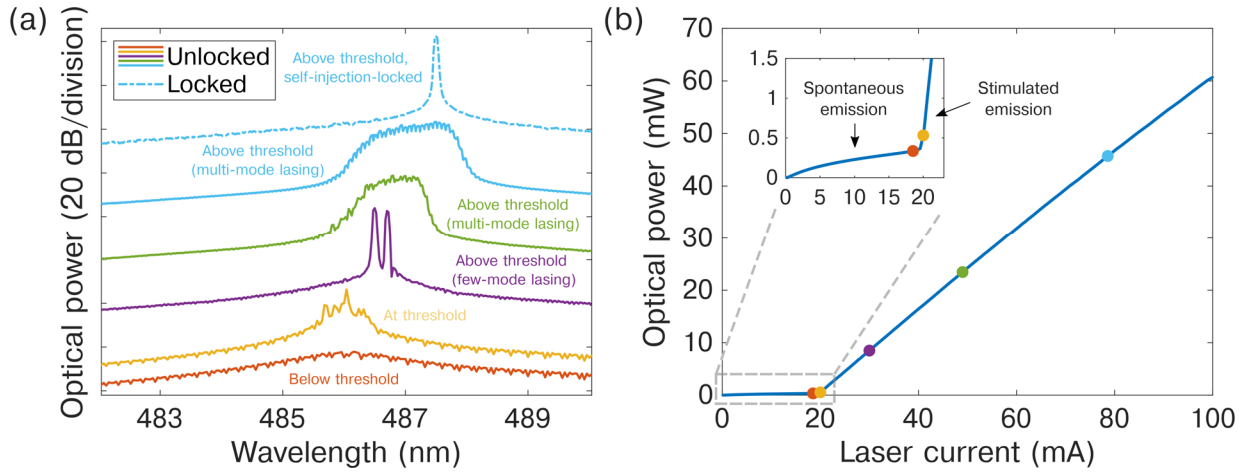


Figure S10: Threshold behavior, linewidth and longitudinal modes evolution of a typical Fabry-Perot laser diode (FPLD). (a) Spectra of the light emitted by the FPLD (QPhotonics QLD-488-50S) at different pump currents (solid curves) and after self-injection locking via our photonic chip (dashed curve). The colors correspond to the points in (b) and the spectra are displaced vertically for easy comparison. We

measure the spectra with an optical spectrum analyzer (OSA, Ando AQ6314A, 0.05 nm resolution) (b) Light-current curve of the FPLD depicting the pump currents and emitted optical powers for the spectra in (a). The inset highlights the transition from spontaneous to stimulated emission around the threshold current.

2.2. Self-injection locking and linewidth narrowing

Self-injection locking of a multi-mode laser is a technique that converts its multi-frequency, broad-linewidth emission into single-frequency, narrow-linewidth light. This transformation is achieved by collapsing all the optical power of the laser into a single longitudinal mode via an external frequency-selective optical feedback. The external feedback element selects a narrow portion of the laser’s multi-mode spectrum and reflects a fraction of its power back to the laser cavity. The reflected light acts as an effective additional gain for the selected frequency, causing the longitudinal mode at that frequency to win the mode competition. Consequently, the optical power distributed among the other longitudinal modes collapse into the selected mode, as illustrated in Fig. S10 by the solid and dashed blue curves or in Fig. 2b-c (main text). The mode collapse itself provides some narrowing of the laser linewidth by suppressing the nonlinear gain dynamics and the mode partition noise. In addition, if the optical feedback is provided by a narrow-linewidth external cavity, the single lasing mode experiences further linewidth reduction^{64,65}. We present a quantitative description of the linewidth narrowing caused by self-injection locking in section 2.13. By tuning the external cavity, we can also coarse and fine tune the lasing wavelength^{64,66}.

Self-injection locking is different from simply filtering a single longitudinal mode of a multi-mode laser, as the latter does not cause mode collapse or linewidth narrowing. Fig. S11 contrasts these two concepts in the context of our particular chip-scale laser platform, where the multi-mode laser is a FPLD and the external cavity is a micrometer-scale, high quality factor ring resonator with a feedback loop at the drop port. The output of the chip-scale laser is the transmission port of the microring. We radiate to free space any leftover light from the feedback loop that is not directed to the FPLD by tapering the waveguide

width down to zero on the right side of the ring's drop port. With the microring detuned from any FPLD mode, no light is dropped to the feedback loop (or radiated by the drop port). In the absence of reflected light, the FPLD lases with multiple longitudinal modes and the chip-scale laser output is multi-mode (Fig. S11a). By aligning the microring to a longitudinal mode of the FPLD, we drop a fraction of the optical power of that mode to the feedback loop and reflect it back to the FPLD. If the phase of the light is not correct to cause constructive interference inside of the FPLD, the laser dynamics is perturbed, but self-injection locking does not occur. While the perturbation often causes instabilities in the FPLD, it can also result in a situation where the laser output resembles a filtered FPLD. In this case, the chip-scale laser output contains all the multiple longitudinal modes of the FPLD except for the one selected by the microring. A small fraction of the selected mode's power, which is single-frequency but broad linewidth, is radiated by the drop port (Fig. S11b). By adjusting the phase of the reflected light using the microheater on the waveguide, we cause self-injection locking, which collapses all the modes of the FPLD into a single lasing mode with narrow linewidth. The light emitted by the FPLD itself and consequently the chip-scale laser output become single-frequency and narrow linewidth (Fig. S11c).

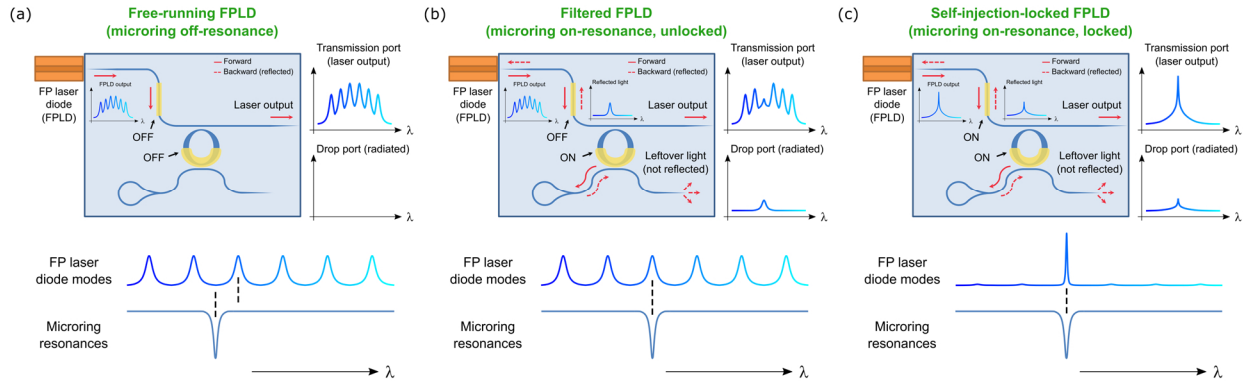


Figure S11: Comparison among a free-running Fabry-Perot laser diode (FPLD), a filtered FPLD and a self-injection-locked FPLD in the context of our chip-scale laser platform and its characteristic optical signals. (a) Free-running FPLD. The microring is detuned from all FPLD modes and consequently there is no optical feedback. The laser output is multi-mode. (b) Filtered FPLD. We align the microring to a FPLD mode, but the phase of the reflected light is not correct to cause self-injection locking. The output

of the FPLD remains multi-mode and the laser output contains all of the FPLD modes except for the one on-resonance with the microring. (c) Self-injection-locked FPLD. We align the microring to a FPLD mode and adjust the phase of the reflected light using the waveguide microheater to cause self-injection locking. Due to the mode collapse and linewidth reduction, the output of the FPLD and consequently the laser output become single-frequency and narrow linewidth.

2.3. Light-current-voltage (LIV) curves and threshold current reduction due to self-injection locking

Figs. S12 to S17 show the light-current-voltage (LIV) curves of the free-running FPLDs and the chip-scale lasers from near-IR to near-UV. The figures also depict microscope images of the FPLDs and highlight the operating points used for all the measurements of the chip-scale lasers unless otherwise stated (see section 2.5). For the free-running FPLDs, we measure the LIV curves by sweeping the voltage applied to the lasers while monitoring the current (using a Keithley 2220G-30-1) and the emitted optical power (using a Thorlabs S130C). For the chip-scale lasers, for each current applied to the FPLD we sweep the voltage applied to the ring microheater to cause self-injection locking at different longitudinal modes of the FPLD. We keep the waveguide microheater fixed throughout the measurement, so that the self-injection locking is controlled only by the ring microheater, and we record the fiber-coupled output power for each self-injection-locked wavelength. Each lasing wavelength may present slightly different power levels due to factors such as the intrinsic gain profile of the FPLDs and the exact detuning between the FPLD mode and the microring resonance at the self-injection-locking. We represent the distributions of measured fiber-coupled powers for each chip-scale laser by the shaded regions in Figs. S12b-c to S17b-c. All the FPLDs were TO can-packaged and we opened their cans with a can opener (Thorlabs WR1) to allow for edge-coupling to the photonic chip.

A well-known signature of self-injection locking is the reduction of the threshold current of the laser, since the optical feedback offsets part of the optical gain that the current needs to provide to cause

lasing^{67–71}. We observe a threshold current reduction due to self-injection locking from 29.5 mA to 19.25 mA for the near-IR laser (Fig. S12), from 28 mA to 24 mA for the red laser (Fig. S13), from 48 mA to 43 mA for the green laser (Fig. S14), from 19.8 mA to 19 mA for the blue laser (Fig. S15), from 16.3 mA to 14.74 mA for the deep blue laser (Fig. S16), and from 27.05 mA to 27 mA for the near-UV laser (Fig. S17). While the exact threshold current reduction depends not only on the amount of optical feedback, but also its phase⁵⁸ and the lasing wavelength⁷¹, we confirm that larger reductions correlate with stronger feedbacks by comparing the measured threshold currents (Figs. S12 to S17, Table S2) with the estimated amounts of optical feedback for each laser (Table S4, section 2.13). Table S2 summarizes the operating points and threshold currents of the FPLDs and chip-scale lasers.

In terms of density values, the threshold current densities before and after self-injection locking are respectively around 1.35 kA/cm² and 0.88 kA/cm² for near-IR, 1.21 kA/cm² and 1.04 kA/cm² for red, 1.73 kA/cm² and 1.55 kA/cm² for green, 1.12 kA/cm² and 1.07 kA/cm² for blue, 0.97 kA/cm² and 0.87 kA/cm² for deep blue, and 4.14 kA/cm² and 4.14 kA/cm² for near-UV. In order to estimate the current flow areas for the density calculation, we multiply the length of the FPLDs extracted from the microscope images in Figs. S12 to S17 (summarized in Table S4) by the measured horizontal mode field diameter of the beam profiles (Fig. S18), which we take as an approximation for the width of the lateral confinement of the current. The resulting current density values on the order of kA/cm² are consistent with typical values for semiconductor laser diodes^{72–82}.

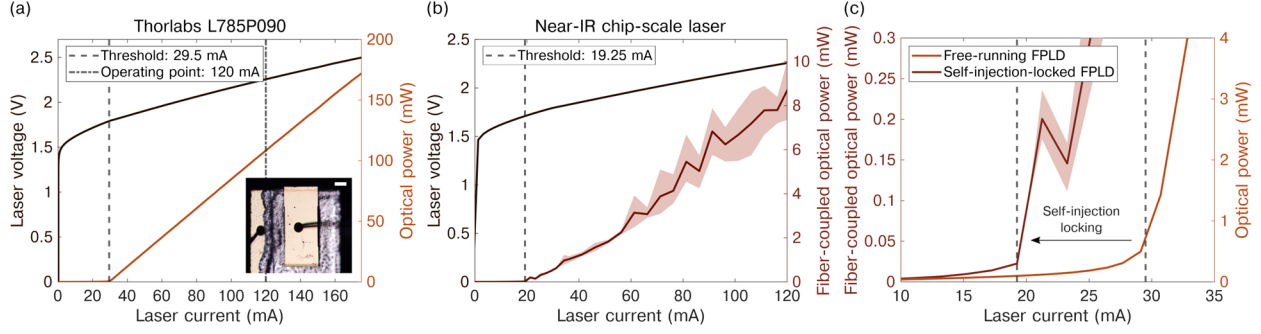


Figure S12: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the near-infrared (near-IR) spectral range (785 nm). (a) LIV curve of the near-IR FPLD Thorlabs L785P090. The threshold current is 29.5 mA and we operate it at a current of 120 mA, corresponding to an optical power of 108 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale near-IR laser up to the operating current of 120 mA. The threshold current is 19.25 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 29.5 mA for the free-running FPLD to 19.25 mA for the chip-scale laser.

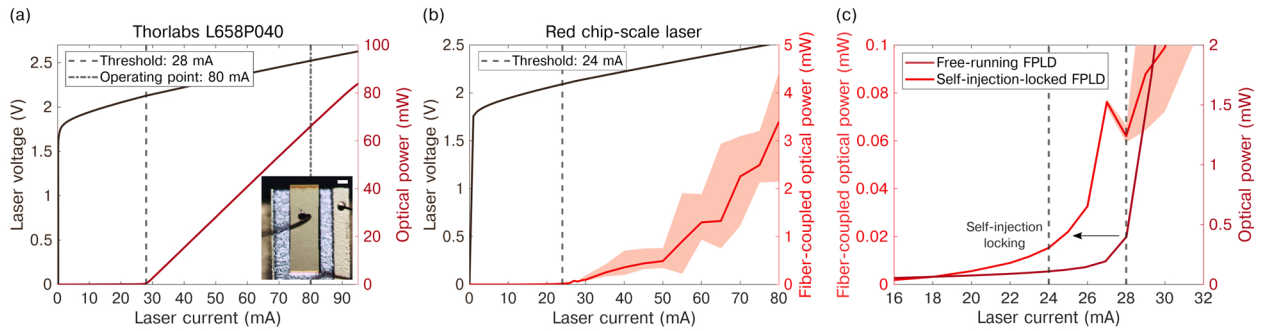


Figure S13: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the red spectral range (660 nm). (a) LIV curve of the red FPLD Thorlabs L658P040. The threshold current is 28 mA and we operate it at a current of 80 mA, corresponding to an optical power of 66 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale red laser up to the operating current of 80 mA. The threshold current is 24 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 28 mA for the free-running FPLD to 24 mA for the chip-scale laser.

shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 28 mA for the free-running FPLD to 24 mA for the chip-scale laser.

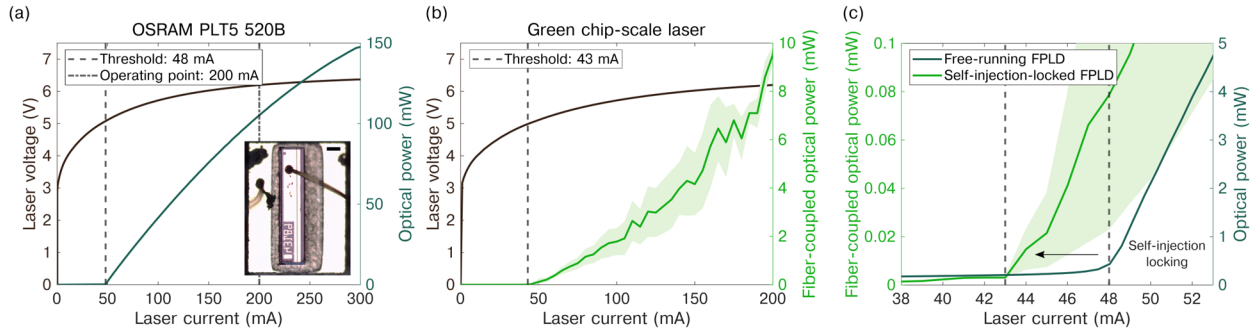


Figure S14: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the green spectral range (520 nm). (a) LIV curve of the green FPLD OSRAM PLT5 520B. The threshold current is 48 mA and we operate it at a current of 200 mA, corresponding to an optical power of 105 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale green laser up to the operating current of 200 mA. The threshold current is 43 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 48 mA for the free-running FPLD to 43 mA for the chip-scale laser.

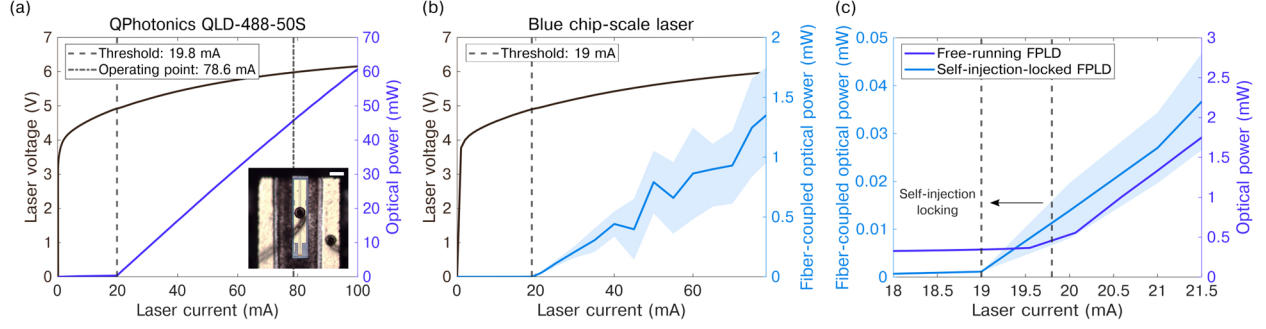


Figure S15: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the blue spectral range (488 nm). (a) LIV curve of the blue FPLD QPhotonics QLD-488-50S. The threshold current is 19.8 mA and we operate it at a current of 78.6 mA, corresponding to an optical power of 46 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale blue laser up to the operating current of 78.6 mA. The threshold current is 19 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 19.8 mA for the free-running FPLD to 19 mA for the chip-scale laser.

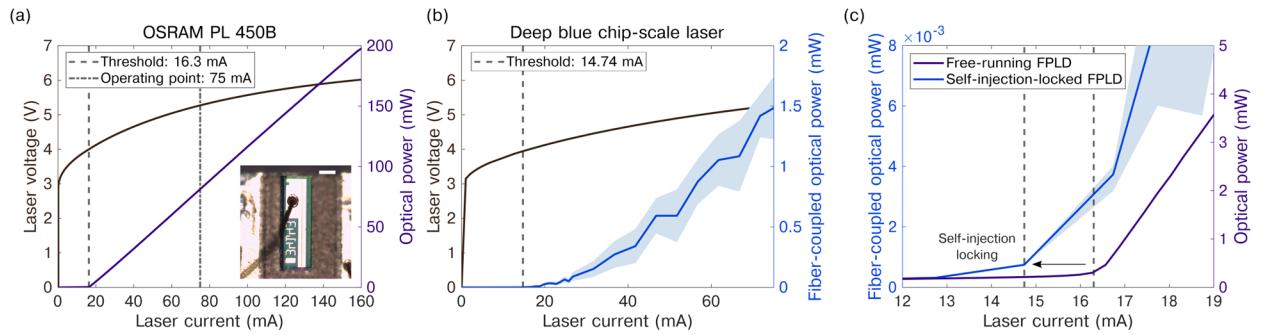


Figure S16: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the deep blue spectral range (450 nm). (a) LIV curve of the deep blue FPLD OSRAM PL 450B. The threshold current is 16.3 mA and we operate it at a current of 75 mA, corresponding to an optical power of 81 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale deep blue laser up to the operating current of 81 mA. The threshold current is

14.74 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 16.3 mA for the free-running FPLD to 14.74 mA for the chip-scale laser.

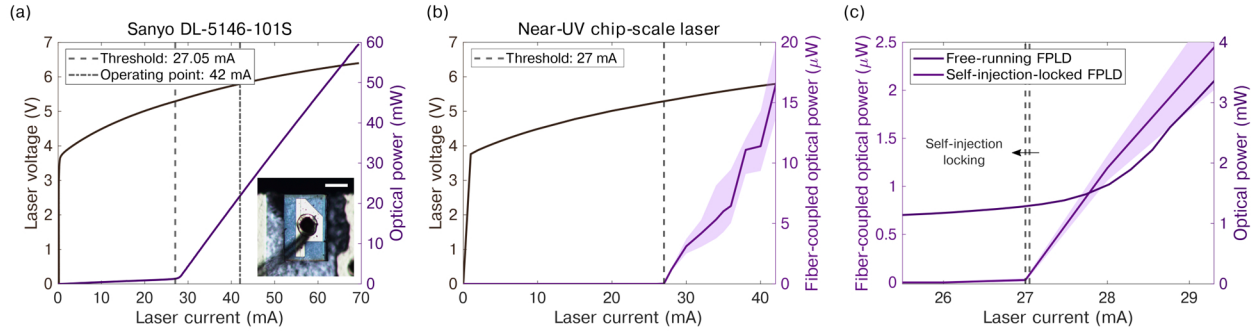


Figure S17: Light-current-voltage (LIV) characteristics of the Fabry-Perot laser diode (FPLD) and the chip-scale laser for the near-ultraviolet (near-UV) spectral range (404 nm). (a) LIV curve of the near-UV FPLD Sanyo DL-5146-101S. The threshold current is 27.05 mA and we operate it at a current of 42 mA, corresponding to an optical power of 21.85 mW. The inset shows a microscope image of the FPLD (scale bar: 100 μm). (b) LIV curve of our chip-scale near-UV laser up to the operating current of 42 mA. The threshold current is 27 mA. The shaded region represents the distribution of measured fiber-coupled optical powers for several self-injection-locked lasing wavelengths at each pump current. (c) Threshold current reduction due to self-injection locking. The threshold current reduces from 27.05 mA for the free-running FPLD to 27 mA for the chip-scale laser.

Table S2: Summary of the operating conditions and threshold currents of the Fabry-Perot laser diodes (FPLDs) and the chip-scale lasers. The smaller threshold currents of the chip-scale lasers when compared to the free running FPLDs are a characteristic signature of the self-injection locking.

Wavelength (nm)	FPLD model	FPLD operating current (mA)	Free-running FPLD optical power (mW)	Chip-scale laser fiber-coupled optical power (mW)	FPLD threshold current (mA)	Chip-scale laser threshold current (mA)
785	Thorlabs L785P090	120.0	108.00	10.00	29.50	19.25
660	Thorlabs L658P040	80.0	66.00	4.42	28.00	24.00
520	OSRAM PLT5 520B	200.0	105.00	9.89	48.00	43.00
488	QPhotonics QLD-488-50S	78.6	46.00	1.75	19.80	19.00
450	OSRAM PL 450B	75.0	81.00	1.74	16.30	14.74
404	Sanyo DL-5146-101S	42.0	21.85	0.02	27.05	27.00

2.4. Beam spatial profile

Fig. S18 exhibits the measured spatial profiles of the emission from the FPLDs, showing well-defined beams above threshold. The figure also depicts vertical and horizontal slice cuts through the center of the beam profiles, as well as Gaussian fittings to extract their mode field diameters (MFDs) in each direction. Within the photonic chip, the light coupled from the FPLDs remains in the fundamental TE mode of the integrated waveguides. We measure the beam spatial profiles by shining the light emitted from the FPLDs directly into an imaging column containing a 50X objective (50X Mitutoyo Plan Apo Infinity Corrected Long WD Objective), a 12X zoom lens system (Navitar) and a CMOS camera (Thorlabs

DCC1645C-HQ). In order to calibrate the imaging system, we image a reference structure on a photonic chip with known dimensions (Fig. S19), from which we extract the correspondence between each camera pixel and the real distance on the imaged plane.

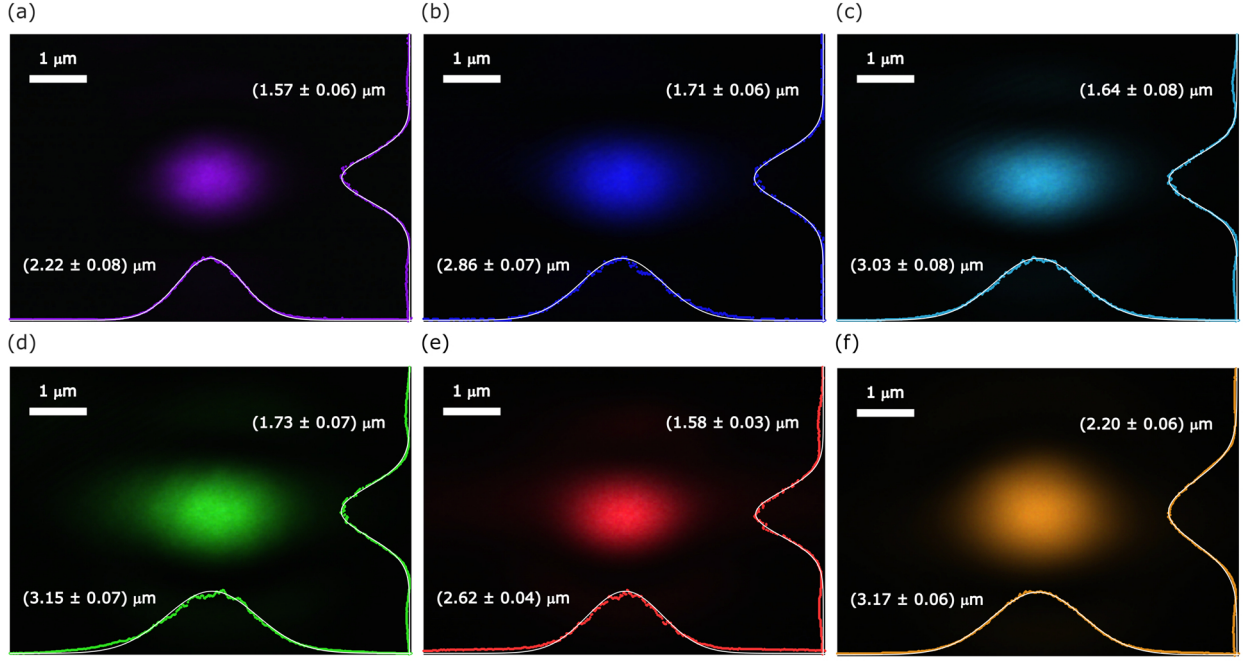


Figure S18: Spatial profiles of the Fabry-Perot laser diodes (FPLDs) from near-ultraviolet to near-infrared. We fit the horizontal and vertical slice cuts through the center of the beams with Gaussian functions (solid white lines) and we show the extracted mode field diameters (MFDs) next to them. The error bars represent the standard deviation of the fitted MFDs for 400 measurements of each mode profile. Scale bars: 1 μm . (a) Near-UV (404 nm) FPLD Sanyo DL-5146-101S. The vertical and horizontal MFDs are $(1.57 \pm 0.06) \mu\text{m}$ and $(2.22 \pm 0.08) \mu\text{m}$ respectively. (b) Deep blue (450 nm) FPLD OSRAM PL 450B. The vertical and horizontal MFDs are $(1.71 \pm 0.06) \mu\text{m}$ and $(2.86 \pm 0.07) \mu\text{m}$ respectively. (c) Blue (488 nm) FPLD QPhotonics QLD-488-50S. The vertical and horizontal MFDs are $(1.64 \pm 0.08) \mu\text{m}$ and $(3.03 \pm 0.08) \mu\text{m}$ respectively. (d) Green (520 nm) FPLD OSRAM PLT5 520B. The vertical and horizontal MFDs are $(1.73 \pm 0.07) \mu\text{m}$ and $(3.15 \pm 0.07) \mu\text{m}$ respectively. (e) Red (660 nm) FPLD Thorlabs L658P040. The vertical and horizontal MFDs are $(1.58 \pm 0.03) \mu\text{m}$ and $(2.62 \pm 0.04) \mu\text{m}$ respectively. (f) Near-IR (785 nm) FPLD Thorlabs L785P090. The vertical and horizontal MFDs are $(2.20 \pm 0.06) \mu\text{m}$ and $(3.17 \pm 0.06) \mu\text{m}$ respectively.

$(3.17 \pm 0.06) \mu\text{m}$ respectively. The real color of the light is dark red, but it looks yellow in the captured image because of the built-in filter and the sensor sensitivity of the camera (Thorlabs DCC1645C) at this wavelength.

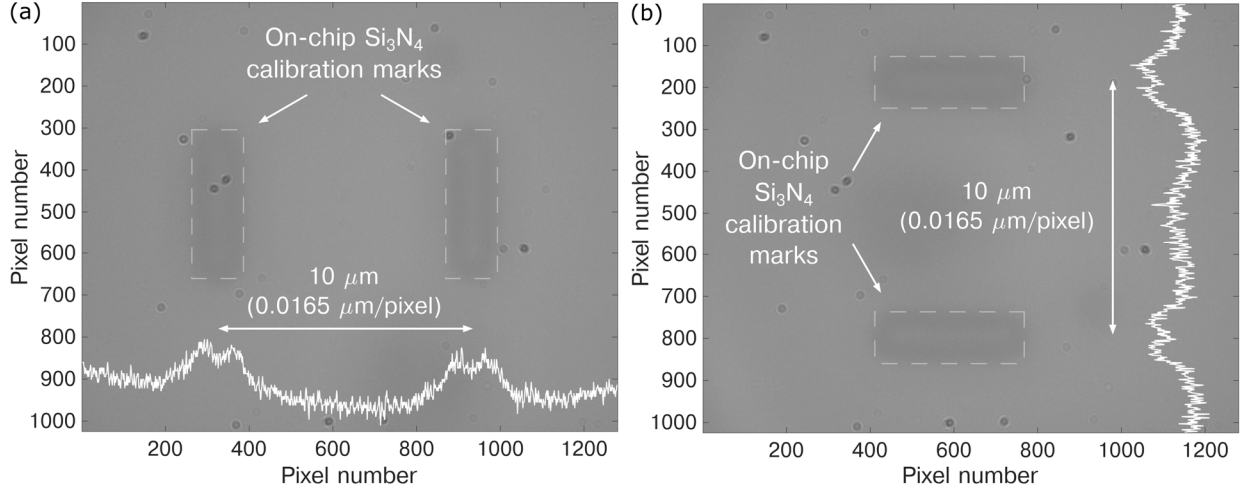


Figure S19: Calibration of the imaging system. (a) Horizontal direction. (b) Vertical direction. The calibration reference is a pair of marks on a Si₃N₄ photonic chip spaced by 10 μm pitch. The white curves represent slices through the center of the marks in each direction. From the pitch, we extract that each pixel corresponds to about 0.0165 μm in either direction. We convert the images to grayscale before slicing.

2.5. Operating conditions

Unless otherwise stated, we operate the FPLDs at room temperature with constant pump currents (applied by a Keithley 2220G-30-1 power supply) corresponding to the values in Table S2 (120 mA for near-IR, 80 mA for red, 200 mA for green, 78.6 mA for blue, 75 mA for deep blue and 42 mA for near-UV). We only change the FPLD temperature for the heterodyne beat note measurements in the red and deep blue spectral ranges (Fig. 5, main text and section 2.10), in which case we keep their currents at the operating points of Table S2. For the laser current-based frequency tuning measurements (Fig. 4b, main text and section 2.9.2), we keep the FPLD at room temperature and sinusoidally modulate its current around

the operating point of Table S2. We add the sinusoidal modulation from an arbitrary waveform generator (Keysight 33512B) to the DC current through a bias tee (Fairview MW SB65000A).

We drive the microheaters on the photonic chip with two separate sources (Keithley 2220G-30-1 for the bus waveguide phase shifter and Keithley 2400 for the resonator phase shifter) with varying voltages to cause self-injection locking and frequency pulling at different wavelengths. For the microheater-based frequency tuning speed (Fig. 4a, main text and section 2.9.1) and the microheaters time constants (section 2.9.1.2) measurements, we replace the DC source of the ring microheater with an arbitrary waveform generator (Keysight 33512B).

2.6. Self-injection locking in near-infrared, red, green, deep blue and near-ultraviolet

Figs. S20 to S24 show the microscope images of the ring resonator-based feedback loops and the optical spectra of the chip-scale lasers output before and after self-injection locking for near-infrared (785 nm), red (660 nm), green (520 nm), deep blue (450 nm) and near-ultraviolet (404 nm). Similarly to the results in the blue (488 nm) spectral range (Fig. 2, main text), we observe clear mode collapse with high side-mode suppression ratios (SMSR) for all wavelengths, above 35 dB from near-infrared to deep blue and around 21 dB for near-ultraviolet. While the linewidth narrowing that accompanies the mode collapse is not apparent in Figs. S20 to S24 due to the limited resolution of the optical spectrum analyzer (0.05 nm), the self-injection-locked lasers do possess narrow linewidth, as we prove via the measurements in Fig. 5 (main text).

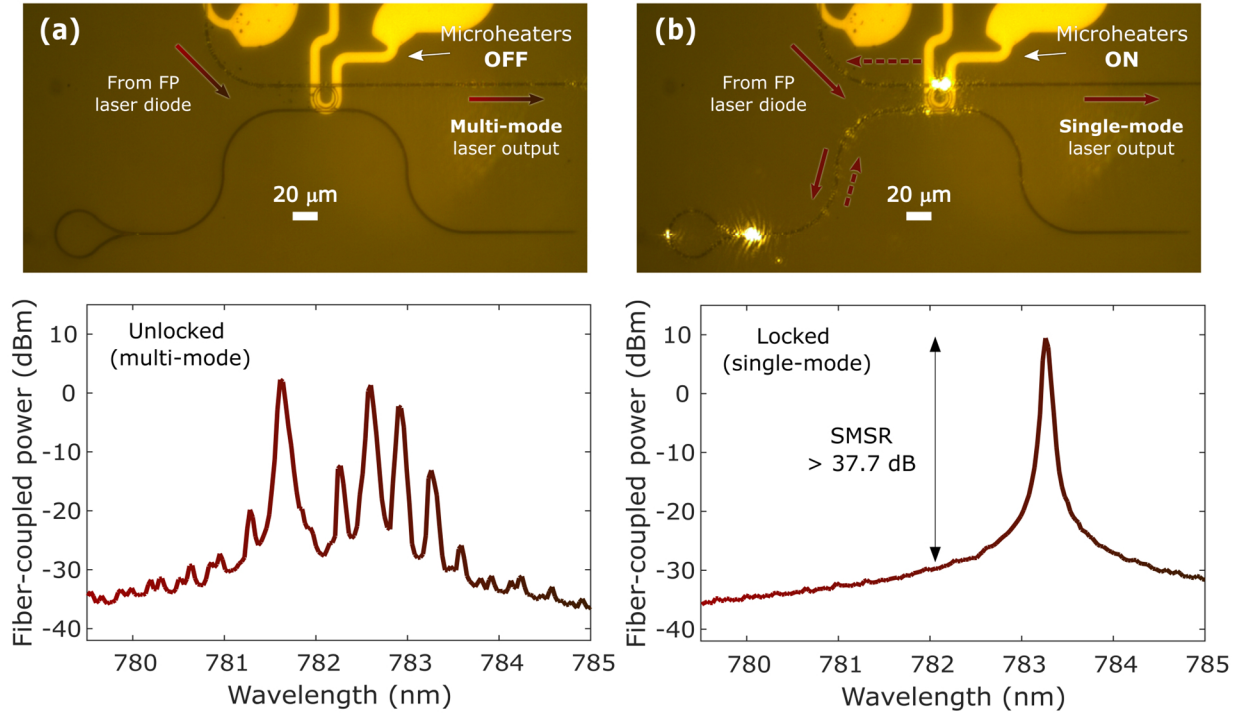


Figure S20: Microscope images of the ring resonator-based feedback loop and optical spectra of the chip-scale laser output in the near-infrared (785 nm) wavelength range. (a) Before self-injection locking. (b) After self-injection locking. The real color of the light is dark red, but it looks yellow in the captured images because of the built-in filter and the sensor sensitivity of the camera (Thorlabs DCC1645C) at this wavelength.

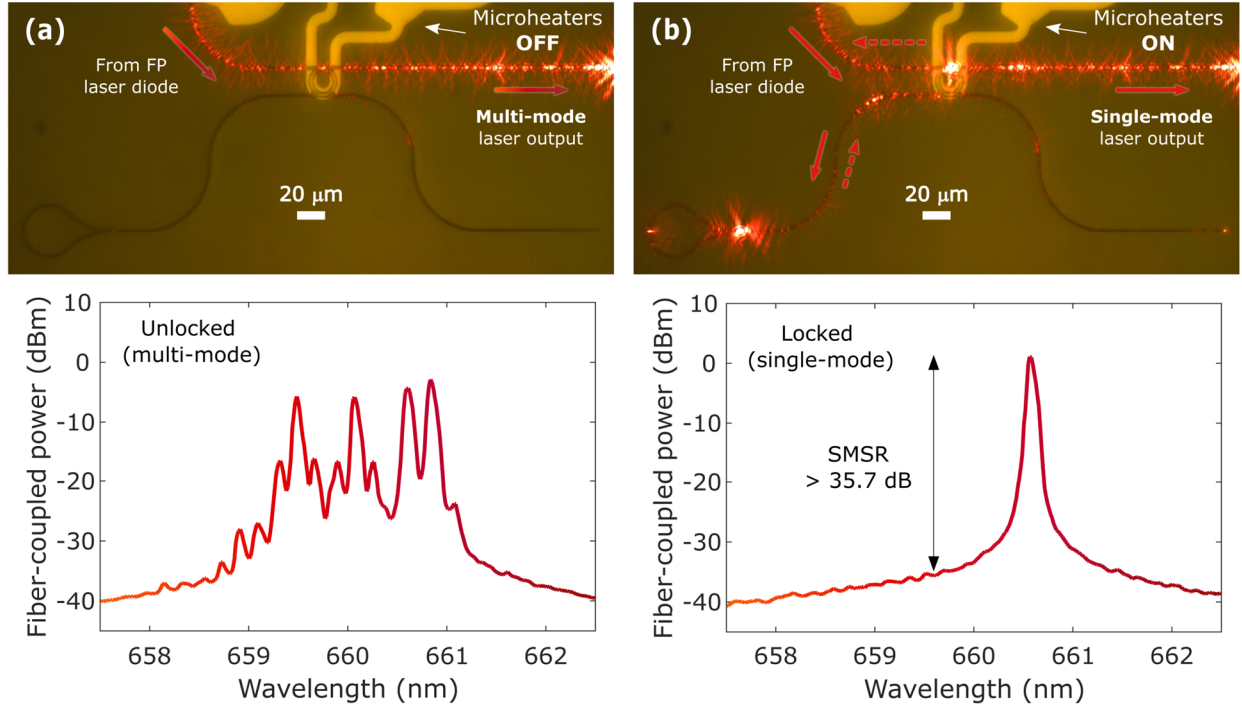


Figure S21: Microscope images of the ring resonator-based feedback loop and optical spectra of the chip-scale laser output in the red (660 nm) wavelength range. (a) Before self-injection locking. (b) After self-injection locking.

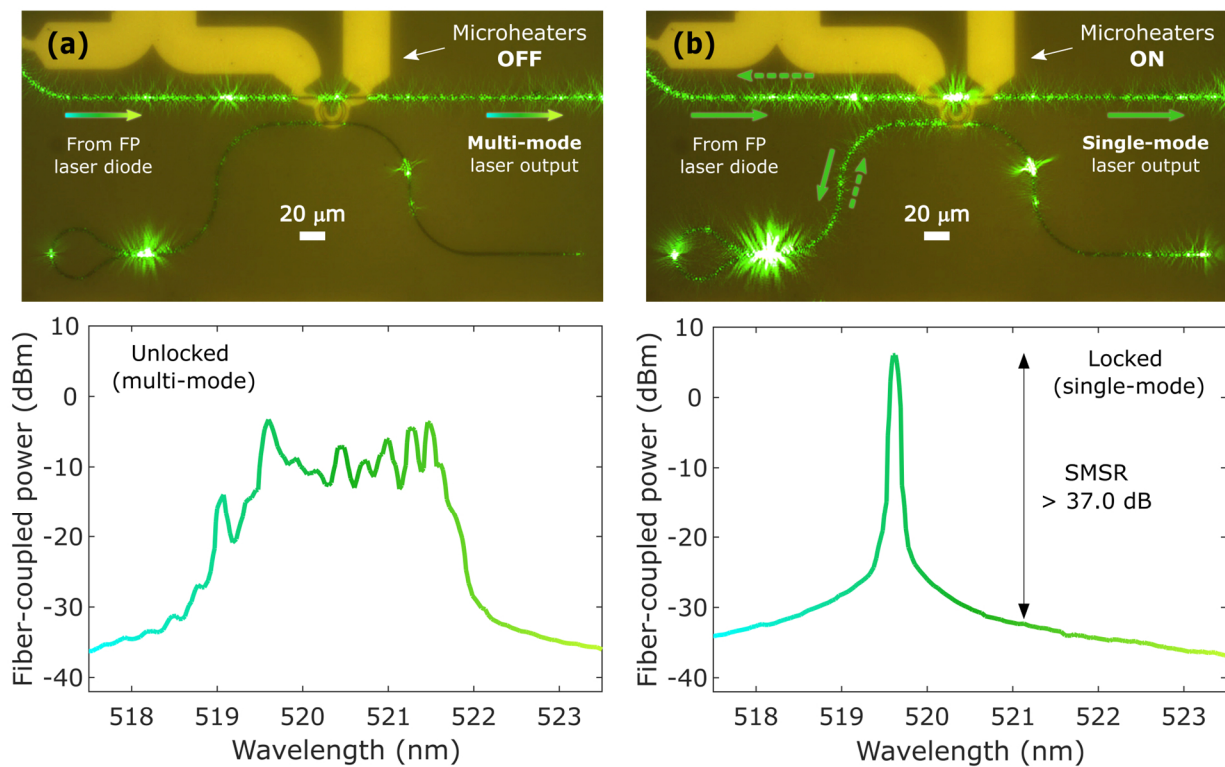


Figure S22: Microscope images of the ring resonator-based feedback loop and optical spectra of the chip-scale laser output in the green (520 nm) wavelength range. (a) Before self-injection locking. (b) After self-injection locking.

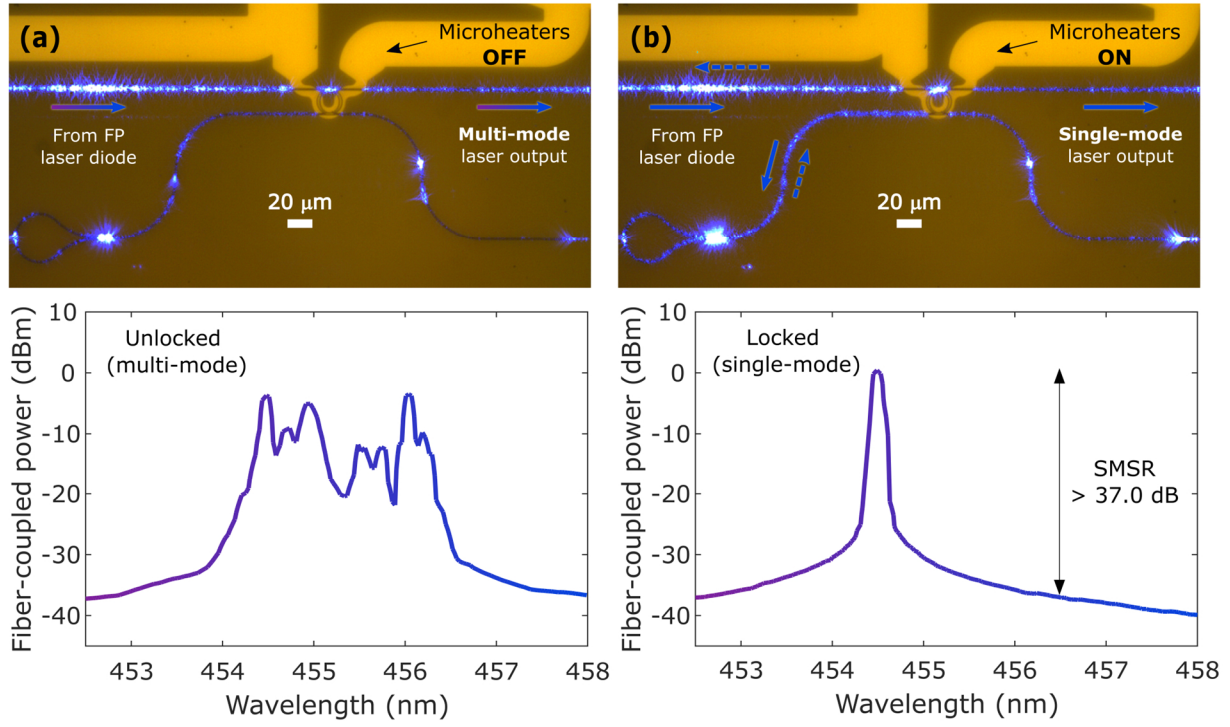


Figure S23: Microscope images of the ring resonator-based feedback loop and optical spectra of the chip-scale laser output in the deep blue (450 nm) wavelength range. (a) Before self-injection locking. (b) After self-injection locking.

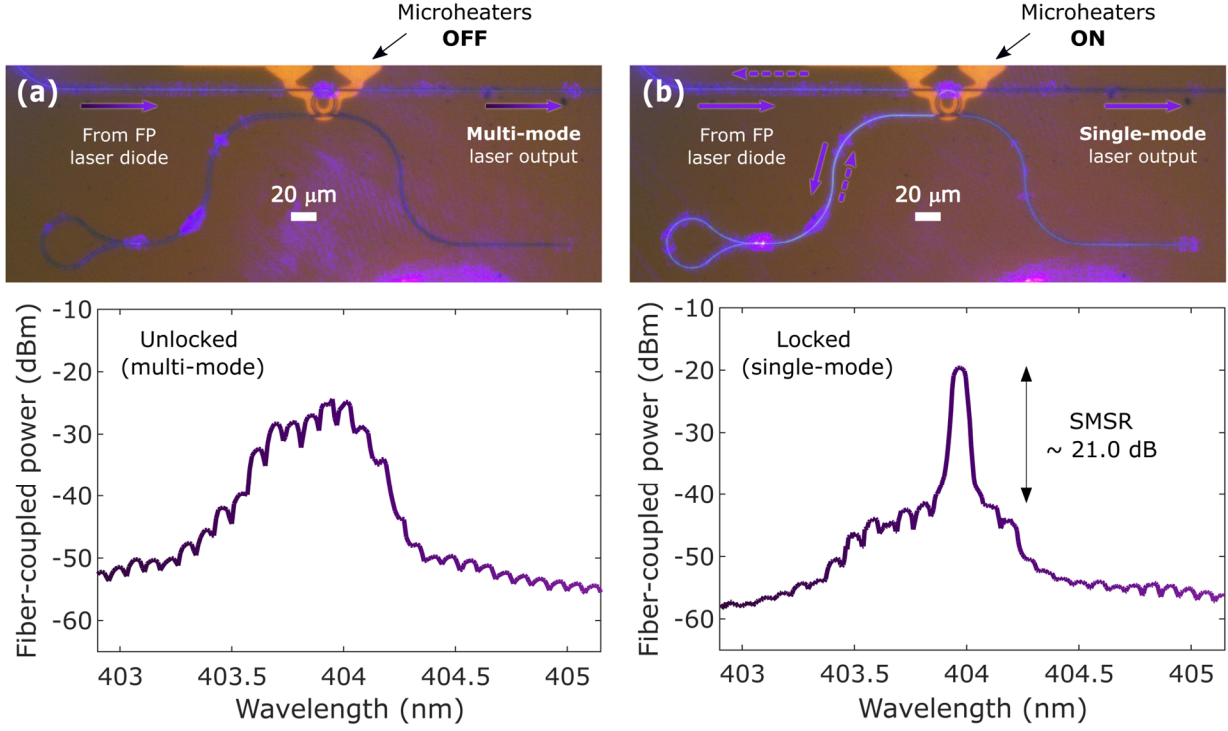


Figure S24: Microscope images of the ring resonator-based feedback loop and optical spectra of the chip-scale laser output in the near-ultraviolet (404 nm) wavelength range. (a) Before self-injection locking. (b) After self-injection locking.

2.7. Coarse tuning and frequency pulling measurements

We measure the coarse tuning ranges of our chip-scale lasers by self-injection locking the FPLDs at different longitudinal modes via the microheaters in the photonic chip. We drive the FPLDs at the operating points presented in Figs. S12 to S17 (or Table S2) and apply different sets of voltages to the bus waveguide and ring resonator microheaters. Each combination of voltages causes self-injection locking at a different wavelength, despite the fact that the FPLDs are driven with constant pump current and remain at room temperature. We monitor the spectra of the lasing wavelengths with an optical spectrum analyzer (Ando AQ6314A) and overlay them to obtain Fig. 3 (main text). In addition to controlling the on-chip microheaters, by changing the pump current and the temperature of the FPLDs we can achieve gap-free coverage^{83,84} of the whole coarse tuning range. We can also extend it several nanometers towards both

shorter and longer wavelengths, as illustrated by the linewidth measurements in deep blue and red (section 2.10) where we self-injection locked the lasers around 2 nm above and 5 nm below the edges of their tuning ranges at room temperature (Fig. 3), respectively.

We measure the frequency pulling ranges of our chip-scale lasers by changing the power in the ring microheater while monitoring the lasing wavelength with a wavemeter after self-injection locking. We start by driving the FPLDs at the operating points presented in Figs S12 to S17 (or Table S2) and setting the voltages applied to the microheaters to cause self-injection locking at some wavelength. Then, we vary the voltage applied to the ring microheater and monitor the change in lasing wavelength with a wavemeter while keeping the bus waveguide microheater unchanged. Since the wavemeters have long minimum exposure times, we vary the voltage applied to the ring microheater in discrete steps, allowing enough acquisition time for the accurate measurement of the lasing wavelengths. At each voltage, we record the lasing wavelength and the electrical power delivered to the microheater (by the Keithley 2400). The frequency pulling measured in this way is discrete, but we also show continuous pulling via modulation of the microheater in Fig. 4a (main text) and section 2.9.1. We chose to keep the bus waveguide microheater and the driving current of the FPLDs fixed throughout the frequency pulling measurements, but we can achieve even broader tuning ranges by driving the microheaters and the FPLDs in a feed-forward fashion^{70,85,86}.

For the near-IR spectral range, the error bars in Fig. 3b (main text) correspond to the resolution of the wavemeter used (Advantest Q8326, with fixed exposure time of 200 ms and resolution of 1 pm). From the near-UV to the red spectral ranges, since the wavemeter used is faster and has better resolution (HighFinesse WSU-10, with minimum exposure time of 1 ms and resolution ≤ 5 MHz), we determine the error bars from the standard deviation of the measured wavelength at each voltage step. In Fig. 3b (main text), the error bars are comparatively more noticeable for the shorter wavelengths because their total frequency pulling ranges are smaller. An example measurement is shown in Fig. S25 for the chip-scale blue (488 nm) laser. The vertical dashed lines indicate the moments when we vary the microring heater voltage,

which are accompanied by clear rapid changes in the lasing wavelength due to the frequency pulling. Closer to the edge of the pulling range (time ≥ 97.3 s), we notice that the lasing wavelength started drifting slowly, indicating that the microring detuning was too large to sustain stable self-injection locking at that particular longitudinal mode of the FPLD. At the following voltage increment (time = 116.19 s), the laser mode-hopped to a different longitudinal mode.

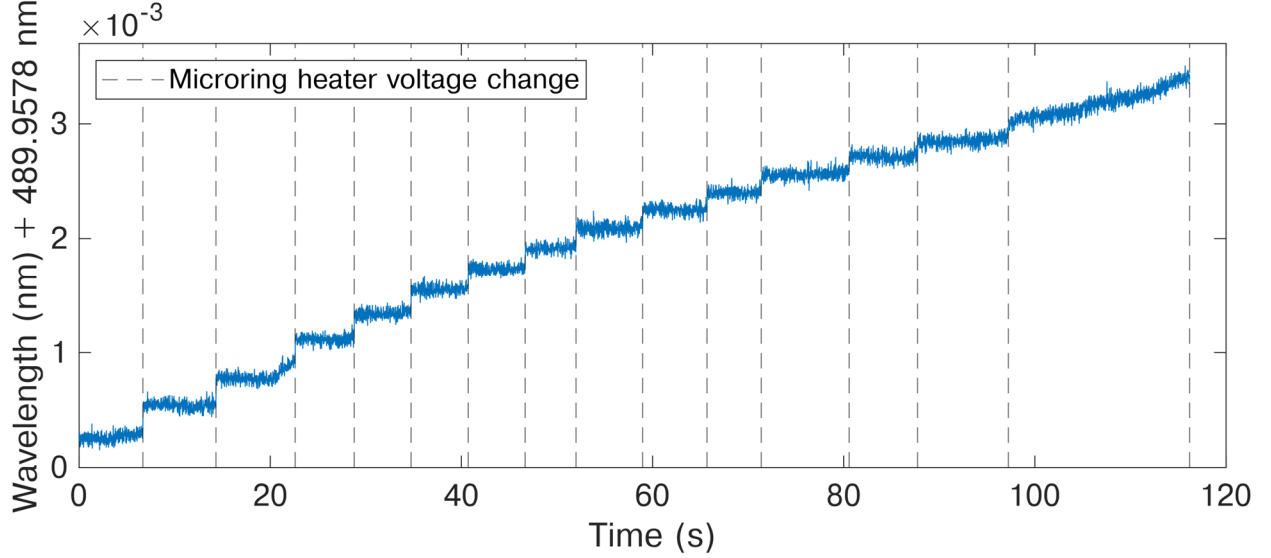


Figure S25: Example of frequency pulling measurement for the blue (488 nm) chip-scale laser. At time zero, we self-injection lock the laser to the wavelength of approximately 489.961 nm by setting the voltages of the integrated microheaters. We keep the voltages fixed for a few seconds in order to record the fluctuations of the lasing wavelength around that point before monotonically incrementing the voltage of the microring heater. This process is repeated until the laser unlocks. The vertical dashed lines indicate the moments when we vary the microring heater voltage. The clear rapid changes in the lasing wavelength whenever the voltage is set corresponds to the frequency pulling caused by the microring tuning.

2.8. Long-term mode-hop-free operation and frequency stability

In order to demonstrate the long-term mode-hop-free operation and frequency stability of our chip-scale lasers, we measure up to 5.69 hours of continuous mode-hop-free operation (Fig. S26), limited by the

coupling stage drift between the FPLD and the photonic chip. We self-injection lock our chip-scale near-IR and blue lasers and monitor the frequency drift of the lasing wavelength over time while keeping the driving sources of the FPLD and the photonic chip at fixed set voltages. We confirm the absence of mode-hops by the fact that the observed peak-to-peak frequency drifts of 7.07 GHz and 8.37 GHz for near-IR and blue lasers, respectively, are about nine times smaller than the estimated 62 GHz and 75 GHz free spectral ranges (FSR) of the near-IR and blue FPLDs, respectively (Table S4). If mode hops had happened, the lasing wavelength would present abrupt jumps on the order of the FSR of the FPLDs or larger. Throughout the measurements, the coupling between the FPLDs and the photonic chip slowly drifted and ultimately caused enough coupling loss to unlock the lasers at the times of 5.34 hours and 5.69 hours for the near-IR and blue chip-scale lasers respectively. Since the long-term behavior of the chip-scale lasers at short (488 nm) and long (785 nm) wavelengths are very similar and limited by the coupling stage drift, we expect the other chip-scale lasers to have a similar performance. By stabilizing the FPLD-to-chip coupling, we could achieve longer periods of continuous mode-hop-free operation.

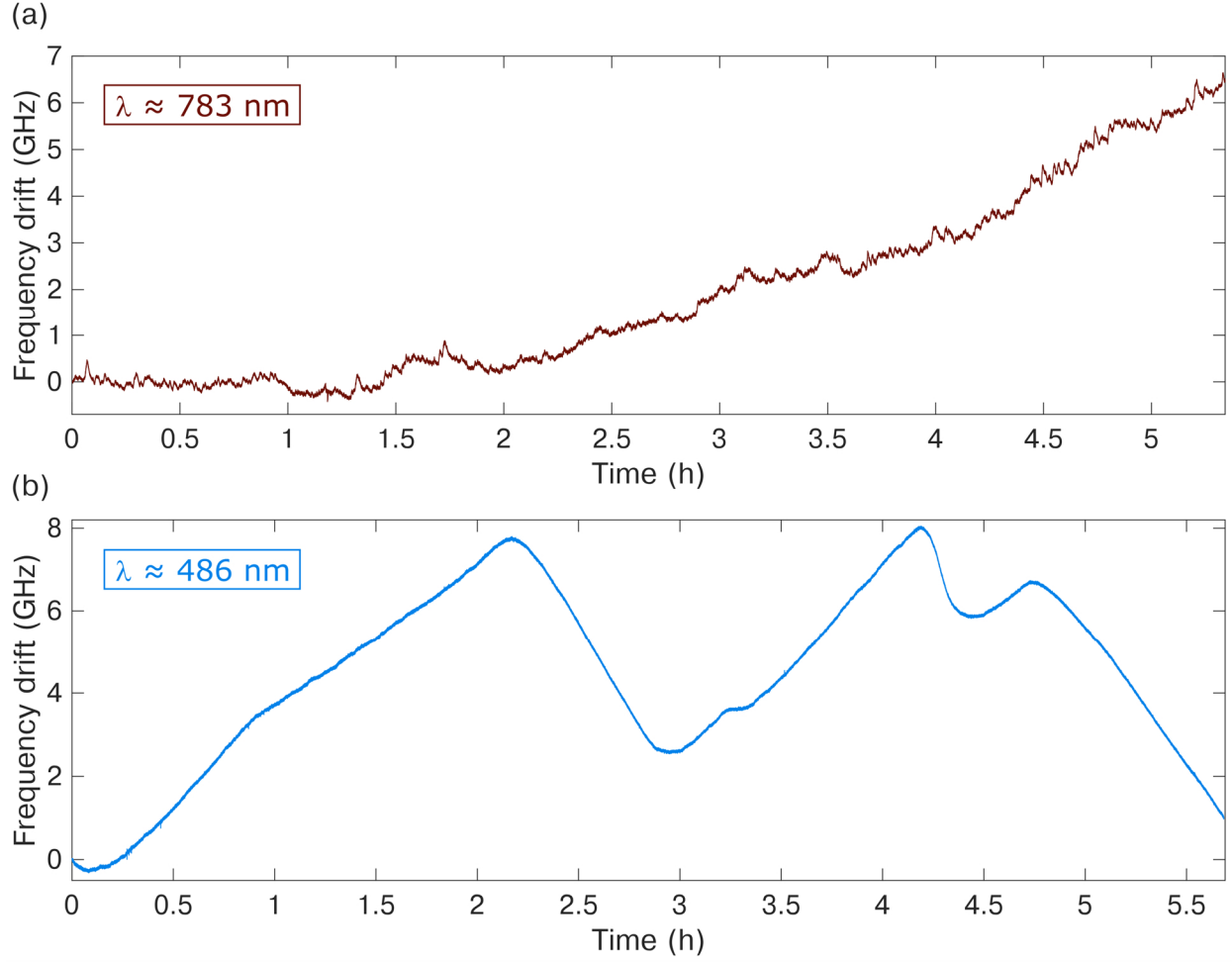


Figure S26: Long-term mode-hop-free operation and frequency stability of our chip-scale lasers. (a) Near-infrared chip-scale laser. We measure up to 5.34 hours of continuous mode-hop-free operation with a peak-to-peak frequency drift of 7.07 GHz after self-injection locking the laser to an initial wavelength of 782.8863 nm. (b) Blue chip-scale laser. We measure up to 5.69 hours of continuous mode-hop-free operation with a peak-to-peak frequency drift of 8.37 GHz after self-injection locking the laser to an initial wavelength of 486.3603 nm. We monitor the frequency drift of the lasing wavelength over time with a wavemeter (HighFinesse WSU-10) while keeping the driving sources of the FPLDs and the photonic chip at fixed set voltages. Throughout the measurements, the coupling between the FPLDs and the photonic chip slowly drifted and ultimately caused enough coupling loss to unlock the lasers.

While there are many sources of lasing frequency drift, we believe that the two most important are temperature variations and FPLD-to-chip coupling drift. Consequently, the frequency stability of our chip-scale lasers can be further improved with temperature stabilization of the FPLD and the photonic chip, optical packaging or active alignment for a more stable FPLD-to-chip coupling, and Pound-Drever-Hall stabilization^{87–92} enabled by the fast tuning responses of our chip-scale lasers via both microheater and laser current modulation (Fig. 4, main text and section 2.9). The fact that the peak-to-peak frequency drift for the blue chip-scale laser is larger than its measured frequency pulling range (Fig. 3, main text) indicates that both the FPLD longitudinal mode and the microring resonance drifted in tandem during at least part of the time, which is explained as follows. Room temperature fluctuations, which are typically on the order of 0.5 °C for the span of several hours in our laboratory, affect the FPLD temperature and the operation of microring resonators⁹³. Due to the thermo-optic effect, a temperature increase (decrease) causes a red (blue) shift of the FPLD cavity modes and the microring resonances. The temperature fluctuations also have an adverse effect on the FPLD-to-chip coupling due to thermal expansion/contraction of the chips and the coupling stage, which adds to the systematic slow drift of the coupling. When the interface between the FPLD and the photonic chip changes, it affects not only the optical power coupled to the chip but also the phase of the optical feedback from the microring, thus causing some unintended frequency pulling. Both of these effects also alter the built-up optical power inside of the microring, which in turn changes the resonator temperature due to intrinsic material absorption (photothermal effect). The resonance shift caused by this combination of photothermal and thermo-optic effects is a well-known phenomenon observed routinely at telecom wavelengths^{94–97} and is expected to be more intense at visible wavelengths due to the higher material absorption. Due to these same effects, a change in the amount of optical feedback to the FPLD also alters its temperature. Additional sources of lasing frequency drift include potential variations in microheater resistance during long-term operation⁹⁸ and drift of the driving sources.

2.9. Frequency tuning speed

We achieve frequency pulling by controlling either the microheater on the ring resonator or the FPLD current once we self-injection lock the laser, since both can change the relative detuning between the longitudinal mode of the FPLD and the ring resonance that provides the external feedback. The microheater shifts the ring resonance, while the laser current shifts the FPLD mode. By keeping either one fixed, we cause frequency pulling by tuning the other within the locking range⁶⁴.

In the following subsections, we provide the detailed characterization of each frequency pulling scheme. The microheater-based tuning enables wide pulling ranges up to tens of GHz with modulation bandwidths up to tens of kHz, making it suitable for applications that require fast long-range tunings such as tunable diode laser absorption spectroscopy⁹⁹ and frequency-modulated continuous wave (FMCW) Lidar^{100–102}. The laser current-based tuning, on the other hand, provides shorter pulling ranges up to several GHz but with modulation bandwidths up to at least tens of MHz, making it suitable for applications that require ultra-fast short-range tunings such as the laser frequency stabilization via negative electronic feedback^{87–92}. By controlling both the microheater and the laser current simultaneously via a feed-forward scheme^{70,85,86}, we can also significantly increase the frequency pulling range of the self-injection-locked laser, since we can keep the detuning between the longitudinal mode and the ring resonance within the locking range while we finely tune the lasing wavelength.

Since wavemeters have long minimum exposure times (≥ 1 ms), they are not able to measure the fast tuning speeds enabled by the microheater and the FPLD current modulations. In order to characterize the frequency modulation performance of our chip-scale lasers, we use two different experimental setups (Fig. S27), each able to address a different scale of modulation bandwidth and tuning speed.

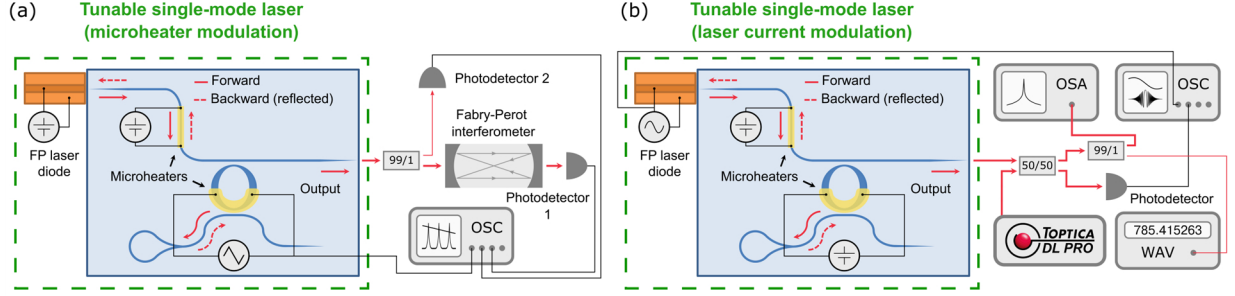


Figure S27: Experimental setups for the frequency tuning speed measurements. (a) Frequency tuning via microheater modulation. We modulate the microring phase shifter with a triangular voltage waveshapes of increasing frequencies (Keysight 33512B) while we keep the Fabry-Perot (FP) laser current and the bus waveguide phase shifter fixed. We couple the chip-scale laser output to a fixed Fabry-Perot interferometer (FPI) with 1.5 GHz free spectral range (Thorlabs SA200-6A), whose transmission we measure with a photodetector (Thorlabs APD120A2). The distance between consecutive FPI resonances provides the laser frequency detuning as a function of time and a linear fit yields the frequency tuning rate. We tap a small portion of the laser output to monitor the mode-hop free tuning ranges (Fig. S28). The oscilloscope is a Picoscope 5444B. (b) Frequency tuning via laser current modulation. We modulate the FP laser current with a sinusoidal voltage waveshape of increasing frequencies (Keithley 2220G-30-1 and Keysight 33512B combined via a bias tee) while we keep the on-chip phase shifters fixed. We combine the chip-scale laser output with the light from a fixed wavelength laser (Toptica DL Pro 780 nm) and send it to a photodetector (Thorlabs DET025AFC, 2 GHz bandwidth). The instantaneous frequency of the beat note provides the detuning between the two lasers as a function of time, from which we extract the tuning rate of the chip-scale laser. We send a fraction of the chip output to an optical spectrum analyzer (Ando AQ6314A) and a wavemeter (HighFinesse WSU-10) to monitor the self-injection locking and to align the center wavelength of the lasers, respectively. The oscilloscope is a Tektronix MSO5204B.

2.9.1. Microheater-based frequency tuning

Fig. S27 shows the setup we use for characterizing the frequency pulling speed via microheater modulation. We apply triangular voltage waveshapes with varying frequencies to the ring microheater and

couple the output of the chip-scale laser to a fixed Fabry-Perot interferometer (FPI) with 1.5 GHz free spectral range (FSR). Since the wavemeter is too slow to monitor the sweeping lasing wavelength, we determine the extremes of the frequency pulling ranges using the sharp transitions of the chip-scale laser output, which are a characteristic signature of mode-hopping^{103–105} (Fig. S28). Within the pulling range, the distance between consecutive peaks of the FPI transmission corresponds to a frequency change of one FSR of the FPI. Since the FSR is known (1.5 GHz), the peak positions represent an accurate frequency calibration for determining the tuning rate. If the laser is swept through the FPI resonances fast enough compared to their photon lifetime, the FPI response exhibits characteristic fringes on top of an exponential decay, as typically seen in cavity ring-down spectroscopy experiments¹⁰⁶. Despite the change in shape, the peaks are still spaced by one FSR. We present the characterization of the FPI, the modeling of its transmitted signal and the tuning rate results for the near-IR and red spectral ranges in the following subsections.

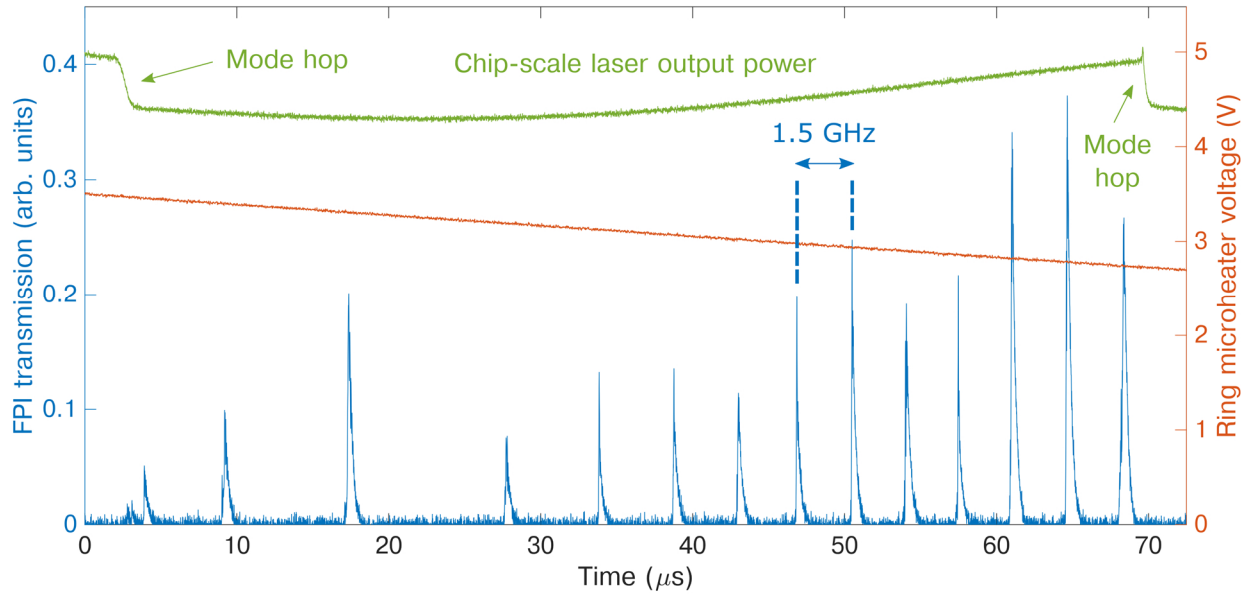


Figure S28: Microheater-based frequency tuning example measurement. The triangular voltage driving the ring microheater has 1 kHz frequency and 5.5 V peak-to-peak. The green curve represents the behavior of the chip-scale laser output power as the lasing wavelength is tuned. The power changes abruptly at the edges of the frequency pulling range (green arrows), which is a characteristic signature of mode-hopping.

2.9.1.1. Fabry-Perot interferometer (FPI) characterization

We measure the linewidths of the FPI resonances around 785 nm and 660 nm wavelengths to be ~ 1.17 MHz and ~ 5.11 MHz, respectively (Fig. S29b-c). The broader linewidth at 660 nm indicates a lower reflectivity of the cavity mirrors in the red spectral range than in the near-IR. The measurement setup is similar to the one in Fig. S38b, where we keep the lasers at fixed wavelengths and drive the FPI with a voltage source (Thorlabs SA201) to sweep its resonances (Fig. S29a). The interferometer we use is a confocal FPI with 1.5 GHz confocal FSR and wavelength range from 650 nm to 800 nm (Thorlabs SA200-6A). The input laser light to the FPI comes from a Toptica DL Pro 780 nm for the near-IR range and from our red chip-scale laser for the red range. Since these lasers have linewidths much narrower than the linewidth of the FPI, the transmitted peaks are direct measurements of the FPI lineshapes. The slight asymmetry of the lineshapes is due to imperfect illumination of the FPI^{107–109}.

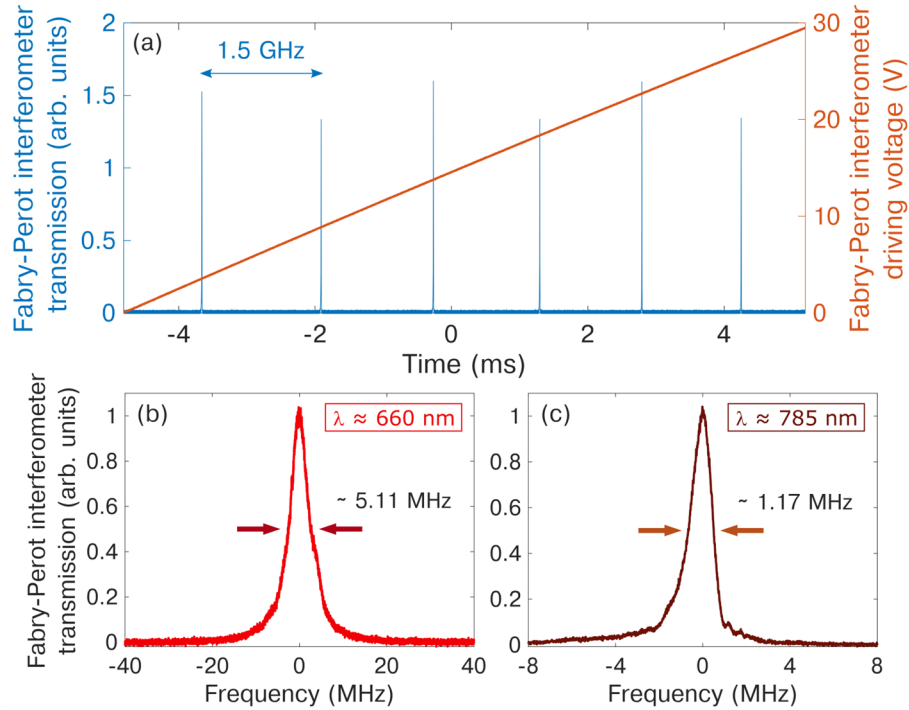


Figure S29: Scanning Fabry-Perot interferometer (FPI) calibration. (a) FPI transmission when we keep the input narrow linewidth laser at a fixed wavelength and drive the FPI with a triangular voltage waveshape. The distance between two consecutive peaks corresponds to one free spectral range (FSR) of

the FPI (1.5 GHz, Thorlabs SA200-6A). The resonant peaks serve as a ruler to determine the frequency detuning as a function of time. (b) Lineshape of the FPI at 660 nm wavelength. The linewidth is ~ 5.11 MHz. (c) Lineshape of the FPI at 785 nm wavelength. The linewidth is ~ 1.17 MHz. The slight asymmetries of the lineshapes and consequent linewidth broadening, in particular in (c), are due to imperfect alignment between the input beam and the FPI cavity^{107–109}. In terms of lifetimes, the measured linewidths Δf correspond to cavity lifetimes $\tau = 1/(\pi\Delta f)$ of approximately 62 ns at 660 nm and 271 ns at 785 nm, which agree with the experimentally measured time constants in Figs. S30 and S31.

If we sweep the narrow linewidth laser through the FPI resonance fast enough compared to its photon lifetime, the transmitted lineshape exhibits characteristic fringes on top of an exponential decay. The period of the fringes is a measurement of the frequency tuning rate and the exponential time constant corresponds to the lifetime of the cavity. The dynamic response of the FPI is described by the set of equations¹¹⁰:

$$\frac{dE_{cav}}{dt'} = -(1 - i v_{\omega} t') E_{cav} + i \frac{\sqrt{T} \mathcal{F}}{\pi} E_0 \quad (9)$$

$$I_{trans} = \frac{c\epsilon_0}{2} T |E_{cav}|^2 \quad (10)$$

where E_{cav} is the field inside of the cavity and I_{trans} is the transmitted optical intensity, which is proportional to the experimentally measured signal. c is the speed of light in vacuum, ϵ_0 is the vacuum permittivity, T is the power transmission coefficient of the mirrors, $\mathcal{F}$ is the finesse of the cavity, v_{ω} is the normalized angular frequency scan rate, t' is the normalized time and E_0 is the input field amplitude. The normalized quantities are defined by

$$t' = \frac{t}{\tau_s} \quad (11)$$

$$v_{\omega} = \frac{2\mathcal{F}L\dot{\omega}\tau_s}{\pi c} \quad (12)$$

where τ_s is field time constant of the cavity, L is the cavity length and $\dot{\omega}$ is the angular frequency scan rate. L is related to the confocal FSR of the FPI via $L = c/(4FSR)$, which yields $L = 50$ mm for $FSR = 1.5$ GHz and matches the radius of curvature of the mirrors. From the known FSR and the distance between measured transmitted peaks, we calculate $\dot{\omega}$. The power time constant τ , which is the time constant of the measured transmitted signal, is related to τ_s via $\tau = \tau_s/2$. From τ , we calculate $\mathcal{F}$ via $\mathcal{F} = 2\pi\tau FSR$ and we obtain the power reflection coefficient of the mirrors R from $\mathcal{F} = \pi\sqrt{R}/(1 - R)$. The power transmission coefficient follows from $T = 1 - R$. The time constant τ is also related to the linewidth Δf of the resonance when the scanning speed is low (as in Fig. S29b-c) via $\Delta f = 1/(\pi\tau)$.

We characterize the dynamic response of the FPI by sweeping the frequency of the input field at various rates and fitting the FPI transmission using Eqs. 9 and 10. We obtain time constants of 308 ns and 63 ns, corresponding to linewidths of 1.03 MHz and 5.05 MHz for the near-IR and red spectral ranges respectively. The linewidths extracted from the time constants agree well with the values measured directly from the lineshapes in Fig. S29b-c. The former are slightly larger due to the broadening induced by the asymmetry of the lineshapes, which is caused by the imperfect illumination of the FPI. Throughout the measurements, we keep the FPI resonances fixed and tune the frequency of the input lasers (Toptica DL Pro 780 nm and our red chip-scale laser for the near-IR and red spectral ranges, respectively). We tune the commercial near-IR laser via its piezoelectric control and our chip-scale laser via the ring microheater. We fit the experimental data to the theoretical model by solving Eq. 9 via fourth order Runge-Kutta method and normalize both the measured and fitted curves by their peak values for ease of visualization.

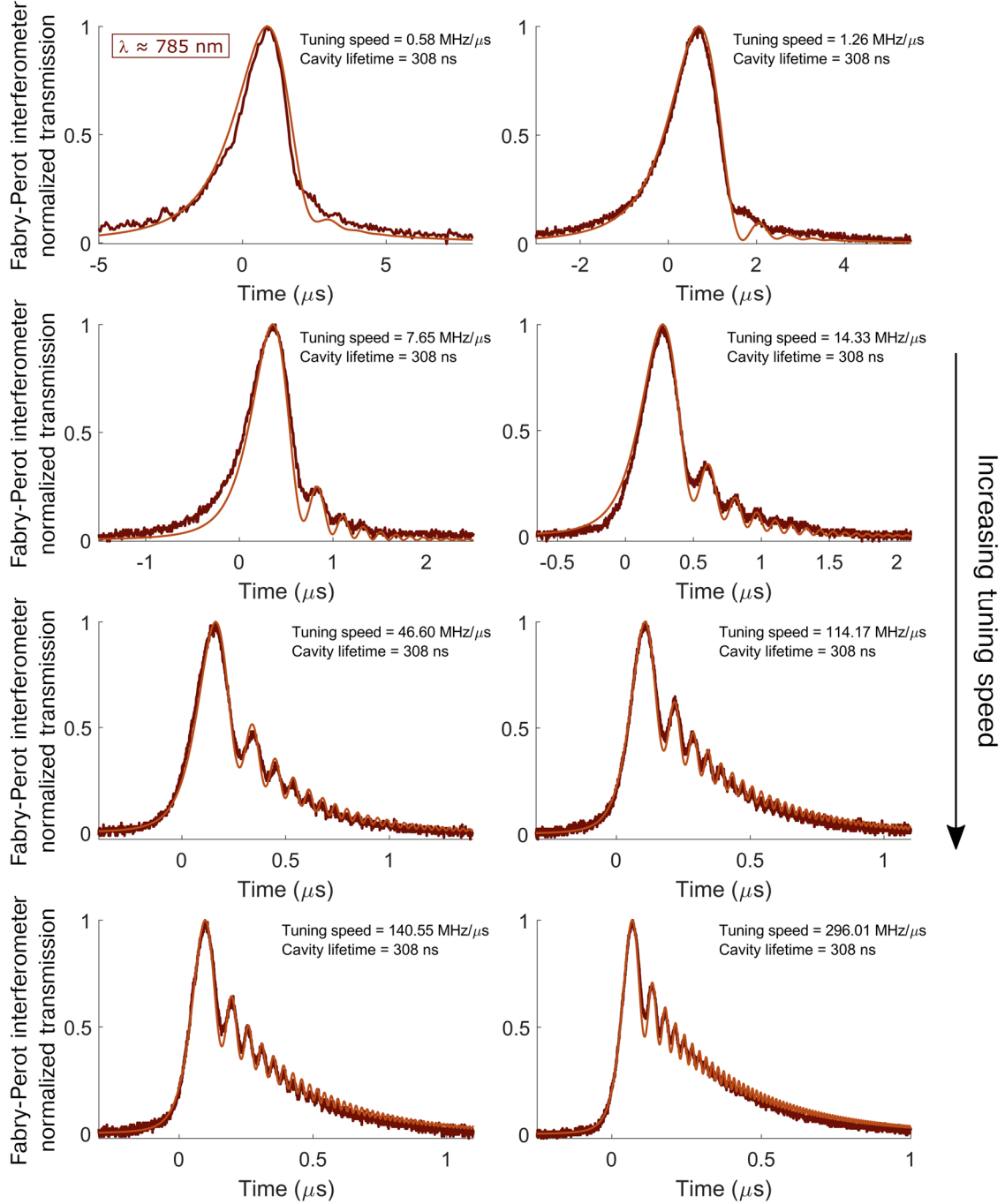


Figure S30: Dynamic response of the Fabry-Perot interferometer (FPI) under frequency-swept input at 785 nm wavelength. From the top left to the bottom right, we increase the frequency tuning rate of the input laser (Toptica DL Pro 780 nm), making the cavity lifetime apparent and causing interference fringes on the transmitted field. The extracted lifetime of 308 ns implies a linewidth of 1.03 MHz, which agrees well with the linewidth of ~ 1.17 MHz measured directly from the lineshape at slow tuning rate (Fig. S29b).

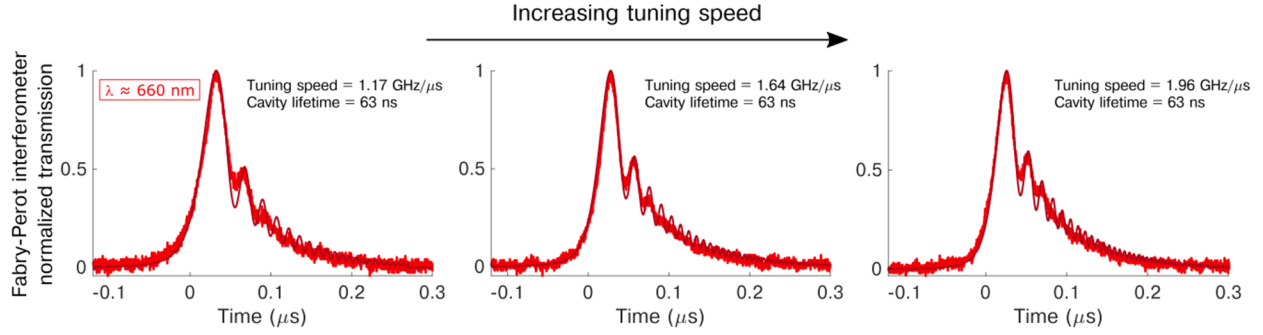


Figure S31: Dynamic response of the Fabry-Perot interferometer (FPI) under frequency-swept input at 660 nm wavelength. From the top left to the bottom right, we increase the frequency tuning rate of the input laser (our red chip-scale laser), making the cavity lifetime apparent and causing interference fringes on the transmitted field. The extracted lifetime of 63 ns implies a linewidth of 5.05 MHz, which agrees well with the linewidth of ~ 5.11 MHz measured directly from the lineshape at slow tuning rate (Fig. S29c).

2.9.1.2. Microheater-based frequency tuning rates and modulation bandwidth

We measure microheater-based frequency tuning speeds up to $7.1 \text{ GHz}/\mu\text{s}$ (Fig. 4a, main text) and $3.4 \text{ GHz}/\mu\text{s}$ (Fig. S34) in the near-IR and red spectral ranges respectively, with a 3-dB modulation bandwidth around 32 kHz across the whole visible spectrum (Fig. S35). The relationship between the microheater modulation frequency and the resulting chip-scale laser frequency tuning rate is mostly linear below the modulation bandwidth of the microheater and saturates beyond it. The results for the near-IR and red spectral ranges indicate that the maximum achievable tuning speed is only limited by the microheater modulation bandwidth. Since this bandwidth is wavelength-independent and around 32 kHz across the whole visible spectrum (Fig. S35), we expect the chip-scale lasers of all colors to have similar tunability behavior.

The measurements supporting Figs. 4a (main text) and S34 are presented in Figs. S32 and S33. The dynamic response of the FPI described in section 2.9.1.1 becomes apparent as we increase the laser frequency tuning speed. We characterize the modulation bandwidth of the microheaters from deep blue to near-IR by measuring their rise and fall time constants. We couple light from commercial lasers (section 3)

to the photonic chip and tune their wavelengths to the linear region of the ring resonances (Fig. S1). We then apply a voltage square wave to the microheater and monitor the output of the chip with a photodetector. The driving signal is small enough to keep the laser wavelength within the linear region of the resonance to prevent any nonlinear distortion from the Lorentzian lineshape. We fit the rising and falling exponentials caused by the microheater time response and extract their rise (τ_r) and fall (τ_f) time constants. Since τ_f is always slightly larger than τ_r and the microheater behaves approximately as a first-order system¹¹¹, we calculate the 3-dB modulation bandwidth of the microheaters using $f_{3dB} = 1/(2\pi\tau_f)$. We obtain the average values and error bars from the statistics of 300 consecutive measurements and fittings.

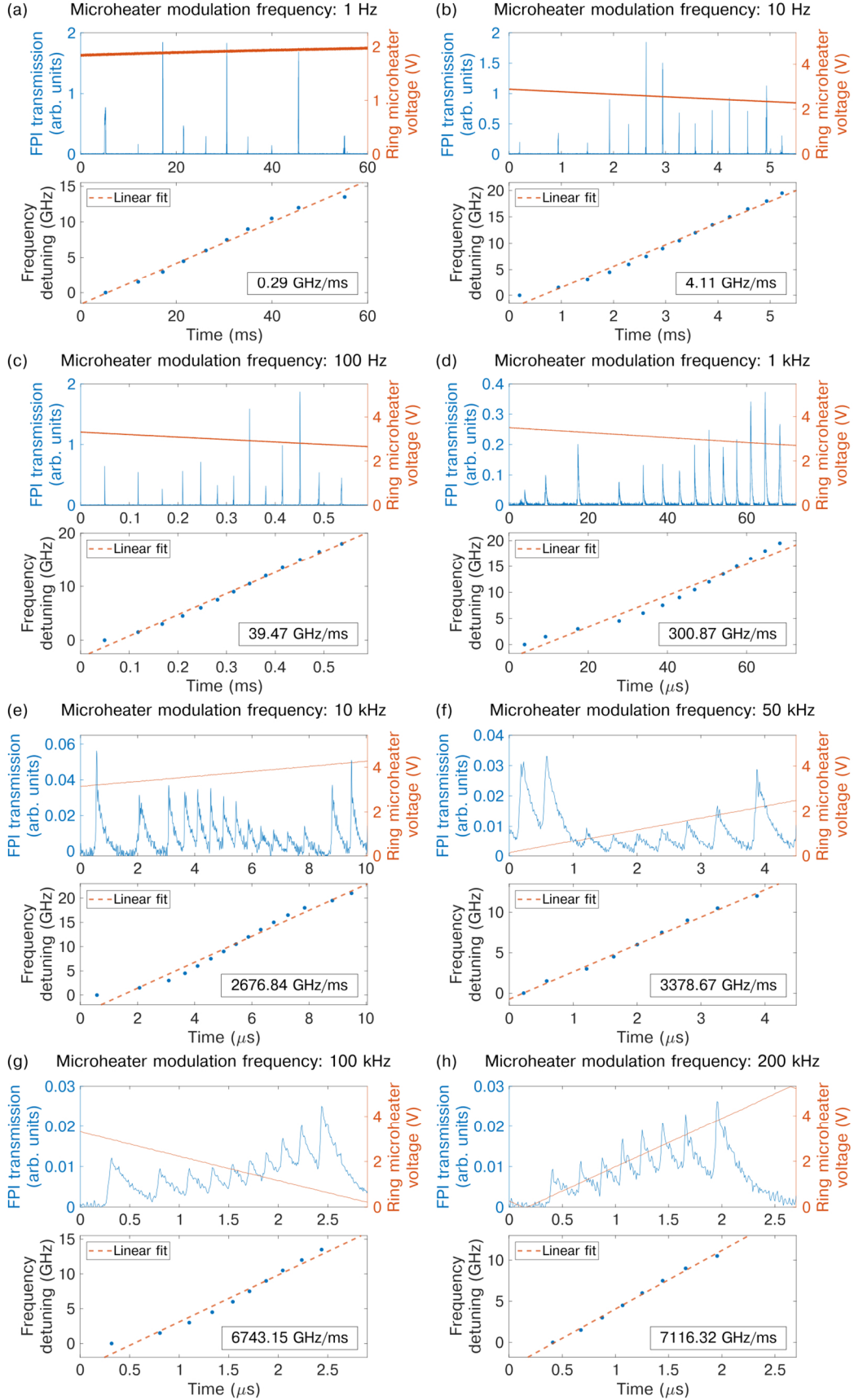


Figure S32: Microheater-based frequency tuning rate measurements for the near-infrared chip-scale laser. Using the experimental setup of Fig. S27a, we modulate the microheater of the ring resonator with triangular voltages of different frequencies and monitor the resulting laser frequency tuning using a Fabry-Perot interferometer (FPI) (top figures). Each free spectral range of the FPI corresponds to a detuning of 1.5 GHz of the laser frequency. From a linear fit of the detuning as a function of time (bottom figures), we extract the frequency tuning rate corresponding to each microheater modulation frequency. In (d)-(h), the frequency tuning speeds of the chip-scale laser are fast enough to expose the dynamic behavior of the FPI (section 2.9.1.1), which has a characteristic exponential decay stemming from the FPI cavity lifetime. In (f)-(h), we notice a saturation of the frequency tuning rate despite the increase in the modulation frequency, which is due to the (34.1 ± 0.3) KHz 3-dB modulation bandwidth of the microheater (Fig. S35).

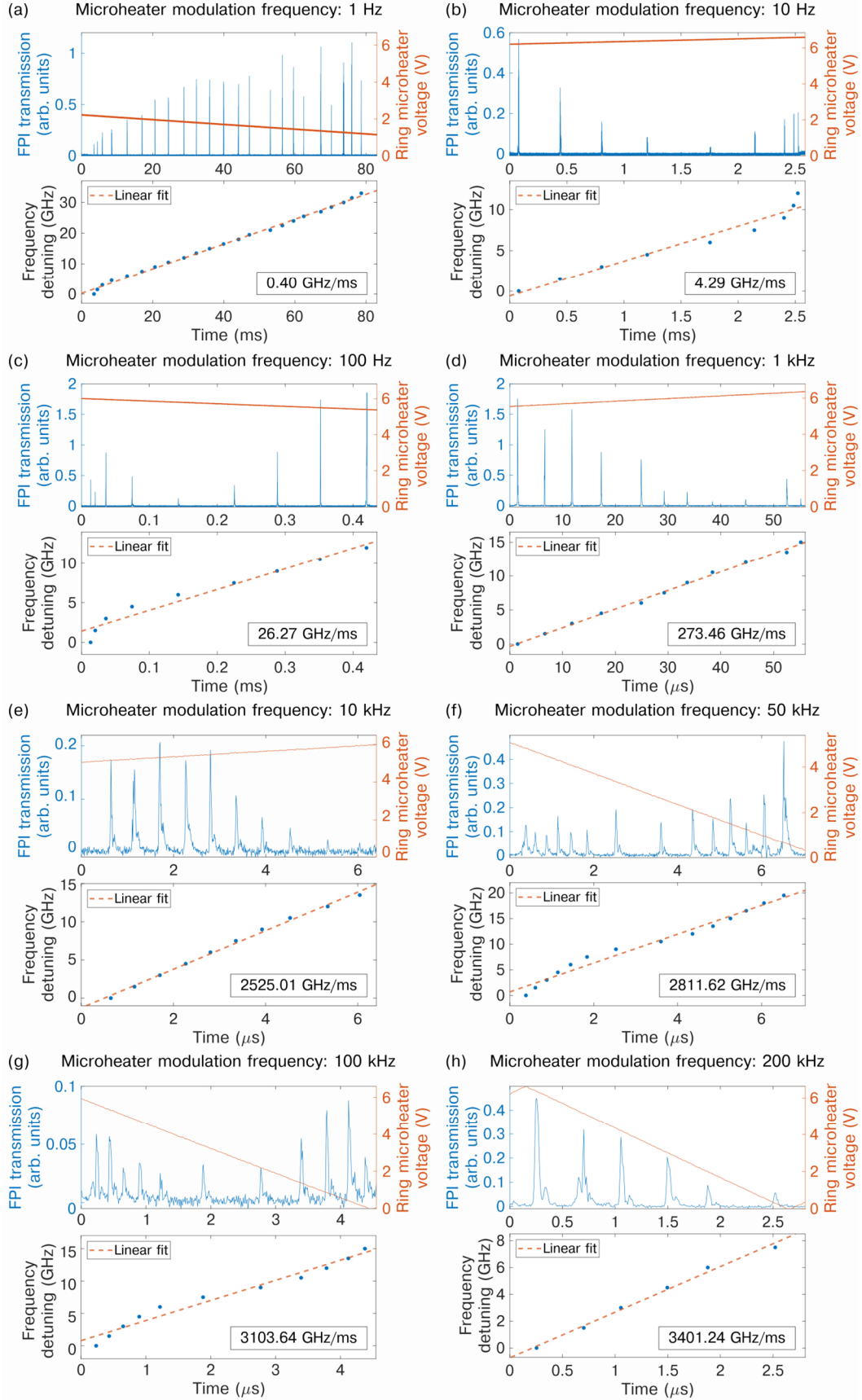


Figure S33: Microheater-based frequency tuning rate measurements for the red chip-scale laser.

using experimental setup of Fig. S27a, we modulate the microheater of the ring resonator with triangular voltages of different frequencies and monitor the resulting laser frequency tuning using a Fabry-Perot interferometer (FPI) (top figures). Each free spectral range of the FPI corresponds to a detuning of 1.5 GHz of the laser frequency. From a linear fit of the detuning as a function of time (bottom figures), we extract the frequency tuning rate corresponding to each microheater modulation frequency. In (e)-(h), the frequency tuning speeds of the chip-scale laser are fast enough to expose the dynamic behavior of the FPI (section 2.9.1.1), which has a characteristic exponential decay stemming from the FPI cavity lifetime. In (f)-(h), we notice saturation of the frequency tuning rate despite the increase in the modulation frequency, which is due to the (31.1 ± 0.2) KHz 3-dB modulation bandwidth of the microheater (Fig. S35).

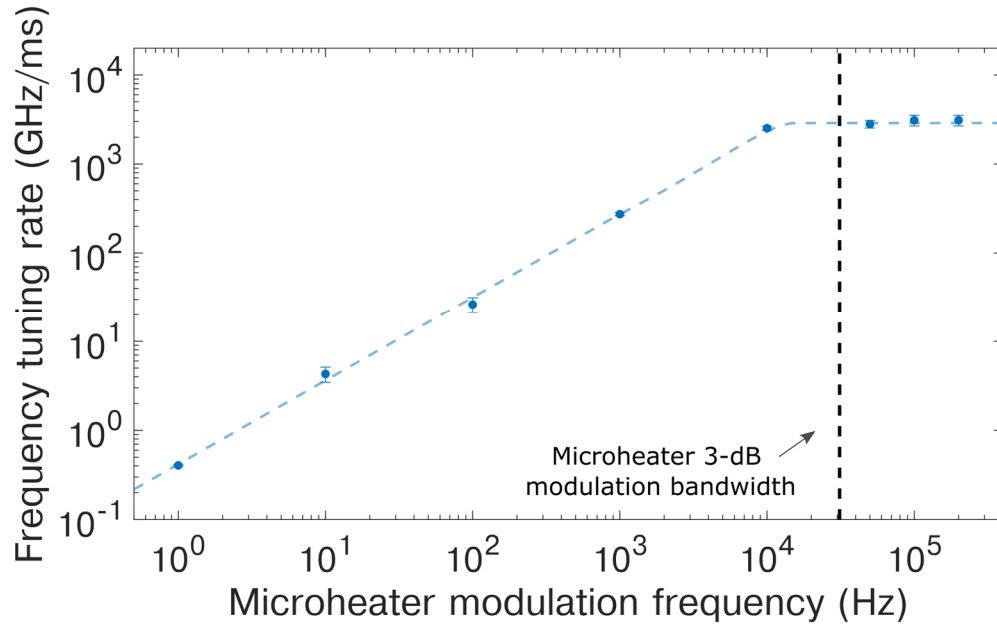


Figure S34: Microheater-based frequency tuning rate as a function of the microheater modulation frequency for the red chip-scale laser. The functional relationship is mostly linear below the microheater 3-dB modulation bandwidth of (31.1 ± 0.2) KHz (black dashed line), beyond which the frequency tuning rate saturates. The maximum measured tuning rate is 3401 GHz/ms. The error bars correspond to the mean $\pm$ s.d. of 50 consecutive measurements and the blue dashed line is a guide to the eye.

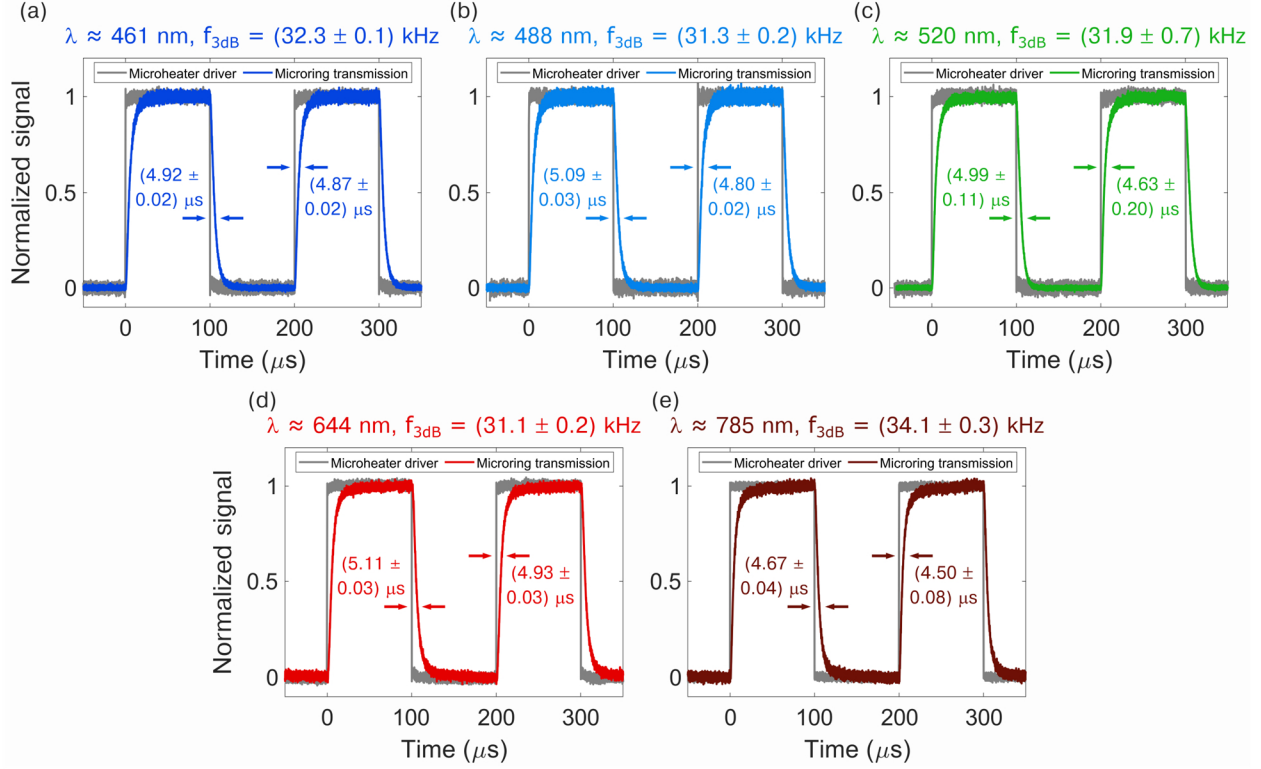


Figure S35: Microheater time response from deep blue to near-infrared, showing a 3-dB modulation bandwidth of around 32 kHz throughout the whole visible spectrum. (a) Microheater response in deep blue (461 nm) corresponding to the resonance in Fig. S1a. The rise and fall time constants are $(4.87 \pm 0.02) \mu\text{s}$ and $(4.92 \pm 0.02) \mu\text{s}$ respectively, implying a 3-dB modulation bandwidth of $(32.3 \pm 0.1) \text{ kHz}$. (b) Microheater response in blue (488 nm) corresponding to the resonance in Fig. S1b. The rise and fall time constants are $(4.80 \pm 0.02) \mu\text{s}$ and $(5.09 \pm 0.03) \mu\text{s}$ respectively, implying a 3-dB modulation bandwidth of $(31.3 \pm 0.2) \text{ kHz}$. (c) Microheater response in green (520 nm) corresponding to the resonance in Fig. S1c. The rise and fall time constants are $(4.63 \pm 0.20) \mu\text{s}$ and $(4.99 \pm 0.11) \mu\text{s}$ respectively, implying a 3-dB modulation bandwidth of $(31.9 \pm 0.7) \text{ kHz}$. (d) Microheater response in red (644 nm) corresponding to the resonance in Fig. S1d. The rise and fall time constants are $(4.93 \pm 0.03) \mu\text{s}$ and $(5.11 \pm 0.03) \mu\text{s}$ respectively, implying a 3-dB modulation bandwidth of $(31.1 \pm 0.2) \text{ kHz}$. (e) Microheater response in near-IR (785 nm) corresponding to the resonance in Fig. S1e. The rise and fall time constants are $(4.50 \pm 0.08) \mu\text{s}$ and $(4.67 \pm 0.04) \mu\text{s}$ respectively, implying a 3-dB modulation

bandwidth of (34.1 ± 0.3) KHz. Since the microheater frequency response is wavelength-insensitive, we expect a similar behavior in the near-UV spectral range. We measure the time responses by coupling light from commercial laser systems (section 3) to the photonic chip and aligning the wavelength to the linear section of the resonances (Fig. S1). We apply a small square voltage driving signal (gray curves) to the microheater and monitor the transmitted optical signal (colored curves) with a photodetector (Thorlabs APD120A2). We fit the rising and falling exponentials to extract the rise (τ_r) and fall (τ_f) time constants, from which we calculate the 3-dB modulation bandwidth $f_{3dB} = 1/(2\pi\tau_f)$. We obtain the average values and error bars from the statistics of 300 consecutive measurements and fittings.

2.9.2. Laser current-based frequency tuning

We measure laser current-based frequency tuning speeds up to 267 GHz/ μ s (Fig. 4b, main text) in the near-IR spectral range with modulation frequencies up to 20 MHz, limited only by our experimental setup. The relationship between the laser current modulation frequency and the resulting chip-scale laser frequency tuning is linear up to 20 MHz modulation. The lack of any saturation behavior indicates that the laser current can be modulated even faster, resulting in faster frequency tuning speeds. In fact, it is well known that FPLDs can be current-modulated with frequencies up to many GHz^{58,68,112–114}. For frequencies lower than about 10 MHz, the tuning mechanism is dominated by thermal effects, whereas for larger frequencies it is dominated by carrier effects^{58,113}. Within the frequency range dominated by thermal effects, the transfer function $\Delta\nu/\Delta I$ relating the laser frequency detuning $\Delta\nu$ caused by a change in the laser current ΔI as a function of the current modulation frequency f is described by⁵⁸

$$\frac{\Delta\nu}{\Delta I} = \frac{C_{th}}{1 + i\left(\frac{f}{f_{th}}\right)} \quad (13)$$

where C_{th} is a negative constant that depends on the thermal resistance of the laser and f_{th} is the thermal cut-off frequency. For our near-IR chip-scale laser, we obtain $C_{th} = -(0.62 \pm 0.02)$ GHz/mA and $f_{th} = (1.4 \pm 0.2)$ MHz (Fig. S36). Since the laser current-based frequency tuning characteristics are dominated

by the properties of the FPLDs themselves, we expect all the chip-scale lasers to have a similar performance across the visible spectrum⁶⁸.

We present the measurements supporting Fig. 4b (main text) and S36 in Fig. S37, and the experimental setup in Fig. S27b. For such fast frequency tuning rates, the FPI cannot respond quickly enough due to its long cavity lifetime, precluding the use of the setup in Fig. S27a. In addition to the exponential decay, the peak amplitude of the FPI transmission decreases as the frequency tuning rate increases, eventually becoming too small to be detected. Instead, we perform a time-domain heterodyne beat note measurement to extract the instantaneous detuning of our chip-scale laser. We combine the light from our integrated laser and from a commercial laser (Toptica DL Pro 780 nm) into a single optical fiber using a splitter and measure it with a fast photodetector (Thorlabs DET025AFC, 2 GHz bandwidth) connected to a real-time oscilloscope (Tektronix MSO5204B). For the nominal driving current of 120 mA of the FPLD (Fig. S12), we self-injection lock the chip-scale laser to the same wavelength of the commercial laser and then apply a sinusoidal modulation to the FPLD current. As the current modulation tunes the chip-scale laser frequency, the instantaneous frequency of the beat note matches the exact detuning between the two lasers. Since we keep the commercial laser at a fixed wavelength, a linear fit of the instantaneous frequency as a function of time corresponds directly to the frequency tuning rate of the chip-scale laser. Due to the limited photodetector bandwidth of 2 GHz, the total measurable detuning is 4 GHz, with 2 GHz corresponding to a red-shift and another 2 GHz to a blue-shift from the commercial laser. Because the tuning mechanism is dominated by thermal effects, larger FPLD currents correspond to a red-shift (C_{th} is negative), which we confirmed by the fact that red-shifting the commercial laser moved the center of the beat note signal (zero detuning) towards larger FPLD currents. We monitor the FPLD current in real-time via the voltage drop on a 10.3 Ω resistor connected in series with the FPLD. For modulation speeds larger than 20 MHz, the tuning speed is too fast to be reliably extracted from the beat note, as indicated by the larger error bar at this frequency in Figs. 4b (main text) and S36.

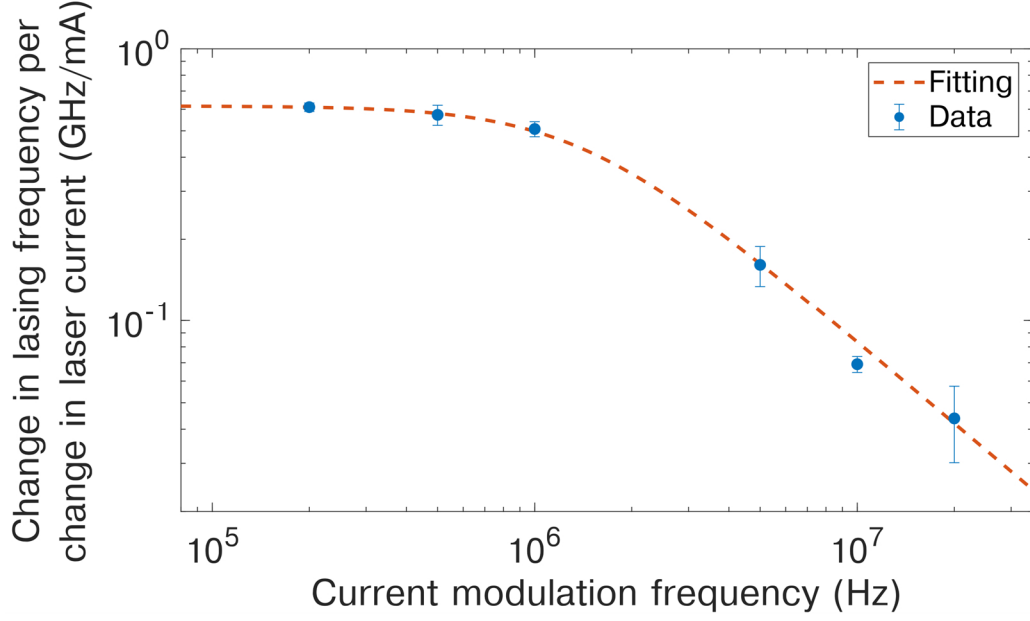


Figure S36: Transfer function ($\Delta\nu/\Delta I$) relating the laser frequency detuning ($\Delta\nu$) caused by a change in the FPLD laser current (ΔI) for the near-IR chip-scale laser as a function of the current modulation frequency. The dashed line represents a fitting using Eq. 13 and shows that the tuning mechanism up to a modulation frequency of 20 MHz is dominated by thermal effects. The fitting parameters are $C_{th} = -(0.62 \pm 0.02)$ GHz/mA and $f_{th} = (1.4 \pm 0.2)$ MHz. The data points (average and error bars) are obtained from the ratio between the linear fit slopes of the frequency detuning and the laser current from Fig. S37.

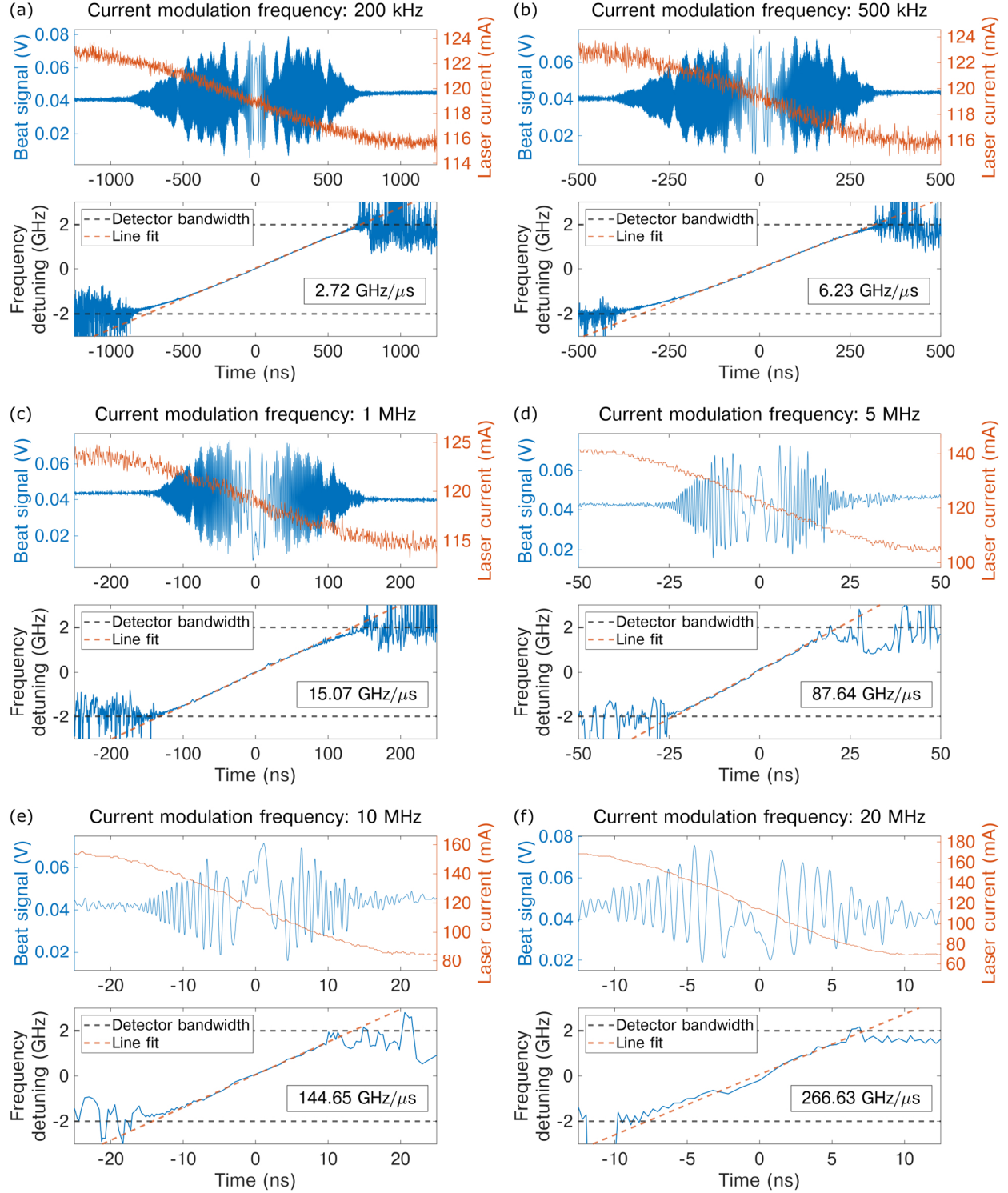


Figure S37: Laser current-based frequency tuning measurements for the near-IR chip-scale laser.

Using the experimental setup of Fig. S27b, we modulate the FPLD current with sinusoidal voltages of different frequencies and beat the chip-scale laser output with a commercial laser (Toptica DL Pro 780 nm).

We extract the instantaneous frequency of the beat note signal (top figures), which corresponds to the detuning as a function of time, and use a linear fit to obtain the frequency tuning rate (bottom figures) for each current modulation frequency. The total measurable detuning is bounded by the photodetector bandwidth (2 GHz, dashed gray lines in the bottom plots). The relationship between frequency tuning rate and modulation frequency is mostly linear for all the measurements. The lack of any saturation indicates that the laser current can be modulated faster, in agreement with the literature that shows that modulation frequencies up to many GHz are possible^{58,68,112–114}.

2.10. Linewidth measurements

In order to assess the linewidths of our chip-scale lasers, we perform heterodyne beat note measurements for the integrated lasers in the deep blue to near-infrared spectral ranges (Fig. S38a) and a scanning Fabry-Perot interferometer (SFPI)-based measurement for the integrated near-ultraviolet laser (Fig. S38b). We summarize the measured linewidths of the chip-scale lasers in Table S3.

In the heterodyne beat note measurements, we set our chip-scale lasers and a narrow linewidth commercial laser system (section 3) to lase within a small detuning from each other (typically hundreds of MHz). We combine their emissions into a single optical fiber and measure the beat note signal using a photodetector connected to a spectrum analyzer. We fit a Voigt profile to the beat note and extract the Lorentzian (intrinsic) and Gaussian components of the lineshape, as detailed in section 2.12.

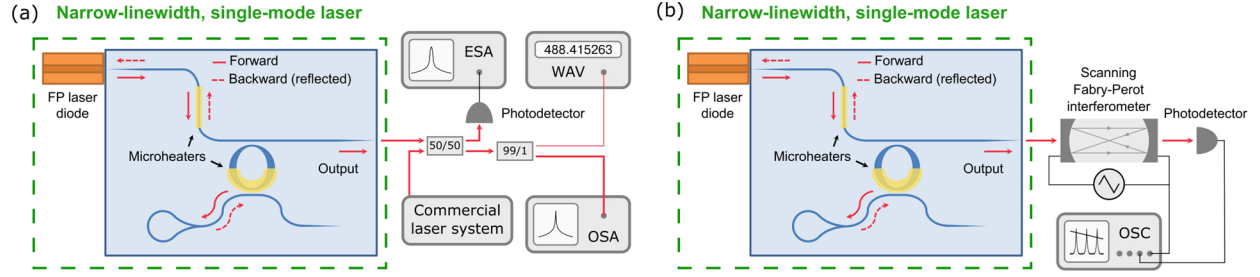


Figure S38: Experimental setups for the linewidth measurements of the chip-scale lasers. (a)

Heterodyne beat note measurement for the deep blue to near-IR integrated lasers. The output of each chip-scale laser is combined with the emission from a narrow linewidth commercial laser system (section 3) into a single optical fiber using a 50%/50% splitter. One end of the splitter goes to a photodetector (Thorlabs DET025AFC, 2 GHz bandwidth) connected to a spectrum analyzer (ESA, Agilent E4407B) configured with a sweep time of 4.990 ms, a resolution bandwidth of 30 kHz and a scanning window of 100 MHz around the beat note. The other end of the splitter goes to a 99%/1% splitter for monitoring the self-injection locking of the chip-scale laser via an optical spectrum analyzer (OSA, Ando AQ6314A) and the lasing wavelengths of both lasers with a wavemeter (WAV, HighFinesse WSU-10). (b) Scanning Fabry-Perot interferometer (SFPI)-based linewidth measurement for the near-ultraviolet integrated laser. The output of the chip-scale laser is free-space-coupled to a scanning Fabry-Perot interferometer (Toptica FPI 100) driven by a triangular voltage source (Toptica SC 110). The resonances of the interferometer scan through the laser and the transmitted signal is recorded with a photodetector (Thorlabs APD120A2) connected to a photodiode amplifier (Toptica PDA-S). The measured signal and the driving voltage of the interferometer are simultaneously monitored with an oscilloscope (Picoscope 5444B).

Since the coarse tuning ranges of our red and deep blue chip-scale lasers at room temperature (Fig. 3, main text) do not overlap with the tuning ranges of the available commercial laser systems (section 3), we extend the spectral coverage of our integrated lasers by varying the temperature of the FPLDs. When we heat up (cool down) the FPLD, we cause a red shift (blue shift) of its emission spectrum, which consequently shifts the center wavelength of our chip-scale lasers by the same amount. By cooling down

our red FPLD and heating up our deep blue FPLD, we are able to self-injection lock them around 652 nm and 459 nm respectively, allowing for the heterodyne beat notes with the commercial laser systems (Fig. 5, main text). We expect the change in FPLD temperature to not affect the linewidth of the self-injection-locked lasers significantly when compared to room temperature, similarly to what has been observed in external-cavity diode lasers (ECDLs)^{115,116}. Figs. S39 and S40 show the measured emission spectra of the free-running red and deep blue FPLDs for different laser case temperatures and their extracted temperature tuning coefficients. We cool down the red FPLD by attaching it to a thermoelectric cooler (TEC) element with a center hole (Thorlabs TEC1.4-6) using thermally conductive epoxy (MG Chemicals 8330D and 8331) and driving the TEC with a power supply (Keithley 2400). We heat up the deep blue FPLD by attaching a metal ceramic heater (Thorlabs HCS013) to the laser mount using thermally conductive epoxy (MG Chemicals 8331) and driving the heater with a power supply (Keithley 2400). The measurements in Figs. S39, S40 and 5b,e (main text) show that by controlling the temperature of the FPLDs we can extend the spectral coverage of our chip-scale lasers several nanometers beyond their typical coarse tuning ranges towards both shorter and longer wavelengths while keeping the linewidth narrow, similarly to benchtop ECDL systems^{117–119}.

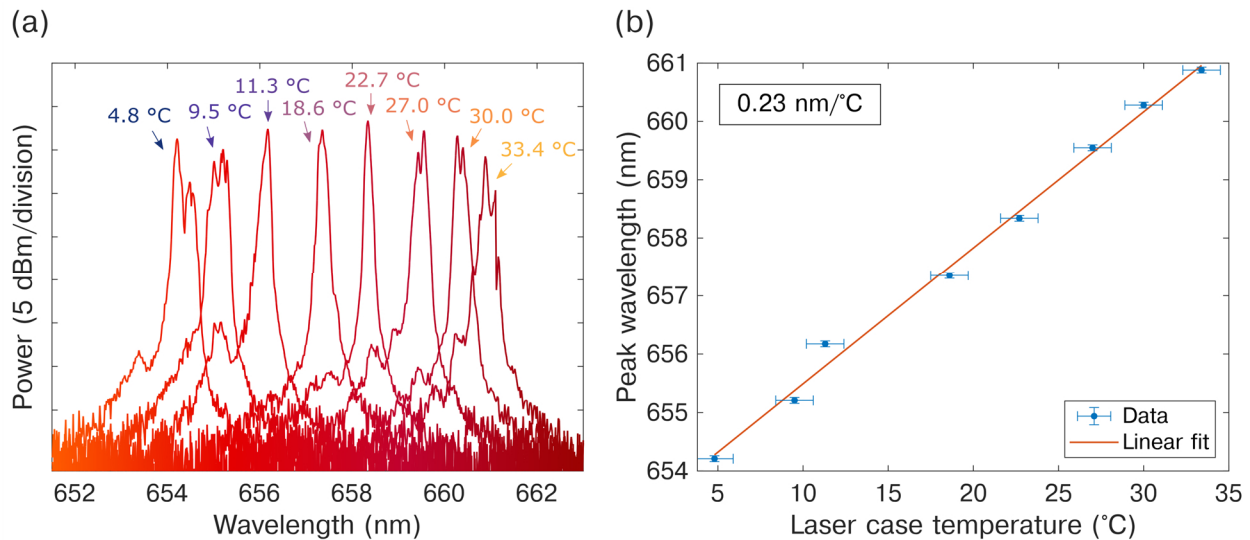


Figure S39: Thermal tuning characteristics of the red Fabry-Perot laser diode (Thorlabs L658P040).

(a) Emission spectra for different laser case temperatures. We overlay the spectra for ease of visualization

and use arrows to indicate the corresponding temperature of each. (b) Peak lasing wavelength as a function of the laser case temperature. The temperature tuning coefficient of $0.23 \text{ nm}/^{\circ}\text{C}$ that we extract from the linear fit of the experimental data matches closely that of a similar FPLD of the same wavelength (Opnext HL6501MG, $0.25 \text{ nm}/^{\circ}\text{C}$). We measure the emission spectra with an optical spectrum analyzer (OSA, Ando AQ6314A), and the laser case temperature with a Fluke 51 II Thermometer and a thermocouple (K type). The horizontal error bars correspond to the accuracy of the thermometer, and the vertical error bars correspond to the OSA resolution (0.05 nm).

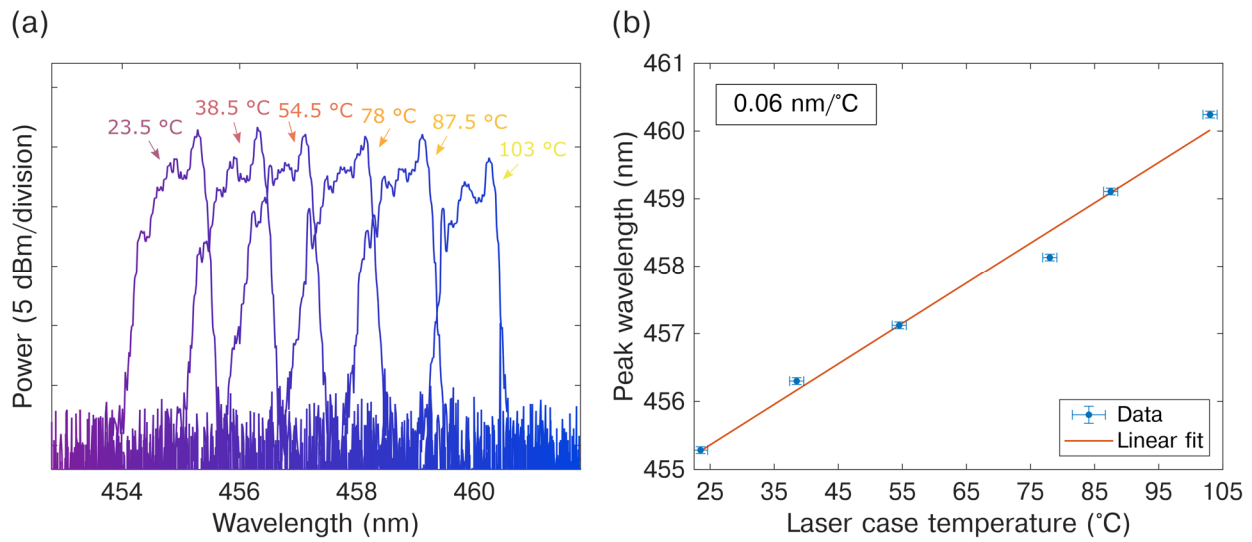


Figure S40: Thermal tuning characteristics of the deep blue Fabry-Perot laser diode (OSRAM PL 450B). (a) Emission spectra for different laser case temperatures. We overlay the spectra for ease of visualization and use arrows to indicate the corresponding temperature of each. (b) Peak lasing wavelength as a function of the laser case temperature. The temperature tuning coefficient of $0.06 \text{ nm}/^{\circ}\text{C}$ that we extract from the linear fit of the experimental data matches closely the typical value provided in the laser's datasheet ($0.04 \text{ nm}/^{\circ}\text{C}$). We measure the emission spectra with an optical spectrum analyzer (OSA, Ando AQ6314A), and the laser case temperature with a Fluke 51 II Thermometer and a thermocouple (K type). The horizontal error bars correspond to the accuracy of the thermometer, and the vertical error bars correspond to the OSA resolution (0.05 nm).

In the near-ultraviolet spectral range, due to the lack of access to a narrow linewidth laser at this wavelength and the lower fiber-coupled power of our integrated laser, we use a SFPI to estimate the effective linewidth of our chip-scale laser. We couple the light from our self-injection-locked near-UV laser to the SFPI via free-space and sweep the SFPI resonances by applying a triangular voltage signal to its piezoelectric actuators. We collect the transmission of the SFPI with a photodetector and simultaneously measure its signal and the SFPI driving voltage with an oscilloscope, resulting in a measurement similar to Fig. S29. Due to the low optical power and consequently noisy signal, we average 186 measured lineshapes to improve the signal-to-noise ratio. From the known free spectral range of the SFPI, we extract the width of the measured peaks, which represent the convolution between the laser lineshape and the SFPI lineshape¹²⁰. Since the nominal linewidth of the SFPI (2.5 MHz) is comparable to the expected linewidth of our chip-scale laser (Table S4, section 2.13), the measured lineshape is significantly broader than both individual lineshapes. In order to properly estimate the laser linewidth, we deconvolve the measured signal using the nominal lineshape of the SFPI (Fig. S41). The transmitted lineshape of the SFPI is given by¹²⁰

$$T = \frac{1}{1 + \frac{\sin^2\left(\frac{\pi}{FSR}\Delta f\right)}{\sin^2\left(\frac{\pi}{FSR}\frac{FWHM}{2}\right)}} \quad (14)$$

where FSR and $FWHM$ are the interferometer's free-spectral range and full-width at half-maximum, respectively, and Δf is the detuning from the center of the resonance. The result of the deconvolution, also presented in Fig. 5a (main text), is a conservative estimate of the laser linewidth, since it attributes all the measurement imperfections to the laser lineshape. The imperfections are from artificial linewidth broadening due to the averaging, and from a non-nominal SFPI lineshape due to imperfect cavity spacing and illumination of the interferometer, which is indicated by the asymmetry of the measured lineshape^{107–109}. Since we do not have access to a more powerful, narrower linewidth near-UV laser, we are not able to either further optimize the illumination of the SFPI or measure its exact lineshape under our experimental conditions. Therefore, when performing the deconvolution we assume the nominal SFPI lineshape, which

inaccurately but conservatively transfers the averaging-induced broadening and experimental imperfections to the estimated laser lineshape. From the deconvolved signal, we extract an effective FWHM linewidth of ~ 3.27 MHz for the near-UV chip-scale laser. While this SFPI-based measurement provides the effective linewidth of the laser, the noise level of the detector and the insufficient signal-to-noise ratio of the signal preclude the extraction of the individual linewidth components (Lorentzian and Gaussian). Therefore, the intrinsic linewidth of the laser is most likely narrower than the ~ 3.27 MHz effective linewidth.

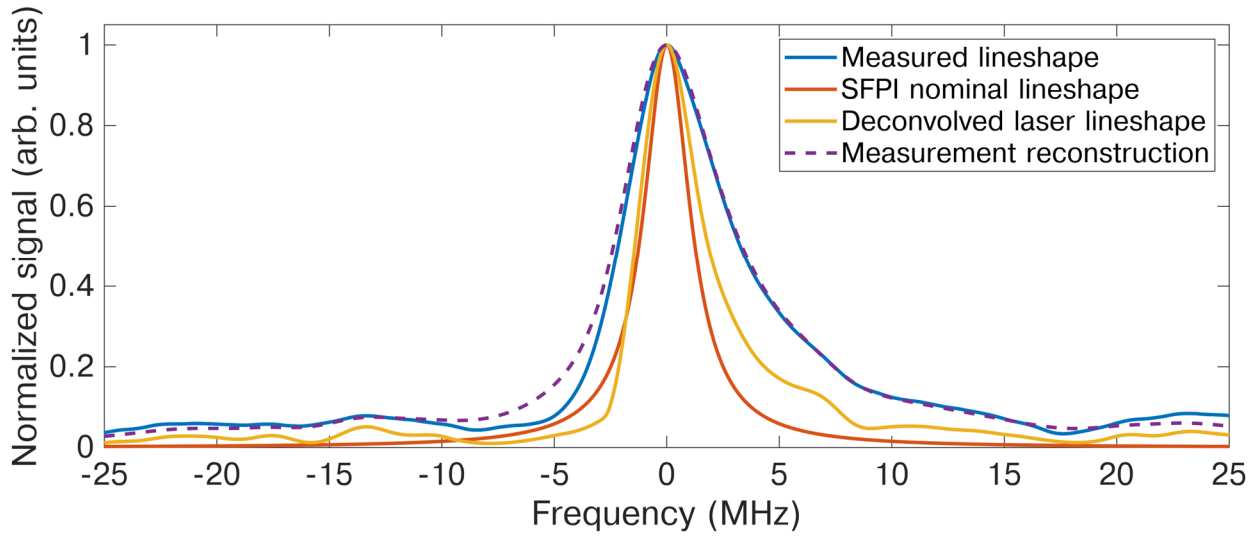


Figure S41: Measured transmission of the scanning Fabry-Perot interferometer (SFPI) and deconvolution of the near-ultraviolet chip-scale laser lineshape. The SFPI transmission (blue solid line) is the convolution between the lineshapes of the laser and the SFPI resonance. Although the actual SFPI lineshape is broadened and asymmetrical due to imperfect operation of the interferometer, we assume the nominal SFPI lineshape with 2.5 MHz linewidth (solid red line) for the deconvolution. Consequently, all the measurement imperfections, which also include an artificial broadening from averaging, are inaccurately but conservatively transferred to the estimated laser lineshape (solid yellow line). The reconstruction of the measured signal by convolving the SFPI nominal lineshape and the deconvolved laser lineshape (dashed purple line) is close to the experimental measurement, demonstrating the validity of the deconvolution procedure.

Table S3: Measured linewidths of the chip-scale lasers. The error bars correspond to the 95% confidence intervals of the Voigt fitting parameters.

Wavelength (nm)	Intrinsic (Lorentzian) linewidth (kHz)	Gaussian linewidth (kHz)	Effective (Voigt) linewidth (kHz)
785	7 ± 2	285 ± 69	289 ± 70
652	8 ± 2	356 ± 66	360 ± 67
520	16 ± 10	225 ± 135	233 ± 140
488	5 ± 3	368 ± 132	371 ± 134
459	6 ± 3	274 ± 94	278 ± 96
404	N/A	N/A	3270

2.11. Frequency noise measurement

The frequency noise of a laser gives information about all the noise sources that cause the lasing wavelength to fluctuate. The low frequency components are due to slow variations caused by flicker and technical noise such as mechanical vibrations and thermal fluctuations. They cause a broadening of the effective linewidth of the laser and are related to the Gaussian linewidth. The high frequency components, on the other hand, are caused by white noise and determine the intrinsic (Lorentzian) linewidth of the laser¹²¹.

We measure the frequency noise of both our near-IR chip-scale laser and the near-IR commercial laser system (Toptica DL Pro 780 nm) using a commercial linewidth analyzer (LWA, HighFinesse LWA-1k 780). The LWA is based on a stable, pre-characterized optical frequency discriminator that converts the frequency deviations of the input laser light into intensity deviations that are detected by a low-noise detector and processed by ultra-low-noise electronics. We use the recorded optical frequency fluctuations as a function of time ($\Delta\nu(t)$) to calculate the frequency noise (S_N) of the laser via^{121,122}

$$S_N(f) = \mathcal{F}(\Delta v(t)) \times \mathcal{F}^*(\Delta v(t))$$

where $\mathcal{F}(\cdot)$ is the Fourier transform. We extract the intrinsic linewidth of the laser from the white noise plateau of the frequency noise at high frequencies (π times the value of the intrinsic linewidth lines in Fig. S42)¹²¹. We also estimate the effective linewidth Δf_{eff} of the laser directly from the frequency noise spectrum by using the β -separation approach¹²², which states that Δf_{eff} is approximated by the surface under the portions of S_N that exceed the β -separation line $8 \ln(2) f / \pi^2$. Mathematically, this means

$$\Delta f_{eff} = \sqrt{8 \ln(2) \int H\left(S_N(f) - \frac{8 \ln(2) f}{\pi^2}\right) S_N(f) df} \quad (15)$$

where $H(x) = 1$ for $x \geq 0$ and $H(x) = 0$ for $x < 0$.

Fig. S42 shows the measured frequency noise spectra of the near-IR commercial (Toptica DL Pro 780 nm) and chip-scale lasers when operated at wavelengths close to 785 nm. For the commercial laser (Fig. S42a), the intrinsic and effective linewidths of 9.5 kHz and 224 kHz, respectively, agree with the specifications provided by the laser manufacturer (section 3). For our chip-scale laser, the frequency noise at high frequencies is below the sensitivity of the LWA, whose noise floor is in between 100 Hz²/Hz and 400 Hz²/Hz (Fig. S42b). This noise floor places an upper limit for the intrinsic linewidth between 314 Hz and 1.26 kHz. While the exact numerical values extracted from the frequency noise in this case (0.7 kHz and 205 kHz for intrinsic and effective linewidths, respectively) may not be completely accurate due to the limited sensitivity of the LWA, they show that the chip-scale laser has an effective linewidth comparable to that of the commercial laser and an intrinsic linewidth that is around an order of magnitude narrower. The theoretical estimate of the intrinsic linewidth of our chip-scale laser (~ 1.1 kHz, Table S4, section 2.13) further supports the latter conclusion, as the predicted linewidth is within the noise floor range of the LWA. These measurements also show that the heterodyne beat note between the two lasers (Fig. 5f, main text) has similar Gaussian contributions from both of them but a Lorentzian component that is mostly dominated by the commercial laser's intrinsic linewidth.

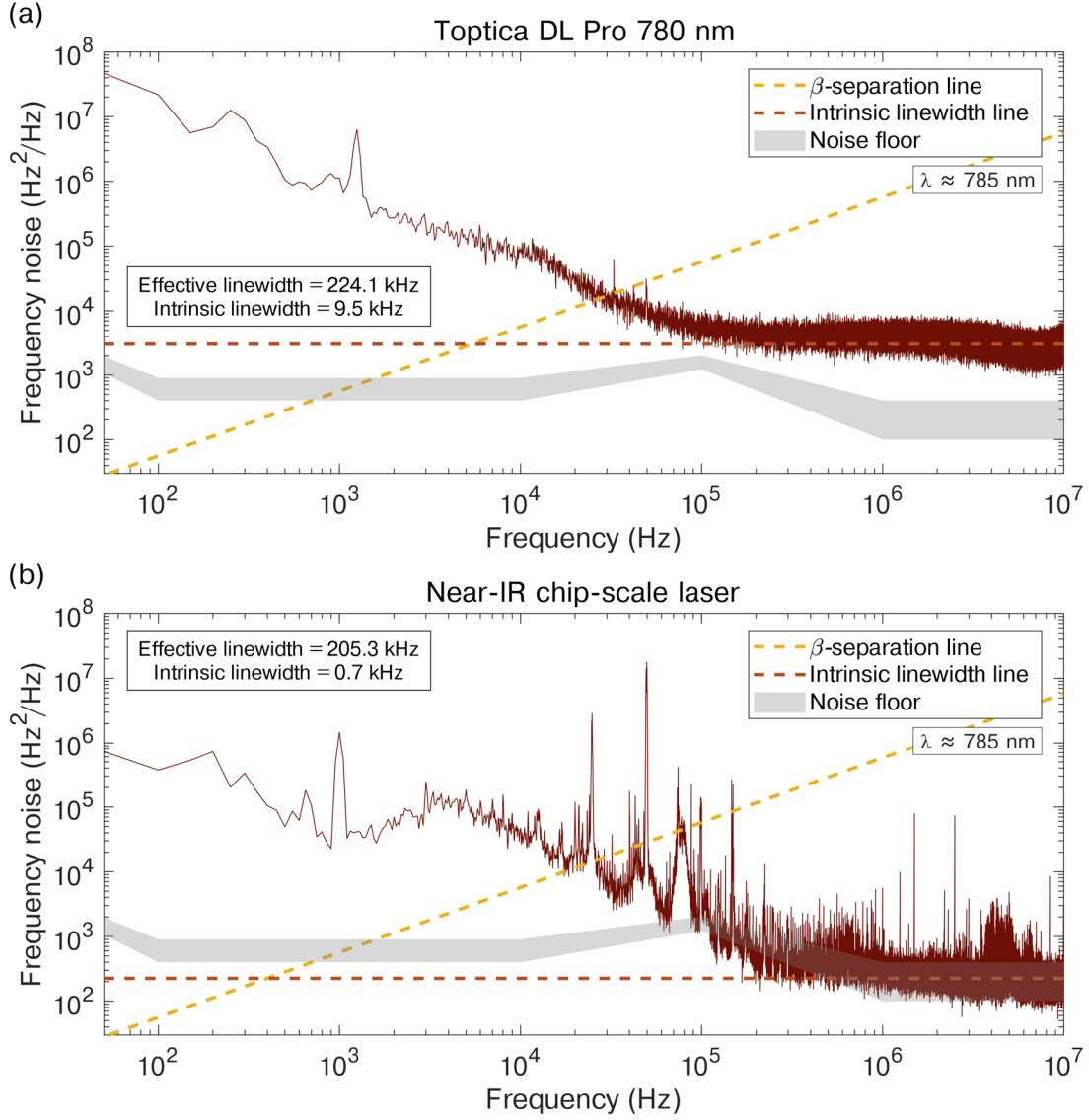


Figure S42: Frequency noise spectra of the near-infrared commercial and chip-scale lasers. (a) Frequency noise of the near-IR commercial laser (Toptica DL Pro 780 nm). We extract intrinsic and effective linewidths of 9.5 kHz and 224 kHz, respectively, which agree with the specifications provided by the laser manufacturer (section 3). (a) Frequency noise of the near-IR chip scale laser. We extract intrinsic and effective linewidths of 0.7 kHz and 205 kHz, respectively. While the values may not be completely accurate since the frequency noise is below the sensitivity of the linewidth analyzer (LWA) at high frequencies, the measurement shows that the chip-scale laser has an effective linewidth similar to that of the commercial laser and an intrinsic linewidth that is around an order of magnitude narrower. These

conclusions are also supported by the theoretical analysis of section 2.13. The noise floor level (shaded gray region) corresponds to the specifications provided by the LWA manufacturer.

2.12. Voigt lineshape fitting

As discussed in the main text and in section 2.11, the frequency noise of a laser is generally composed of a white noise and a flicker noise, giving rise to Lorentzian and Gaussian contributions to the laser lineshape, respectively. The total laser lineshape is well described by a Voigt profile^{123,124} ($V(f)$), which represents the convolution between Lorentzian ($L(f)$) and Gaussian ($G(f)$) lineshapes. Mathematically, the lineshapes are given by

$$V(f; \sigma, \psi) = \int_{-\infty}^{+\infty} G(f'; \sigma) L(f - f'; \psi) df' \quad (16)$$

$$G(f; \sigma) = \frac{e^{-\frac{f^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \quad (17)$$

$$L(f; \psi) = \frac{\gamma}{\pi(f^2 + \psi^2)} \quad (18)$$

where f is the detuning from the center of the lineshape, and σ and ψ relate to the full-width at half-maximum (FWHM) of the Gaussian and Lorentzian lineshapes via $\Delta f_G = 2\sigma$ and $\Delta f_L = 2\psi$ respectively. The total Voigt linewidth Δf_V , which represents the effective linewidth of the laser, relates to Δf_G and Δf_L via⁵⁹

$$\Delta f_V = \frac{1}{2} \left[c_1 \Delta f_L + \sqrt{c_2 (\Delta f_L)^2 + 4(\Delta f_G)^2} \right] \quad (19)$$

where $c_1 = 1.0692$ and $c_2 = 0.86639$.

Each heterodyne beat note of Fig. 5 (main text) can be described by a Voigt lineshape that is a combination of the lineshapes of the chip-scale laser and the commercial laser. The Lorentzian and

Gaussian linewidths of the beat note (Δf_L^B and Δf_G^B respectively) relate to the corresponding individual lasers linewidths ($\Delta f_L^{(i)}$ and $\Delta f_G^{(i)}$, $i = 1, 2$) via the relations^{59,71}

$$\Delta f_L^B = \Delta f_L^{(1)} + \Delta f_L^{(2)} \quad (20)$$

$$(\Delta f_G^B)^2 = \left(\Delta f_G^{(1)}\right)^2 + \left(\Delta f_G^{(2)}\right)^2 \quad (21)$$

Since the exact linewidth components of the commercial laser systems are not known and are comparable to the expected linewidths of our chip-scale lasers, we estimate the latter by assuming that $\Delta f_L^{(1)} \approx \Delta f_L^{(2)}$ and $\Delta f_G^{(1)} \approx \Delta f_G^{(2)}$. Consequently, we obtain the linewidth components Δf_L and Δf_G of our integrated lasers from their corresponding beat note linewidths via

$$\Delta f_L = \frac{\Delta f_L^B}{2} \quad (22)$$

$$\Delta f_G = \frac{\Delta f_G^B}{\sqrt{2}} \quad (23)$$

and we calculate the total linewidths Δf_V using Eq. 19.

2.13. Linewidth estimation

In order to estimate the intrinsic linewidth $\Delta\nu$ of our chip-scale lasers, we treat the self-injection locking process as two subsequent steps: a mode collapse that converts the multi-mode FPLD into a single-mode laser, and a linewidth narrowing of the single-mode laser^{64,65,124}. The mode-collapsed FPLD resulting from the first step has an intrinsic linewidth $\Delta\nu_0$ given by Eqs. 24 to 26⁵⁸

$$\Delta v_0 = \Delta v_s + \Delta v_m \quad (24)$$

$$\Delta v_s = \frac{1 + \alpha^2}{4\pi P_{out}} K_{tot} n_{sp} (h\nu) v_g^2 \alpha_m (\alpha_m + \alpha_s) \quad (25)$$

$$\Delta v_m = \frac{1}{2\pi} \frac{\alpha^2 \Delta \kappa^2 P_1^3}{K_{tot} n_{sp} h\nu \eta_e} \quad (26)$$

where Δv_s is the linewidth of a corresponding ideal single-mode laser and Δv_m is a correction to account for the finite side-mode suppression ratio (SMSR) of the actual mode-collapsed laser, which causes additional linewidth broadening through mode partition noise and the nonlinear gain dynamics⁵⁸. α is the linewidth enhancement factor of the FPLD, P_{out} is the optical power emitted by the free-running FPLD, K_{tot} is the spontaneous emission enhancement factor, n_{sp} is the inversion factor, h is the Planck constant, ν is the optical frequency of the emitted light, v_g is the group velocity, α_m is the FPLD mirror loss, α_s is the FPLD distributed loss coefficient, $\Delta \kappa$ is the gain compression coefficient difference between the lasing mode and the weak side-mode with average power P_1 , and η_e is the external quantum efficiency of the FPLD. We estimate P_1 Eq. 27, which approximates the total power P_{out} as the sum of the power $P_1 SMSR$ in the main lasing mode and the power P_1 in the weak side-mode:

$$P_1 = \frac{P_{out}}{SMSR + 1} \quad (27)$$

The linewidth narrowing of the single-mode laser corresponding to the second step is described by Eq. 28⁶⁴

$$\Delta v = \Delta v_0 \left[\frac{Q_d^2}{Q_m^2} \frac{1}{16C^2 \Gamma_m^2 (1 + \alpha^2)} \right] \quad (28)$$

where Q_d is the FPLD quality factor, Q_m is the ring resonator loaded quality factor, C is the power coupling coefficient between the FPLD and the photonic chip and Γ_m is the amplitude reflection coefficient of the ring resonator-based feedback loop. $C^2 \Gamma_m^2$ represents the fraction of the light emitted from the FPLD that is effectively reflected back to it by the photonic chip.

We determine the values for the parameters in Eqs. 24 to 28 as follows. We assume α based on typical values found in the literature for each wavelength^{125–127}. We calculate α_m from Eq. 29⁵⁸

$$\alpha_m = \frac{1}{2L} \log \left(\frac{1}{R_f R_b} \right) \quad (29)$$

where L is the FPLD cavity length, R_f is the FPLD front-facet power reflectivity (assumed based on typical values for commercial FPLDs) and R_b is the FPLD back-facet power reflectivity (assumed to be 1 for all FPLDs). We measure L with a calibrated optical microscope (insets of Figs. S12a to S17a). We extract P_{out} from the measured LIV curves at the operating points (Figs. S12a to S17a). We obtain α_s from Eq. 30⁵⁸

$$\alpha_s = \alpha_m \left(\frac{1 - \frac{\eta_e}{\eta_i}}{\frac{\eta_e}{\eta_i}} \right) \quad (30)$$

where η_i and η_e are the internal and external quantum efficiencies of the FPLD, respectively. We assume η_i to be 1 and we obtain η_e by fitting the P vs I curves of the FPLDs (Figs. S12a to S17a) using Eq. 31¹²⁸

$$P = \eta_e \frac{h\nu}{q} (I - I_{th}) \quad (31)$$

where I_{th} is the threshold current of the free-running FPLD and q is the elementary charge. We calculate $v_g = c/n_g$ assuming typical group index n_g values of FPLDs¹²⁹, and we estimate their free spectral ranges (FSR) via $FSR = v_g/(2L)$. We assume $K_{tot}n_{sp}$ and $\Delta\kappa$ to be 3 and 10^{-3} mW^{-1} respectively for all lasers⁵⁸. We extract $SMSR$ directly from the optical spectra measurements in Figs. 2c (main text) and S20 to S24.

We calculate the FPLD quality factor Q_d using Eq. 32¹³⁰

$$Q_d = \frac{2\pi\nu L}{v_g} \frac{\exp\left(-\frac{\alpha_r L}{2}\right)}{1 - \exp(-\alpha_r L)} \quad (32)$$

where $\alpha_r = \alpha_m + \alpha_s$ is the effective overall distributed loss coefficient. We obtain Q_m from the resonance measurements (Fig. S1) for deep blue to near-IR and conservatively assume it to be 1.5×10^3 for near-UV,

based on the lowest quality factor measured in deep blue (Fig. S7). We estimate C from numerical simulations of the optical overlap between the measured FPLD mode profiles (Fig. S18) and the on-chip edge couplers modes, as presented in section 1.3 and summarized in Table S1. We calculate Γ_m via Eq. 33:

$$\Gamma_m = \Gamma_{loop}(1 - ER) \quad (33)$$

where ER is the extinction ratio of the ring resonance and Γ_{loop} is the amplitude reflection coefficient of the optical feedback loop. In deriving Eq. 33, we approximate that the self-injection locking happens when the FPLD mode is exactly on-resonance with the ring resonance, and we consider that the extrinsic quality factors of the rings are much larger than the intrinsic quality factors (section 1.1). The latter implies that nearly all the optical power on-resonance with the ring is dropped to the drop port instead of being absorbed by the material or scattered to free-space. We extract ER from the ring resonance measurements in Fig. S1 for the deep blue to near-IR spectral ranges. For near-UV, we assume ER to be 15%, based on the typical ER values for small coupling gaps seen across all the visible wavelengths (Figs. S3 to S7). We estimate Γ_{loop} from numerical simulations, as presented in section 1.2.

Table S4 summarizes the numerical values of the parameters for Eqs. 24 to 33 and the estimated linewidths for the chip-scale lasers. The estimates agree well with the experimental values, supporting the validity of our results. For the near-UV chip-scale laser, we notice that both $\Delta\nu_s$ and $\Delta\nu_m$ are orders of magnitude larger than for the other wavelengths, which results in a larger $\Delta\nu$. $\Delta\nu_s$ is larger due to the shorter length and larger overall loss (α_r) of the FPLD, and $\Delta\nu_m$ is larger due to the smaller SMSR of the self-injection-locked laser. By optimizing the FPLD-to-chip coupling, we can further narrow the linewidth $\Delta\nu$, since more optical feedback to the FPLD reduces $\Delta\nu_m$ (Eq. 26) through a larger SMSR and intensifies the linewidth reduction (Eq. 28).

Table S4: Summary of the measured and estimated parameters of the chip-scale lasers used in Eqs. 24 to 33 and their expected intrinsic linewidths.

Wavelength (nm)	404	450	488	520	660	785
Q_d	3.2×10^3	6.7×10^3	2.1×10^3	2.6×10^3	7.3×10^3	2.0×10^3
Q_m	1.5×10^4	4.0×10^4	5.5×10^4	3.6×10^4	1.6×10^4	8.1×10^3
$C^2 \Gamma_m^2$ (%)	0.6	3.2	2.4	6.0	54.2	46.8
FSR (GHz)	150	75	75	49	49	62
$SMSR$ (dB)	21	37	37	37	35	37
L (μm)	294	590	585	880	880	690
α_m (m^{-1})	5.1×10^3	2.5×10^3	2.6×10^3	1.7×10^3	2.2×10^3	5.0×10^3
α_s (m^{-1})	6.0×10^3	2.5×10^3	6.0×10^3	4.4×10^3	1.0×10^3	1.6×10^3
η_i	1	1	1	1	1	1
η_e	0.46	0.51	0.30	0.28	0.67	0.76
n_g	3.4	3.4	3.4	3.5	3.5	3.5
α	4.1	4.1	4.1	4.1	2.2	2.5
$K_{tot} n_{sp}$	3	3	3	3	3	3
$\Delta\kappa$ (mW^{-1})	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
ER	0.15	0.36	0.50	0.22	0.13	0.12
P_{out} (mW)	21.9	81.0	46.0	105.0	66.0	108.0
$\Delta\nu_s$ (kHz)	4.2×10^4	2.3×10^3	6.5×10^3	1.2×10^3	3.4×10^2	9.8×10^2
$\Delta\nu_m$ (kHz)	2.1×10^4	1.8×10^1	5.6×10^0	7.8×10^1	1.2×10^1	1.8×10^1
$\Delta\nu_0$ (kHz)	6.3×10^4	2.3×10^3	6.5×10^3	1.3×10^3	3.5×10^2	9.9×10^2
$\Delta\nu$ (kHz)	1.5×10^3	6.9	1.4	0.4	1.4	1.1

3. Commercial laser systems used in the experiments

Table S5 summarizes the specifications of the narrow linewidth commercial laser systems that we use the experiments to characterize our chip-scale lasers. All of the systems are based on external cavity diode lasers (ECDLs). For near-IR, red, blue and deep blue, the ECDLs emit directly at the desired wavelength range. For green, we use an ECDL at 1 μm wavelength to pump a second harmonic generation (SHG) system. While the manufacturers do not specify exact linewidths for their lasers, they provide typical ranges of values for their effective linewidths, which are all on the order of a few hundreds of KHz. Since the effective linewidths of our measured heterodyne beat notes (Fig. 5, main text) are also in this range, we conclude that our chip-scale lasers must have linewidths comparable to or narrower than the commercial systems. In particular for the near-IR spectral range, we demonstrate this with the frequency noise measurements in Fig. S42.

Table S5: Summary of the narrow linewidth commercial laser systems that we use in the experiments to characterize our chip-scale lasers.

Spectral range	Model	Wavelength coverage (nm)	Typical effective linewidth
Near-infrared	Toptica DL Pro 780 nm	765 - 795	10 – 500 kHz (5 μ s integration time) *
Red	Sacher Lion Series TEC-500-0645-010-M	640.5 - 653.1	< 100 kHz (1 μ s integration time) < 300 kHz (1 ms integration time) < 1 MHz (1 s integration time)
Green	Velocity TLB-6721-P 1 μ m seed laser + Azurlight 532 nm SHG system	Seed laser: 1030 - 1070 SHG signal: 515 - 535	Seed laser: < 200 kHz (50 ms integration time) SHG signal: < 400 kHz (50 ms integration time)
Blue	Toptica DL Pro HP 488 nm	482.6 - 490.2	10 – 500 kHz (5 μ s integration time)
Deep blue	Toptica DL Pro HP 461 nm	457 - 461	10 – 500 kHz (5 μ s integration time)

* See Fig. S42 for the measured frequency noise

4. Comparison to existing technologies

In this section, we describe the existing academic and commercial state-of-the-art technologies for tunable and narrow linewidth visible lasers and compare them to our chip-scale lasers in terms of the key performance metrics. While we provide detailed analyses in the following subsections, we summarize the comparisons in Table S6.

Table S6: Summary of the state-of-the-art in narrow linewidth and tunable electrically-pumped visible lasers. Given the large number of commercial lasers, we choose representative ones at wavelengths close to our chip-scale lasers. For a thorough list of commercial lasers, see Table S11.

Center wavelength (nm)	Reference or company	Integration scale	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	SMSR (dB)	Output power (mW)	Estimated footprint
785	This work	Chip-scale	12.47	33.9	7.1 (microheater) 267 (laser current)*	34 kHz (microheater) > 20 MHz (laser current)*	< 9* (< 1.1 estimated)	< 359* (effective)	≥ 37	10.00 (fiber-coupled)	0.6 x 2 mm ²
785	Toptica DLC DL Pro 780	Free space	40	≥ 20 (typical)	N/A	N/A	≥ 10 (datasheet), 9.5 (measured, section 2.13)	50 (datasheet) 224 (measured, section 2.13)	N/A	22 - 57.75 (fiber-coupled)	90 x 90 x 240-300 mm ³
780.25	131	WGM	0.026	12.8 (via current)	N/A	N/A	5×10^{-3} (stabilized)	N/A	N/A	4.6 (fiber coupled), 10 (free space)	< 20000 mm ³
660	This work	Chip-scale	4.89	33.0	3.4 (microheater)	31 kHz (microheater)	< 10* (< 1.4 estimated)	< 427*	≥ 35	4.42 (fiber-coupled)	0.6 x 2 mm ²
660	Sacher TEC-100-0660-025	Free space	> 6	> 30	N/A	N/A	N/A	< 1000 (1 ms integration time)	40	25 (free space)	Benchtop
684.4	31	Chip-scale	10.8 (at 50 mA, 4 mA)	N/A	N/A	N/A	2.3 ± 0.2 (90 mA current)	56.3 ± 0.4 (Gaussian)	40	4.8 (90 mA current)	28 x 8 mm ²

Center wavelength (nm)	Reference or company	Integration scale	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	SMSR (dB)	Output power (mW)	Estimated footprint
			above threshold)					(90 mA current)		(fiber-coupled)	
520	This work	Chip-scale	5.98	8.2	N/A	32 kHz (microheater)	< 26 * (< 0.4 estimated)	< 373 *	≥ 37	9.89 (fiber-coupled)	0.6 x 2 mm ²
520	No standard commercial lasers available										
518.6	35	Chip-scale	N/A	N/A	N/A	N/A	N/A	N/A	36	1.9 (fiber-coupled)	16.8 x 9.6 mm ²
488	This work	Chip-scale	5.62	3.7	N/A	31 kHz (microheater)	< 8 * (< 1.4 estimated)	< 505 *	≥ 37	1.75 (fiber-coupled)	0.6 x 2 mm ²
488.5	Sacher TEC-100-0488-020	Free space	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms integration time)	40	20 (free space)	Benchtop
492.1	104	WGM	1.37	20 (via current)	N/A	N/A	1	N/A	> 40	100 (free space)	N/A
455	This work	Chip-scale	4.49	2.0	N/A	32 kHz (microheater)	< 9 * (< 6.9 estimated)	< 374 *	≥ 37	1.74 (fiber-coupled)	0.6 x 2 mm ²
452.5	Sacher TEC-100-0455-015	Free space	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms integration time)	40	15 (free space)	Benchtop

Center wavelength (nm)	Reference or company	Integration scale	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	SMSR (dB)	Output power (mW)	Estimated footprint
461.8	103	WGM	N/A	2 (via current)	N/A	N/A	N/A	< 1000 (estimated, not measured)	N/A	3 (free space)	2 cm cavity length
446.5	35	Chip-scale	0.57 (via current)	2.4 (via current)	4.1×10^{-7}	N/A	> 1100	> 3500 (0.1 ms integration time)	32	1.1 (fiber-coupled)	16.8 x 9.6 mm ²
404	This work	Chip-scale	1.61	1.6	N/A	N/A	< 1500 (estimated)	< 3300 *	≤ 21	0.02 (fiber-coupled)	0.6 x 2 mm ²
404.5	Sacher TEC-100-0405-030	Free space	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms integration time)	40	30 (free space)	Benchtop
410.3	35	Chip-scale	N/A	N/A	N/A	N/A	N/A	N/A	20	0.185 (fiber-coupled)	16.8 x 9.6 mm ²

* Limited by the measurements

4.1. Academic literature

We can group the academic research on tunable and narrow linewidth visible lasers in three different categories based on their footprint and level of integration: free space, whispering gallery mode (WGM) resonators and integrated stimulated Brillouin scattering, and chip-scale.

4.1.1. Free space

External cavity diode lasers (ECDLs) based on free-space cavities are commonly built and used in research laboratories due to their simplicity and lower cost when compared to similar commercial laser systems. While a basic ECDL is somewhat simple to assemble, its overall performance is strongly dependent on the mechanical and optical characteristics of the system^{86,132–134}, such that achieving performance similar to commercial ECDLs can be difficult. An ECDL can, for example, be made out of simple reflectors and splitters^{68,69,135,136}, in which case the resulting linewidth is extremely broad and the tuning is achieved via careful control of the cavity length and the angle of the feedback light⁶⁹. Most ECDLs, however, have a grating as a frequency-selective feedback element, which allows for narrower linewidths in the range of hundreds of kHz to a few MHz and simplified tuning via the grating angle^{62,70,71,86,115,116,137–141}. The coarse and fine tunings are typically performed via a motor and a piezoelectric actuator connected to the grating, respectively. Since the tuning relies on the mechanical movement of a massive grating, the modulation bandwidth is typically limited to a few kHz. Alternatively, larger modulation bandwidths of at least a few MHz can be achieved via direct laser current modulation, but the tuning is limited to short ranges unless appreciable change in the optical output power can be tolerated. This kind of fast tuning is routinely employed in atomic research for frequency stabilization and linewidth narrowing of home-built ECDLs to probe narrow atomic transitions.

While convenient, these ECDLs are benchtop in size and require very careful mechanical construction and optical alignment. Since the laser's performance depends strongly on its construction

details, the linewidth, tuning and optical power characteristics of ECDLs found in literature vary vastly (Table S7).

Table S7: Representative summary of the state-of-the-art academic external cavity diode lasers (ECDLs) based on free-space cavities for visible wavelengths. We list the tuning ranges for constant temperature (room temperature) and fixed laser current unless stated otherwise.

Reference	Center wavelength (nm)	Pump type	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
86	780	Electrical	Benchtop	N/A	137 (with feed-forward)	N/A	N/A	N/A	2×10^3 (effective)	10 – 60 (free space, current-dependent)	N/A
117	780	Electrical	Benchtop	11	15 (with feed-forward)	N/A	N/A	26	73 (Gaussian)	50 (free space)	N/A
133	780	Electrical	Benchtop	10	40	N/A	N/A	40 (with active stabilization)	87 (Gaussian, with active stabilization)	50 (free space)	N/A
62	661	Electrical	Benchtop	6 – 11 (different ECDLs)	4 – 6 (different ECDLs)	N/A	N/A	N/A	6×10^3 – 1.1×10^4 (effective, different ECDLs)	26 – 40 (free space, different ECDLs)	N/A
137	646	Electrical	Benchtop	12	20	N/A	N/A	70	4×10^3 (effective)	20 (free space)	N/A
68	642	Electrical	Benchtop	N/A	N/A	N/A	2 (current)	N/A	1.7×10^8 (effective)	≤ 1 (free space)	8
68	525	Electrical	Benchtop	N/A	N/A	N/A	2.41 (current)	N/A	1.8×10^8 (effective)	≤ 10 (free space)	6
69	525	Electrical	Benchtop	2.4 - 4.73 (current- and temperature-dependent)	N/A	N/A	N/A	N/A	$\geq 7.7 \times 10^7$ (effective)	≤ 16.7 (current-dependent)	$\geq 6, \leq 27.8$ (current-dependent)

Reference	Center wavelength (nm)	Pump type	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
115	518	Electrical	Benchtop	4.9 - 10.5 (current-dependent)	N/A	N/A	N/A	N/A	$6.9 \times 10^3 - 1.0 \times 10^4$ (effective, current-dependent)	2.5 - 42 (free space, current-dependent)	N/A
71	495	Electrical	Benchtop	8	N/A	N/A	N/A	88	1.22×10^3 (Gaussian)	≤ 60 (free space)	≥ 30
135	450	Electrical	Benchtop	≤ 12.11	N/A	N/A	N/A	N/A	$\leq 1.6 \times 10^8$ (effective)	3 (free space)	$\geq 15, \leq 40$
68	450	Electrical	Benchtop	N/A	N/A	N/A	2.26 (current)	N/A	1.9×10^8 (effective)	≤ 8 (free space)	20
116	445	Electrical	Benchtop	2.6 - 4.3 (current-dependent)	N/A	N/A	N/A	N/A	$4.7 \times 10^3 - 8.0 \times 10^3$ (effective, current-dependent)	1 - 20 (free space, current-dependent)	N/A
62	413	Electrical	Benchtop	2 - 5 (different ECDLs)	5 - 6 (different ECDLs)	N/A	N/A	N/A	$4.0 \times 10^3 - 9.0 \times 10^3$ (effective, different ECDLs)	1 - 13 (free space, different ECDLs)	N/A
70	410	Electrical	Benchtop	N/A	93 (with feed-forward)	< 0.002	N/A	N/A	N/A	2 (free space)	N/A
138	410	Electrical	Benchtop	4 (grating Plus temperature)	50 (with feed-forward)	N/A	N/A	N/A	8×10^2 (effective)	≤ 17.2	35
139	405	Electrical	Benchtop	6.01 (only at threshold, TE)	N/A	N/A	N/A	N/A	5.5×10^7 (effective)	< 35 (free space, TE, current-dependent)	N/A

Reference	Center wavelength (nm)	Pump type	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
136	403	Electrical	Benchtop	< 4 (current-dependent)	N/A	N/A	N/A	N/A	3.5×10^8 (effective)	≤ 5 (free space, current dependent)	23
140	398	Electrical	Benchtop	6.3	0.8	N/A	N/A	N/A	1.1×10^4 (effective)	5 (free space)	N/A
141	392	Electrical	Benchtop	2.7	6	N/A	N/A	N/A	5.0×10^3 (effective)	3.5 (free space)	N/A

4.1.2. Whispering gallery mode resonators and integrated stimulated Brillouin scattering

One approach for miniaturizing ECDLs consists in replacing the bulky free-space cavities with compact and low-loss whispering gallery mode (WGM) resonators, whose intrinsic Rayleigh backscattering provides the frequency-selective optical feedback for the self-injection locking^{103–105,131}. Although this strategy represents a significant step towards integration, the resulting systems are still not chip-scale and their demonstrated tunability has been extremely limited. The total footprint is comprised not only by the millimeter-scale WGM cavities, but also by the free space components such as lenses and prisms necessary for coupling light to the resonator and thermoelectric coolers/piezoelectric actuators for wavelength tuning. In these systems, the coarse tuning is typically achieved by changing the temperature of the WGM resonator, making it very slow due to the large size of the resonator. Fine tuning, on the other hand, is typically achieved by changing the FPLD current or via a piezoelectric actuator attached to the WGM cavity. Since the free space components need to be precisely aligned and the actuators need to be carefully positioned and controlled, the packaging of the whole system is complex.

Another approach for achieving narrow-linewidth lasing using centimeter-scale resonators is via stimulated Brillouin scattering (SBS), which has been recently demonstrated in the red spectral range (674 nm) using a photonic chip³³. While providing narrow intrinsic linewidth emission, this approach requires an external optical pump source with high power and narrow linewidth close to the desired emission wavelength, making the system not integrated. In addition to not being electrically pumped, the SBS laser is not tunable unless the optical pump is as well.

Table S8: Summary of the state-of-the-art on visible lasers based on whispering gallery mode (WGM) resonators and integrated stimulated Brillouin scattering.

Reference	Center wavelength (nm)	Pump type	Platform	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
131	780.25	Electrical	DBR + WGM + Rb cell	$< 20 \text{ cm}^3$	0.026	12.8 (via current)	N/A	N/A	5×10^{-3} (stabilized)	N/A	4.6 (fiber coupled), 10 (free space)	N/A
33	674	Optical (external)	High power benchtop Ti-Sapphire laser + Low-confinement silicon nitride	$18 \times 18 \text{ mm}^2$ + benchtop Ti-Sapphire laser	N/A	N/A	N/A	N/A	2.69×10^{-1}	N/A	9.28 (on-chip)	N/A
104	492.1	Electrical	FPLD + WGM	N/A	1.37	20 (via current)	N/A	N/A	1	N/A	100 (free space)	> 40
103	446.5	Electrical	FPLD + WGM	2 cm-long cavity	N/A	2 (via current)	N/A	N/A	$< 1 \times 10^3$ (estimated, not measured)	N/A	3 (free space)	N/A
105	370	Electrical	FPLD + WGM	N/A	N/A	< 4.5 (via current)	N/A	N/A	$< 1 \times 10^2$	N/A	10 (free space)	N/A

4.1.3. Chip-scale

A traditional strategy for achieving single-frequency lasing from a laser diode is to include a frequency-selective element either continuously inside of the laser cavity (distributed feedback laser, DFB) or right outside of it as a mirror (distributed Bragg reflector laser, DBR). Both approaches alter the lasing dynamics such that only one longitudinal mode of the laser experiences enough gain to lase. Although the emitted light is single-frequency, the linewidths are wide due to the high loss of the lasing cavity, ranging from a few MHz to hundreds of MHz^{82,142–152} (Table S9). In addition, the wavelength tunability is extremely limited, and achieved via either temperature or laser current. Since the frequency-selective element is a diffraction grating and the required pitch is comparable to the wavelength of the light, fabricating such structures is also challenging at short wavelengths.

Instead of using gratings as a frequency-selective element, an alternative approach is to use a high-quality factor and tunable reflector placed outside of the gain medium. Such reflector not only provides frequency selection and tunability, but also extends the effective laser cavity length beyond the gain medium to result in narrow linewidth. This strategy has been recently employed to demonstrate narrow linewidth and tunable lasing in the red spectral range (684 nm), where the gain medium is a reflective semiconductor optical amplifier (RSOA) and the reflector is an on-chip photonic system comprised of waveguides and resonators³¹. The disadvantages of this approach when compared to our chip-scale lasers are detailed in the main text. Notably, the footprint of the system is centimeter-scale despite being chip-scale, the demonstration has been limited only to the red spectral range and the wavelength tunability has been shown only very close to threshold, where the emitted optical power is low (0.4 mW).

While this article was under review, Siddharth et al. publish a work showing self-injection locking of FPLDs in the violet (410 nm), deep blue (461 nm) and green (518 nm) spectral ranges using low-confinement silicon nitride resonators³⁵. However, they show restricted tunability and over-MHz intrinsic linewidth only measured at 461 nm and lack any on-chip control for the resonators, thus relying on the FPLD current to achieve self-injection locking. The absence of on-chip phase shifters precludes precise and

gap-free tuning, since the FPLDs can only be self-injection locked to the static positions of the ring resonances, which are fixed by fabrication. The authors did not show any tuning for the violet and green lasers, and they measured only 0.57 nm of coarse tuning in deep blue, full of spectral gaps. In terms of coherence, they measured linewidth only for the deep blue laser, showing a broad intrinsic linewidth > 1.6 MHz and an effective linewidth > 3.5 MHz. The stability of their lasers is also not clear, with a demonstration of only 25 seconds of continuous mode-hop-free operation for the deep blue laser only. Their large resonator footprint (200 μm radius) results in a ring FSR (~ 100 GHz) almost equal to the FPLD FSR (~ 94 GHz, extracted from their Fig. 3), suggesting that their laser is susceptible to mode-hops under both static and modulated operation.

Table S9: Summary of the state-of-the-art on DFB lasers at visible wavelengths.

Reference	Center wavelength (nm)	Pump type	Platform	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
142	780	Electrical	DFB	N/A	N/A	80 (temperature tuning)	N/A	N/A	N/A	2.15×10^3 (effective)	70 (free space)	> 40
143	660	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	N/A	5 (free space)	N/A
144	513.85	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	3.5×10^4 (effective)	14 (free space)	36.9
82	479.712	Electrical	DFB	N/A	0.375 (via current)	N/A	N/A	1.1 GHz (current)	N/A	4.5×10^5 (effective)	14 (free space)	≤ 42.4
145	443	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	7.6×10^3 (effective)	80 (free space)	≤ 29
146	434	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	2.7×10^5 (effective)	40 (free space)	22
147	408.6	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	1.2×10^4 (effective)	20 (free space)	35
148	408.5	Electrical	DFB	N/A	N/A	N/A	N/A	< 2.3 GHz (current)	N/A	N/A	15 (free space)	35
149	408	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	3.6×10^4 (effective)	90 (free space)	26
118	404.63	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	4.2×10^5 (effective)	60 (free space)	N/A
119	404.6	Electrical	DFB	N/A	N/A	N/A	N/A	N/A	N/A	7.3×10^4 (effective)	46 (free space)	23
152	400	Electrical (pulsed)	DFB	N/A	N/A	N/A	N/A	N/A	N/A	4.3×10^5 (effective)	90 (free space)	N/A

Table S10: Summary of state-of-the-art on tunable and narrow linewidth chip-scale lasers for visible wavelengths.

Reference	Center wavelength (nm)	Pump type	Platform	Estimated footprint	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning 3-dB modulation bandwidth	Intrinsic linewidth (kHz)	Gaussian or effective linewidth (kHz)	Output power (mW)	SMSR (dB)
31	684.4	Electrical	RSOA + Low-confine ment silicon nitride	28 x 8 mm ²	10.8 (4 mA above threshold, 50 mA current)	N/A	N/A	N/A	2.3 $\pm$ 0.2 (90 mA current)	56.3 $\pm$ 0.4 (Gaussian) (90 mA current)	4.8 (90 mA current) (fiber-coupled)	40
35	518.6	Electrical	FPLD + Low-confine ment silicon nitride	16.8 x 9.6 mm ²	N/A	N/A	N/A	N/A	N/A	N/A	1.9 (fiber-coupled)	36
35	461.8	Electrical	FPLD + Low-confine ment silicon nitride	16.8 x 9.6 mm ²	0.57 (via laser current)	2.4 (via laser current)	4.1 $\times 10^{-7}$	N/A	1.2 $\times 10^3$	1.6 $\times 10^3$ (Gaussian), 3.6 $\times 10^3$ (effective, 0.1 ms integration time)	1.1 (fiber-coupled)	32
35	410.3	Electrical	FPLD + Low-confine ment silicon nitride	16.8 x 9.6 mm ²	N/A	N/A	N/A	N/A	N/A	N/A	0.185 (fiber-coupled)	20

4.2. Commercial lasers

The most widely used commercial lasers systems providing narrow linewidth and tunability at visible wavelengths are based on free-space ECDLs. While the exact performance metrics vary among different manufacturers and wavelength ranges (Table S11) and more often than not only typical values are provided instead of actual ones, common specifications are several nanometers of course tuning, few to tens of GHz of fine tuning, effective linewidths of several hundreds of kHz and output powers of a few to tens of mW. The frequency pulling ranges are typically reported considering that feed-forward is employed to extent the mode-hop-free continuous tuning range. Despite their state-of-the-art performance, these lasers are benchtop in size and expensive, with a single ECDL typically costing tens of thousands of dollars.

Table S11: Summary of state-of-the-art commercial external-cavity diode lasers (ECDLs) for visible wavelengths. For the effective linewidths, the times in between parenthesis correspond to the integration time.

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
801	Velocity TLB-6713	Newport	Benchtop, 114 x 114 x 197 mm ³	18	80	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	20 (free space) 10 (fiber-coupled @ 795 nm)	N/A
794-810	Vortex Plus TLB-6814	Newport	Benchtop, 114 x 95 x 95 mm ³	0	100	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	30 (free space) @ 795 nm	N/A
790	TEC-500-0780-100	Sacher	Benchtop	40	> 30	N/A	N/A	N/A	< 100 (1 ms) < 1000 (1 s)	> 100 at maximum gain (free space)	> 50
785	DLC DL Pro 780	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	40	≥ 20 (typical)	N/A	N/A	≥ 10 (datasheet), 9.5 (measured, section 2.13)	50 (5 μ s) (datasheet), 224 (measured, section 2.13)	40 - 105 (free space) 22 - 57.75 (fiber-coupled) (wavelength-dependent)	40
785	TEC-100-0780-060	Sacher	Benchtop	> 20	> 30	N/A	N/A	N/A	< 1000 (1 ms)	60 (free space)	40
785	TEC-100-0780-150	Sacher	Benchtop	> 20	> 30	N/A	N/A	N/A	< 1000 (1 ms)	150 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
782.5	TEC-500-0780-030	Sacher	Benchtop	25	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 30 (free space)	> 40
780	TEC-150-0785-120	Sacher	Benchtop	20	> 30	N/A	N/A	N/A	< 100 (1 ms) < 2000 (1 s)	> 120 at maximum gain (free space)	> 40 at gain maximum
778.5	TEC-150-0780-120	Sacher	Benchtop	> 20	> 30	N/A	N/A	N/A	< 100 (1 ms) < 2000 (1 s)	> 120 at maximum gain (free space)	> 40 at gain maximum
773.5	Vantage TLB-7113-01	Newport	Benchtop, 165 x 152 x 92 mm ³	17	50	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	90 (free space) @ 780 nm	N/A
773	Velocity TLB-6712	Newport	Benchtop, 114 x 114 x 197 mm ³	16	80	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	50 (free space) 25 (fiber-coupled) @ 780 nm	N/A
770-790	OE4078	OEWaves	Benchtop, 59 x 153 x 25 mm ³	0	10 (thermal tuning)	1×10^{-7}	100 kHz	$< 8 \times 10^{-2}$	N/A	2 (fiber-coupled)	50
770	TEC-500-0770-060	Sacher	Benchtop	25	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 1000 (1 s)	> 60 (> 80 at maximum gain (free space))	> 50
767.5	TEC-100-0770-030	Sacher	Benchtop	> 20	> 30	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
765-781	Vortex Plus TLB-6813	Newport	Benchtop, 114 x 95 x 95 mm ³	0	100	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	70 (free space) @ 780 nm	N/A
765	TEC-500-0770-040	Sacher	Benchtop	20	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 100 (1 ms) < 1000 (1 s)	> 20 (free space)	> 40
765	TEC-150-0770-100	Sacher	Benchtop	20	> 30	N/A	N/A	N/A	N/A	100 (free space)	N/A
765	TEC-100-0770-100	Sacher	Benchtop	> 20	> 30	N/A	N/A	N/A	< 1000 (1 ms)	100 (free space)	40
734.5	Velocity TLB-6711	Newport	Benchtop, 114 x 114 x 197 mm ³	9	80	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	20 (free space) 10 (fiber-coupled) @ 737 nm	N/A
732.5	TEC-500-0730-010	Sacher	Benchtop	15	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 10 (free space)	> 40
730	TEC-500-0730-030	Sacher	Benchtop	10	> 30	N/A	N/A	N/A	N/A	30 (free space)	N/A
730	TEC-150-0730-020	Sacher	Benchtop	10	> 30	N/A	N/A	N/A	N/A	20 (free space)	N/A
730	TEC-100-0730-020	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	20 (free space)	40
729	DLC DL Pro FL 729	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	2	≥ 5 (typical)	N/A	N/A	≥ 10 (typical)	60 (5 μ s)	28 (free space) 15.4 (fiber-coupled)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
725-741	Vortex Plus TLB-6811	Newport	Benchtop, 114 x 95 x 95 mm ³	0	100	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	20 (free space) @ 737 nm	N/A
703	TEC-100-0705-025	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	20 (free space)	40
701.5	TEC-150-0705-030	Sacher	Benchtop	10	> 30	N/A	N/A	N/A	< 100 (1 ms) < 2000 (1 s)	> 30 at maximum gain (free space)	> 40 at gain maximum
699	TEC-500-0705-010	Sacher	Benchtop	18	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 10 (free space)	> 40
699	Vantage TLB-7113-02	Newport	Benchtop, 165 x 152 x 92 mm ³	8	10	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	25 (free space) @ 702 nm	N/A
687.5	TEC-500-0690-010	Sacher	Benchtop	15	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 8 (free space)	> 40
686	TEC-100-0690-025	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
680	TEC-100-0680-025	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	25 (free space)	40
677.5	TEC-500-0680-010	Sacher	Benchtop	15	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 10 (free space)	> 40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
673	DLC DL Pro FL 674	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	2	≥ 5 (typical)	N/A	N/A	≥ 10 (typical)	100 (5 μ s)	14 (free space) 7.7 (fiber-coupled)	40
672.5	TEC-500-0670-010	Sacher	Benchtop	15	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 10 at maximum gain (free space)	> 40
672	Vantage TLB-7104-01	Newport	Benchtop, 165 x 152 x 92 mm ³	2	15	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	20 (free space) @ 671 nm	N/A
670	TEC-150-0670-020	Sacher	Benchtop	10	> 30	N/A	N/A	N/A	< 100 (1 ms) < 2000 (1 s)	> 20 at maximum gain (free space)	> 40 at gain maximum
670	TEC-100-0670-010	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
670	TEC-100-0670-015	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	15 (free space)	40
670	TEC-100-0670-020	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	20 (free space)	40
666.5	DLC DL Pro 670	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	13	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	200 (5 μ s)	25 (free space) 13.75 (fiber-coupled)	40
660	TEC-100-0660-025	Sacher	Benchtop	> 6	> 30	N/A	N/A	N/A	< 1000 (1 ms)	25 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
654.5	Vantage TLB-7104-02	Newport	Benchtop, 165 x 152 x 92 mm ³	9	10	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	15 (free space) @ 655 nm	N/A
651.5	TEC-500-0650-010	Sacher	Benchtop	9	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 8 (free space)	> 40
647	TEC-100-0650-005	Sacher	Benchtop	> 6	> 30	N/A	N/A	N/A	< 1000 (1 ms)	5 (free space)	40
645	TEC-500-0645-010	Sacher	Benchtop	10	> 30	0.02 (piezo)	> 2 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 300 (1 ms) < 100 (1 μ s) < 1000 (1 s)	> 8 (free space)	> 40
670	TEC-100-0670-020	Sacher	Benchtop	> 8	> 30	N/A	N/A	N/A	< 1000 (1 ms)	20 (free space)	40
643	TEC-100-0640-100	Sacher	Benchtop	> 6	> 30	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
637	DLC DL Pro HP 637	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	4	$\geq$ 20 (typical)	N/A	N/A	$\geq$ 10 (typical)	500 (5 μ s)	45 (free space) 24.75 (fiber-coupled)	40
636.5	Velocity TLB-6704	Newport	Benchtop, 114 x 114 x 197 mm ³	3	80	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	8 (free space) 2.5 (fiber-coupled) @ 638 nm	N/A
636.5	TEC-500-0635-005	Sacher	Benchtop	7	> 30	N/A	N/A	N/A	< 100 (1 ms) < 1000 (1 s)	> 5 (free space)	> 40
635.5	TEC-100-0635-010	Sacher	Benchtop	> 5	> 30	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
633	DLC DL Pro 633	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	4	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	200 (5 μ s)	40 (free space) 22 (fiber-coupled)	40
632.5-640	Vortex Plus TLB-6804	Newport	Benchtop, 114 x 95 x mm ³	0	120	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	8 (free space) @ 638 nm	N/A
488.5	TEC-100-0488-020	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	20 (free space)	40
471.5	TEC-100-0470-010	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
459-461	Vortex Plus TLB-6802	Newport	Benchtop, 114 x 95 x mm ³	0	25	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	<200 (50 ms)	40 (free space) @ 461 nm	N/A
459	DLC DL Pro HP 461	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	4	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	150 (5 μ s i)	250 (free space) 137.5 (fiber-coupled)	40
455-457	Vortex Plus TLB-6802-455	Newport	Benchtop, 114 x 95 x mm ³	0	25	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	40 (free space) @ 455 nm	N/A
452.5	TEC-100-0455-015	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	15 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/ μ s)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
443	TEC-100-0440-015	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	15 (free space)	40
422	Vantage TLB-7102-02	Newport	Benchtop, 165 x 152 x 92 mm ³	2	5	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	10 (free space) @ 423 nm	N/A
421.5	DLC DL Pro HP 420	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	3	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	150 (5 μ s)	70 (free space) 38.5 (fiber-coupled)	40
421	TEC-100-0420-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	15 (free space)	40
416.5	TEC-100-0415-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40
409.5	Velocity TLB-6702	Newport	Benchtop, 114 x 114 x 197 mm ³	5	120	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	< 200 (50 ms)	4 (free space) @ 411 nm	N/A
408	TEC-100-0408-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40
404.5	TEC-100-0405-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40
404	TEC-150-0405-030	Sacher	Benchtop	4	> 30	N/A	N/A	N/A	< 100 (1 ms) < 2000 (1 s)	> 30 at maximum gain (free space)	> 40 at gain maximum
401.5	TEC-100-0402-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40

Center wavelength (nm)	Laser name	Company	Dimensions	Coarse tuning range (nm)	Mode-hop-free fine tuning range (GHz)	Fine tuning rate (GHz/μs)	Fine tuning modulation bandwidth	Intrinsic linewidth (kHz)	Effective linewidth (kHz)	Output power (mW)	SMSR (dB)
399.75	DLC DL Pro HP 399	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	1.5	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	150 (5 μs)	110 (free space) 60.5 (fiber-coupled)	40
397.5	DLC DL Pro HP 397	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	2	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	150 (5 μs)	65 (free space) 35.75 (fiber-coupled)	40
396	Vantage TLB-7102-01	Newport	Benchtop, 165 x 152 x 92 mm ³	8	10	N/A	> 100 Hz (100% amplitude), 1.5 kHz (20% amplitude) (piezo); 100 MHz (current)	N/A	300 (50 ms)	15 (free space) @ 397 nm	N/A
395	TEC-100-0395-030	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	30 (free space)	40
377.5	TEC-100-0378-010	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
374.5	TEC-100-0375-010	Sacher	Benchtop	> 2	> 10	N/A	N/A	N/A	< 1000 (1 ms)	10 (free space)	40
372.5	TEC-100-0372-010	Sacher	Benchtop	> 2	> 30	0.005 (piezo)	> 0.5 kHz @ 10 GHz tuning amplitude (piezo)	N/A	< 2000 (1 ms) < 10000 (1 s)	> 10 at maximum gain (free space)	> 35 at gain maximum
369.7	DLC DL Pro HP 369	Toptica	Benchtop, 90 x 90 x 240-300 mm ³	1	≥ 20 (typical)	N/A	N/A	≥ 10 (typical)	150 (5 μs)	13 (free space) 7.15 (fiber-coupled)	40

5. References for Figure 1d of the main text

We present the extended version of the “Quantum optics, clocks and cold atoms physics” portion of Fig. 1d (main text) with the names of the species corresponding to each line in Fig. S43. The references we used for each species are: Ba^{153,154}, Ba^{+153–156}, Ca^{157–160}, Ca^{+161,162}, Cr^{163,164}, Cs^{165–168}, Diamond NV center^{169,170}, Dy^{171,172}, Er^{172,173}, Eu³⁺¹⁷⁴, Fr¹⁷⁵, Hg⁺¹⁶², K^{176–181}, Li^{182–184}, Li⁺¹⁸⁴, Na^{185,186}, Pr³⁺¹⁸⁷, Rb^{167,168,177,188–191}, Sr^{160,192–207}, Sr^{+162,202,208,209}, Yb^{162,182,191,210–216} and Yb^{+160,162,217–221}.

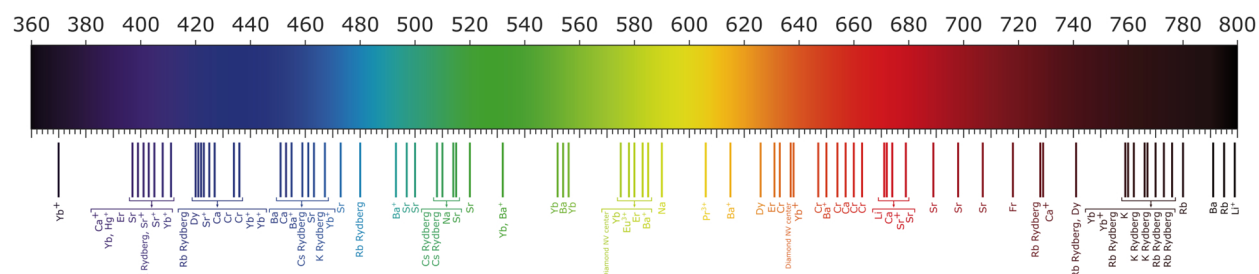


Figure S43: Expanded version of the “Quantum optics, clocks and cold atoms physics” portion of Fig. 1d (main text) with the same of the species corresponding to each line.

The field of ultracold molecules, represented in the Fig. 1d (main text) by the portion “Cold molecules physics” is an ever-expanding field^{222,223}, with new molecules being proposed and studied frequently²²⁴. Cooling down and trapping molecules requires typically more laser sources than in the case of atoms or ions due to their much more complex structure, and the wavelengths needed spread all across the visible to infrared spectral ranges. In addition, changing the molecules compositions also change the wavelengths of the lasers required to manipulate them²²⁵. Therefore, we represent this field of research as a continuous line across the whole spectrum. Particular wavelengths of interest can be found for example in the extensive list of references of these two review articles^{222,223}.

The references we used to support the other portions of the Fig. 1d (main text) are: “Li-Fi”^{68,136,226,227}, “Underwater LIDAR”^{228,229}, “Fluorescent imaging and optogenetics”^{230–234} and “Laser displays”^{235–238}.

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