

High-gain and high-speed wavefront shaping through scattering media

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Supplementary Note 1: Review on the state-of-the-art wavefront shaping techniques

The system speeds, control DOFs and energy gains in representative publications in the realm of WFS are summarized in Supplementary Table 1.

The progress outlined in the Introduction of the main manuscript and Supplementary Table 1 shows that current WFS modalities satisfactorily achieve only one or two of the desired performance metrics: system speed, control DOF, and energy gain. Digital WFS (feedback-based, transmission-matrix-based, and DOPC-based WFS) can realize a high energy gain by increasing the energy of the modulated light, as long as the light intensity is below the damage threshold of the SLM. However, digital WFS suffers from low system speed or low control DOF, limited by the speeds and available pixel numbers of such digital devices as SLMs and cameras. Especially as manifested in state-of-the-art digital WFS techniques, the system speed and the control DOF are irreconcilable contradictions: improving one of them compromises the other. As for analog WFS (AOPC-based WFS), its largest advantage is the sufficiently large control DOF that is independent of the system speed. Although the fastest system latency of current AOPC techniques is still several milliseconds, this could be further improved by employing new PRC materials. In comparison, the low energy gain of AOPC is a disappointing disadvantage that is much more difficult to resolve, because conjugation reflectivity is an intrinsic property of a nonlinear PRC.

We note that a gain medium was also used in Ref. 11 of the main manuscript [*Nature Photonics* 7, 919 (2013)] to realize high-speed WFS through a scattering medium having a speckle correlation time as short as 620 ns, where the scattering medium and a pinhole were placed into a multimode laser cavity composed of a gain medium and two cavity mirrors. Since the lasing process naturally selects the lasing state with minimal loss in the laser cavity, the

complex wavefront that can pass through the pinhole with minimal loss starts to resonate in the cavity quickly and all other lasing modes are suppressed via the process of mode competition. Therefore, an optical focus is formed in the position of the pinhole. Our HGHS-WFS is quite different to this earlier work in principle, although both methods employed gain media for WFS. In fact, the method in Ref. 11 of the main manuscript works well only for focusing through a thin scattering layer, because thick scattering media would destroy the degeneracy of the laser cavity, which is an important precondition for the method to work. As pointed out by the authors, thick multiply scattering samples preserves the degeneracy of the laser cavity only within the isoplanatic angle, which means that the method is effective for scattering media with a remarkable optical memory effect. In contrast, our HGHS-WFS does not require the sample to have any optical memory effect. Moreover, the method in Ref. 11 can achieve a limited control DOF, on the order of ~ 1000 . Our HGHS-WFS controlled three-order more optical modes, due to the high spatial resolution of the PRC.

Supplementary Note 2: Characterization of gain modules

To characterize the gain of the two gain modules employed in our HGHS-WFS system, we placed a photodiode (PDA36A, Thorlabs) at the output port of the second gain module (see Fig. 2) to measure the energies of the received light signal before and after the gain modules were launched. A typical example of the amplified output signal from the two gain modules is shown in Fig. S1(a1). Notably, it takes about $600\ \mu\text{s}$ for the light energy output from the gain modules to increase to the maximum. The ASE also manifests the same temporal characteristic, as shown in Fig. S1(a2). This phenomenon reflects the time needed to accumulate to the maximum population inversion in the gain crystal, which is determined by the lifetime of the upper energy

level. Therefore, in the experiments, we delayed our wavefront shaping operations by $600\ \mu\text{s}$ so the gain modules reached their maximum gains.

The individual gain of each gain module and the total gain of the two gain modules can both be estimated as $(\text{signal with gain} - \text{ASE}) \times \text{attenuation} \div \text{signal without gain}$, where ASE is the amplified spontaneous emission that can be recorded separately. The relationship between the gain and the driving current of the pump source in each gain module is presented in Fig. S1(b), showing that the two gain modules have similar gain performances. When working with the maximum driving current, both gain modules have a gain of ~ 50 . Note that “gain” here means small-signal gain, which is measured on the precondition that the gain module is not saturated by the incident light. When the two gain modules are placed in series, the optical power output from the first gain module may saturate the second gain module so that the second gain module does not work in a state of maximum gain. To estimate this effect, we measured the total gain of the two gain modules in cases of different incident powers, with the results shown in Fig. S1(c). The total gain stays unchanged when the incident power is below $\sim 10\ \text{mW}$ and is roughly equal to the product of the small-signal gains of the two gain modules, which means that the two gain modules are both working at their maximum gains in this case. When the incident power is larger, the total gain starts to fall because the second gain module is saturated. In these tests, we used a collimated beam with a diameter of $1.5\ \text{mm}$. Therefore, the incident intensity that starts to saturate the gain modules is $10\ \text{mW} / (\pi \times 0.75^2\ \text{mm}^2) \approx 5.7 \times 10^3\ \text{W/m}^2$. As a reference, we also theoretically calculate the incident intensity that would be amplified to the saturation intensity of the laser gain medium. In theory, the saturation intensity of a laser crystal is $I_s = h\nu / \sigma\tau$, where h is the Planck constant, ν is the photon frequency, σ is the effective cross section of the laser transition, and τ is the effective upper-level lifetime. For Nd:YAG crystal, the typical

values of the material parameters include $\sigma = 2.8 \times 10^{-19} \text{ cm}^2$ and $\tau = 0.23 \text{ ms}$ at the laser wavelength of 1064 nm, which results in $I_s \approx 2.9 \times 10^7 \text{ W/m}^2$. Such a saturation intensity corresponds to an incident intensity of $I_s/G \approx 1.16 \times 10^4 \text{ W/m}^2$, where $G = 2500$ is the total gain of our gain modules. In wavefront shaping experiments, the gain medium is filled with scattered photons, thus the beam diameter inside the gain medium is the same as the diameter of the gain medium (6.35 mm). If we want to saturate the gain medium using the scattered light, the incident power should be $5.7 \times 10^3 \text{ W/m}^2 \times [\pi \times (6.35/2)^2 \text{ mm}^2] \approx 180 \text{ mW}$. Generally, the optical power of the collected scattered light from scattering media is far below this value, especially for thick and highly scattering media (for example, such a saturation incident power is two orders higher than the maximum scattered optical power incident on the gain modules in our experiments). We also note that this value is a conservative estimation. Even if the power of the scattered signal incident on the gain modules is doubled, the total gain drops only by $\sim 10\%$, as can be seen from Fig. S1(d).

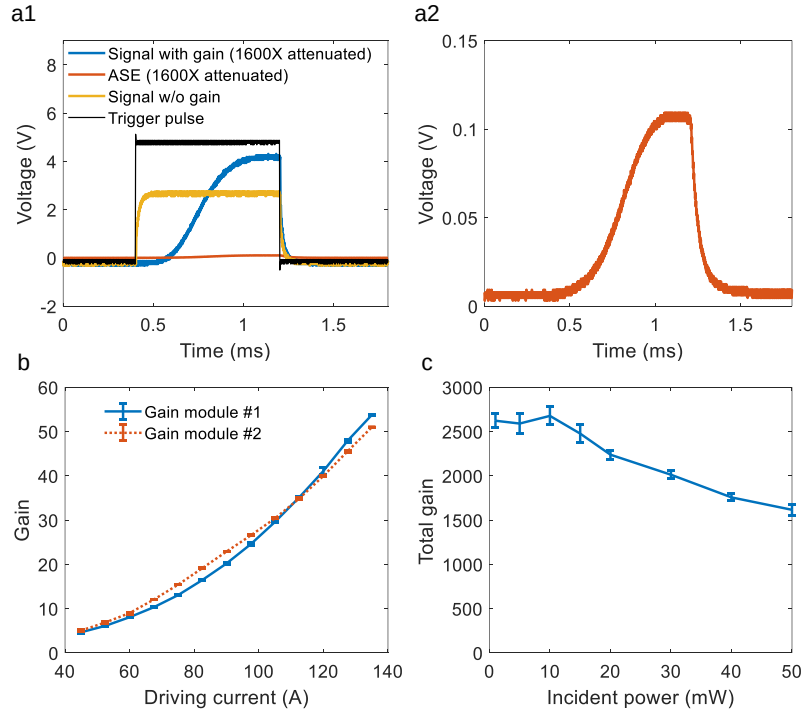


Fig. S1. Characterization of gain modules. **a1**, A typical example of the amplified signal output from the two gain modules in our HGHS-WFS system. **a2**, A zoomed-in view of the ASE in **a1**. **b**, The relationship between the gain and the driving current of the pump source in each gain module. The error bars show the standard deviations of three measurements. **c**, The total gains of the two gain modules in cases of different incident powers, tested using a collimated beam with a diameter of 1.5 mm. The error bars show the standard deviations of three measurements.

Supplementary Note 3: Comparison between time reversal via the separate hologram writing and reading (W/R) mode and the four-wave mixing (FWM) mode for the GaAs PRC

Photorefractive-crystal-based time reversal can be realized via either the separate hologram writing and reading (W/R) mode or the four-wave mixing (FWM) mode. In the W/R mode, the PRC first records the hologram formed by the interference between the signal beam and the reference beam. Then the signal beam and the reference beam are turned off, and the conjugated reference beam starts to read the hologram to generate the time-reversed wavefront at the same time. In the FWM mode, the signal beam and the reference beam are still kept on when the conjugated reference beam is reading the hologram. Figure S2 shows the experimental results for time-reversed foci through a scattering medium achieved via FWM and W/R modes using the GaAs PRC. In this comparative experiment, we set the time for hologram writing and hologram reading to 10 ms and 1 ms, respectively, for the W/R mode. The intensity of the time-reversed focus from the FWM mode is about 50 times larger than that from the W/R mode. The reason may be that, for the W/R mode, the hologram is “washed out” quickly by the reading beam because of the fast response of the PRC used. For the FWM mode, however, the new hologram is still being recorded while the old hologram is being washed out, so that the hologram always

remains strong inside the PRC. In addition, the system speed of the W/R mode is determined by the total time for hologram writing and reading, while that of the FWM mode is determined only by the response time of the PRC. Therefore, using the FWM mode can also maximize the speed of the time reversal system, which is the key to the real-time time reversal achieved in our system.

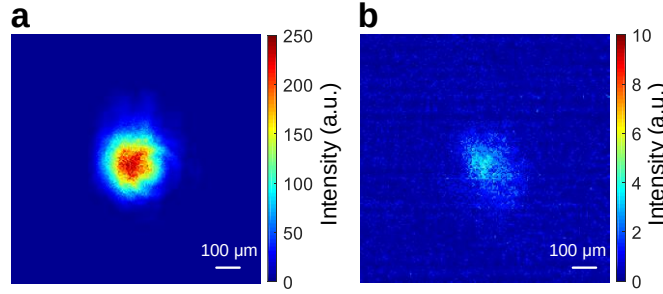


Fig. S2. Time-reversed optical foci through scattering media via (a) FWM mode and (b) W/R mode.

Supplementary Note 4: Energy gain estimation of the time-reversed wavefront

The energy gain of a time reversal system is defined as the ratio of the energy of the corrected wavefront to that of the detected scattered wavefront, which is related to the efficiency of the wavefront modulation system. For an ordinary mirror with 100% reflectivity, the energy gain of the reflected light is 1. However, the energy gain of an analog phase-conjugation mirror (PCM) based on a PRC is generally well below 1, typically 10^{-3} or less. Therefore, it is highly desirable to realize an analog PCM having an energy gain close to or even larger than 1. Our HGHS-WFS makes this goal possible.

To estimate the energy gain of our HGHS-WFS system, we used the same camera to record the speckle pattern incident on the PRC and the time-reversed focus through the scattering medium. Figure S3 shows an example of the recorded results. Next, we calculated respective

total intensities of the speckle pattern and the time-reversed focus by summing all the pixel intensities. Finally, we estimated the energy gain as $\alpha I_{TR}/I_s$, where I_{TR} and I_s are the total intensities of the time-reversed focus and the speckle pattern, respectively, and α is a correction factor that accounts for the pick-up fraction of the time-reversed energy. In our experiments, we used a 50/50 beam splitter to pick up half time-reversed energy on the camera, so $\alpha = 2$ in this case. In the example in Fig. S3, the energy gain was estimated as 1.03.

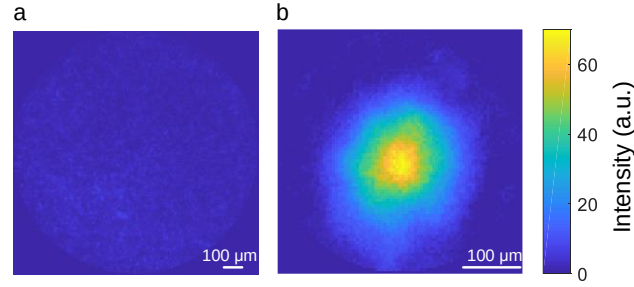


Fig. S3. An example for estimating the energy gain of the time-reversed wavefront. **a**, The recorded speckle pattern on the PRC plane. **b**, The recorded time-reversed focus through the scattering medium.

Supplementary Note 5: Contrast-to-noise ratio (CNR) and Peak-to-background ratio (PBR)

The PBR, which is defined as the ratio between the peak intensity of the focus pattern and the mean of the background, is widely used in digital WFS to evaluate the intensity enhancement on the targeted optical spot. The background around the focus in digital WFS comes from the imperfect shaping for the optical field due to the limited number of control modes and phase measurement errors, which appears as speckle patterns. Therefore, the PBR is a reasonable gauge of the intensity enhancement in the position of the focus relative to the average intensity of the speckle patterns. However, the PBR is rarely used in conventional analog WFS, because analog WFS has several orders higher control degrees of freedom and resolution than digital WFS. The

background around the optical focus is negligible compared to that in digital WFS. Different from conventional analog WFS, our method introduces gain modules in the system, which introduces optical noise to the time-reversed optical focus. Because the optical noise from the gain medium is incoherent, it appears in the image of the time-reversed optical focus as uniform DC-like background rather than speckle patterns. Therefore, the PBR is also not suitable to be used in this case.

The CNR, which is defined as the ratio between the peak intensity of the time-reversed signal above the mean of the background and the standard deviation of the background, is a good gauge of the visibility of the focus pattern in case of the optical incoherent noise from the gain medium, because it evaluates both the peak intensity of the signal and the fluctuation of its background (rather than the background DC value). It is also worth noting that, in digital WFS, the PBR and CNR of the focus have a relationship: $\text{PBR} = \text{CNR} + 1$, for fully developed background speckle patterns. This fixed relationship stems from the fact that the speckle background in digital WFS follows well-defined speckle statistics, where the mean and standard deviation of the background are the same value.

In Fig. S4(a), we presented the simulated cross-section of an optical focus formed by digital wavefront shaping, where the PBR and CNR are 10.9 and 9.9, respectively. It can be seen that the peak of the focus is almost immersed in the background of fully developed speckles in this case. In Fig. S4(b), we show the experimental cross-section of an optical focus (from Fig. 3f) by our HGHS-WFS. We see that the background around the optical focus is a DC-like signal, which originates from the incoherent noise of the gain medium. Visually, the visibility of the focus in Fig. S4(b) is much better than that in Fig. S4(a). But the PBR in Fig. S4(b) is 10.6, which is

similar to that in Fig. S4(a). Therefore, the PBR commonly used in digital WFS is not a reasonable metric for evaluating the contrast of the focus by our HGHS-WFS. The CNR of the focus in Fig. S4(b) is 604, which reveals the contrast of the focus pattern reasonably. Therefore, we adopted CNR instead of PBR as the contrast metric of the time-reversed focus by HGHS-WFS.

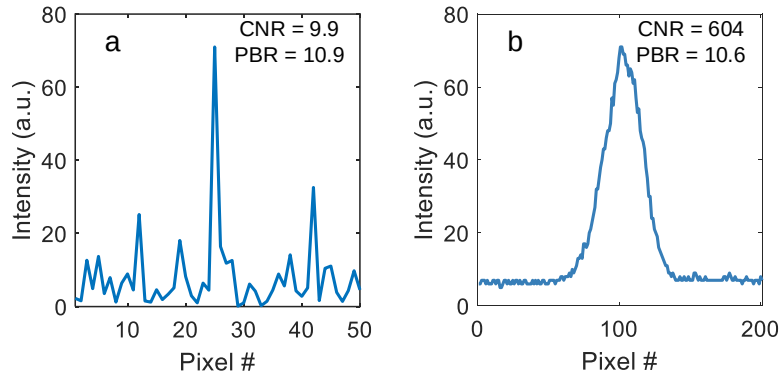


Fig. S4. Comparison of the metrics in wavefront shaping, CNR and PBR. a, The simulated cross-section of an optical focus formed by digital wavefront shaping. b, The experimental cross-section of an optical focus (from Fig. 3f) by our HGHS-WFS.

Supplementary Note 6: Optical focusing through thicker scattering samples

To demonstrate the ability of the HGHS-WFS to focus through thicker scattering samples, we also performed experiments using different numbers of stacked optical diffusers and chicken breast tissue slices as samples. With the same illumination power on the sample, we first replaced one optical diffuser with two stacked optical diffusers (the separation distance between the scattering surfaces of different diffusers is 2 mm). Due to the resulting much higher scattering, the collected light power on the PRC was reduced to 1/3 that of the experiments in Fig. 3. The original time-reversed focus without gain is shown in Fig. S5a, where no obvious optical

focus can be observed because the energy of the time-reversed focus is too weak to be detected. In the HGHS-WFS, however, a bright time-reversed focus with a CNR of ~ 415 through the two optical diffusers immediately appeared when the gain modules started to run, as shown in Fig. S5b. Moreover, even when we replaced the scattering medium with more stacked diffusers, the brightness of the time-reversed focus was still prominent (Figs. S5c and S5d), although the peak intensity of the focus decreased for a thicker scattering medium. The CNRs for Figs. S5c and S5d are calculated to be about 121 and 39, respectively. We note that the optical power of the collected scattered light on the PRC through four optical diffusers was just $40 \mu W$ in our experimental configuration, which is low for efficient time reversal. Even so, our HGHS-WFS successfully addressed the challenge of such an extreme case. If the incident power on the sample and the powers of the reference beam and conjugated reference beam can be increased further, then HGHS-WFS can be expected to realize good time reversal for even thicker samples. Similar experiments were also carried out with chicken breast tissues as samples (the extinction coefficient¹ of the chicken breast tissues is measured to be $\sim 20 \text{ mm}^{-1}$ at the wavelength of 1064 nm, corresponding to a mean free path of 0.05 mm). Without gain, the time-reversed focus through a 2 mm thick tissue slice can barely be discerned for AOPC because of the low conjugation reflectivity of the PRC (Fig. S5e). However, the HGHS-WFS can provide high-fidelity optical focusing through even thicker tissue overburdens (Figs. S5f-S5h). The CNRs for Figs. S5f-S5h are about 322, 159, and 125, respectively.

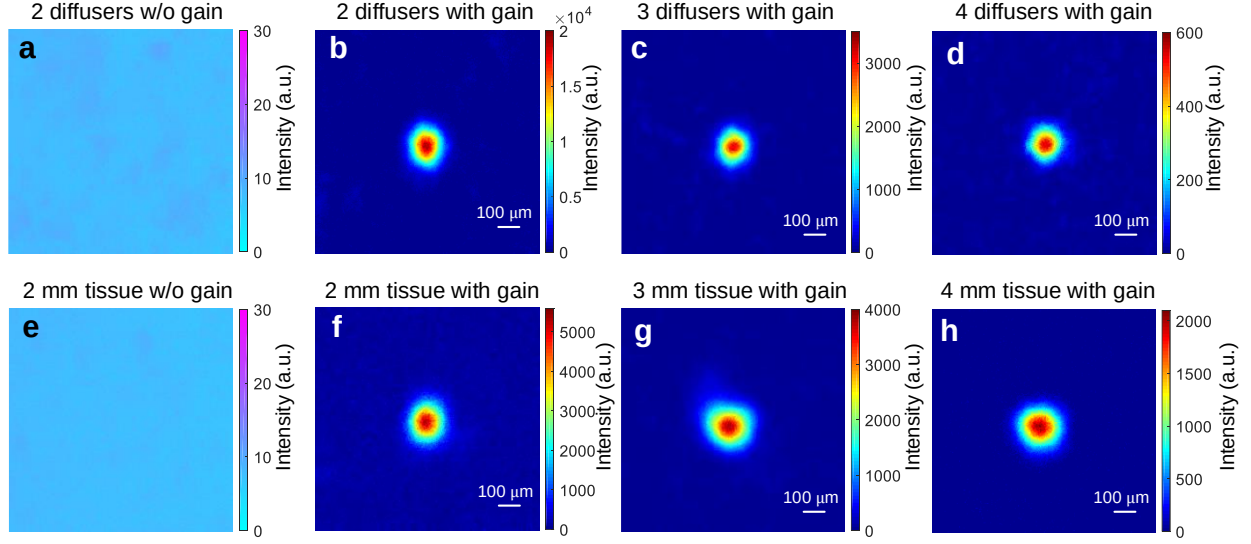


Fig. S5. HGHS-WFS for optical focusing through different scattering media. **a-b**, Results for optical focusing through two optical diffusers with conventional AOPC and the HGHS-WFS, respectively. **c-d**, Focusing through three diffusers and four diffusers with the HGHS-WFS, respectively. **e-f**, Results for optical focusing through a piece of 2 mm thick chicken tissue with conventional AOPC and the HGHS-WFS, respectively. **g-h**, Focusing with the HGHS-WFS through a piece of 3 mm thick chicken tissue and a piece of 4 mm thick chicken tissue, respectively.

Supplementary Note 7: The analysis on further improvement of the energy gain in HGHS-WFS

In our HGHS-WFS, the scattered probe beam collected on the PRC is first amplified by the gain medium and then phase-conjugated by the PRC. The phase-conjugated signal is amplified again by the gain medium before it passes through the scattering medium. Therefore, if we neglect the absorption of the scattering medium and the imperfection of the time reversal, the energy gain of the time-reversed signal in HGHS-WFS can be estimated as G^2R , where G is the total single-pass gain of the gain modules and R is the phase conjugation reflectivity of the PRC. It can be seen that increasing G and R is the most straightforward way to improve the energy gain in

HGHS-WFS. The R is a highly nonlinear function of the material parameters of the PRC, the optical intensities involved in the phase conjugation generation, and the experimental configuration, which can be modeled by a set of coupled-wave equations based on the theory of the photorefractive effect ². Cronin-Golomb *et al.* ^{3,4} obtained the solution of R under the approximation of negligible linear absorption for the PRC as

$$R = \frac{4|C|^2 |\tanh(\mu l)|^2}{\left| \Delta \cdot \tanh(\mu l) + \sqrt{\Delta^2 + 4|C|^2} \right|^2}. \quad (S1)$$

In Eq. (S1), l is the effective crystal length of the PRC, and

$$\mu = \frac{\gamma \sqrt{\Delta^2 + 4|C|^2}}{2(I_s + I_r + I_{r*})}, \quad (S2)$$

$$\Delta = I_{r*} - I_s - I_r, \quad (S3)$$

$$\gamma = \frac{i\pi n_1 \exp(-i\varphi)}{\lambda \cos(\theta)}, \quad (S4)$$

where I_s , I_r , and I_{r*} are the intensities of the signal beam, reference beam and conjugated reference beam incident on the PRC, respectively, λ is the wavelength of the beams, and θ is the half angle between the signal beam and reference beam. The n_1 and φ in the expression of the coupling constant γ [Eq.(S4)] are respectively given by

$$n_1 = -\frac{n_0^3 r_{eff} E_q}{2} \left[\frac{E_0^2 + E_D^2}{E_0^2 + (E_D + E_q)^2} \right]^{1/2}, \quad (S5)$$

and

$$\tan\varphi = \frac{E_D(E_D + E_q) + E_0^2}{E_0 E_q}, \quad (S6)$$

where r_{eff} and n_0 are the effective electro-optic coefficient and the original refractive index of the PRC, respectively, $E_D = (kt/q)(2\pi/\Lambda)$ is the diffusion field (t , temperature; k , Boltzmann constant; q , electron charge; $\Lambda = \lambda/2 \sin \theta$, period of the refractive index grating within the PRC), $E_q = (N_A q / \epsilon_0 \epsilon)(\Lambda/2\pi)$ is the maximum space charge field (N_A , the acceptor density within the PRC material; ϵ_0 , vacuum permittivity; ϵ relative permittivity), and E_0 is the external applied electric field. The parameter $|C|^2$ is given by the roots of the following transcendental equation:

$$\begin{aligned} & (|C|^2 - I_r I_{r*}) \left| \Delta \cdot \tanh(\mu l) + \sqrt{\Delta^2 + 4|C|^2} \right|^2 + \\ & 4|C|^2 |\tanh(\mu l)|^2 I_s I_{r*} + \\ & 2|C|^2 I_s \sqrt{\Delta^2 + 4|C|^2} \times [\tanh(\mu l) + \tanh^*(\mu l)] = 0. \end{aligned} \quad (S7)$$

Finally, the influence of absorption of the PRC is approximated by multiplying R by a factor $\exp(-\alpha L)$, where α and L are the absorption coefficient and physical length of the crystal, respectively ⁵.

The material parameters of the GaAs PRC are $N_A = 1.3 \times 10^{21}/m^3$, $\epsilon = 12.9$, $n_0 = 3.48$, $r_{eff} = 1.43 \times 10^{-12} m/V$, and $\alpha = 0.87 cm^{-1}$ ⁵. In our experiments, the temperature $t = 18^\circ C$ and the laser wavelength $\lambda = 1064 nm$. The diameters of the beams on the PRC are 4 mm.

Since the signal beam incident on the PRC is diffusing, we used the average angle between the signal beam and the reference beam to estimate θ in Eq. (S4) as $\theta \approx 25^\circ$. The physical length L of the PRC we used is 7 mm. The effective crystal length l is the average length of the overlap area between the signal beam and the reference beam in the crystal, which is estimated to be about 2.7 mm. Substituting these parameters into Eqs. (S1-S7), we can investigate how the energy gain of the time-reversed signal could be improved further by improving the phase conjugation reflectivity of the PRC.

In Fig. S6(a), we show the phase conjugation reflectivity of the PRC at different optical powers of the reference beam and the conjugated reference beam. Here, the optical power of the scattered probe beam collected on the PRC is set to be 1 mW, which is similar to the case in Fig. 3. Note that, because the scattered probe beam is amplified by the gain modules before impinging on the PRC, the practical optical power of the probe beam on the PRC that is used in the model above should be multiplied by a factor G . The black cross in Fig. S6(a) corresponds to the optical powers we used in our experiments, i.e., 100 mW for both the reference beam and conjugated reference beam, which results in a phase conjugation reflectivity of about $10^{-6.34}$. The theoretical estimation can provide some insights on why we only achieved a maximum energy gain of 1.05 experimentally for the time-reversed signal in Fig. 3. Since the total single-pass gain G of our gain modules is ~ 2500 , the maximum theoretical energy gain should be $G^2 R = 2500^2 \times 10^{-6.34} \approx 2.86$. The discrepancy between the theoretically estimated energy gain and the practical one may come from the approximated treatment for the absorption of the PRC in the employed model, the statistical estimations for the angle θ and effective crystal length l for diffusing light illumination, and the ignorance of the phase conjugation imperfection.

This calculation shows that it is the low phase conjugation reflectivity of the PRC that limits the energy gain of the time-reversed signal in our current configuration, which is mainly due to the low optical powers of the reference and conjugated reference beams available.

According to Fig. S6(a), it is obvious that more powerful reference and conjugated reference beams tend to produce a larger phase conjugation reflectivity for the PRC, which is a well-known fact in wavefront shaping based on PRCs. For example, if the powers of the reference beam and conjugated reference beam can both be increased to 1000 mW, the phase conjugation reflectivity of the PRC can be 36 times larger than the current one. However, Fig. S6(a) also reveals that when the powers of the reference beam and the conjugated reference beam are large enough, the phase conjugation reflectivity would approach a stable value, as can be seen from the upper right area of Fig. S6(a). Moreover, it can be observed that the high phase conjugation reflectivity tends to be achieved at the diagonal of Fig. S6(a), which corresponds to the case of equal optical powers for the reference and conjugated reference beams. In Fig. S6(b), we present the line profile along the diagonal of Fig. S6(a), which clearly shows how the phase conjugation reflectivity of the PRC increases with the increasing powers of the reference and conjugated reference beams. When the optical powers exceed 10^4 mW, the phase conjugation reflectivity of the PRC can be increased by ~ 200 times compared to the current one, which means that the energy gain of the time-reversed signal in our HGHS-WFS is expected to be enlarged by the same order of magnitude. This is the maximum enlargement for the energy gain we could have by increasing the illumination powers on the PRC in theory.

The maximum phase conjugation reflectivity of a PRC can be further enhanced by applying an AC electric field at a sufficiently high frequency to the crystal to augment the space-charge

field with the crystal when the photorefractive effect occurs ^{5,6}. The required condition for the applied electric field is that its period must be greater than the recombination time of the carriers in the PRC and less than the photorefractive response time ⁵. In Fig. S6(c), we present the relationship between the phase conjugation reflectivity of the PRC and the amplitude of the external electric field, where the optical powers of the reference and conjugated reference beams are both set to 1000 mW. Obviously, a higher electric field amplitude contributes to a higher phase conjugation reflectivity. For example, a 10 kV/cm electric field can improve the phase conjugation reflectivity by one order of magnitude relative to that without an external electric field. It is also notable that the growth rate of the phase conjugation reflectivity would gradually slow down when the amplitude of the electric field is high enough. In Fig. S6(d), we show similar plots with Fig. S6(b) but for the phase conjugation reflectivity at several typical amplitudes of external electric fields, where the black line corresponding to the case in Fig. S6(b) is also added for comparison. These results indicate that sufficiently high optical powers and external electric fields together can increase the phase conjugation reflectivity of the PRC to $\sim 10^{-3}$, which approaches the maximum that can be achieved in theory (this result also theoretically illustrates why the energy gain in conventional wavefront shaping based on PRCs is generally well below unity, as pointed out in the Introduction section of our manuscript). In such a limiting case, the energy gain of the time-reversed signal in our HGHS-WFS can be expected to have a ~ 2000 times increase compared to the current one.

In addition, a higher energy gain could be obtained with higher pump powers for the gain modules. However, it is worth noting that the gain of the gain modules cannot be arbitrarily high, which is clamped by the self-oscillation of the gain medium. For our current gain modules with a total single-pass gain of 2500, we avoid the formation of any cavity and the effect of self-

oscillation by slightly tilting the optics near the gain medium. Encouraged by the reports on high-gain optical amplifiers in the literatures^{7,8}, we believe that it is possible to enhance the gain of the gain modules to 10^4 if enough pumping power is available, that is, a four times increase from our current one. If so, the energy gain of the phase conjugation could be further improved by 16 times because of the round-trip amplification.

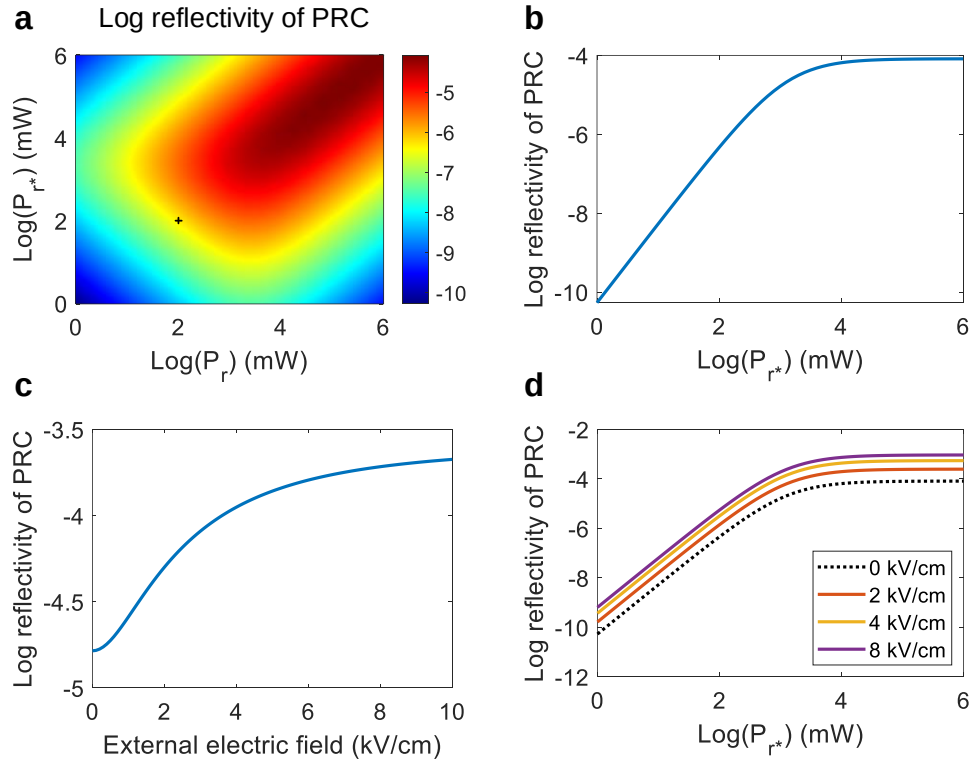


Fig. S6. **a**, The phase conjugation reflectivity of the PRC at different optical powers of the reference and conjugated reference beams. Note that the reflectivity and the optical powers are both denoted on a base-10 logarithmic scale. The black cross corresponds to the optical powers we used in our experiments. **b**, The phase conjugation reflectivity of the PRC with respect to the optical power of the conjugated reference beam when the optical powers of the reference beam and the conjugated reference beam are kept the same. **c**, The phase conjugation reflectivity of the PRC with respect to the amplitude of an external electric field superposed to the

PRC. **d**, Similar to **b** but for the phase conjugation reflectivity at several typical amplitudes of external electric fields.

Supplementary Note 8: System speed when the gain modules are turned off

We also characterized the speed of our system when the gain modules were turned off in a similar way to that introduced in Fig. 4 of the main manuscript. The results are shown in Fig. S7. Here, using a moving optical diffuser as the test sample, we find that the brightness of the time-reversed focus at a sample movement speed of 300 mm/s drops to about 25% of that when the sample is static, as shown in Fig. S7(c). With a higher sample movement speed, we could rarely observe the time-reversed focus during the experiments. In contrast, in Fig. 4 of the main manuscript, when the gain modules are turned on, the time reversal performance does not have obvious variation when the sample (chicken tissue) movement speed is no larger than 350 mm/s. These comparative results demonstrate that the extremely high speed of our HGHS-WFS system benefits from two aspects: one is the intrinsic fast response of the semiconductor PRC we used, and the other is the enhanced signal intensity on the PRC due to the energy amplification from the gain modules (a PRC tends to have faster response for a higher illumination intensity).

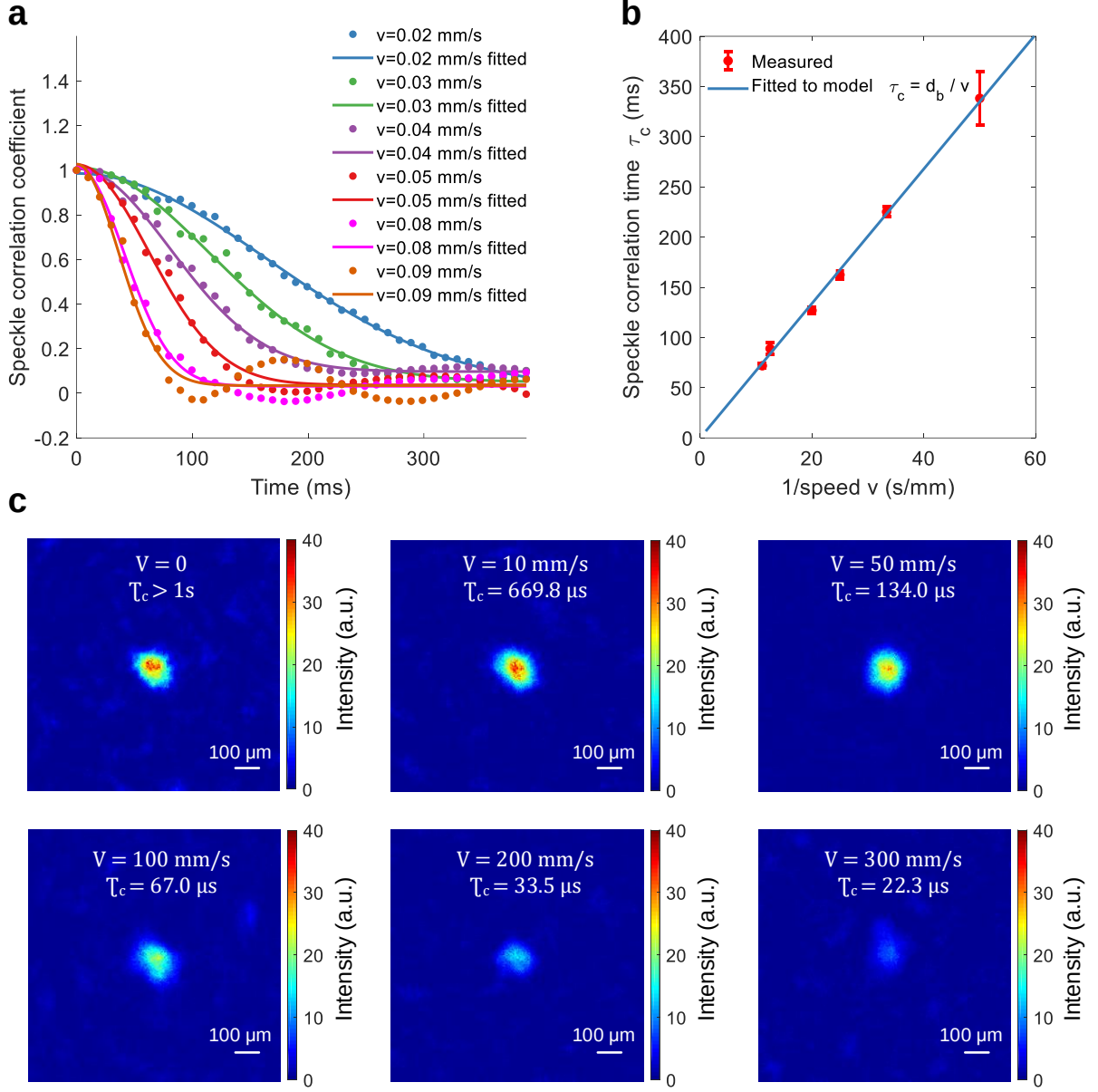


Fig. S7. The system speed without gain modules. **a**, Several examples of speckle correlation coefficients as a function of time when an optical diffuser was moved at different speeds. **b**, The relationship between the speckle correlation time and the sample movement speed, where the fitted parameter $d_b = 6.698 \mu\text{m}$. The error bars show the standard deviations of τ_c measured when light illuminated four different locations on the optical diffuser. **c**, The time-reversed foci when the sample was static ($\tau_c > 1$ s), moved at $v = 10 \text{ mm s}^{-1}$

$(\tau_c = 669.8 \mu s), v = 50 \text{ mm s}^{-1}$ $(\tau_c = 134.0 \mu s)$, $v = 100 \text{ mm s}^{-1}$ $(\tau_c = 67.0 \mu s)$, $v = 200 \text{ mm s}^{-1}$
 $(\tau_c = 33.5 \mu s)$, and $v = 300 \text{ mm s}^{-1}$ $(\tau_c = 22.3 \mu s)$.

Supplementary Note 9: Demonstration of the time-reversal speed via a heterodyne method

In the main manuscript, we presented direct results for focusing through dynamic scattering media with different speckle correlation times and demonstrated that our HGHS-WFS system could tolerate speckle correlation times as short as ~ 10 microseconds. Here, via an indirect experiment, we show another demonstration of the time-reversal speed. In this demonstration, we employed two AOMs to shift the frequency of the probe beam such that there was a frequency detune, Δf , between the probe beam and the reference/conjugation reference beams [Fig. S8(a)]. As a result, the hologram inside the PRC was a heterodyne signal with a frequency Δf . If the response speed of the time reversal cannot follow the variation speed of the hologram inside the PRC, the time-reversed focus will disappear. Figure S8(b) shows the normalized peak intensities of the time-reversed foci using our HGHS-WFS system at different detuned frequencies. We can see that even at a detuned frequency of 0.2 MHz, the HGHS-WFS system still generates a time-reversed focus, although the peak intensity of the focus decreases to about 35% of the maximum when $\Delta f = 0$. Such a detuned frequency corresponds to a variation period of 5 microseconds for the heterodyne signal. Therefore, these results indicate that the HGHS-WFS system is fast enough to address signal variation on the timescale of several microseconds, which is consistent with the conclusion obtained from Fig. 4. A similar plot is shown in Fig. S8(c), when the gain modules are off. We note that the scattering medium was removed when we performed the experiments for Fig. S8(c), because the time-reversed energy is very weak when the gain modules are off, which is not beneficial to these tests. As can be seen, the peak intensity of the time-reversed focus decreased to $\sim 30\%$ of the maximum at a detuned frequency of 0.02

MHz, and the time-reversed focus almost disappears when the detuned frequency is close to 0.05 MHz. Comparing the results of Figs. S8(b) and S8(c) indicates that the energy amplification from the gain modules can promote the response speed of the PRC further, which is also consistent with the conclusion obtained from Fig. S7.

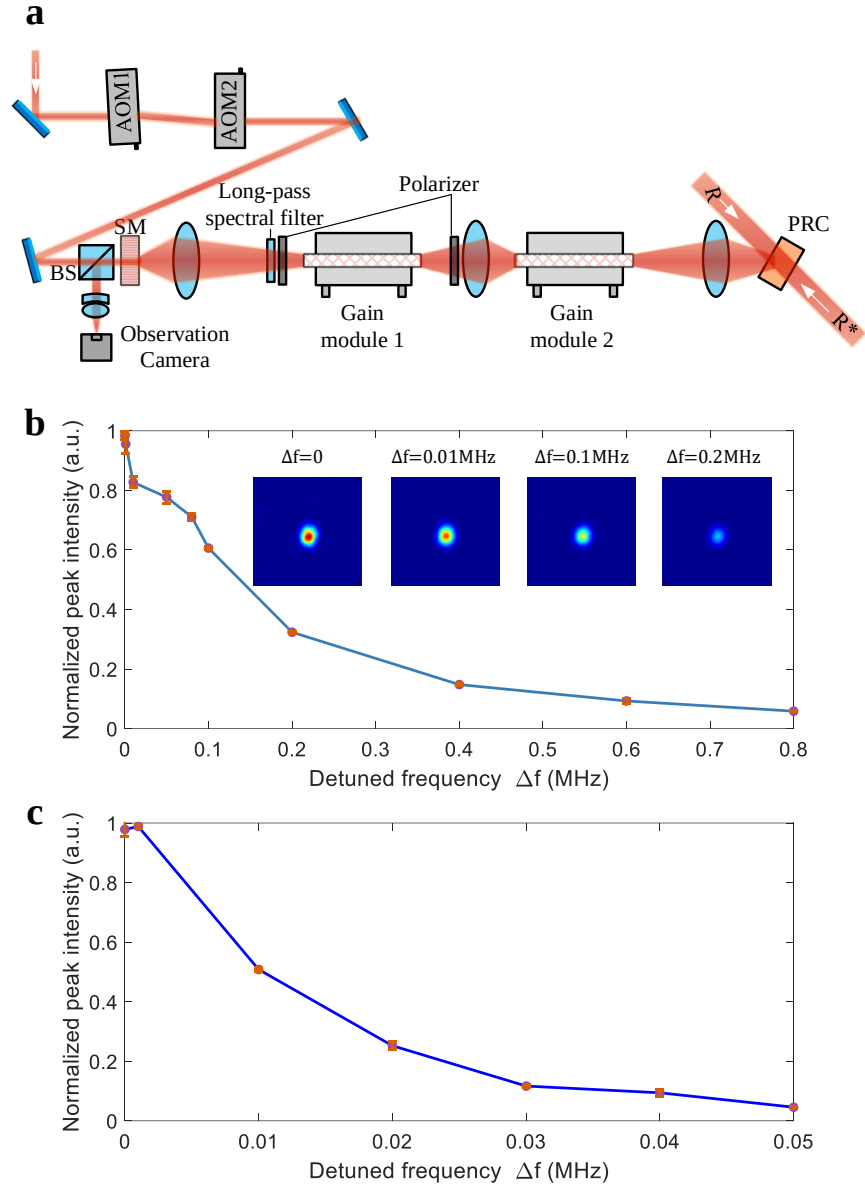


Fig. S8. Demonstration of the time-reversal speed achieved via a heterodyne method. **a**, The simplified schematic for this experiment. **b**, The peak intensities of the time-reversed optical foci via the HGHS-WFS

system working at different detuned frequencies between the probe beam and the reference beam. Inset shows several examples of the time-reversed optical foci. The error bars show the standard deviations of the peak intensities of three time-reversed foci. **c**, Same as **b**, but while keeping the gain modules off.

Supplementary Note 10: Discussion on HGHS-WFS at other laser wavelengths

Although we only demonstrated the HGHS-WFS at 1064 nm, the concept can be adapted to other laser wavelengths by employing corresponding gain media. Generally, we can choose the same gain material as used in the laser cavity that generates the given laser wavelength (e.g., Ti:Al₂O₃ crystal: 650 nm~1100 nm; ruby crystal: 694.3 nm). Dye gain media can also be considered because a dye can usually be used for a wider range of wavelengths. For example, we can use a fluorescein dye cell (530–560 nm) as the gain medium for operation at the wavelength of 532 nm.

Supplementary Note 11: The ASE spectrum

We measured the ASE spectrum using a spectrometer (ULS3648, Avantes). The full spectrum of the ASE from 600 nm to 1100 nm is shown in Fig. S9(a), where three main spectral lines are located at about 938 nm, 946 nm, and 1064 nm. In our setup, a long-pass filter with a cut-on wavelength of 950 nm rejected the 938 nm and 946 nm lines. Since the 1064 nm line of the ASE has the same central wavelength as the incident signal, great care must be taken when using a spectral filter to reject the ASE at 1064 nm. From the zoomed-in view of the 1064 nm line shown in Fig. S9(b), we can find that the spectral width of the ASE at 1064 nm is about 1 nm. Considering that the spectral width of our signal light is less than 10^{-5} nm, we note that the ASE could be further suppressed through narrowband spectral filters that have sub-nanometer bandwidths. For example, if a spectral filter with a bandwidth of 0.3 nm as reported in Ref. 43 of

the main manuscript is available, it can be expected that the background signal of the gain medium would be reduced by a factor of 55%, which is estimated by the measured spectrum of the background signal and the given spectral function of the filter in Ref. 43.

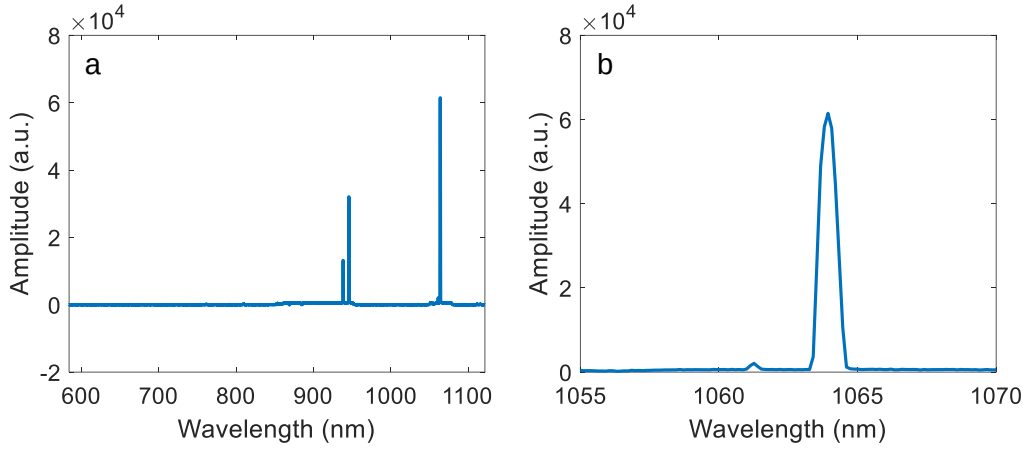


Fig. S9. **a**, The full spectrum of the ASE. **b**, A zoomed-in view of the 1064 nm line of the ASE.

Supplementary Note 12: The control flowchart of the HGHS-WFS system

As shown in Fig. S10(a), the control signal for launching the HGHS-WFS system starts from a customized program [Fig. S10(b)], through which a DAQ (NI 6251, National Instruments) generates a pulse to trigger a delay generator (DG645, Stanford Research Systems). The outputs of the delay generator, whose delay times and pulse widths can be set flexibly, are then used to trigger different devices, such as the gain modules and camera. The status of the pump lasers in the gain modules is monitored in real time by the customized program to ensure the safety of the gain modules.

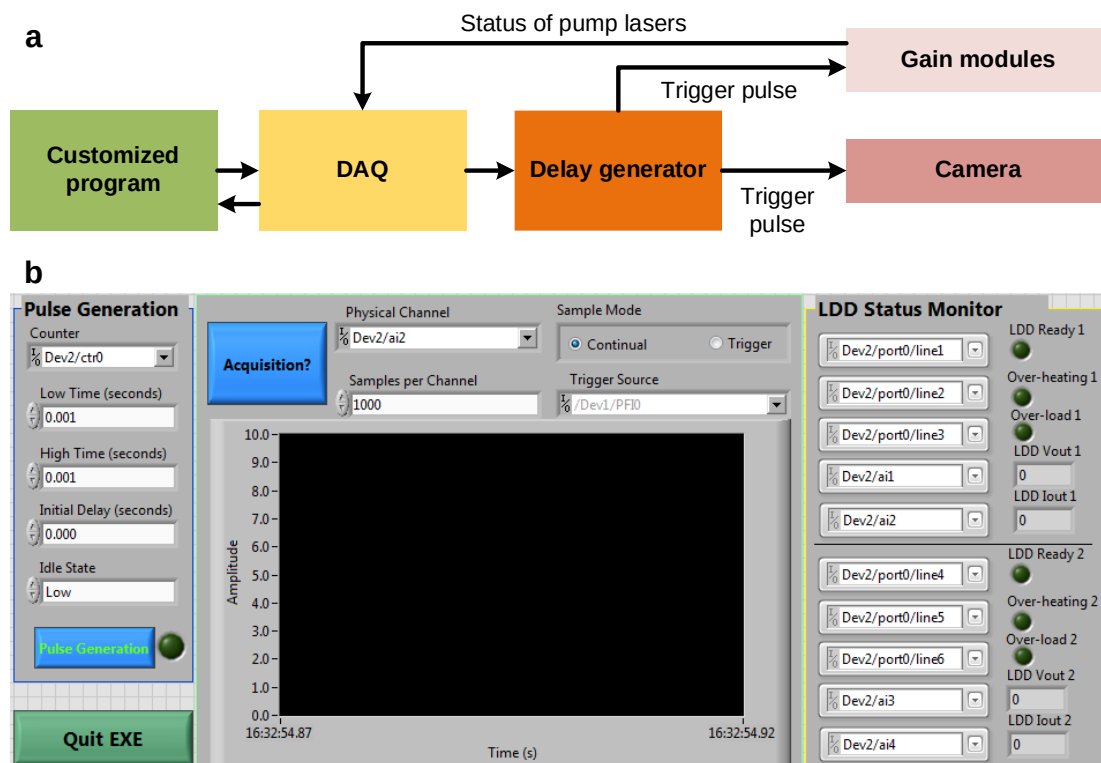


Fig. S10. a, Control flowchart of the HGHS-WFS system. **b,** The customized program.

Supplementary Figure

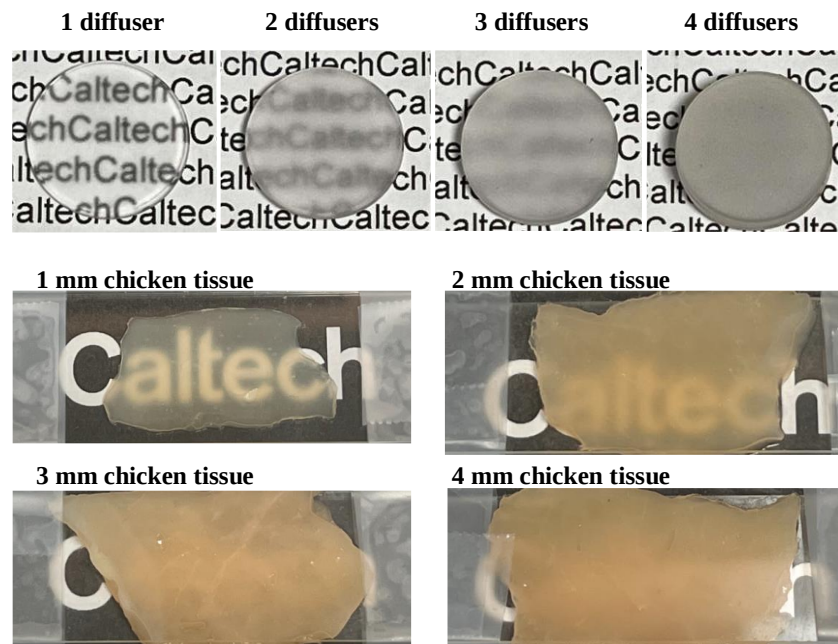


Fig. S11. Photos of different samples.

Supplementary Table

No.	Publication	System latency	Control DOF	Energy gain
1	<i>Optica</i> 2 , 728-735 (2015)	5.3 ms	10^5	Theoretically it can be >1 . ^a
2	<i>Nature Photonics</i> 13 , 788-793 (2019)	2.4 ms	256	
3	<i>Science Advances</i> 6 , eaay1192 (2020)	1 ms	1000	
4	<i>Nature Photonics</i> 5 , 154-157 (2011)	200 ms	—	$<10^{-3}$
5	<i>Nature Communications</i> 6 , 5904 (2015)	5.6 ms	10^7	
6	<i>Nature Photonics</i> 7 , 919-924 (2013)	620 ns	1000	—
7	Our approach	10 μ s	10^6	1.05 achieved

^a For digital WFS, the energy gain could be larger than one as long as the energy of the playback beam is sufficiently high.

Supplementary Table 1. Summary of the system speeds, control DOFs and energy gains in representative publications in the realm of WFS. Current methods can satisfy only one or two of the desired performance metrics: system speed, control DOF, and energy gain. Digital WFS techniques can realize high energy gain theoretically, but they suffer from a limited system speed or control DOF. Analog WFS techniques based on nonlinear crystals have high control DOFs, but the energy gains of the corrected wavefronts are well below unity because of the weak nonlinear effect. Also, the fastest analog WFS system still has a runtime of several milliseconds. Our HGHS-WFS can simultaneously achieve high speed, high control DOF, and high energy gain.

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