

Fully on-chip photonic turnkey quantum source for entangled qubit/qudit state generation

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1 Miniaturization factor

A photograph of the electrically-pumped laser-integrated qubit/qudit source is shown in Fig. S.1a. The entire chip size is less than one Euro coin. The indium phosphide (InP) gain-section and rest of the circuit consisting of silicon nitride (Si_3N_4) Vernier filtering and the spontaneous four-wave mixing (SFWM) photon-pair generation stages are shown in the enlarged Fig. S.1b with the three rings, viz. R1, R2, and R3, visible prominently in Fig. S.1c. This is so far the first known scalable quantum photonic source with laser and Vernier filter integrated on-chip, which generates with a very high generation rate near-perfect frequency-bin entangled high-dimensional Bell-states. Compared to similar studies [1], which used a nonlinear chip, external laser, and a filter occupying roughly a space of two to three shoe-boxes or about one-third of a standard optical bench, this chip is at least (300×200) times smaller. Further, due to the light-weight of this chip, it is perfect for out-of-the-lab applications, e.g., quantum satellite communications.

2 Microring resonator design parameters

As described in the Methods, the experimental values of the Q -factors are determined from the measured transmissions of R1, R2, and R3 by fitting Lorentzian curves. An example is given in Fig. S.2 at the pumping wavelength. All obtained values are listed in the table S.1.

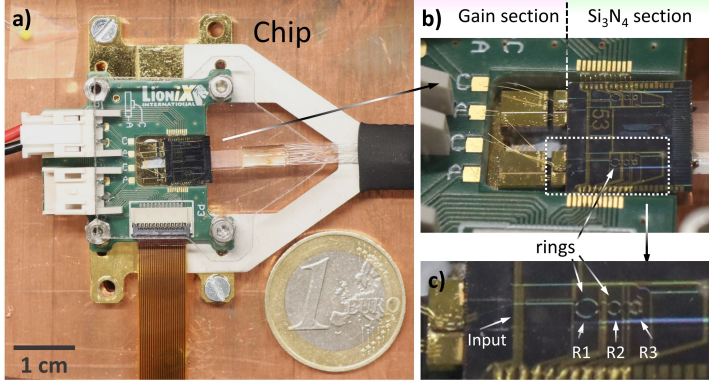


Fig. S.1 Photographs of the designed chip. a) Size of the laser-integrated chip is compared to a one Euro coin. b) Enlarged photograph, both the InP gain section and the rest of the circuit, i.e., where the Si_3N_4 Vernier filtering and the SFWM photon-pair generation process occurred, are marked. c) Further magnified image, where the three rings R1, R2, and R3, are visible. The perimeters of R1 and R2 are $1579\ \mu\text{m}$, and $1005\ \mu\text{m}$, respectively, whereas the entangled photon pair generation took place in the smallest ring R3 having a circumference of about $850\ \mu\text{m}$.

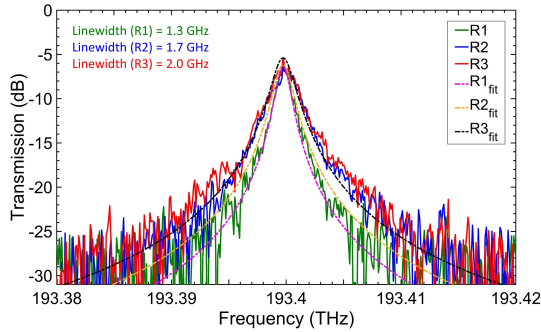


Fig. S.2 Linewidth calculation from transmission characteristics of R1, R2, and R3. Transmissions of R1, R2, and R3 at 193.4 THz and their Lorentzian fits. From the Lorentzian, the resonator linewidths and the Q -factors were calculated.

3 Experimental setup for the quantum frequency comb (QFC) emission characterization

A schematic of the QFC characterization experiments determining the frequency-time correlation of the photon-pairs is shown in Fig. S.3a. The idler and the signal photons generated through a SFWM process in the third microring resonator (MRR), exhibit correlations in the frequency-time domain over several frequency modes due to energy ($2\omega_p = \omega_s + \omega_i$) and momentum conservation ($2\kappa_p = \kappa_s + \kappa_i$) [2]. These photon pairs (see Fig. S.3a) were out-coupled from the chip and transmitted to a high rejection

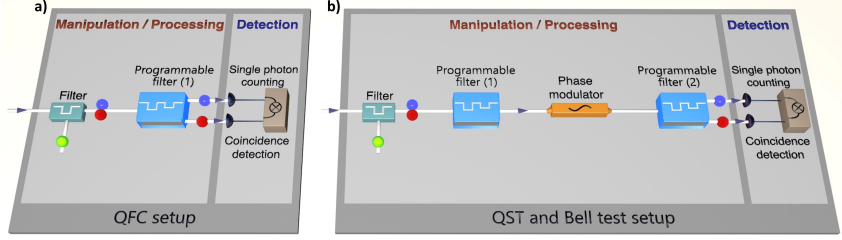


Fig. S.3 a) **Experimental setup for the quantum frequency comb (QFC) characterization.** The signal and idler photons, being superposition of the frequency modes defined by the resonances of QFCs, are collected from the chip. After passing the high rejection notch filter (NF) and programmable filter (PF) the photon pairs were detected by two superconducting nanowire single-photon detectors (SNSPDs) prior to determining their time correlations by a time-tagger (TT). b) **Experimental setup for the frequency-bin entanglement characterization.** Programmable filters (1) and (2) and an electro-optic phase modulator (EOM) were used to manipulate and characterize the frequency entangled quantum states before the signal and idler photons were detected by two SNSPDs and the correlation measured by the time-tagger.

Table S.1 FWHM, loaded (Q_L), and intrinsic Q -factors (Q_{int}) of the rings R1, R2, and R3 both from theoretical calculations and experimentally measured transmissions.

Ring	Perimeter (μm)	Q_L		Q_{int}	FWHM (GHz)	
		Theory	Expt.		Theory	Expt.
R1	1579	1.74×10^5	1.49×10^5	1.97×10^6	1.11	1.3
R2	1005	1.14×10^5	1.137×10^5	1.97×10^6	1.69	1.7
R3	850	9.78×10^4	9.67×10^4	1.97×10^6	1.98	2.0

notch filter (NF) with an isolation bandwidth of 200 GHz and an out-of-band suppression of 100 dB in order to separate the excitation field from the single/idler photons. Subsequently, the signal and idler photons were split into different spatial modes and the photons frequency components were selected by a programmable filter with a bandwidth of 25 GHz. Finally, the photon pairs at the selected comb modes were detected by two superconducting nanowire single photon detectors (SNSPDs). The time correlations between the photons at the output of the two channels were determined by a time tagger (TT), working in a 5 ps resolution mode.

4 Experimental setup for frequency-bin entanglement characterization

For the quantum state tomography (QST) and the Bell test, it is necessary to manipulate the photonic states by mixing the individual frequency modes to realize quantum interference and perform projection measurements [3–5]. The

setup for this is shown in Fig. S.3b. The photon pairs were sent to the first (left-most) programmable filter (1), where suitable attenuation- and phase masks were applied to the QFC, specifically selecting with a 25 GHz bandwidth the corresponding frequency modes. As shown in Fig. 2a of the main manuscript, for Bell-tests and QST on qubits we have chosen frequency modes 2 and 3, as well as 3 and 4. For the same measurements on qutrits we have selected modes 2, 3 and 4.

Then the photons were passed through an electro-optic phase modulator (EOM) driven by a single sinusoidal radio-frequency (RF) tone. Sidebands created through this process allowed the mixing of the frequency modes. It is a crucial prerequisite to perfectly adjust the EOM driving signal in such a way that integer values of its driving frequency match the free spectral range (FSR) of ring R3 (i.e., 198.9 GHz). The RF-source amplifier combination used to drive the EOM in our experiments had a maximum RF bandwidth of 25 GHz, which allowed us to adjust it to 24.863 GHz providing an integer ring FSR to EOM frequency ratio of eight. Specifically, the 8th sideband is required to mix a resonance with its neighboring mode(s). The EOM frequency adjustment and the phase offset compensation (EOM offset compensation) procedures are described in the next paragraphs.

The amplitude of the n^{th} sideband at a particular RF power can be described by the n^{th} order Bessel function of the first kind, where the RF power is the argument of the Bessel function. For measurements on the two-dimensional (2D) state, we set the RF signal power to -7.5 dBm, where the amplitude of the 4th sideband was maximal. This setting caused a mixing of two adjacent modes $|\bar{k}\rangle$, and $|\bar{k} + 1\rangle$ in a frequency mode centered between the two frequency modes. For projection measurements on the three-dimensional (3D) state, we applied a signal power of -4.85 dBm to the EOM, at which the 0th and the 8th sideband were created with the same amplitudes. This was set by comparing the single count rates of these sidebands for different values of the EOM power. As a result, the modes $|\bar{k}\rangle$, and $|\bar{k} + 2\rangle$ were mixed to the frequency mode $|\bar{k} + 1\rangle$, which itself remained at this position with the same amplitude as the other two. As the intensity of the sidebands created by the EOM decreased with the order of the band we were currently limited to perform entanglement measurements and successful tomography for a dimension of three. In future, this limitation can be circumvented by using another RF-drive with a different, e.g., 50 GHz driving frequency.

At last, a second programmable filter (2) collected the photons within a bandwidth of 25 GHz and routed the individual signal/idler photons to the respective SNSPDs for coincidence measurements with the time tagger. The coincidence measurement time was 5 min per setting for the quantum interference of two-dimensional states and 10 min per setting for the interference of three-dimensional states as well as for all tomography measurements.

Due to the resonance linewidth of the microring R3, the dispersion of the waveguides, and the timing jitter of the detectors, the time difference between the detection of the two photons of a coincidence event was distributed in a

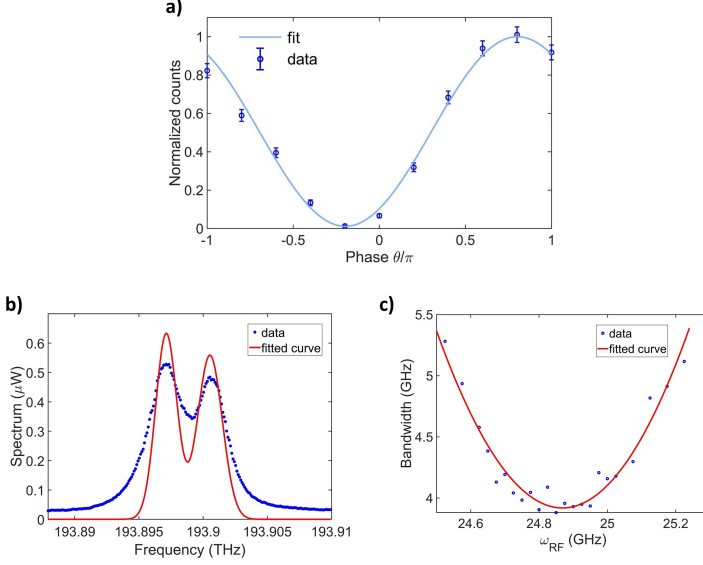


Fig. S.4 Adjustment of the EOM. **a)** Interference measurement of the EOM induced offset. The EOM mixed two frequency modes and the phase of one mode was continuously changed from 0 to 2π via PF1 while coincidence counts were measured. The maximum of the curve corresponds to a superposition without a resulting phase difference between mixed modes, so that this phase was used as a reference. The error bars show the square root of the absolute count numbers. **b)** Spectrum of two modes mixed by the EOM with a mismatch in FSR and ω_{RF} . The FWHM of this curve was determined. **c)** Interpolation of the FWHM of the mixed modes for different ω_{RF} to find the optimum.

histogram following a Lorentz curve. We selected those counts that lie in a predefined symmetric window around the center.

4.1 Setting of EOM driving frequency

For optimal quantum interference and projection measurements, perfect overlap of the mixed modes is required. This was achieved by setting the EOM driving frequency to a multiple of the FSR. We used an erbium-doped fiber amplifier (EDFA) as a broadband light source to obtain the transmission spectra of the MRR R3. Next, the transmitted signal of the ring (see Fig. 1a) passed the EOM and was then sent to a wave analyzer, where the full width at half maximum (FWHM) of the mixed sidebands was measured for different EOM driving frequencies (Fig. S.4b). As a result, the interpolated minimum of the measured FWHM values defined the RF frequency providing the maximum possible overlap for the modes (Fig. S.4c).

4.2 EOM offset compensation

Driving the EOM introduces an additional phase to the sidebands, which depends on the RF-source’s power and frequency and scales proportional to the order of the sidebands. We measured this phase between two resonant modes by creating the superposition of both signal- and both idlers and rotating the phase of only one mode in the first programmable filter (PF1) (see Fig. S.3b). Thereby, the measured coincidence counts follow a sine curve, where its maximum marks the phase of PF1 that compensates for the phase of the EOM. Once this phase was determined, it was kept fixed for the entire measurement. This was done for the combination of modes 2 and 3 as well as modes 3 and 4 for the two-dimensional states, resulting in phases of $+0.84\text{ rad}$ and -0.51 rad for mode 3 and mode 4, respectively (Fig. S.4a). For three-dimensional state characterization, the phases of both the resonances 2 and 4 were corrected with respect to the resonant mode 3 at PF1.

5 Fidelity

Fidelity is a measure of the distance between two quantum states, where a unit fidelity implicates the two states are perfectly identical. For two density matrices $\hat{\rho}_1$ and $\hat{\rho}_2$, the fidelity reads as follows:

$$F(\hat{\rho}_1, \hat{\rho}_2) = \left(\text{tr} \sqrt{\sqrt{\hat{\rho}_1} \hat{\rho}_2 \sqrt{\hat{\rho}_1}} \right)^2 \quad (\text{S.1})$$

We calculated the fidelities from the measured quantum states with respect to the pure 2D and 3D Bell states as $F_{2-3}^{(2)} = (0.988 \pm 0.01)$ and $F_{3-4}^{(2)} = (0.981 \pm 0.01)$ for qubit and $F_{2-3-4}^{(3)} = (0.806 \pm 0.02)$ for qutrit, respectively. The differences are explained by the lower coincidence rates for 3D than for 2D, as the intensity of the created EOM sidebands decreases in general with the order (4^{th} vs. 8^{th}), in turn increasing the noise from up to 0.1 % for qubits to 6 % for qutrits, as predicted by the linear model of uncolored noise [6]. Taking this noise into account yields a fidelity of $F_{2-3-4}^{(3), \text{ noise}} = (0.857 \pm 0.02)$, while the improvement for the qubit case was only up to 0.001. In addition, the number of measurements required for 3D QST is 81, significantly higher than the 16 measurements for the 2D case, thereby making 3D measurements more prone to be exposed to the laser instabilities due to the much longer measurement time.

6 Bell inequality

The Bell test measurements are directly related to the S -parameters of Bell inequalities by the multiplication of the parameter ϵ_D with the respective maximal value of the inequality, which is $2\sqrt{2} \approx 2.8284$ for qubits and

$4/(6\sqrt{3}-9) \approx 2.8729$ for qutrits [6]. Our measurements yield values of $S = 2.758$, and $S = 2.760$ for 2D and $S = 2.735$ for 3D, where $S > 2$ means a violation of the respective inequality. We can also derive the S -parameters from the reconstructed density matrices, yielding values of up to $S = 2.793$, which is in good agreement with the Bell tests. Just as with the Bell tests, the violation of a Bell inequality proves the presence of entanglement.

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