

Wide-band frequency modulation of a terahertz intrinsic Josephson junction emitter of a cuprate superconductor

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Contents of Supplementary Information

Supplementary Note 1. FM Josephson oscillation of an IJJ stack.....	2
Supplementary Figure 1: Ideal FM spectrum for $m_f = 6$	3
Supplementary Note 2. Calculation of transmission loss.....	3
Supplementary Figure 2: Equivalent circuit of the measurement circuit.	4
Supplementary Figure 3: V_m^{-1} derived from the measured bandwidth B versus squared modulation frequency f_m^2	4
Supplementary Note 3. Possible maximum bandwidth.....	5
Supplementary Figure 4: Comparison between experimental spectrum and virtual one(half)-octave FM spectra.	5
Supplementary Figure 5: Expectation of expanded spectrum intensity.....	6
Supplementary Note 4. Empirical reduction of contact resistance.....	6
Supplementary Figure 6: Enlarged current-voltage characteristics of the device and fitting to extract mesa voltage.	7

Supplementary Note 1. FM Josephson oscillation of an IJJ stack

In a single Josephson junction with the gauge-invariant phase difference θ between the

superconducting electrodes, $I = I_c \sin \theta$ and $V = \frac{\hbar}{2e} \frac{d\theta}{dt}$ hold for tunneling current I and biased

voltage V , where I_c is the maximum tunneling Josephson current, $\hbar$ is the Dirac constant, and e is the elementary charge. Now, let us consider the case applying a time-dependent voltage $V(t) = V_0 + V_m \cos \omega_m t$ to a structure composed of N -stacked IJJs. V_0 is the DC voltage, V_m is the amplitude of the modulating voltage, ω_m is the angular frequency of the modulation voltage. In this case, the instantaneous gauge-invariant phase difference $\theta(t)$ is the integral value of the applied voltage as,

$$\theta(t) = \frac{2e}{\hbar} \int \frac{V(t)}{N} dt = \frac{2eV_0}{N\hbar} t + \frac{2eV_m}{N\hbar\omega_m} \sin \omega_m t \quad (S1)$$

Equation (S1) is equivalent to the instantaneous angle of the FM electromagnetic wave with the

center angular frequency $\omega_c = \frac{2eV_0}{N\hbar}$, modulation index $m_f = \frac{2eV_m}{N\hbar\omega_m}$, and modulation angular

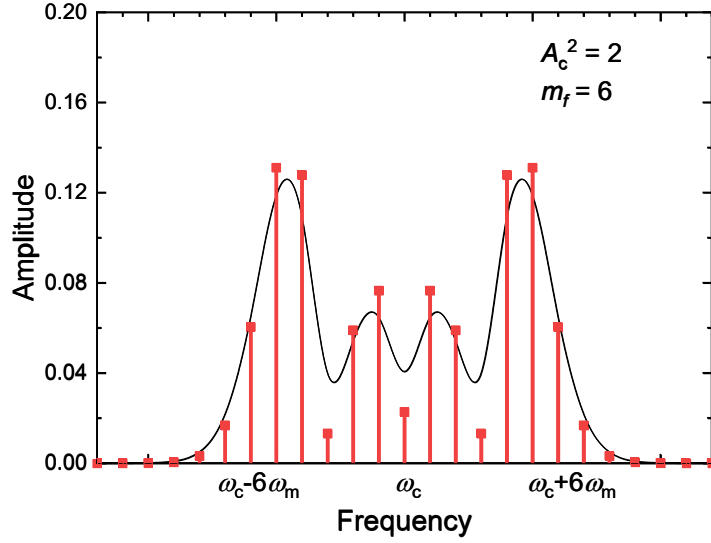
frequency ω_m , and if the AC Josephson current is radiated as it is as an electromagnetic wave, the signal $s(t) = m_f \sin \omega_m t$ can be propagated. The intensity spectrum of the FM electromagnetic wave is expressed by the following,

$$I_{\text{FM}}(\omega_n) = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(m_f), \quad (S2)$$

where A_c is the amplitude of the carrier wave and $J_n(x)$ is an n -th order Bessel function of the first kind with n being an integer. From this, the power spectrum of the FM signal has peaks with heights of $J_n^2(m_f)$ at angular frequencies of $\omega_n = \omega_c + n\omega_m$. As shown in Supplementary Figure 1, the spectrum has multiple peaks in a comb-like pattern at intervals of ω_m centered at ω_c and its bandwidth is determined by m_f . The envelope of the spectrum has maximum intensities near $\omega_c \pm (m_f - 1)\omega_m$ and decreases slowly on the side toward ω_c and decreases sharply away from ω_c . In other words, when the spectrum corresponding to Eq. (S2) is obtained, the communication at $f_c = \omega_c/2\pi$ with the bandwidth of

$$B = 2(m_f + 1)\omega_m/2\pi, \quad (S3)$$

is demonstrated. As practical values, applying $V_0 = 1.6$ V with $V_m = 0.037$ V and $f_m = \omega_m/2\pi = 3$ GHz to a mesa of 1000 IJJs give rise to generate FM signal with $f_c = 800$ GHz with the bandwidth $B = 42$ GHz, as plotted in **Supplementary Figure 1**.



Supplementary Figure 1: Ideal FM spectrum for $m_f = 6$. The solid curve is a spline connecting the peak of comb teeth, which simulates data taken by low-resolution spectrometer.

Supplementary Note 2. Calculation of transmission loss

From the Kirchhoff's current law on the equivalent circuit for the transmission line depicted in Supplementary Figure 2,

$$\tilde{I}_s(t) = \tilde{I}_c(t) + \tilde{I}_m(t), \quad (S4)$$

where $\tilde{I}_s(t) = I_s e^{i\omega_m t}$, $\tilde{I}_c(t) = I_c e^{i\omega_m t}$, and $\tilde{I}_m(t) = \tilde{I}_m e^{i\omega_m t}$ denote complex source, leak, and modulation currents, respectively. The voltage law is

$$\tilde{V}_s(t) = -L \frac{d\tilde{I}_s}{dt} + R\tilde{I}_s(t) + \tilde{V}_m(t), \quad (S5)$$

where $\tilde{V}_s(t) = V_s e^{i\omega_m t}$ and $\tilde{V}_m(t) = \tilde{V}_m e^{i\omega_m t}$ are source and modulation voltages, and L and R are inductance and resistance of the transmission line, respectively. These two equations are reformulated as

$$V_s = (-i\omega_m L + R)(i\omega_m C \tilde{V}_m + \tilde{I}_m) + \tilde{V}_m, \quad (S6)$$

where C is the capacitance of the transmission line. The radiating JPE device, which is a series connection of a thousand of IJJs oscillating synchronously, can be considered as a single RCSJ circuit. Since we here focus on current at frequency of $\frac{\omega_m}{2\pi}$ which applies a finite bias to the JPE, the current flowing across the shunted junction can be zero. Then, the current conservation law in the JPE part is given by

$$\tilde{I}_m = (i\omega_m C_{\text{JPE}} + R_{\text{JPE}}^{-1})\tilde{V}_m, \quad (S7)$$

where C_{JPE} and R_{JPE} are capacitance and resistance of the JPE device. From Equations (S6) and

(S7), we obtain

$$V_s = \left\{ \omega_m^2 LC' + \frac{R}{R_{JPE}} + 1 + i\omega_m \left(RC' - \frac{L}{R_{JPE}} \right) \right\} \tilde{V}_m, \quad (S8)$$

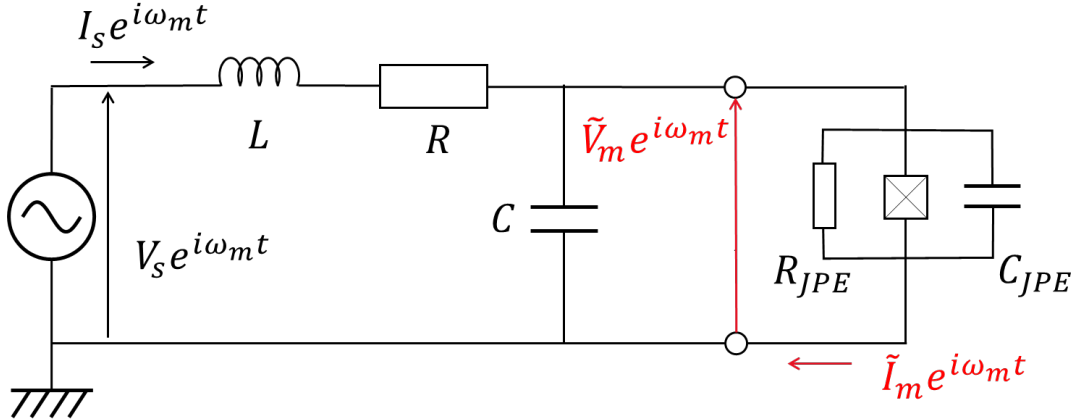
where $C' = C + C_{JPE}$ is the coupled capacitance of the system. Therefore, the reciprocal of the product of modulation index and modulation angular frequency $(m_f \omega_m)^{-1}$ is proportional to

$$\frac{V_s}{V_m} = \left| \frac{V_s}{\tilde{V}_m} \right| = \left(\omega_m^2 LC' + \frac{R}{R_{JPE}} + 1 \right)^2 + \omega_m^2 \left(RC' - \frac{L}{R_{JPE}} \right)^2 = c_4 \omega_m^4 + c_2 \omega_m^2 + c_0, \quad (S9)$$

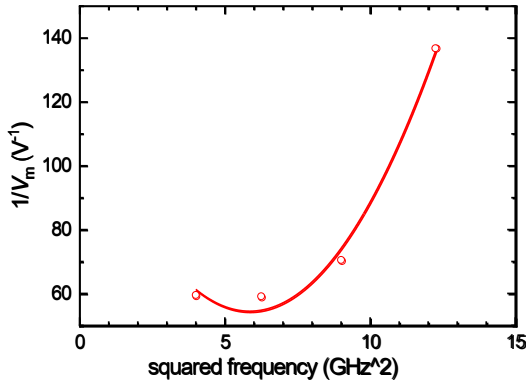
where $c_4 = (LC')^2$, $c_2 = 2LC' + (RC')^2 + \left(\frac{L}{R_{JPE}}\right)^2$, and $c_0 = \left(\frac{R}{R_{JPE}} + 1\right)^2$. Using the total impedance of the circuit $Z_{\text{total}} = Z + Z_{\text{mesa}}$ mentioned in Main, the equation

$$V_m^{-1} = \frac{c_4 \omega_m^4 + c_2 \omega_m^2 + c_0}{\sqrt{2Z_{\text{total}} P_m}}, \quad (S10)$$

holds. **Supplementary Figure 3** shows that experimentally obtained $V_m^{-1} = 2e/(hNB/2)$ does not have a parabolic f_m^2 dependence, suggesting that Z_{total} strongly depends on f_m in this region.



Supplementary Figure 2: Equivalent circuit of the measurement circuit. $\tilde{V}_m e^{i\omega_m t}$ is voltage applied to the IJJ stack.

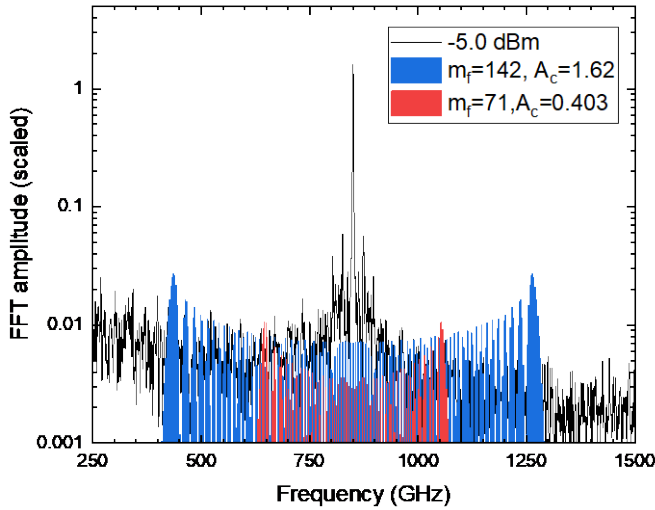


Supplementary Figure 3: V_m^{-1} derived from the measured bandwidth B versus

squared modulation frequency f_m^2 .

Supplementary Note 3. Possible maximum bandwidth

Detectable maximum bandwidths have been estimated in the following manners. First, A_c in Eq. (S2) is determined from the experimental unmodulated spectrum height ($P_m = -5$ dBm in Figure 2a of the main text) with $m_f = 0$. Then an ideal FM spectrum is drawn with increasing m_f while keeping the determined A_c . Supplementary Figure 4 is the comparison between the ideal FM spectrum with $m_f = 142$, $f_m = 3$ GHz, and $f_c = 852$ GHz. The highest peaks at 441 and 1264 GHz are several-times higher than the floor noise level, thus one-octave modulation is possible. Note that this argument works under the assumption the reduction in integral intensity with increasing modulation amplitude discussed in the main text is negligible.



Supplementary Figure 4: Comparison between experimental spectrum and virtual one(half)-octave FM spectra. Blue is for $m_f = 142$ and $A_c = 1.62$ with neglecting the intensity decrease derived from Eq. (S2) and red is for $m_f = 71$ and $A_c = 1.62$ with taking the intensity decrease into consideration. Here $f_m = 3$ GHz, and $f_c = 852$ GHz are not changed.

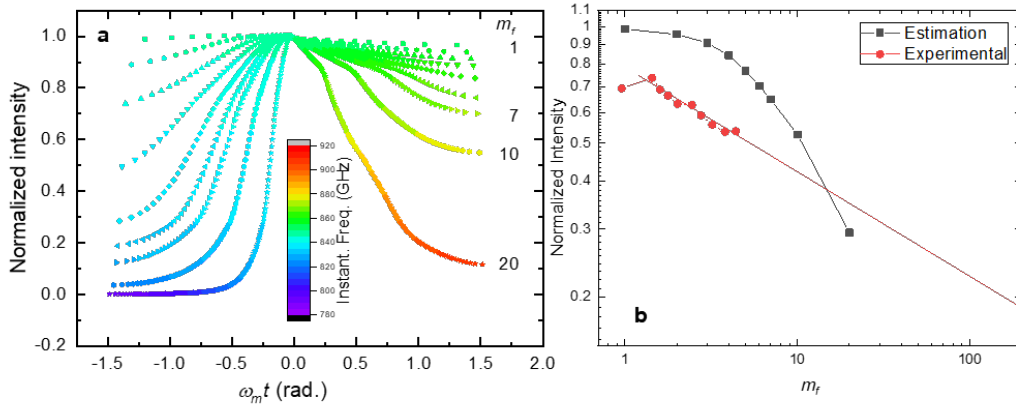
Next, let us estimate FM spectrum intensity $S_{\text{FM}}(m_f, \omega_c) = \int_{\omega_L}^{\omega_U} I(\omega) d\omega$ with taking the frequency dependence of radiation intensity into consideration. Assuming that $S_{\text{FM}}(m_f, \omega_c)$ is proportional to the sum of temporal distribution of unmodulated radiation intensity $I_{\text{um}}(\omega)$ per a sinusoidal frequency modulation

$$\omega = \omega_c + m_f \omega_m \sin \omega_m t, \quad (\text{S11})$$

with a period of $T_m = 2\pi/\omega_m$. For a parabolic frequency dependence of $I_{\text{um}}(\omega) = I_0 - a(\omega - \omega_0)^2$ in the vicinity of ω_0 , the intensity maximum, we obtain

$$S_{FM}(m_f) \propto \langle I_{um}(\omega) \rangle_t = T_m^{-1} \int_0^{T_m} I_{um}(t) dt = I_0 - a \left\{ \frac{m_f^2 \omega_m^2}{2} + \Delta \omega_0^2 \right\}, \quad (S12)$$

where $\Delta \omega = \omega_c - \omega_0$. The modulated spectrum intensity also shows a parabolic m_f dependence. To estimate $S_{FM}(m_f)$ for a large m_f , experimental $S_{FM}(P_m = 25 \text{ dBm}, \omega_c)$ is used instead of $I_{um}(\omega)$ in Eq. (9). Supplementary Figure 5a shows temporal variation of the expected radiation intensity for a bias modulation period. $S_{FM}(m_f, \omega_c)$ is considered to be equivalent to the average of the plot for m_f , thus the normalized $S_{FM}(m_f, \omega_c)$ decreases with increasing m_f as shown in Supplementary Figure 5b. Experimental values evaluated from Figure 2c are also plotted. Although the measured reduction of the spectrum intensity is larger within the experimental m_f range, the measured reduction rate at larger m_f is smaller than the simulated result. For the half-octave modulation of $m_f = 71$, the linear extrapolation in the log-log plot implies that spectrum intensity decreases to 0.25 of the spectrum intensity at $P = -5 \text{ dBm}$. Under this assumption, the highest peak of the half-octave modulated spectrum is almost twice as the noise level because $0.4 A_c$ is used to estimate the modulated spectrum height as shown in Supplementary Figure 4. Therefore, half-octave ($B \sim 400 \text{ GHz}$) FM will be the largest limit of wideband transmission in this case.



Supplementary Figure 5: Expectation of expanded spectrum intensity. **a**, Intensity distribution per a modulation period $2\pi/\omega_m$ estimated by center frequency dependence of spectrum intensity. With increasing m_f , instantaneous frequency $\hat{\theta}(t)$ expands further from the center frequency, then above $m_f = 10$, $\hat{\theta}(t)$ goes away from the radiation range. **b**, Estimated spectrum intensity and experimental spectrum intensity as functions of m_f . The experimental m_f is estimated from the bandwidth B plotted in **Figure 2c** with adjusting the B -interception of the linear fitting to satisfy $B = 2(m_f + 1)f_m$. The red line is the guide for a linear extrapolation.

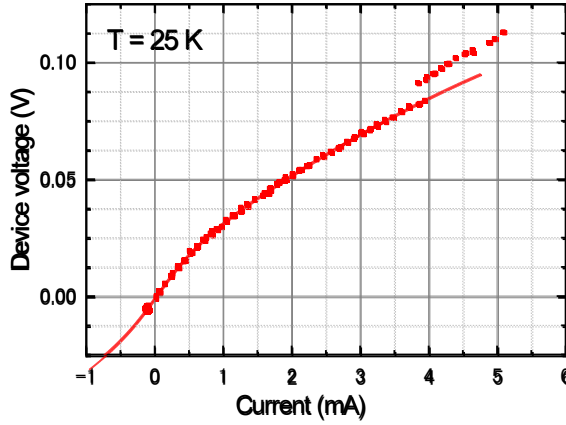
Supplementary Note 4. Empirical reduction of contact resistance

To evaluate voltage applied to the series of IJJs, voltage contributions due to wiring inside and outside the cryostat and contact resistance have to be removed because voltage terminals are

connected on the way of bias line outside the cryostat. Linear fittings to the current-voltage characteristics in the vicinity of the origin, where all IJJs are considered to be zero resistance, have been usually employed so far. However, the reduction often turns to be more overestimated at higher current region probably because of a non-ohmic contact resistance between the top IJJ and the evaporated electrode. Here, we suggest to apply an antisymmetric non-linear function of

$$V_{\text{lead}}(I) = c_0 \operatorname{atan} \left\{ \frac{p(I - I_0)}{\pi} \right\} + bI + dI^3, \quad (\text{S13})$$

to the lowest voltage (closest to the $V = 0$) branch of the I-V characteristics. **Supplementary Figure 6** shows that the agreement of the lowest branch and the fitted function is excellent. The mesa voltage is the reduction of V_{lead} from the device voltage V_{dev} defined by **Figure 6 d**, whose values are collected by data acquisition system.



Supplementary Figure 6: Enlarged current-voltage characteristics of the device and fitting to extract mesa voltage.