

Metasurface-enabled single-shot and complete Mueller matrix imaging

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Metasurface-enabled single-shot and complete Mueller matrix imaging: Supplementary Information

Aun Zaidi,^{1,*} Noah A. Rubin,¹ Maryna L. Meretska,¹ Lisa W. Li,¹ Ahmed H. Dorrah,¹ Joon-Suh Park,¹ and Federico Capasso^{1,†}

¹*Harvard John A. Paulson School of Engineering and Applied Sciences,
Harvard University, Cambridge, MA 02138, USA*

In this supplement, we provide additional details related to the theory, design and measurements of our device, to support the work presented in the main text.

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S1. ANALYTICAL AND COMPUTATIONAL ANALYSIS OF SINGLE-SHOT MUELLER MATRIX IMAGING

A. Explanation of Our Choice of Structured Polarization Illumination

For single-shot Mueller matrix (MM) imaging, we are required to spatially, instead of temporally, sample the polarization properties of the object. This is done through the generation and incidence of a spatially varying polarization pattern. The choice of this pattern is not unique – that is, different polarization states in different juxtapositions can be used to sample the object. Our choice of the spatially varying polarization (or ‘structured polarization illumination’), is governed by a need to assign maximal bandwidth available to each component, while

* zaidi01@g.harvard.edu

† capasso@seas.harvard.edu

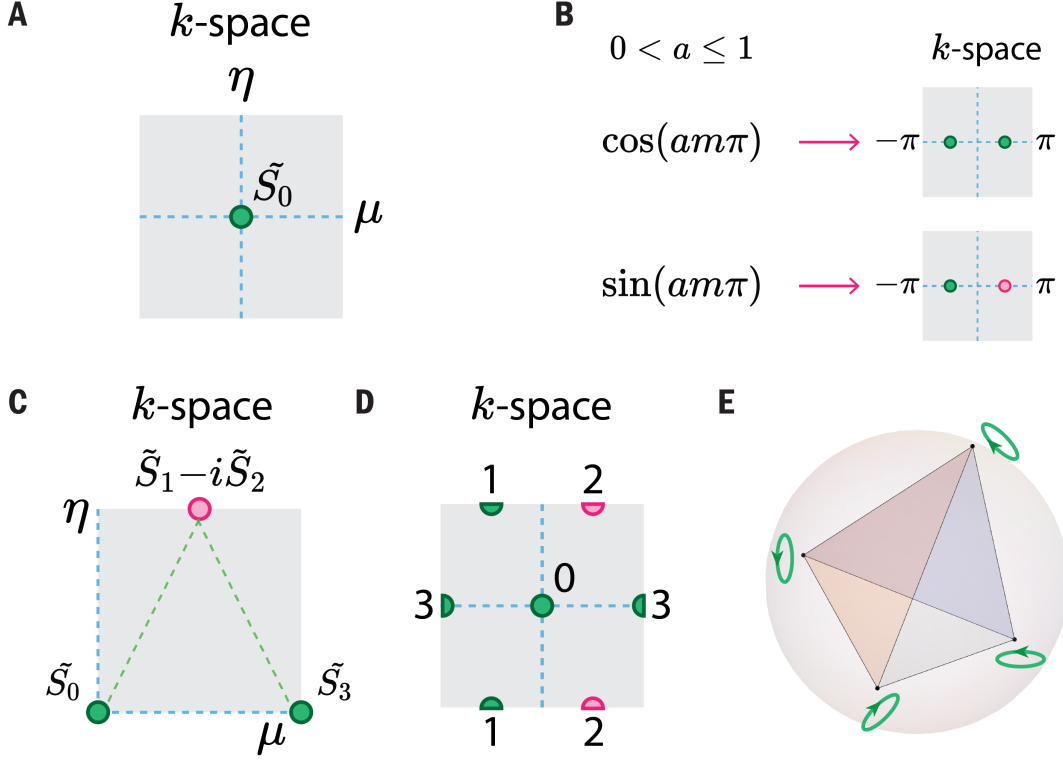


FIG. S1. (A) S_0 cannot vary spatially, with respect to S_1, S_2 and S_3 , because the degree of polarization has to always equal 1. This confines $\tilde{S}_0$ to the origin. (B) The Fourier spectrum of cosine is symmetric, while the Fourier spectrum of sine is anti-symmetric. (C) The channel locations of the spectra of the Stokes components, in a quadrant in k -space, form a triangle. (D) The four channel locations displayed on the entire k -space, and labeled with channel numbers 1, 2, 3 and 4. (E) The four polarization states correspond to the vertices of a tetrahedron inscribed within the Poincare sphere.

satisfying constraints surrounding structured polarization generation. In particular, we generate the polarization distribution defined by the following equation:

$$\vec{S}(m, n) = \begin{pmatrix} 1 \\ \sqrt{2/3} \cos(0.5m\pi) \cos(n\pi) \\ \sqrt{2/3} \sin(0.5m\pi) \cos(n\pi) \\ \sqrt{1/3} \cos(m\pi) \end{pmatrix} \quad (1)$$

where (m, n) are discrete spatial coordinates. There are two main constraints that any structured polarization illumination has to satisfy, for it to be practical:

- Constraint 1: Only the state of polarization, and not the degree of polarization, is allowed to vary spatially. This is because currently, there is no technology that can **deterministically** modulate the degree of polarization, **locally in space**, in the far-field. This is a technological, and not a fundamental, constraint that can perhaps be overcome in the future with emerging technologies.
- Constraint 2: Stokes (and Mueller) components are always real. This means that the spatially varying Stokes components, composed of *cosines* (even) and *sines* (odd), have either symmetric or anti-symmetric Fourier-spectra, which limits the space available in the Fourier domain for channel separation. This is a fundamental constraint.

Constraint 1 means that the **normalized** S_0 component cannot vary spatially (with respect to S_1, S_2 and S_3) as $S_0 = \sqrt{S_1^2 + S_2^2 + S_3^2}$ has to equal 1 for all (m, n) – that is, the degree of polarization (DOP) has to equal 1. In the Fourier-space, the spectrum ($\tilde{S}_0$) of S_0 is thus a delta function centered at the origin, as shown in Fig. S1A. S_1, S_2 and S_3 which define the state of polarization of light, and thus can spatially vary, are real-valued (Constraint 2). Their spectra ($\tilde{S}_1, \tilde{S}_2$ and $\tilde{S}_3$) can either be symmetric and/or anti-symmetric, which, in two dimensions (considering the

product of two sinusoids) reduces the available bandwidth for channel placement to a single quadrant in the Fourier domain (Fig. S1C). $\tilde{S}_0$ is confined to the origin (Constraint 1), while $\tilde{S}_1$ and $\tilde{S}_3$ are placed at the edges to form a triangle (Fig. S1C). The last remaining element $\tilde{S}_2$, is designed to be *odd*, so it can be placed in the same channel as the *even* $\tilde{S}_1$, as it is orthogonal to $\tilde{S}_1$. (Note that $\tilde{S}_2$ can neither be placed at the origin with $\tilde{S}_0$ ($\sin(0) = 0$), nor be placed with $\tilde{S}_3$ ($\sin(\pm\pi) = 0$).) The placement of the delta functions (or ‘channel locations’) that creates maximal separation between the different channels, while satisfying both Constraint 1 and Constraint 2, is the one shown Fig. S1D which corresponds to our structured polarization illumination of choice (Eq. 1). We can prove that Eq. 1 satisfies Constraint 1 at each discrete (m, n) :

$$\begin{aligned} S_0^2 &= S_1^2(m, n) + S_2^2(m, n) + S_3^2(m, n) \\ S_0^2 &= (\sqrt{2/3} \cos(0.5m\pi) \cos(n\pi))^2 + (\sqrt{2/3} \sin(0.5m\pi) \cos(n\pi))^2 + (\sqrt{1/3} \cos(m\pi))^2 \\ S_0^2 &= (2/3) \cos^2(n\pi) (\cos^2(0.5m\pi) + \sin^2(0.5m\pi)) + (1/3) \cos^2(m\pi) \\ S_0^2 &= (2/3) \cos^2(n\pi) + (1/3) \cos^2(m\pi) = 2/3 + 1/3 = 1 \end{aligned} \quad (2)$$

Physically speaking, Eq. 1 defines four different states of polarization. These polarization states, are maximally separate on the Poincare sphere, and form the edges of a tetrahedron inscribed within the Poincare sphere (Fig. S1E). Using four maximally separate polarization states for sampling (either spatially or temporally) provides the most efficient (only four – the minimum number required – states are used) and effective (smallest condition number for the inverse matrix) sampling solution [1, 2].

B. A Numerical Example of a Band-limited Object

In Fig. S2, we show an artificially constructed numerical example of a spatially varying MM of an object. The Mueller components in this example, are defined over a small lattice of 40 pixels by 40 pixels. The Mueller components are designed to have spatial features that vary much more ‘slowly’ in comparison to the sampling rate of the incident Stokes components (Fig. S2B). This means that the Fourier spectra of all the Mueller components are completely within the confines of the filters applied in the Fourier space (Fig. S2C), and there is no aliasing. Resultantly, all the Mueller components are retrieved from $\tilde{S}_{out}$ without *any* loss of information (Fig. S2D).

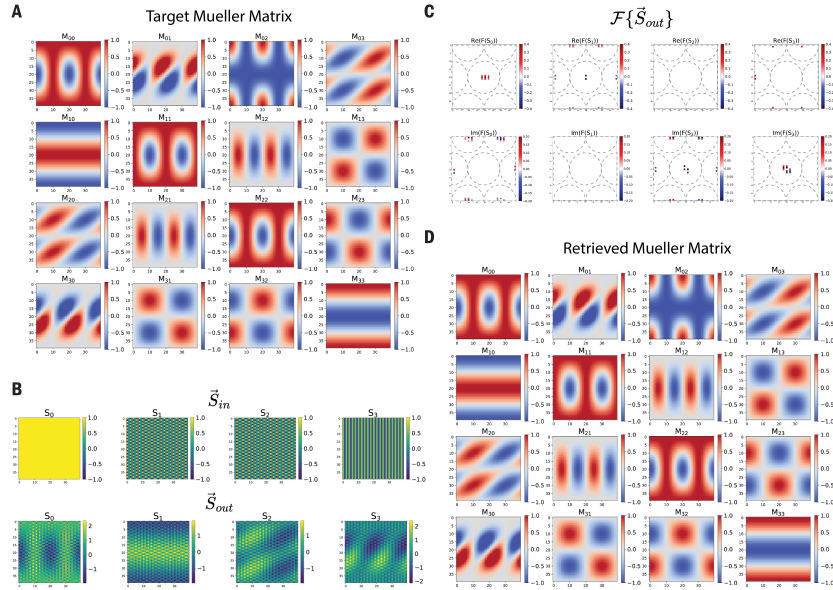


FIG. S2. **A Numerical Analysis of a Band-limited Object.** (A) A numerical example of a spatially varying Mueller matrix. The Fourier spectra of all the Mueller components in this example are band-limited. (B) Plots of the spatially varying Stokes components of $\tilde{S}_{in}$, and plots of the spatially varying Stokes components of the resulting $\tilde{S}_{out}$. (C) The real and imaginary parts of Fourier spectra of the Stokes components of $\tilde{S}_{out}$. (D) The retrieved Mueller matrix exactly matches the target Mueller matrix in (A). Because the spectra of the Mueller components of the target are sufficiently band-limited, with respect to the incident illumination, there is no loss of information while filtering.

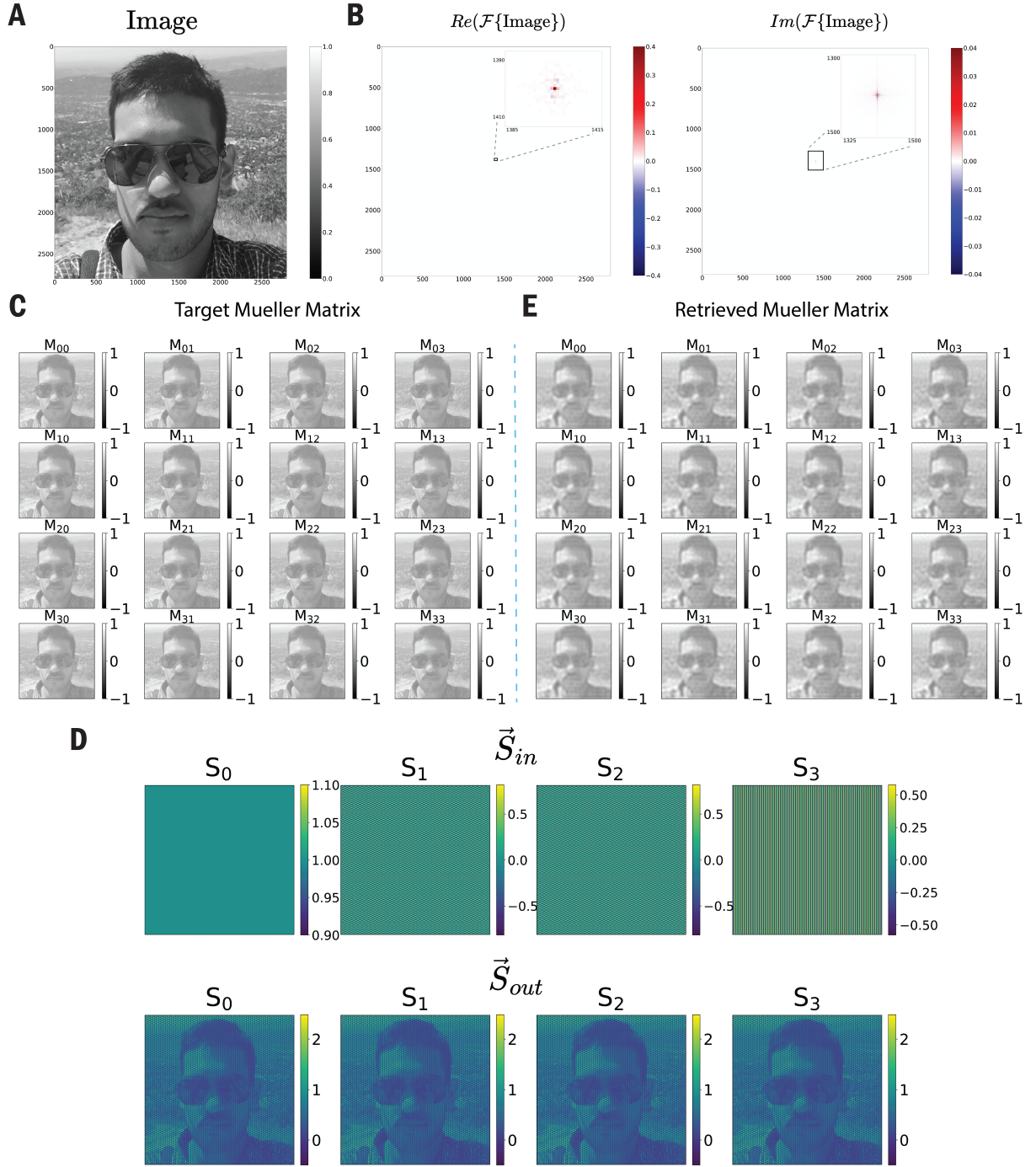


FIG. S3. **A Numerical Analysis of a Non-bandlimited Object.** (A) A spatially rich image, defined over 2800 pixels by 2800 pixels. (B) The real and imaginary parts of the Fourier spectrum of the image in (A) show that the majority of information is concentrated at low spatial frequencies. (C) We use the image in (A) to numerically define a spatially varying MM object. (D) Plots of spatially varying $\vec{S}_{in}$ and $\vec{S}_{out}$. (E) The retrieved MM image from $\vec{S}_{out}$ shows accurate (low-pass) reconstruction – close to the target MM object in (A). Furthermore, we see little, if any, signs of aliasing degrading the reconstructed MM image, even though spectra of the target MM object are not band-limited within the confines of the Fourier filters.

C. A Numerical Example of a Non-bandlimited Object

The analysis in the previous subsection (Sec. S1B) may invite the question: what if the object is non-bandlimited with respect to the sampling rate of the incident structured polarization illumination? It is true that images in real life contain features (such as sharp edges) composed of very high spatial frequencies, yet – as we show in the following analysis – these high spatial frequency components, in many instances, have amplitudes much smaller than the low spatial frequency components. This may, depending on the application and its requirements, allow the use of a lower sampling rate for the illumination. Consider, for instance, the image shown in Fig. S3A, inspired by the real world application of facial recognition. The image, defined over a lattice of 2800 pixels by 2800 pixels, has rich spatial features with many sharp edges and transitions. What we notice in Fig. S3B is that even for this relatively rich and complicated image, the Fourier spectrum is dominated by low spatial frequencies.

We use Fig. S3A to construct an artificial MM object where each component takes the form of Fig. S3A, as shown in Fig. S3C. Consider now, the structured polarization illumination ($\vec{S}_{in}$) show in Fig. S3D: instead of the highest sampling rate allowed by Eq.1, we purposely choose a sampling rate that is 20 times lower, to try and exaggerate the effects of aliasing. The Mueller components are filtered out from $\vec{S}_{out}$ (Fig. S3D), to reconstruct the MM image, shown in Fig. S3E. We see that the reconstructed images of MM components look like low-pass filtered images of the target MM images. This is to be expected, as we are using low-pass filters in the filtering process, and a much lower sampling rate. The thing to note, however, is that the effects of aliasing are hardly noticeable. This is because the high spatial frequency components from other channels, crossing into the low pass filter of a given channel, have much smaller amplitudes than the low spatial frequency components of that channel, and are thus eclipsed. This allows us – of course depending on the application and its requirements – to relax the constraints on the spatial resolution needed to design an effective MM imaging system.

S2. MATERIALS AND METHODS: DETAILS

A. Phase Retrieval Optimization of Metasurface Hologram for Structured Polarization Illumination

Algorithm S1 Polarization Phase Retrieval

```

1: procedure METASURFACE PHASE RETRIEVAL
2:   Inputs:
3:    $E_x^{meta,init}(m, n), E_y^{meta,init}(m, n)$   $\triangleright$  Initial guess of electric field distribution right after the metasurface
4:    $|E_x^{targ}(k_m, k_n)|, |E_y^{targ}(k_m, k_n)|, \phi^{targ}(k_m, k_n)$   $\triangleright$  Desired far-field polarization distribution
5:   iter  $\triangleright$  Number of iterations
6:
7:
8:    $(E_x^{meta}(m, n), E_y^{meta}(m, n)) \leftarrow (E_x^{meta,init}(m, n), E_y^{meta,init}(m, n))$   $\triangleright$  Initialization
9:   for  $i = [1, iter/2]$  do
10:     $E_x^{far}(k_m, k_n) \leftarrow \mathcal{F}\{E_x^{meta}(m, n)\}, E_y^{far}(k_m, k_n) \leftarrow \mathcal{F}\{E_y^{meta}(m, n)\}$   $\triangleright$  F.T. to far-field
11:     $E_x^{far}(k_m, k_n) = |E_x^{targ}(k_m, k_n)| \exp(j \angle \mathcal{F}\{E_x^{meta}(m, n)\})$   $\triangleright$  Update the  $x$ -field component
12:     $E_y^{far}(k_m, k_n) = |E_y^{targ}(k_m, k_n)| \exp(j \angle \mathcal{F}\{E_y^{meta}(m, n)\} + j \phi^{targ}(k_m, k_n))$   $\triangleright$  Update the  $y$ -field component
13:     $E_x^{meta}(m, n) \leftarrow \mathcal{F}^{-1}\{E_x^{far}(k_m, k_n)\}, E_y^{meta}(m, n) \leftarrow \mathcal{F}^{-1}\{E_y^{far}(k_m, k_n)\}$   $\triangleright$  I.F.T. to “near-field”
14:   for  $i = [iter/2, iter]$  do
15:     $E_x^{far}(k_m, k_n) \leftarrow \mathcal{F}\{E_x^{meta}(m, n)\}, E_y^{far}(k_m, k_n) \leftarrow \mathcal{F}\{E_y^{meta}(m, n)\}$   $\triangleright$  F.T. to far-field
16:     $E_x^{far}(k_m, k_n) = |E_x^{targ}(k_m, k_n)| \exp(j \angle \mathcal{F}\{E_y^{meta}(m, n)\} - j \phi^{targ}(k_m, k_n))$   $\triangleright$  Update the  $x$ -field component
17:     $E_y^{far}(k_m, k_n) = |E_y^{targ}(k_m, k_n)| \exp(j \angle \mathcal{F}\{E_x^{meta}(m, n)\})$   $\triangleright$  Update the  $y$ -field component
18:     $E_x^{meta}(m, n) \leftarrow \mathcal{F}^{-1}\{E_x^{far}(k_m, k_n)\}, E_y^{meta}(m, n) \leftarrow \mathcal{F}^{-1}\{E_y^{far}(k_m, k_n)\}$   $\triangleright$  I.F.T. to “near-field”
19:    $\mathbf{J}_{meta}(m, n) \leftarrow E_x^{meta}(m, n), E_y^{meta}(m, n)$   $\triangleright$  Compute metasurface Jones matrix profile
20:    $\phi_X(m, n), \phi_Y(m, n), \theta(m, n) \leftarrow \mathbf{J}_{meta}(m, n)$   $\triangleright$  Compute nanopillar phases and orientations from Jones matrix profile
21:    $D_X(m, n), D_Y(m, n) \leftarrow \phi_X(m, n), \phi_Y(m, n)$   $\triangleright$  Compute nanopillar dimensions from phases
22: return  $D_X(m, n), D_Y(m, n), \theta(m, n)$ 

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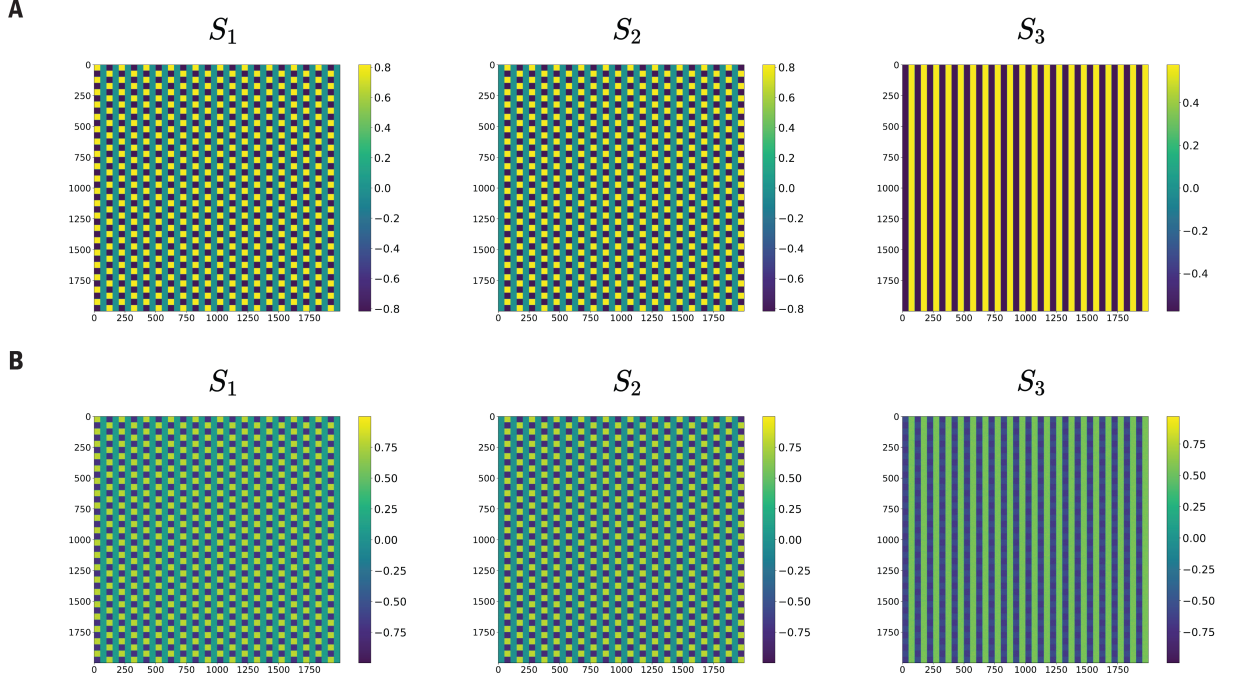


FIG. S4. **Polarization Phase Retrieval Optimization – Numerical Result.** (A) The desired S_1 , S_2 , and S_3 components defined over a lattice of 2000 pixels $\times$ 2000 pixels. This target distribution follows Eq. 1, where ‘superpixels’ m and n are composed of 100 pixels $\times$ 100 pixels, each. (B) The resulting S_1 , S_2 , and S_3 components following 50 iterations of the phase retrieval optimization, described in Algorithm S1.

For our application, we desire a far-field polarization distribution, given by Eq. 1: four different polarizations, juxtaposed in a 2×4 repeating unit cell. The Stokes vectors of these polarizations are:

$$\vec{S}_i = \begin{pmatrix} 1 \\ \sqrt{2/3} \\ 0 \\ \sqrt{1/3} \end{pmatrix}, \vec{S}_{ii} = \begin{pmatrix} 1 \\ 0 \\ \sqrt{2/3} \\ -\sqrt{1/3} \end{pmatrix}, \vec{S}_{iii} = \begin{pmatrix} 1 \\ -\sqrt{2/3} \\ 0 \\ \sqrt{1/3} \end{pmatrix}, \vec{S}_{iv} = \begin{pmatrix} 1 \\ 0 \\ -\sqrt{2/3} \\ -\sqrt{1/3} \end{pmatrix} \quad (3)$$

As seen in Algorithm S1, the CGH is designed using Jones calculus, instead of Stokes calculus, since the phases between different polarizations are important in the optimization of the CGH. We can convert the Stokes vectors in Eq. 3 – fully polarized states with DOP = 1 – to Jones vectors. These Jones vectors are:

$$\vec{j}_i = \begin{pmatrix} 0.953 \\ i0.303 \end{pmatrix}, \vec{j}_{ii} = \begin{pmatrix} 0.577 + i0.408 \\ 0.707 \end{pmatrix}, \vec{j}_{iii} = \begin{pmatrix} -i0.303 \\ 0.953 \end{pmatrix}, \vec{j}_{iv} = \begin{pmatrix} -0.577 + i0.408 \\ 0.707 \end{pmatrix} \quad (4)$$

We first numerically define a target distribution of polarization states composed of the four states in Eq. 4. This target distribution is then used in the iterative phase retrieval to optimize a CGH that can then be implemented using a metasurface. As an example, we show a 2000 pixels $\times$ 2000 pixels CGH in Fig. S4. Fig. S4A shows the target polarization of the CGH, defined in terms of the S_1 , S_2 , S_3 components; S_0 can be ignored in the target as it is assumed to be uniform (DOP = 1). Fig. S4B shows the optimized numerical response of the CGH after convergence of the GS algorithm. The GS algorithm converges quickly (≤ 50 iterations), results of which are shown in Fig. S4B.

One thing to note in Fig. S4 is that we define ‘super-pixels’ that correspond to the coordinates (m, n) so our structured illumination has a lower spatial resolution (or spatial sampling rate), than the maximum resolution allowed by the GS algorithm. This is because of practical reasons. As the *modulation transfer function* (MTF) of any imaging system shows, the contrast achievable at higher spatial frequencies is lower than the one achievable at lower spatial frequencies. We have to be cognizant of the fact that our full-Stokes imaging system may not be able to sufficiently resolve the structured polarization illumination if we choose a very high sampling rate. Therefore, in our proof of concept work, we settle on a sampling rate for our structured polarization illumination that can be easily imaged by our full-Stokes camera. (See also Sec. S2 E.)

B. Gradient-Descent optimization of Metasurface Gratings for Full-Stokes Imaging

Algorithm S2 Gradient Descent Optimization

```

1: procedure METASURFACE GRADIENT-BASED OPTIMIZATION
2:   Inputs:
3:    $\phi_X^{init}(m, n), \phi_Y^{init}(m, n), \theta^{init}(m, n)$   $\triangleright$  Initial guess of parameters defining metasurface Jones matrix profile
4:    $G$   $\triangleright$  Define the set of chosen diffraction orders
5:    $j_{targ}(k_m, k_n)$  for all  $(k_m, k_n) \in \{G\}$   $\triangleright$  Preferred polarizations for analyzers in the set of chosen diffraction orders  $G$ 
6:
7:
8:    $\mathbf{J}_{meta}(m, n) \leftarrow \phi_X^{init}(m, n), \phi_Y^{init}(m, n), \theta^{init}(m, n)$   $\triangleright$  Initialization
9:   while  $\vec{\nabla} \neq 0$  do
10:     $\tilde{\mathbf{J}}_{far}(k_m, k_n) \leftarrow \mathcal{F}\{\mathbf{J}_{meta}(m, n)\}$   $\triangleright$  F.T. to far-field
11:     $\eta, \xi, \sigma \leftarrow \tilde{\mathbf{J}}_{far}(k_m, k_n)$   $\triangleright$  Compute merit figure and constraints
12:     $\vec{\nabla} \leftarrow \eta, \xi, \sigma$   $\triangleright$  Compute gradient
13:     $\phi_X(m, n), \phi_Y(m, n), \theta(m, n) \leftarrow \vec{\nabla}$   $\triangleright$  Update parameters using gradient
14:     $\mathbf{J}_{meta}(m, n) \leftarrow \phi_X(m, n), \phi_Y(m, n), \theta(m, n)$   $\triangleright$  Compute metasurface Jones matrix profile
15:     $D_X(m, n), D_Y(m, n) \leftarrow \phi_X(m, n), \phi_Y(m, n)$   $\triangleright$  Compute nanopillar dimensions from phases
16:  return  $D_X(m, n), D_Y(m, n), \theta(m, n)$ 

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C. Full-Stokes Imaging Calibration

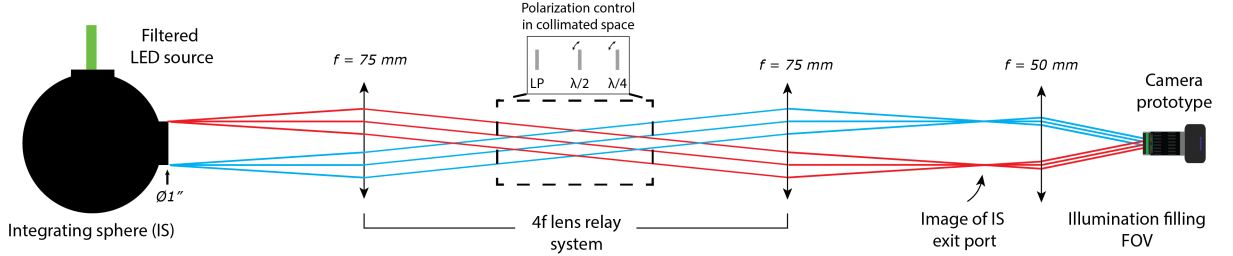


FIG. S5. **Full-Stokes Camera Calibration.** The camera is calibrated over its entire FOV, by using light from an extended source: the exit port of an integrating sphere (IS). The 4f setup allows the light from the IS to be imaged onto the focal plane of the lens just before the full-Stokes camera, providing a collimated space for rotating polarization optics to be placed. The final (50 mm) lens ensures uniform illumination over the camera's FOV. The polarization optics are rotated, and images are acquired for known incident states.

As mentioned in the main text, it is important to independently design and calibrate a robust metasurface full-Stokes camera as the reconstruction accuracy of the MM image is directly related to the accuracy of the full-Stokes camera. In this subsection, we summarize the calibration procedure used to calibrate our full-Stokes camera.

Our full-Stokes camera consists of a metasurface diffraction grating (Metasurface 2) that diffracts light into the first four diffraction orders, while simultaneously analyzing for four different polarization states. These polarization states (shown in main text Fig.3A), also, form the vertices of tetrahedron inscribed within the Poincare sphere, and are thus maximally separate on the Poincare sphere. The Stokes vectors of these polarization states are:

$$\vec{S}_A = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \vec{S}_B = \begin{pmatrix} 1 \\ -\sqrt{2}/3 \\ -\sqrt{2}/3 \\ -1/3 \end{pmatrix}, \vec{S}_C = \begin{pmatrix} 1 \\ -\sqrt{2}/3 \\ \sqrt{2}/3 \\ -1/3 \end{pmatrix}, \vec{S}_D = \begin{pmatrix} 1 \\ 2\sqrt{2}/3 \\ 0 \\ -1/3 \end{pmatrix} \quad (5)$$

(Note that the states described in Eq. 3 and Eq. 5, both form the vertices of a tetrahedron inscribed within the Poincare sphere – the only difference is in the absolute coordinates of the vertices for these two otherwise equivalent tetrahedrons. The choice of the ‘reference frame’ for the coordinates is arbitrary.)

The ideal 4×4 instrument matrix $\hat{A}$ consisting of these four states is given by:

$$\hat{A} = \begin{pmatrix} \text{---}\vec{S}_A\text{---} \\ \text{---}\vec{S}_B\text{---} \\ \text{---}\vec{S}_C\text{---} \\ \text{---}\vec{S}_D\text{---} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & -\sqrt{2}/3 & -\sqrt{2}/3 & -1/3 \\ 1 & -\sqrt{2}/3 & \sqrt{2}/3 & -1/3 \\ 1 & 2\sqrt{2}/3 & 0 & -1/3 \end{pmatrix} \quad (6)$$

A well-conditioned (small condition number) matrix system, is much less sensitive to errors in input. The instrument matrix in Eq. 6 made of the four ‘tetrahedron states’ has a condition number of 1.732, which is the smallest one can have for a 4×4 matrix using states on the Poincare sphere. In practice, however, the actual instrument matrix will differ, to some extent, from the desired instrument matrix, because of fabrication errors and polarization-dependent effects that can happen inside the imaging system (e.g., Fresnel reflections off lens surfaces). It is important, thus, to calibrate the system and determine the actual instrument matrix over the entire field-of-view (FOV).

Our scheme, as shown in Fig. S5, uses an extended light source that uniformly fills the FOV of the camera. The exit port of an integrating sphere (IS) illuminated with green LED light is imaged using a $4f$ system consisting of two lenses of the same focal length which provides a collimated space for the linear polarizer, quarter-wave plate and half-wave plate to reside. The IS’ exit port, and its image created following the $4f$ system, provide an extended source of uniform illumination that, when passed through a final lens one f away, fills the camera’s entrance pupil (coinciding with the metasurface grating). This final lens is chosen to fill the instrument’s FOV. The quarter-wave and half-wave plates are sequentially rotated to generate different known states of polarization and the response of the grating, over the entire FOV of the imaging system, is recorded. We use a minimum of 25 rather arbitrarily generated (and relatively uniformly distributed) polarization states for our calibration. The instrument matrix is computed pixel-by-pixel over the entire FOV, and is then averaged to give the final instrument matrix to be used in our computation [3, 4].

D. Mueller Matrix Imaging Calibration and Demodulation

We compute the complete MM image through amplitude demodulation of the full-Stokes image. We use the ‘product detector’ method, to demodulate the signal(s) from the carrier wave(s) [5]. This method can be intuitively understood by a simple one-dimensional example. Imagine a carrier wave $\cos(kx + \phi)$ amplitude modulated by a slowly varying signal of interest $A(x)$ resulting in the output signal $A(x) \cos(kx + \phi)$. If we multiply the output signal by the original carrier wave signal, we get the signal $A(x) \cos^2(kx + \phi) = \frac{A(x)}{2} + \frac{A(x) \cos(2(kx + \phi))}{2}$. Applying a low pass filter, centered on the origin and with a radius of $k/2$, on the (Fourier) spectrum of $A(x) \cos^2(kx + \phi)$, gives us the spectrum of $\frac{A(x)}{2}$, given that the spectrum of $A(x)$ lies within the $k/2$ radius. Thus, the desired signal $A(x)$ is fully and completely retrieved. This method requires the knowledge of the carrier wave, which necessitates a reference measurement. In our case, the ‘air’ image serves as a reference measurement, since air has a (spatially uniform) MM of identity. The reference measurement also helps us calibrate out any variations in the amplitude of our structured polarization illumination. Metasurface 1 is designed to produce spatially varying polarization, with uniform intensity; however, in practice, there are variations (both fast and slow) in intensity across the hologram that should be calibrated out. The fast intensity variations are clipped off by the low pass filter, and though they contribute to aliasing, their contribution is small, if not negligible. In the case of slow intensity variations, we can define the carrier to be $\alpha(x) \cos(kx + \phi)$ where $\alpha(x)$ is a slowly varying amplitude envelope. For a signal of interest $A(x)$, the amplitude modulated signal can be written as $A(x)\alpha(x) \cos(kx + \phi)$. The demodulation described above can be carried out as is, resulting in the retrieved signal $\frac{A(x)\alpha^2(x)}{2}$, which when divided by $\alpha^2(x)$, gives us our signal of interest $A(x)$.

This 1D description of amplitude demodulation can be easily extended to 2D, to demodulate the 16 Mueller component images, from the 4 Stokes component images, as shown in Fig. S6. To prepare the air Stokes image as a reference measurement to be used in demodulation, we fill in the parts of the image with no signal (corresponding to the regions where light is blocked from the iris and the beam block), through extrapolation (Fig. S6A-B). This ensures that the reference Stokes components are close to the desired sinusoid signals. Using the *channeled polarimetry* scheme described in Sec. S1A and Fig. S1D, we prepare the reference for S_0 by filtering the spectrum (of the S_0 image) around channel ‘0’, for S_1 by filtering the spectrum (of the S_1 image) around channels ‘1’ and ‘2’, for S_2 by filtering the spectrum (of the S_2 image) around – again – channels ‘1’ and ‘2’, and for S_3 by filtering the spectrum (of the S_3 image) around channel ‘3’ (Fig. S6C). We also compute the intensity variations of the reference images, $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ (Fig. S6D), to divide out during the demodulation step as explained above.

This same reference (Fig.S6), once prepared, is used to demodulate and retrieve MM images for all other samples. To illustrate the computational workflow in computing MM images, we share an example of the quarter-wave plate

(QWP) oriented at 45° , in Fig. S7. From the snapshot raw image obtained (Fig. S7A), we first compute the Stokes images ($\vec{S}_{\text{object}}$, Fig. S7B) using the instrument matrix, following which a ‘filtered’ $\vec{S}_{\text{object}}$ is prepared, for each Stokes component image, by keeping the spectra surrounding the channels 0,1,2 and 3 using bandpass filters, and discarding the rest (which consists of high spatial frequency contributions, and contributions from noise etc). The Mueller components are then demodulated from the filtered $\vec{S}_{\text{object}}$ by multiplying the images with images of the prepared references $\vec{S}_{\text{reference}}$ (Fig. S7C-D), followed by low pass filtering (Fig. S7E), and finally dividing by reference amplitudes and normalization, with respect to the maximum value of the M_{00} image (Fig. S7F).

We acknowledge that the calibration performed in our work is not necessarily the most optimal, and that there is room for further exploration and improvement. In our work, we somewhat independently calibrate the polarization state analyzer and polarization state generator modules, and that a more holistic calibration procedure may involve an ‘end-to-end’ calibration by treating the entire system as one. This could involve the use of other techniques, such as the eigenvalue calibration method [6]. Exploring different calibration procedures to optimize the performance of our device will be the subject of future work.

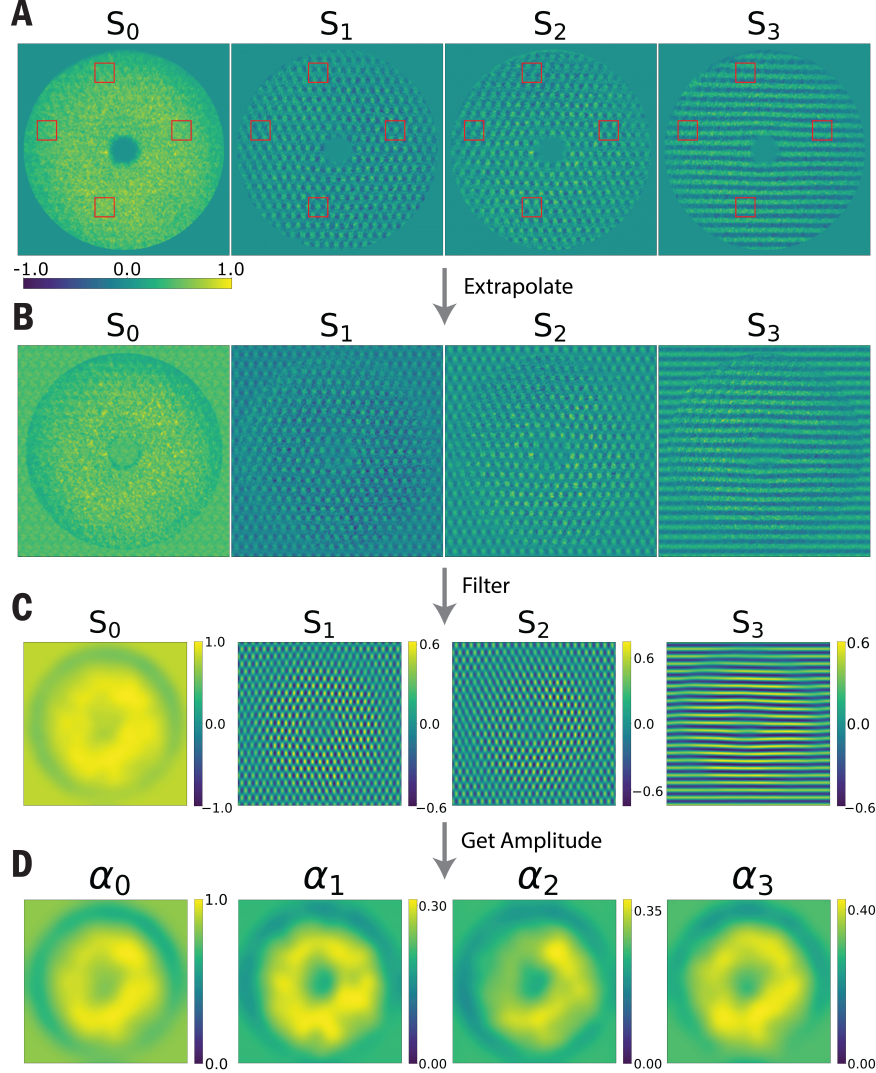


FIG. S6. **Mueller Matrix Imaging – Preparing the Reference.** (A) Periods from four different sections (red boxes) of the periodic Stokes component images are used to extrapolate the signal to parts of the images with no signal. (B) Extrapolated images. (C) The Stokes components in (B) are filtered around the corresponding channels 1, 2, 3 and 4 to prepare references for demodulation. (D) The references prepared in (C) are not pure sinusoids, but have a slowly varying intensity envelope, that can be computed and divided out later after demodulation.

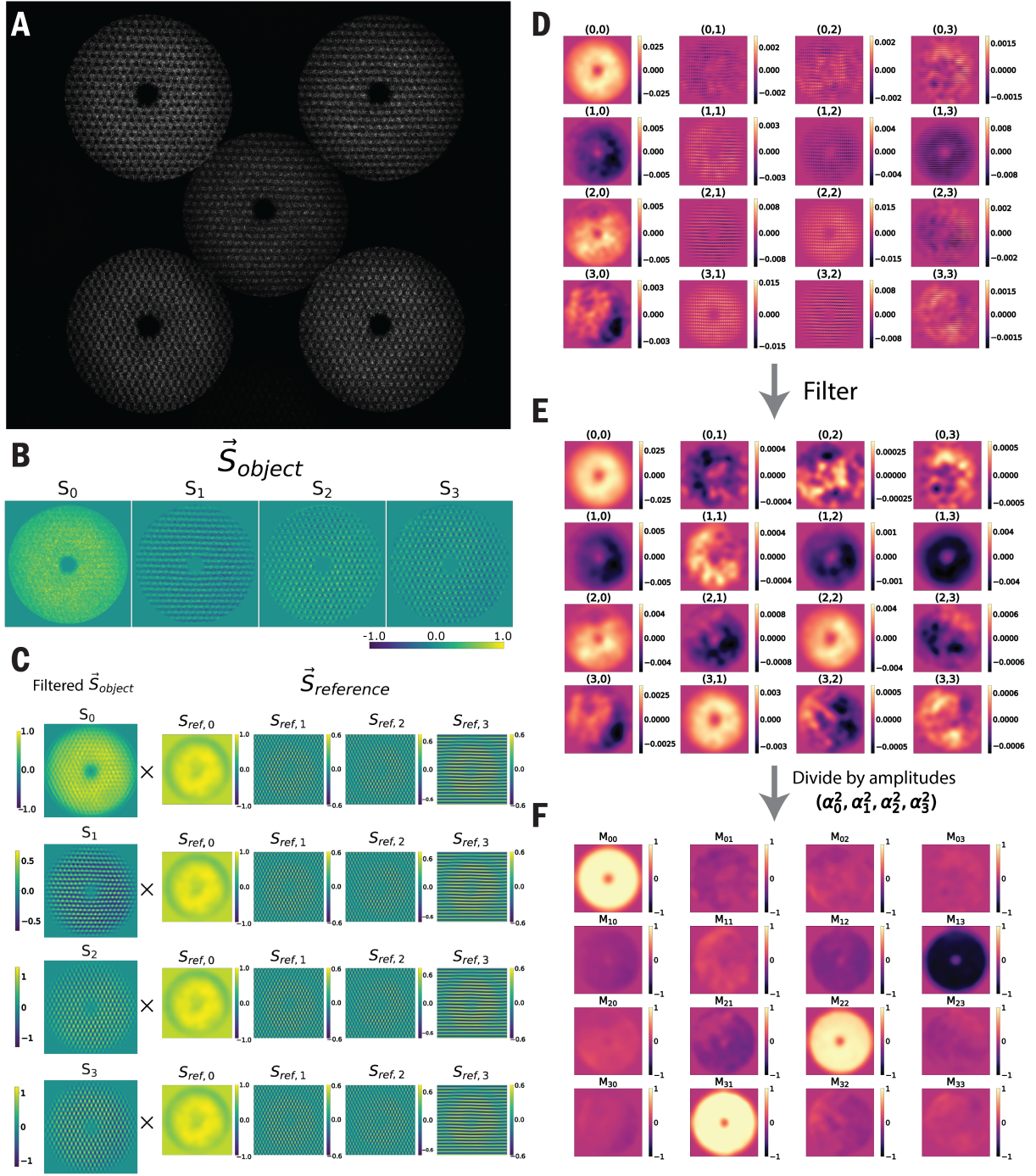


FIG. S7. **Mueller Matrix Imaging Demodulation – Step by Step.** (A) Raw image of a QWP oriented at 45° . (B) The resulting Stokes image. (C) The demodulation step using the product detection method. In this method, the signal is multiplied by the reference. (D) The resulting 4×4 image following (C). (E) After applying low pass filters to the images in (D). (F) The final retrieved Mueller matrix image following division by the intensity envelopes of the reference images (followed by normalization).

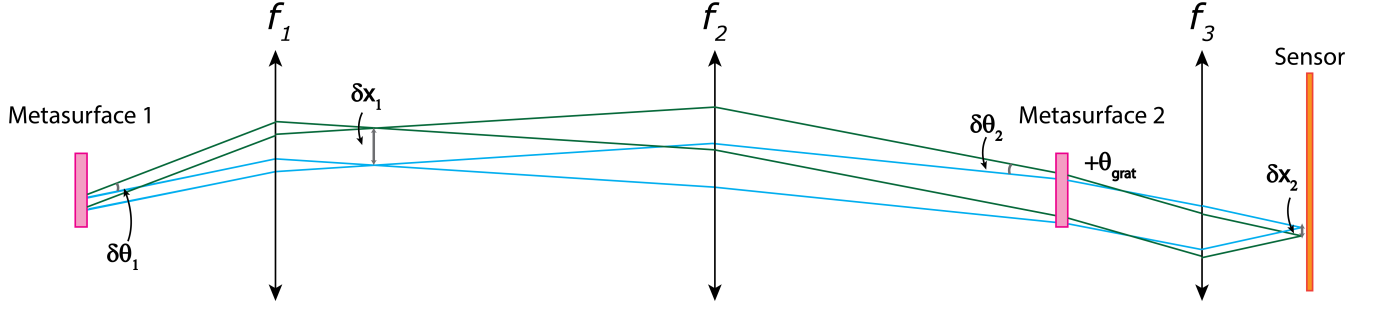


FIG. S8. **System Angular and Spatial Resolution.** A simple ray trace through the system, to describe the propagation of angular spread and spatial resolution through the system. (Not drawn to scale.)

E. Factors Affecting the Resolution of the System: First-order Analysis

1. Spatial resolution analysis of structured polarization illumination

To understand the variables affecting the final resolution of the system, we perform a simple analysis involving ray tracing through the system (in one dimension), as shown in Fig. S8. We start by considering the size (or diameter) D of the first metasurface, Metasurface 1, used to generate the structured polarization pattern. D equals the number of nanopillars (N) times the nanopillar separation (s). Even though the metasurface is aperiodic, one way of thinking about this structured pattern generation is to imagine that the entire metasurface hologram is a bit like a grating whose period is the size of the entire device D . Then, like a grating, the hologram produces discrete orders, but owing to the large period these are close enough together to approximately form a continuum. It is the discreteness of these orders, however, that set a resolution to the finest polarization patterns which can be encoded (set by D).

The ‘discrete’ far-fields orders m of Metasurface 1 – a computer generated hologram (CGH) – are related to the device size D , diffraction angle θ , and the incident wavelength λ by the grating equation:

$$D \sin \theta = m\lambda \quad (7)$$

To find the resolution of the holographic pattern, we consider the difference between two adjacent orders (m_i and m_{i+1}):

$$\begin{aligned} D(\sin \theta_{i+1} - \sin \theta_i) &\approx D \underbrace{(\theta_{i+1} - \theta_i)}_{\delta\theta_1} = \underbrace{(m_{i+1} - m_i)}_1 \lambda \\ \delta\theta_1 &\approx \frac{\lambda}{D} = \frac{\lambda}{Ns} \end{aligned} \quad (8)$$

where, working under the paraxial approximation, we assume $\sin \theta \approx \theta$.

The resulting spatial resolution δx_1 , in the Fourier plane of the first converging lens (Fig. S8) with a focal length f_1 is given as:

$$\delta x_1 = f_1 \delta\theta_1 = f_1 \frac{\lambda}{Ns} \quad (9)$$

Eq.9 gives us a straightforward relation between the spatial resolution (or the sampling rate) of the structured polarization illumination, to the different variables one can control. For instance, a larger metasurface (by increasing the number of pillars N and/or increasing the pillar separation s), a smaller wavelength λ , and a smaller focal length (or a higher numerical aperture) lens, can all increase the resolution of the structured polarization illumination to adequately spatially sample the object of interest. (In case of no lens, the focal length f_1 in Eq.9 can be replaced by the far-field distance d_1 .)

By extending our ray trace analysis, as shown in Fig. S8, we can write the final resolution of the pattern (that occupies the sensor) as:

$$\delta x_2 = \frac{f_3}{f_2} \delta x_1 \quad (10)$$

Ideally, δx_2 is large so that it covers many pixels on the sensor, increasing the accuracy of the image (though that would come at the cost of the ‘range’ of the FOV). The ratio $\frac{f_3}{f_2}$ provides us with a tuning knob to design a large δx_2 , while keeping δx_1 small as desired.

Note that in this first-order analysis, the effect of the ‘shift’ of the angles introduced by the grating, θ_{grat} (Fig. S8), is not relevant to the final resolution. In a more rigorous analysis, however, the grating angles become relevant keeping in mind the MTF of the system and practical concerns like vignetting [3].

2. Resolution of the system presented in the main text

Above, we have given a paraxial estimate for the resolution with which a structured polarization distribution can be prepared in the sample plane of the proposed MM polarimeter. This defines an effective “pixel” pitch to the structured illumination pattern. As depicted in Fig. 2 of the main text, the structured illumination pattern consists of a repeating pattern of 2×4 unique polarization states. So, the minimum resolution achievable by such a system is 2 (vertical) or 4 (horizontal) times the limit derived in Eq. 9, i.e., the spatial period of the repeating polarization pattern.

In experimental practice in the main text, we have not designed the polarization hologram to reach this resolution limit. That is, instead of designing the polarization hologram to vary polarization state with a pitch of $\Delta m = 1$ (the limit in which each “order” of the hologram carries a different polarization state), we have instead opted for $\Delta m = 50$ which relaxes design constraints for this proof-of-concept demonstration.

The spatial resolution of the MM polarimeter described in the main text is most easily obtained by direct inspection of the spatially-varying polarization pattern produced (as viewed through the polarization-analyzing metasurface camera, as in Fig. 3B of the main text) from the scale bar. The area over which the state of polarization of the structured illumination remains constant is observed to be $\approx 0.5\text{mm}$. We know from Eq. 1 above (and Eq. 2 of the main text), how the individual Stokes components of the structured illumination (S_{1-3}) vary as a function of the ‘pixels’ m and n . In the k -space picture, as depicted in Fig. S1C, this means the distance between channels $\tilde{S}_0$ and $\tilde{S}_3$ is $\frac{2\pi}{2(0.5\text{mm})} = 2\pi\text{mm}^{-1}$, and the distance between channels $\tilde{S}_0$ and $\tilde{S}_{1,2}$ (and $\tilde{S}_3 - \tilde{S}_{1,2}$) is $\sqrt{(\frac{2\pi}{2(0.5\text{mm})})^2 + (\frac{(0.5)(2)\pi}{2(0.5\text{mm})})^2} = \sqrt{5}\pi\text{mm}^{-1}$. The filter size (radius) is chosen to be half of the minimum of these distances (from the Nyquist theorem), which in this case would be $\frac{2\pi\text{mm}^{-1}}{2} = \pi\text{mm}^{-1}$. This filter size, then, translates to a spatial resolution of $\frac{2\pi}{\pi\text{mm}^{-1}} = 2\text{mm}$. This means that, roughly speaking, our system can image feature sizes of 2mm or greater, without any aliasing.

3. Effect of broadband illumination

In the main text, monochromatic laser illumination was used. However, it is natural to ask what chromatic bandwidth the illumination $\Delta\lambda$ could occupy. In diffractive optics, both the efficiency and diffraction angle of a grating (in accordance with the grating condition) will vary with λ . In the case of Metasurface 1 in Fig. S9 above (the illumination hologram), the angle of each order will vary with wavelength; for broadband illumination, the pattern will have a tendency to “smear” with angle. In the case of Metasurface 2 (the imaging polarization-analyzing grating), each order will also smear with broadband illumination. This “chromatic smearing” on both sides will become unacceptable for MM imaging when it reaches an extent that “blurs” out the structured polarization pattern being projected (or imaged) by the metasurface(s). This metric allows us to derive a limit on the acceptable bandwidth $\Delta\lambda$ of the illumination.

Suppose we concentrate on a particular section of the polarized illumination (i.e., a particular “order” m of the polarization hologram). We can write

$$\theta_{\lambda_1} = \arcsin \frac{m\lambda_1}{D} \quad (11)$$

and

$$\theta_{\lambda_2} = \arcsin \frac{m\lambda_2}{D} \quad (12)$$

where as above D is the physical size of the polarization hologram. In the paraxial limit we can then write

$$\Delta\theta = \frac{m}{D}\Delta\lambda, \quad (13)$$

with $\Delta\lambda = \lambda_2 - \lambda_1$. In other words, the angular smearing incurred by broadband illumination varies over the field of view of the polarization-structured illumination, becoming inconsequential at the center (i.e., at $m = 0$). We can derive a limit from this picture by asking at which $\Delta\lambda$ does it become true that

$$\theta_{\lambda_1}^{m+\Delta m} = \theta_{\lambda_2}^m. \quad (14)$$

In other words, the chromatic acceptance of the setup is defined by the situation in which a specific polarized “pixel” produced by the hologram at one wavelength λ_2 lands on top of the adjacent polarized pixel Δm diffraction orders away produced at the second wavelength λ_1 due to chromatic dispersion, completely obscuring the desired pattern. Here, we take $\lambda_2 > \lambda_1$ and assume that these wavelengths lie on opposite sides of the chromatic bandwidth $\Delta\lambda$.

Again taking the paraxial limit and using Eqs. 11 and 12 above, we can write that it must be true that

$$(m + \Delta m)\lambda_1 = m\lambda_2. \quad (15)$$

from which we can write

$$\Delta\lambda = \frac{\Delta m}{m}\lambda_1 \quad (16)$$

The condition at which this unacceptable chromatic smearing occurs again depends on FOV coordinate m (at the field center $m = 0$, the two wavelengths could be infinitely far apart without consequence, but for $m \gg \Delta m$, $\Delta\lambda \rightarrow 0$).

We can use this relation to make a back-of-the-envelope calculation of the chromatic acceptance of the MM polarimeter presented in this work. As mentioned above, each “pixel” of the polarized illumination pattern here occupies $\Delta m = 50$ diffraction orders. From inspection of Fig. 3B of the main text, roughly 12 of these pixels bring us to a position about halfway across the field-of-view of the polarimeter (accordingly, we can set $m = 12\Delta m$). Eq. 16 thus gives a $\Delta\lambda$ that is roughly 1/12 of the central wavelength (532 nm in this work), or $\Delta\lambda \sim 40$ nm before unacceptable chromatic blurring is introduced to the structured polarization pattern.

It should be noted that this treatment is only geometrical in nature. Other effects may dominate first, such as a falloff in efficiency away from the design wavelength, though in practice we find that such structured polarization holograms tend to function approximately as-intended even far off their design wavelength (see, e.g., the supplement of [7]).

We also note here that design strategies exist by which the MM polarimeter concept presented here could be generalized for multi-color or broadband MM polarimetry. This is discussed in Sec. S4B.

S3. ANALYSIS OF MEASUREMENTS AND RESULTS

A. Imaging in Transmission: a Compilation of Results

In this section, we present a compilation of the dataset used to generate MM images for Fig.4 in the main text. Fig. S9 show the raw and full-Stokes images used to generate the final MM images, using the demodulation scheme described in Sec. S2D. The MM images presented in Fig.4 of the main text are, following normalization, clipped at the upper and lower bounds of +1 and -1, respectively. This is because (normalized) Mueller component values beyond ± 1 are nonphysical. Nonphysical MM values give us a good way to gauge the accuracy of the overall instrument. In Fig. S10, we show the ‘non-clipped’ (or unsaturated) MM images, and the maximum ‘overshoot’ values. We see that, on average, the overshoot is within 10%, which is a testament to the reasonable efficacy and accuracy of our system and computation. In Fig. S11 below, we plot box plots to quantify the error of our device in predicting the Mueller matrix values for different polarization components. Further improving the accuracy and robustness of the system for more sensitive applications, will be the subject of future work.

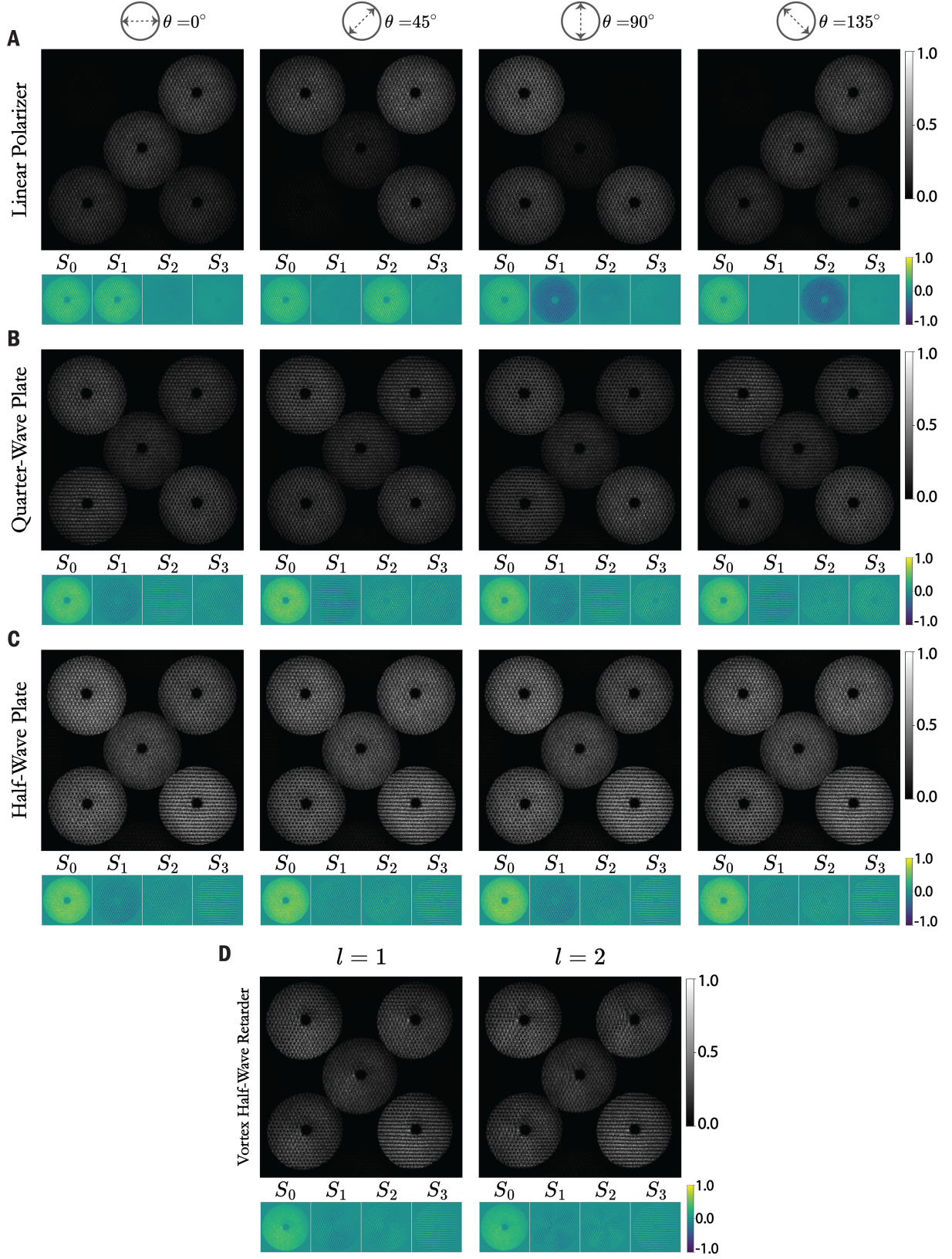


FIG. S9. **Compilation of Raw and Stokes Images.** A compilation of ‘raw’ and full-Stokes images of (A) Linear Polarizer, (B) Quarter-Wave Plate, (C) Half-Wave Plate, and (D) Vortex Half-Wave Retarders, used to generate the Mueller matrix images seen in Fig. 4 of the main text, and Fig. S10 of the supplement.



FIG. S10. **Mueller Matrix Imaging Results – Unsaturated Plots.** Measured MM images following the template of Fig.4 of the main text, plotted without clipping the results at ± 1 (unsaturated).

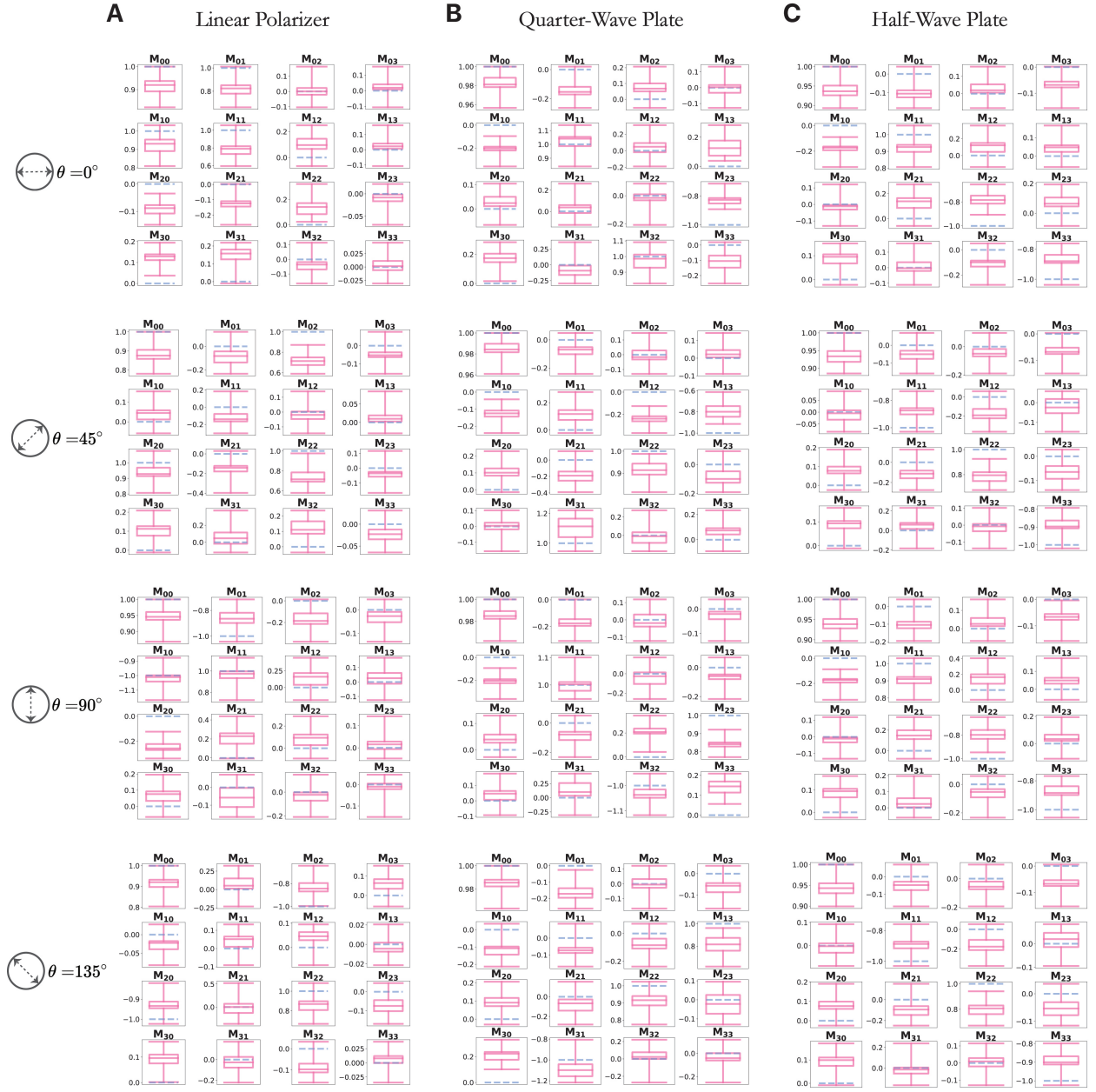


FIG. S11. **Mueller Matrix Imaging Results – Error Plots.** Box plots to quantify error distributions in measured Mueller elements for different configurations of (A) Linear Polarizer, (B) Quarter-Wave Plate and (C) Half-Wave Plate. For these components, we compare the distribution of measured Mueller matrix values across each image (of the polarization active region) with the ideal (theoretical) values, using box plots. For each box plot, the middle line of the box represents the median value, while the top and bottom lines of the box represent the upper and lower quartile values, respectively. The top and bottom lines of the whiskers represent maximum and minimum values in the distribution, respectively. The dotted blue line in each box plot represents the ideal value. The image of each Mueller component is composed of 506×506 pixels, and thus the sample size N for every box plot shown above is $N = 506 \times 506 = 256,036$.

B. Imaging in Reflection: Reference-free Computation

We also use our system to image in reflection: we image *Chrysina gloriosa*, or the ‘chiral beetle’, results of which we display in Fig.5 of the main text. In this section, we explain our methodology to generate the final MM image using the raw and full-Stokes images of the beetle. We explained in Sec. S2D, when imaging in transmission, the use of a reference measurement for demodulation and to calibrate out intensity variations. Due to the lack of a suitable reference for imaging in reflection, we develop and use a ‘reference-free’ method for computing the final MM image.

Consider a one-dimensional carrier-wave, like the one we describe in Sec. S2D: $\alpha(x) \cos(kx + \phi)$. There are three important quantities defining the carrier wave: the period (the wave vector k), the phase (ϕ), and the slowly varying amplitude (α). The period is actually easy to estimate from the raw/full-Stokes image even without a reference measurement – by counting the number of pixels in the repeating ‘lines’ or ‘squares’ projected by the structured illumination. Knowing the period gives us the location of the channels 0,1,2, and 3 (Fig. S1D) in k -space. The phase and the varying amplitude are much harder to directly estimate without a reference measurement. In this example, we actually choose to ignore the effect of the varying amplitude (assuming it to be uniform). To work around the unknown phase, we use the image of one of the Stokes components (S_0 in our case) – which contains information about the first row of MM component images – as a ‘quasi-reference’ to demodulate the Mueller component images for all the other rows. As we explain below in more detail, this method gives reasonable results when used in conjunction with *prior* knowledge about the sample.

After estimating the period from the full-Stokes image, and hence the channel locations (0,1,2 and 3), we filter the spectra around these channels using bandpass filters. The images of the first column of the MM, with spectra centered at the origin of the Stokes images (channel 0), are rather straightforward to filter out: a low pass filter (of an appropriate size) centered at the origin gives us the images for the Mueller components in the first column – $M_{00}, M_{10}, M_{20}, M_{30}$ – as shown in Fig. S13A. The images are normalized with respect to the maximum value of the M_{00} image (Fig. S13B).

Now let us consider the image processing of the last column, for Mueller components $M_{0-3,3}$. The signal corresponding to the spectrum centered at channel 3 of the S_0 image, we can assume, approximately equals $\sim \sqrt{\frac{1}{3}} M_{03}(m, n) \cos(k_m m + \phi)$. (For brevity, we drop constant factors, such as $\sqrt{\frac{1}{3}}$, and the spatial (m, n) dependence of the Mueller components, in this and the following analysis.) We start by taking this signal – $M_{03} \cos(k_m m + \phi)$ – and multiplying it by itself to get: $M_{03}^2 \cos^2(k_m m + \phi) = \frac{M_{03}^2}{2} + \frac{M_{03} \cos(2(k_m m + \phi))}{2}$. Applying a low pass filter to the Fourier spectrum of $M_{03}^2 \cos^2(k_m m + \phi)$, gives us the spectrum of M_{03}^2 . Thus we can get the amplitude $\sqrt{M_{03}^2} = |M_{03}|$ through filtration; however, we lose information about any potential zero crossings which are important as Mueller values can vary between -1 and 1 . We can either assume there are no zero crossings in the sample, or use prior knowledge to fill in this information. In our case, we can assume that there are no zero crossings when it comes to our sample – the beetle’s shell. The signal $M_{03} \cos(k_m m + \phi)$ can serve as a reference to demodulate the signals in other rows of the same MM image column. For instance, consider the signal corresponding to the spectrum centered at channel 3 of the S_1 image: $M_{13} \cos(k_m m + \phi)$. We can multiply it by our reference $M_{03} \cos(k_m m + \phi)$ to get $M_{13} M_{03} \cos^2(k_m m + \phi) = \frac{M_{13} M_{03}}{2} + \frac{M_{13} M_{03} \cos(2(k_m m + \phi))}{2}$. Applying a low pass filter to the Fourier spectrum of $M_{13} M_{03} \cos^2(k_m m + \phi)$, gives us the spectrum of $M_{13} M_{03}$. If we divide this signal by $|M_{03}|$, we get $\frac{M_{13} M_{03}}{|M_{03}|} = \text{sgn}(M_{03}) M_{13}$ which gives us the spectrum of M_{13} (with the relative sign $\text{sgn}(M_{03})$). We can similarly demodulate and retrieve the images of the M_{23} and M_{33} components (Fig. S13A-B).

To demodulate and retrieve images for components in the second column ($M_{0-3,1}$) and the third column ($M_{0-3,2}$), we can similarly prepare quasi-references using spectra surrounding channels 1 and 2 of the S_0 image, but there is an added complication when it comes to these channels, which we address here. The delta functions related to the structured illumination components S_1 and S_2 actually occupy the same channels (1 and 2) in k -space, but are orthogonal to each other, as one is engineered to be symmetric (cosine) and the other one anti-symmetric (sine). Therefore, after we filter the spectra surrounding channels 1 and 2, we need to consider symmetric and anti-symmetric combinations so we can decouple the spectra of the two Mueller components. Once again, this is better understood with an example. Consider the signal corresponding to the spectrum centered at channels 1 and 2 of the S_0 image which, we can assume, approximately equals:

$$(M_{01} \cos(k_m m + \phi) + M_{02} \sin(k_m m + \phi)) \cos(k_n n + \phi) \quad (17)$$

Ignoring the common $\cos(k_n n + \phi)$, we carry on with the symmetric $M_{01} \cos(k_m m + \phi)$ and anti-symmetric

$M_{02} \sin(k_m m + \phi)$ parts:

$$\begin{aligned}
& M_{01} \cos(k_m m + \phi) + M_{02} \sin(k_m m + \phi) \\
&= \frac{M_{01}}{2} [\exp\{i(k_m m + \phi)\} + \exp\{-i(k_m m + \phi)\}] + i \frac{M_{02}}{2} [\exp\{i(k_m m + \phi)\} - \exp\{-i(k_m m + \phi)\}] \\
&= \left(\frac{M_{01}}{2} + i \frac{M_{02}}{2}\right) \exp\{i(k_m m + \phi)\} + \left(\frac{M_{01}}{2} - i \frac{M_{02}}{2}\right) \exp\{-i(k_m m + \phi)\}
\end{aligned}$$

Using the signal above, we create two different copies – a symmetric copy ① and an anti-symmetric copy ②, given as:

$$\begin{aligned}
\textcircled{1} &\Rightarrow \left(\frac{M_{01}}{2} + i \frac{M_{02}}{2}\right) \exp\{i(k_m m + \phi)\} + \left(\frac{M_{01}}{2} + i \frac{M_{02}}{2}\right) \exp\{-i(k_m m + \phi)\} \\
\textcircled{2} &\Rightarrow \left(\frac{M_{01}}{2} - i \frac{M_{02}}{2}\right) \exp\{i(k_m m + \phi)\} + \left(\frac{M_{01}}{2} - i \frac{M_{02}}{2}\right) \exp\{-i(k_m m + \phi)\}
\end{aligned}$$

By taking the sum and difference of ① and ②, we get the two signals (or images) with decoupled M_{01} and M_{02} :

$$\begin{aligned}
\textcircled{1} + \textcircled{2} &\rightarrow M_{01} (\exp\{i(k_m m + \phi)\} + \exp\{-i(k_m m + \phi)\}) = M_{01} \cos(k_m m + \phi) \\
i(\textcircled{2} - \textcircled{1}) &\rightarrow M_{02} (\exp\{i(k_m m + \phi)\} + \exp\{-i(k_m m + \phi)\}) = M_{02} \cos(k_m m + \phi)
\end{aligned}$$

$M_{01} \cos(k_m m + \phi)$ and $M_{02} \cos(k_m m + \phi)$ thus serve as quasi-references to demodulate and retrieve MM images for the remaining components in the second column ($M_{1-3,1}$) and the third column ($M_{1-3,2}$) respectively (Fig.S13A-B), and are processed using the same methodology as described above for the last column ($M_{0-3,3}$).

Fig. S13 shows the different images that correspond to the demodulation and filtration steps described above. While we handle the unknown phase ϕ using the quasi-reference approach, it is also important to appropriately normalize the final MM component images. As a general rule, due to the modulation transfer function (MTF) of an optical system, we know the contrast achievable at higher spatial frequencies is lower than the one achievable at lower spatial frequencies. Without a reference measurement, we would need knowledge of the sagittal and tangential MTFs to compute the normalization coefficients for Mueller components located at different spatial frequency channels. Furthermore, we would also need to make sure that the scalar factors ($1, \sqrt{2/3}, \sqrt{2/3}, \sqrt{1/3}$) introduced by the structured illumination (Eq. 1) are also used when computing normalization factors. Instead, in this proof of concept implementation, we take a more simplistic approach: we assume that the Mueller components located at the ‘DC’ channel 0 ($M_{0-3,0}$) have one normalization factor (the maximum of M_{00}), and the rest of the Mueller components ($M_{0-3,1-3}$) located at higher spatial frequency channels (1,2 and 3) all have another normalization factor (the maximum of M_{03}) same for all high frequency components.

Furthermore, there is an additional source of error that more adversely affects higher spatial frequencies than lower spatial frequencies: the ‘pixel misregistration’. Recall that to generate our full-Stokes image, we use a metasurface grating to create four separate copies of the scene (over some FOV) analyzed for four different polarization states. To compute the full-Stokes vector for each pixel in the image, we need to appropriately match (or ‘register’) the four pixels in the four different copies of the FOV, that correspond to the same pixel in the original scene [4]. This registration is not always perfect, partly due to optical aberrations. Any misregistration in this step, will show up as noise, more so in high spatial frequency channels (1,2 and 3) than the DC channel (0). Assuming our sample is a polarizer, we can perform a final normalization step where we normalize all the components in each row x by the factor $\frac{\max(M_{x0})}{\max(M_{00})}$, resulting in the final MM image shown in Fig. S13C.

The signal processing steps presented above, involving quasi-references and extra normalization steps, can be avoided altogether if an appropriate reference measurement is available, as we showed (Sec. S2D) in the case of transmission measurements. Therefore, it is always recommended to first perform a reference measurement, especially for applications requiring higher accuracies. Still, the concepts discussed in this section can be useful in scenarios where a reference measurement is unfeasible, allowing greater flexibility and enabling more applications. The beetle imaged in this work is a unique imaging sample with its curved geometry and a response that is fully chiral (i.e. S_1 and S_2 components are negligible). As a ‘consistency check’, we also perform a reference-free computation to demodulate and retrieve the MM image of ‘air’ (Fig. S14). In particular, using bandpass filters, we filter the spectra surrounding the channels 0, 1, 2 and 3. The images corresponding to Mueller components M_{00}, M_{10}, M_{20} and M_{30} are retrieved with the use of low-pass filters on S_0, S_1, S_2 and S_3 images respectively. The remaining Mueller components

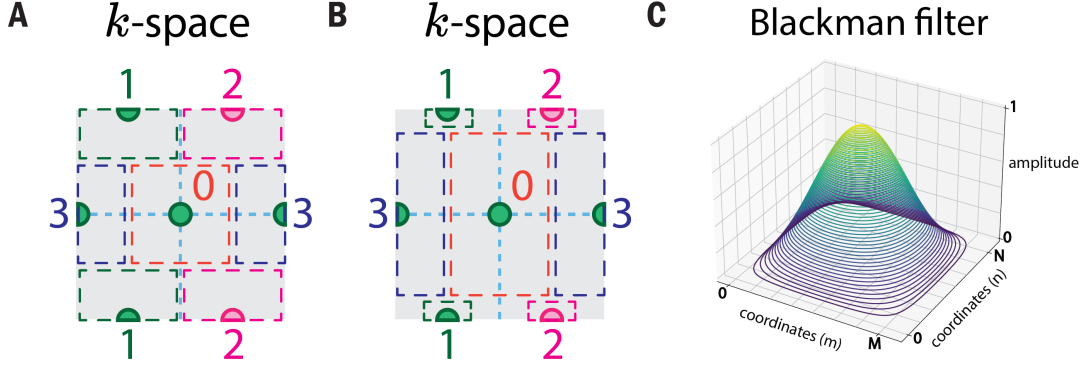


FIG. S12. **Fourier Filters.** (A) Square filters used in the Fourier domain to process MM images in transmission. The square filters ensure each Mueller component is assigned equal bandwidth during demodulation. (B) Rectangular filters used during the processing of the MM image for the beetle (in reflection). The rectangular filters assign more bandwidth to the relevant components at the expense of the components not relevant in this application. (C) Two-dimensional Blackman filter, applied as an intensity envelope, during the filtration process to avoid ‘ripples’ due to the sharp filter cutoffs.

($M_{0-3,1-3}$) are retrieved from channels 1,2 and 3; each signal is multiplied by itself followed by a low-pass filter to give the amplitudes ($|M|$) of the remaining Mueller components. The normalized images – where images in each column are normalized with respect to the maximum value in that column – are shown in Fig. S14B. For comparison, we also show the MM image of air when processed using the reference measurement (Fig. S14C).

Another important aspect to consider during the signal processing of these images is the type and size of filters applied in the Fourier space. Normally – as is the case in the transmission results shown in Sec. S3 A – we assign equal bandwidth to every channel (Fig. S12A). However, depending on the application, we can assign more bandwidth to some channels, at the cost of others. While processing the beetle image, we use rectangular bandpass filters to assign more bandwidth to channels 0 and 3, and much less bandwidth to channels 1 and 2 (Fig. S12B). (Using prior information about the sample, we can safely assume that the spectra surrounding channels 1 and 2 are negligible in comparison.)

Furthermore, for every filter we apply in the Fourier domain, we use a Blackman window (intensity envelope) to avoid sharp filter cutoffs (Fig.S12C). The Blackman window (in one dimension) is defined as:

$$w(n) = 0.42 - 0.5 \cos(2\pi n/N) + 0.08 \cos(4\pi n/N) \quad (18)$$

where N is the size of the window. Engineering the proprieties of the Fourier filters and windows gives us an additional degree of freedom to leverage in processing images for different application. The investigation and use of more sophisticated filters will be the subject of future work.

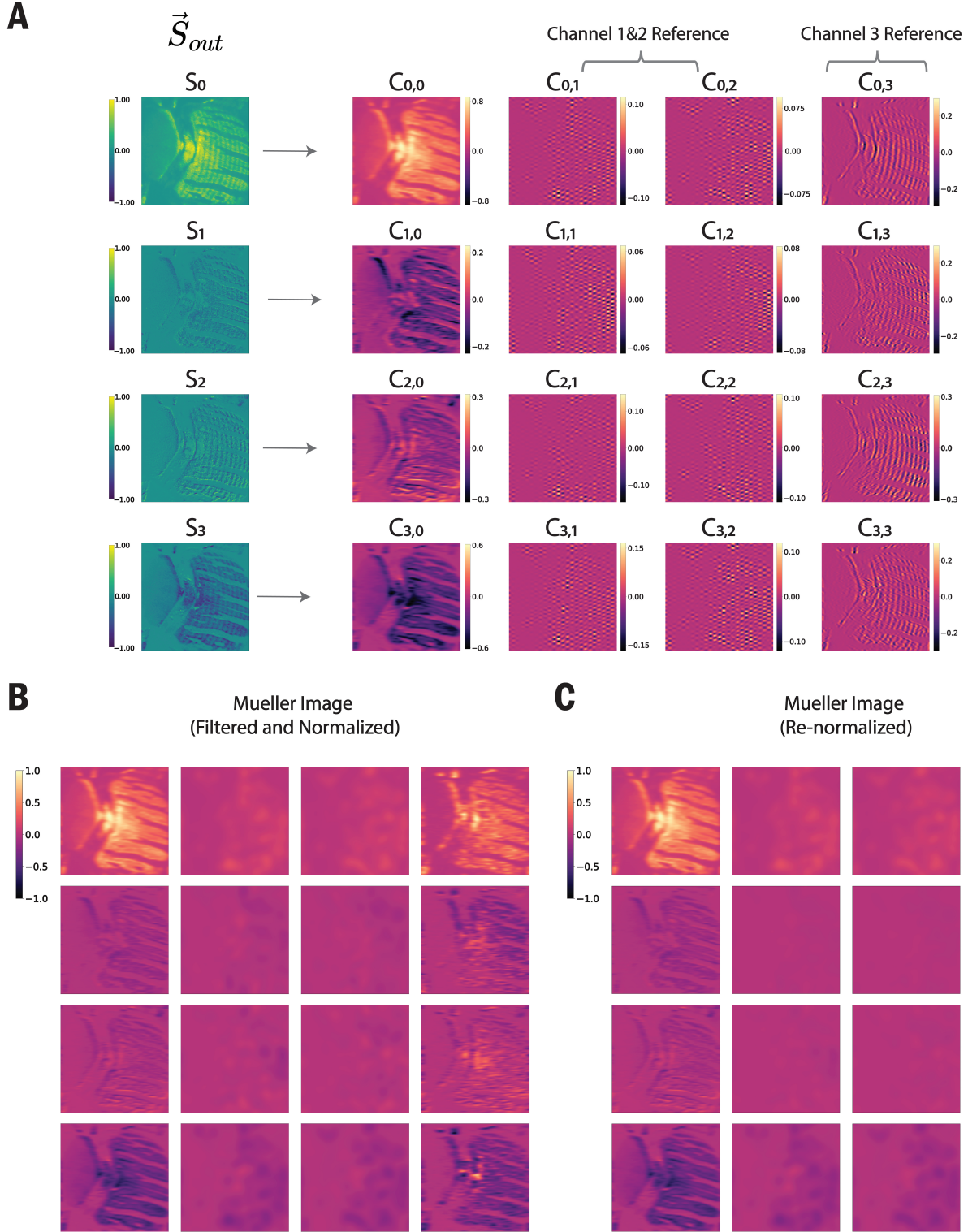


FIG. S13. **Beetle Mueller Matrix Computation – Reference Free Method.** (A) The information centered on the four different channels (0,1,2 and 3) is retrieved, using bandpass filters, from the images of every component of $\vec{S}_{out}$. The images from the first row are used as quasi-references to demodulate and retrieve images for other rows. (B) The MM image following low-pass filtering and normalization. (C) The MM image is re-normalized, assuming the sample being imaged is a polarizer.

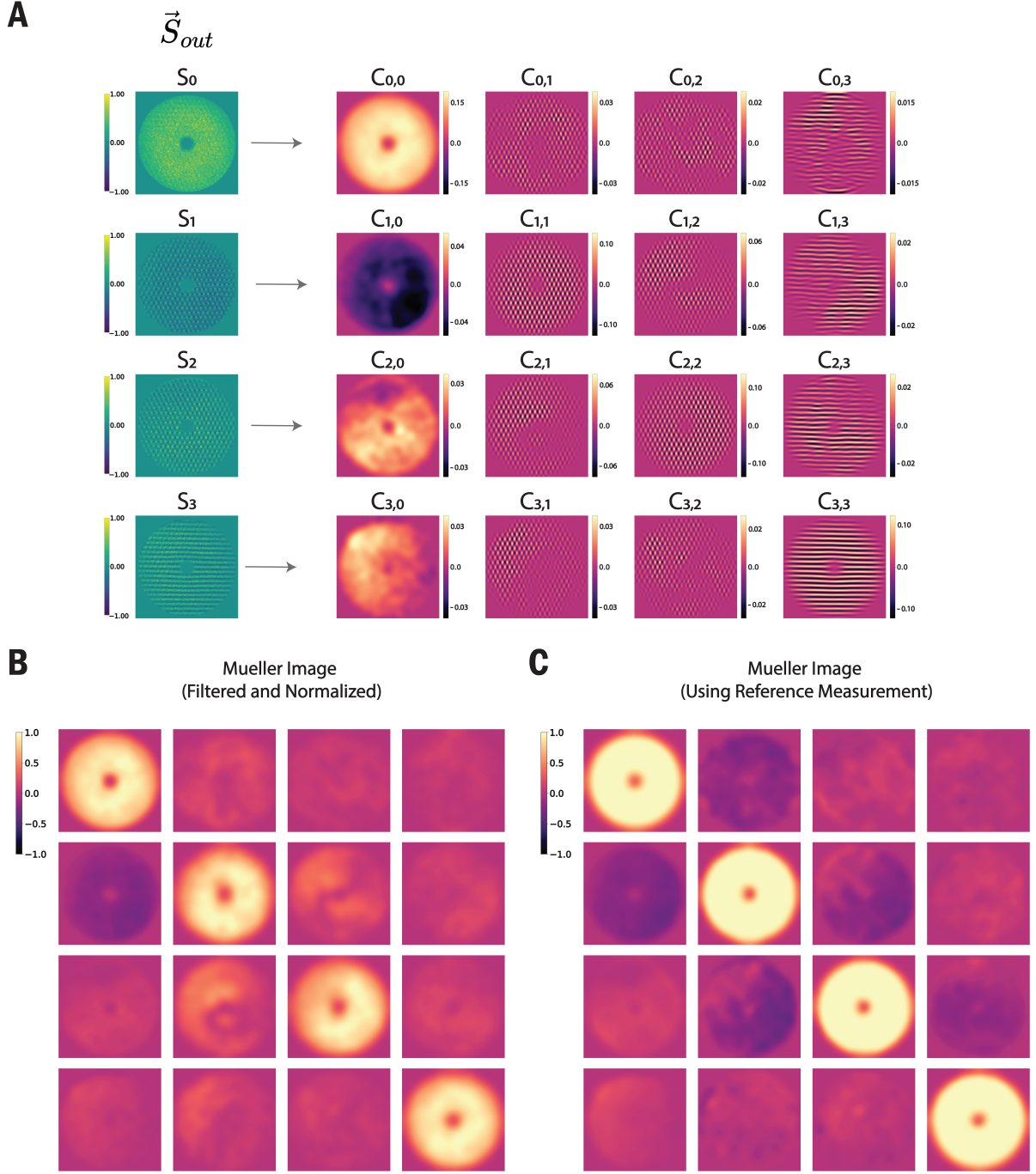


FIG. S14. **Air Mueller Matrix Computation – Reference Free Method.** (A) The information centered on the four different channels (0,1,2 and 3) is retrieved, using bandpass filters, from the images of every component of $\vec{S}_{out}$. (B) The MM image following low-pass filtering and normalization. (C) The MM image when a reference measurement is used.

S4. PREVIOUS WORKS AND FUTURE DIRECTIONS

A. This Work vs. Past Works: A Comparison

Work	Number of Polarization Elements	Types of Polarization Elements	Experimental Validation?	Single-shot?	Complete?	Additional Notes
This work	2	Metasurfaces	Yes	Yes	Yes	(i) Same fabrication for both metasurfaces. (ii) Extremely compact .
Tyo et al. 2023 [8]	2	SLM, DoFP sensor	Limited	Yes	No	Measures linear-only states (9/16 Mueller components).
Angelo et al. 2019 [9]	6	Liquid-crystals, Polarizers	Limited	No	Yes	Long acquisition time (spatial frequency sweep).
Huang et al. 2021 [10]	5	Waveplates, Polarizers, DoFP sensors	Limited	No	Yes	Additional beam splitter use.
Kudenov et al. 2012 [11]	12	Waveplates, Polarizers, Polarization Gratings	No	Yes	Yes	Channel redundancy (limits resolution and increases cross-talk).
Cao et al. 2023 [12]	8	Waveplates, Polarizers, Savart Plates	Limited	Yes	Yes	Channel redundancy (limits resolution and increases cross-talk).
Zaidi et al. 2022 [13]	2-3	Metasurface or SLM, DoFP sensor	No	Yes	Yes	Theory only – practical components not fully elaborated .

TABLE I. A comparative summary of this work and previous works proposing various architectures for Mueller matrix imaging. Strengths are highlighted by green, while shortcomings are highlighted by red.

Table I compares our work with other, relatively recent, attempts at designing a MM imaging system to try to overcome the limitations of a standard division-of-time (rotating polarizers/waveplates dependent) MM imaging system. Most of these proposed architectures require a combination of many different polarization components such as polarization gratings, Savart plates, division-of-focal-plane (DoFP) sensors, spatial light modulators (SLMs), waveplates, polarizers and beamsplitters, making them extremely challenging to implement. This is perhaps one of the main reasons why these works focus entirely on theoretical/computational analysis without an experimental validation. Our work demonstrates a practical single-shot Mueller matrix imaging system that can acquire images for all sixteen Mueller matrix components, and do so compactly by using two extremely thin metasurfaces for polarization generation

and analysis. Both metasurfaces are fabricated involving the exact same fabrication steps, potentially allowing the consolidation of the supply chain in mass manufacturing of MM imaging modules for applications such as smartphone facial recognition and eye-tracking for AR/VR [14].

B. Metasurface Wavelength Dependence and Potential for Spectral Mueller Imaging

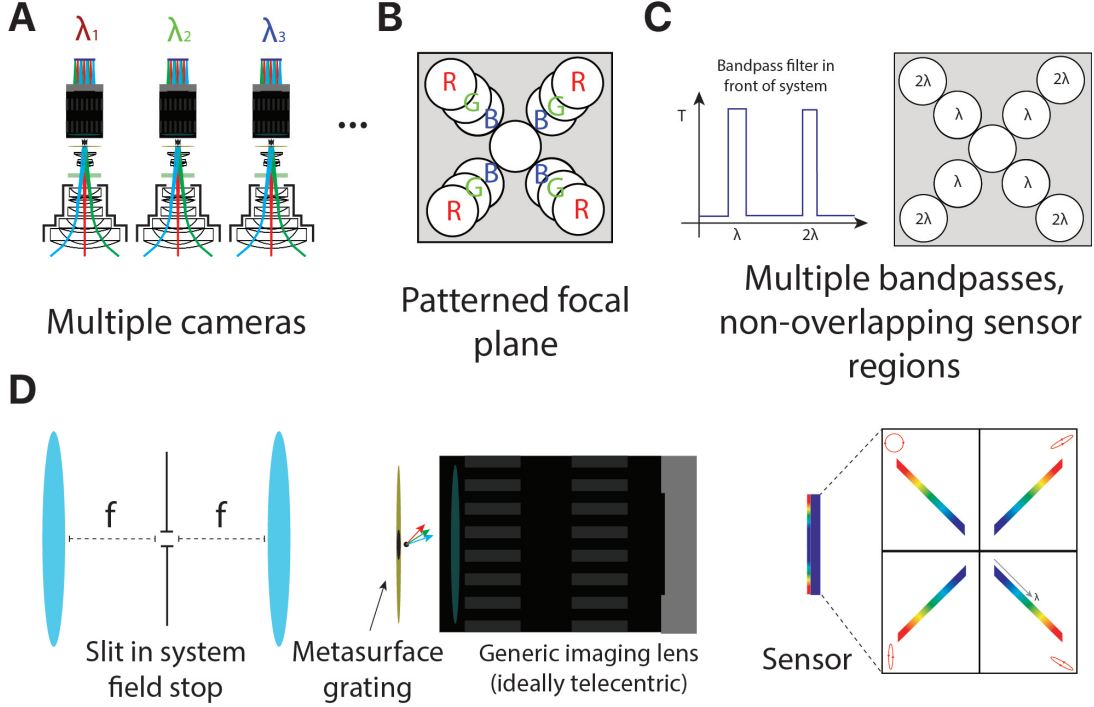


FIG. S15. **Methods for Multispectral or Hyperspectral Polarization Imaging.** Different strategies for multi/hyperspectral polarization imaging, as detailed in the text below.

The wavelength dependence of TiO_2 metasurfaces has been well researched and documented in previous studies [15, 16], especially in the design of achromatic [17] and dispersion-engineered metasurfaces [18]. In the context of metasurface (intensity) holograms, we have empirical evidence that a hologram designed for one particular wavelength (for e.g. the green wavelength), ‘works’ (i.e. displays the holographic pattern) over a broadband of wavelengths (the entire visible spectrum) – this was confirmed for both geometric phase metasurfaces [16] and propagation phase metasurfaces [19] in our previous works. This may be because as a material, (i) TiO_2 is not highly dispersive in the visible, and (ii) the individual nanopillars that make up a metasurface may impart different absolute phases at different wavelengths, but still impart similar relative phase differences (between nanopillars) at different wavelengths over a broad band. (Because the far-field pattern is a result of interference, it is the relative phase difference between responses of the nanopillars that matters.)

A metasurface polarization hologram is designed using the phase retrieval algorithm, similar to the design of a metasurface intensity hologram, and operates also through interference in the far-field. Therefore, we expect a similar broadband response of our designed polarization hologram used for structured polarization illumination. The discrepancies found in the structured polarization illumination at different wavelengths can also be calibrated out using a suitably measured instrument matrix.

Similar to the metasurface hologram, the polarization analyzer grating can also function reasonably well over a broadband of wavelengths [3]. Once again, a suitably measured instrument matrix can be utilized (as a function of wavelength) to calibrate out discrepancies. This means that even if we design our MM imaging system for a given wavelength, the user could be given the option to operate at other wavelengths by choosing from a library of different instrument matrices measured at different (discrete) wavelengths (perhaps provided as part of the accompanying software).

While the method involving different instrument matrices works for single-shot MM images acquired separately at distinct wavelengths, it does not allow the acquisition of single-shot multispectral (simultaneously acquired for

multiple discrete wavelengths) or hyperspectral (simultaneously acquired over a broadband of wavelengths) MM images. For multispectral MM imaging, the structured polarization illumination module can be paired with one of the three different architectures shown in Fig.S15 [3]. The first of these (Fig.S15A) is rather straightforward: separate metasurface-based polarization cameras can be designed for each desired wavelength (using bandpass filters) and co-aligned as a system. However, the use of multiple cameras would make the system bulky. A second (Fig.S15B) architecture involves patterning a color filter array (such as R-G-B) onto the sensor, and using demosaicing techniques to get independent multispectral polarization images from overlapping images focused onto the sensor. A third architecture involves placing a special dual bandpass filter in front of the imaging system. Images centered on the grating angles corresponding to central wavelengths of the first harmonic λ and second harmonic 2λ will have no overlap on the camera sensor. This would allow for dual-band imaging provided that filter with two bandpasses at λ and 2λ is used.

A fourth possibility (Fig.S15D) may allow the acquisition of hyperspectral MM images by explicitly capitalizing on the metasurface grating's inherent chromatic angular dispersion. In this architecture, an afocal system is placed in front of the grating/imaging lens pair with a slit at its center, acting as a field stop. As shown in Fig.S15D, the slit, which is imaged onto each of the four diffraction orders, is smeared radially outward due to the grating's chromatic dispersion, with longer wavelengths diffracting at higher angles. The long edge of the slit (which images along the vertical direction) corresponds to the system's vertical field-of-view (FOV). The short edge of the slit (which images along the horizontal direction here) sets the system's instantaneous field-of-view (IFOV). The vertical direction, then, corresponds to the spatially-resolved FOV, while the radial direction corresponds to wavelength. The imaging of color information along the radial direction means that only one spatial dimension can be simultaneously imaged. To image the other spatial dimension, the camera would need to be rotated or translated. Ideally, the short edge of the slit (IFOV) would be infinitely small. A finite sized IFOV limits the horizontal resolution of the image that can be reconstructed. and the slit's width also limits the chromatic resolution of the imaging system. However, reducing the width of the slit also decreases the system's radiative throughput, so there exists a trade-off between exposure time and resolution.

These proposed architectures to extend our single-shot MM imaging system for multi/hyper spectral MM imaging come with drawbacks including additional bulk, volume, complexity, acquisition time and cost – drawbacks that we set out to eliminate in our current work. Some of these architectures may be feasible in certain applications, or may be the 'best' option with regards to the current state-of-the-art. Our work does provide a solid foundation to build on, but more work needs to be done to design a simple, compact, single-shot multi/hyper spectral MM imager, which is beyond the scope of this work. This would be the subject of future research.

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