

Endlessly self-injection-locked photonic integrated lasers

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I. NUMERICAL SIMULATION OF MULTI-RESONANCE SELF-INJECTION LOCKING

The system consisting of a laser coupled to k longitudinal microresonator modes can be modelled by numerically integrating the corresponding field rate equations [1–3]. Here, the laser is described by the intracavity field amplitude A_L and carrier density N , and the microresonator by k longitudinal modes, for which the clockwise and counterclockwise fields corresponding to the i^{th} resonance are $A_{i,\text{CW}}$ and $A_{i,\text{CCW}}$, respectively:

$$\frac{dN}{dt} = \frac{I_{\text{bias}} + \Delta I}{eV} - \kappa_N N - gV(N - N_0)|A_L|^2, \quad (\text{S1})$$

$$\frac{dA_L}{dt} = \left[(1 + i\alpha)(N - N_0)gV - \frac{\tilde{\kappa}_{\text{LD}}}{2} + i\omega_{\text{LD}} \right] A_L + i\zeta \Delta I A_L + \sqrt{\kappa_{\text{bus}}\kappa_{\text{LD}}} \frac{\tau_0}{\tau_L} \sum_i^k e^{i\omega_i \tau_{\text{bus}}} A_{i,\text{CW}} + F_A, \quad (\text{S2})$$

$$\frac{dA_{j,\text{CCW}}}{dt} = \left(i\omega_j - \frac{\tilde{\kappa}}{2} \right) A_{j,\text{CCW}} + \sqrt{\kappa_{\text{bus}}\kappa_{\text{LD}}} \frac{\tau_L}{\tau_0} e^{i\omega_j \tau_{\text{bus}}} A_L, \quad (\text{S3})$$

$$\frac{dA_{j,\text{CW}}}{dt} = \left(i\omega_j - \frac{\tilde{\kappa}}{2} \right) A_{j,\text{CW}} - R_M \kappa_{\text{drop}} e^{i\omega_j (2\tau_R + 2\tau_{\text{drop}})} A_{j,\text{CCW}}. \quad (\text{S4})$$

In the laser rate equations, I_{bias} is the biased injection current, ΔI the injection current modulation, e is the electron charge, V is the volume of the active region, κ_N is the carrier recombination rate, g is the carrier gain coefficient, N_0 is the carrier density at transparency, α is the linewidth enhancement factor, $\tilde{\kappa}_{\text{LD}} = \kappa_L + \kappa_{\text{LD}}$ includes the intrinsic laser cavity loss rate κ_L and the external coupling rate κ_{LD} , ω_{LD} is the free-running laser frequency, ζ is the current-frequency tuning coefficient, τ_L is the round-trip time of the laser cavity, and F_A is a Langevin noise term describing spontaneous emission into the lasing field.

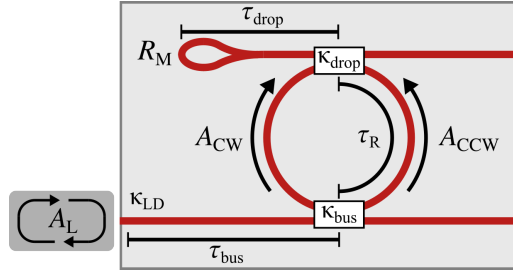


Figure S1. Schematic of the laser-resonator system with a mirror-assisted drop port.

The noise in carrier density fluctuations can be ignored as its contribution is negligible compared to the noise in the optical field [4]. In the coupled resonator mode equations, ω_i is the resonance frequency of the i^{th} longitudinal mode, $\tilde{\kappa} = \kappa_0 + \kappa_{\text{bus}} + \kappa_{\text{drop}}$ is the loaded cavity loss rate including the intrinsic loss rate κ_0 , external coupling to the bus waveguide κ_{bus} and external coupling to the drop port κ_{drop} , τ_0 is the round trip time of the microresonator cavity, R_M is the reflectivity of the drop port mirror, and τ_R and τ_{drop} are the time delays between the bus and drop coupler in the resonator and the drop port and the mirror, respectively.

The Langevin noise term F_A describing spontaneous emission can be considered to originate from a white noise process with a Gaussian probability distribution [4]. Numerically, F_A can thus be implemented in the rate equations as

$$F_A = \xi \sqrt{\frac{2R_{\text{sp}}N}{\Delta t}}, \quad (\text{S5})$$

where ξ is a zero-mean, unit-variance Gaussian distribution, R_{sp} is the spontaneous emission coefficient, N is the carrier density, and Δt is the time step used in the numerical solver. The Langevin noise term is used to initiate stimulated optical amplification in the absence of an initial field.

To investigate the self-injection locking dynamics in this work, we integrate Eqs. S1-S4 using a Runge-Kutta solver. For Figs. 1,2 and 4 (H) in the main text we sweep the injection current offset ΔI , while for Fig. S3 we sweep the time delay τ_{bus} , effectively sweeping the feedback phase, while keeping all other parameters fixed, as given in Table S1.

Table S1. **Parameters used in the numerical simulations.**

Symbol	Value	Unit	Definition
$\hbar$	1.05×10^{-34}	$\text{m}^2\text{kg s}^{-1}$	Reduced Planck constant
e	1.6×10^{-19}	C	Elementary charge
κ_{N}	1×10^9	s^{-1}	Carrier recombination rate
N_0	1×10^{24}	m^{-3}	Carrier density at transparency
I_{bias}	0.25	A	Injection current bias
g	1×10^6	m^3s^{-1}	Carrier gain coefficient
V	1×10^{-16}	m^3	Volume of the active region
α	5		Linewidth enhancement factor
κ_{L}	$2\pi \times 5 \times 10^{10}$	rad s^{-1}	Laser intrinsic loss rate
κ_{LD}	$2\pi \times 1.1 \times 10^{10}$	rad s^{-1}	Laser external coupling rate
ζ	1×10^{13}	$\text{s}^{-1}\text{A}^{-1}$	Current-frequency tuning coefficient
R_{sp}	2×10^5	s^{-1}	Spontaneous emission coefficient
τ_{L}	1×10^{-12}	s	Laser cavity roundtrip time
κ_0	$2\pi \times 17 \times 10^6$	rad s^{-1}	Resonator intrinsic loss rate
κ_{bus}	$2\pi \times 18 \times 10^6$	rad s^{-1}	Resonator-bus coupling rate
κ_{drop}	$2\pi \times 18 \times 10^6$	rad s^{-1}	Resonator-drop-port coupling rate
τ_0	1×10^{-12}	s	Resonator roundtrip time
R_{M}	1		Drop port mirror reflectivity

II. MODE SELECTION BETWEEN SELF-INJECTION LOCKED STATES

A self-injection locked laser can have multiple stable stationary states at any given operating point, as illustrated in Fig. S2. In this sense, the system is similar to an external cavity laser with multiple stable external cavity modes, which is a system that has been studied extensively [5–7].

Generally, all stationary states of a SIL system (with a positive $\partial\omega/\partial\omega_{\text{LD}}$) are stable in the absence of system perturbations, since they represent (at least local) potential minima in a potential landscape that is imposed on a background potential valley centered at the free-running laser frequency, in analogy to external cavity lasers [7]. However, perturbations in the form of noise, for example, can induce mode hopping between these stationary states [8]. For external cavity lasers, the average time T for a mode hop to occur has been shown to scale as $T \propto e^{-\Delta V/2\pi\delta\nu}$, where $\delta\nu$ is the linewidth and ΔV the height of the potential well that the mode occupies [7]. Analogously, for SIL lasers, the ultra-narrow linewidth of a self-injection locked state – multiple orders of magnitude lower than the free-running state – is crucial in enabling SIL lasers with mode spacings of only hundreds of megahertz to stably lase in a self-injection locked state over long time periods [9–11]. In addition to enabling stable laser operation in complex potential landscapes, the ultra-narrow linewidths of stationary SIL states also govern the selection

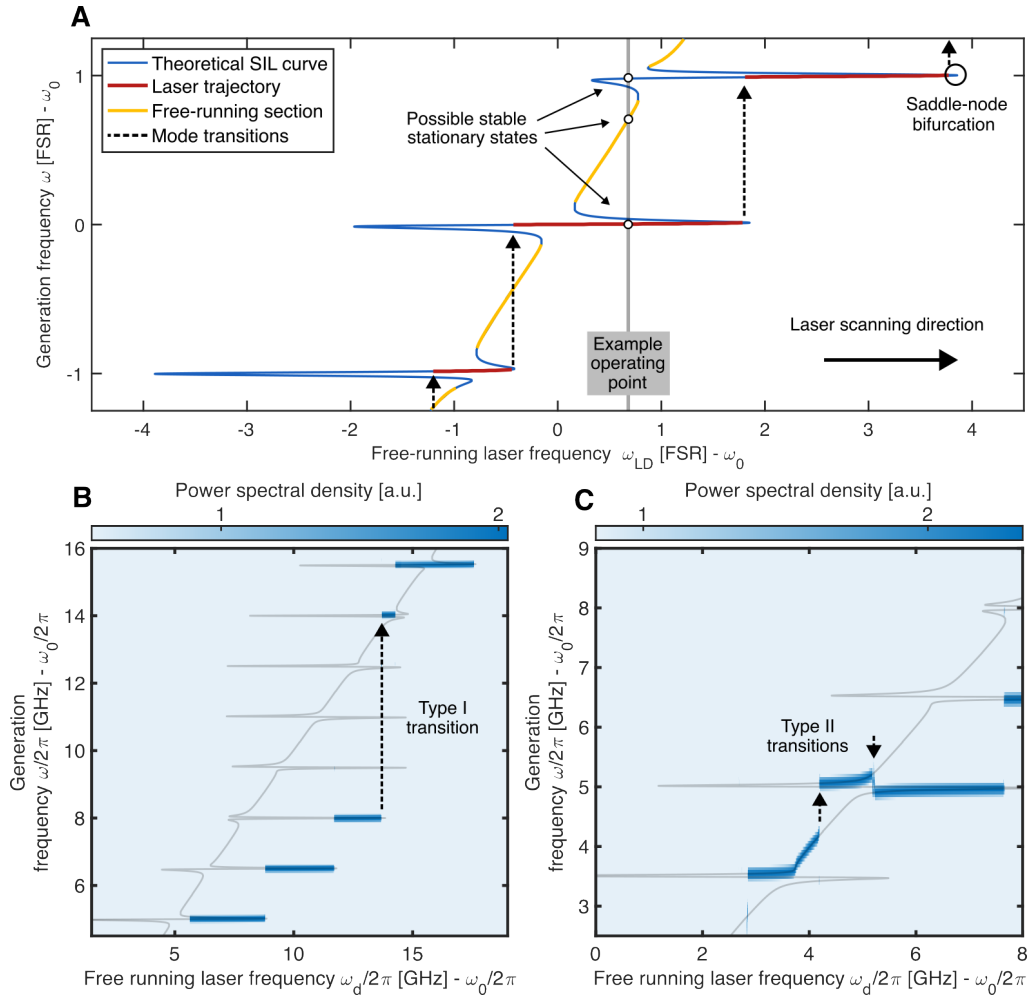


Figure S2. **Stationary states of a multi-resonance self-injection locking curve and numerically simulated laser trajectories during injection current tuning.** (A) Theoretical SIL tuning curve with the simulated laser trajectory, showing mode transitions between self-injection locked states. (B)-(C) Numerically simulated spectrograms of the generation frequency during injection current tuning, with *Type I* and *Type II* transitions visible.

dynamics between different stationary states. Here, we demonstrate this experimentally together with a qualitative explanation of the observed behavior, leaving the details of a quantitative study for future work.

As the operating point of the laser is tuned, for example, by tuning the injection current or feedback phase, the laser trajectory follows the SIL tuning curve, as shown in Fig. S2. Consequently, as the slope of the tuning curve changes, the laser linewidth of a state can grow such that a mode transition becomes probable. Alternatively, an ideal, noiseless laser can reach a saddle-node bifurcation point, where the stable and unstable branches annihilate, as shown in Fig. S2, forcing the laser to mode hop. In any real system, however, mode hopping will occur before the laser can reach such a saddle-node bifurcation point, since the linewidth reduction factor approaches zero when approaching a saddle-node bifurcation. Thus, unlocking events from a self-injection locked state – experimentally observed as sharp power changes or near-instantaneous frequency changes in the spectrum as seen in Fig. 2 in the main text – are always noise-induced.

During a mode transition, the instantaneous laser generation frequency will experience a transient-like change, governed by the underlying potential landscape. In what we denote a *Type I* transition (Fig. S2), the laser is unlocked from a locked state

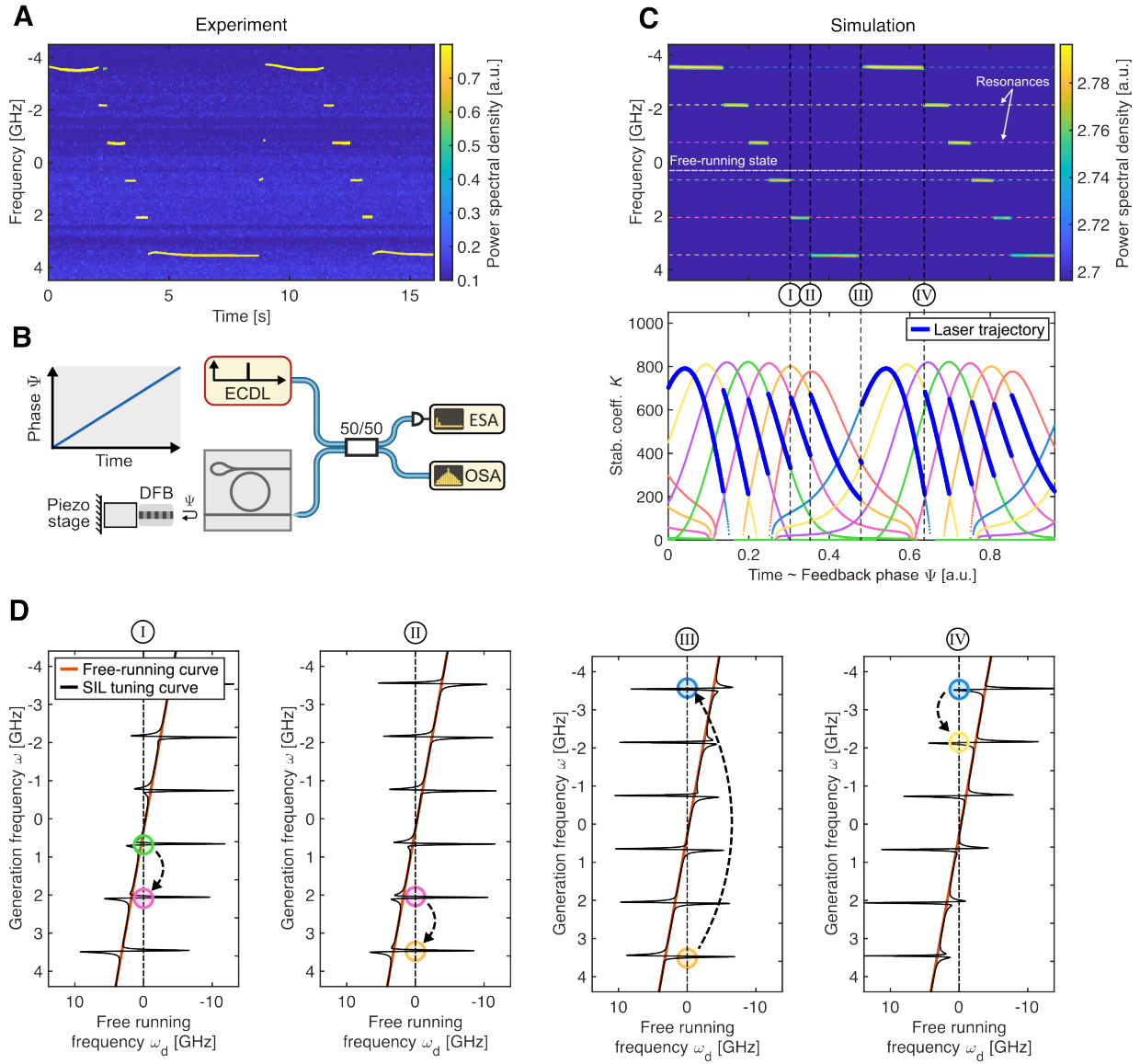


Figure S3. **Demonstration of mode selection between self-injection locked states by tuning the collective feedback phase.** (A) Experimental spectrogram of the generation frequency evolution, as the feedback phase of all resonances is swept in unison. (B) Experimental setup, where the feedback phase to all resonances is changed equally by tuning the laser-to-PIC distance. (C) Simulated spectrogram with identical parameters to the experiment in panel (A), showing close agreement. The stabilization coefficients $K = \partial\omega_{LD}/\partial\omega$ of all the available stationary states are shown, demonstrating mode selection to a neighboring mode with a higher K (i.e. narrower linewidth). (D) Stationary tuning curve at the mode-hopping points highlighted in panel (C), demonstrating, that as the feedback phase of the resonances is tuned, the laser initiates a mode hop when the stabilization coefficient (and with it the linewidth narrowing factor) is reduced sufficiently such that the laser selects a state with a narrower linewidth.

that is far away from the free-running frequency – often close to the edge of a locking range – so that the generation frequency will generally get pulled towards the free-running laser frequency until assuming operation in another stationary state, as seen in the experimental and simulated spectrograms in Figs. 3 (J) and 2 (F) of the main text, respectively. Conversely, a *Type II* mode transition (Fig. S3) is initiated when the laser is pulled from a stable stationary state directly into another stable state, which usually has a narrower linewidth, such as observed in the spectrograms in Figs. 2 (G)-(H) of the main text, where the laser leaves

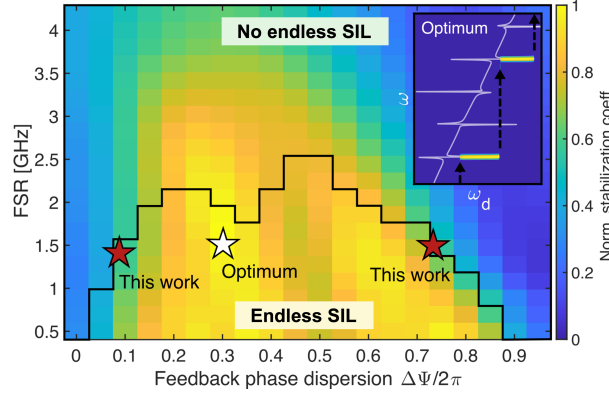


Figure S4. **Optimal device design for endless SIL.** Numerically simulated average stabilization coefficients as a function of FSR and $\Delta\Psi$. The inset shows a portion of the optimal tuning curve with the corresponding numerically simulated current scan. Since $\Delta\Psi$ has a period of 2π , $\Delta\Psi > 0$ corresponds to $\Delta\Psi/2\pi \in (0, 0.5)$ and $\Delta\Psi < 0$ corresponds to $\Delta\Psi/2\pi \in (0.5, 1)$.

an otherwise stable free running state and is pulled directly into a locked state.

These mode transition mechanisms are demonstrated experimentally in Fig. S3, where we use the same laser-microresonator combination as in the endless self-injection locking demonstrations of the main text. Here, the laser is lasing in a self-injection locked state while multiple neighboring states have overlapping locking regimes, together providing multiple simultaneously stable stationary states for the laser. Starting from point (I) in Fig. S3 (B), the collective feedback phase (i.e. the term in the feedback phase that is equal to all the resonances) is tuned such that the locking regime shapes of all resonances change in unison. As a consequence, the linewidth of the initially lasing mode becomes broader and the laser is observed to jump to a neighboring, narrower linewidth mode *away from the free-running frequency*. This process repeats, until the generation frequency ends up far enough from the free-running frequency, such that the background potential gradient becomes steep enough to prevent further mode transitions away from the free-running frequency. These mode hops represent *Type II* transitions as described above, where the linewidth of a lasing stationary state becomes large enough such that a narrower-linewidth neighboring state is able to capture the laser, in this case even pulling the laser further away from the free-running laser frequency. When the collective feedback phase is tuned further, we can observe a *Type I* transition, where the laser is seen to jump over multiple FSRs to a narrow linewidth mode, crossing over multiple stationary states that have relatively large linewidths due to the periodic mode landscape determined by the feedback phase dispersion, and thus are not able to capture the laser during the frequency transient. This transition type can be observed in the main text both in simulations and experiments shown in Figs. 2 (F) and 3 (J), where the laser jumps over multiple non-optimal self-injection locking states, locking only to modes with a optimal linewidth narrowing factor.

III. OPTIMAL DEVICE DESIGN FOR ENDLESS SELF-INJECTION LOCKING

Fig. S4 shows the average stabilization coefficient as a function of FSR and $\Delta\Psi$ during laser scans across multiple microresonator resonances, obtained from numerical simulations matched to the parameters of our experimental setup. As expected, a non-zero feedback phase dispersion is required for endless SIL: for $\Delta\Psi = 0$, the laser becomes unlocked even at a dense mode spacing of 500 MHz, whereas virtually any non-zero $\Delta\Psi$ ensures endless SIL. Furthermore, positive values of $\Delta\Psi$ support endless SIL with a high stabilization coefficient, even for large FSRs extending beyond 2 GHz, while for a negative $\Delta\Psi$ both the maximum

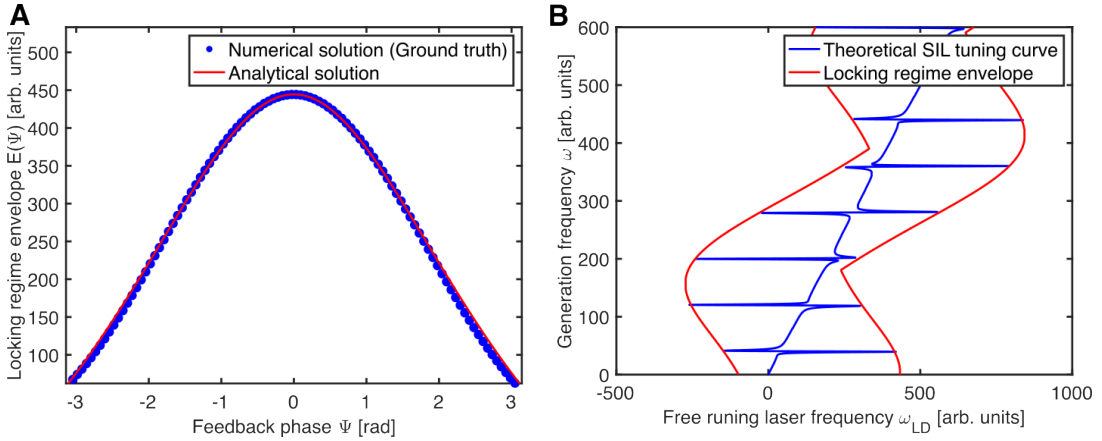


Figure S5. **Analytical solution for the self-injection locking regime envelope.** (A) The analytical solution for the locking regime envelope (Equation S12) shows good agreement with numerically calculated values. (B) The analytically calculated locking regime envelope (Equation S12) of a multi-resonance SIL tuning curve.

allowed FSR and stabilization coefficient are lower.

The stabilization coefficient is maximized at $\Delta\Psi/2\pi \approx 0.3$, because the short feedback phase period, spanning only three microresonator resonances, restricts the laser to lock only onto states with the most favorable feedback phase (see inset in Fig. S4). In contrast, although the shortest feedback phase period, corresponding to $\Delta\Psi/2\pi = 0.5$, enables endless SIL for the largest FSR, it does not prevent locking to low- K resonances, reducing the average stabilization coefficient. We note, however, that maximizing the average stabilization coefficient does not necessarily translate into proportional linewidth improvement in practice, because the stabilized linewidth ultimately approaches the thermorefractive-noise (TRN) floor of the locking cavity. Thus, once the device is near TRN-limited operation, as for the device demonstrated in this work (see Fig. 4C in the main text), further gains from tuning $\Delta\Psi$ toward the optimum primarily yield an improved safety margin with respect to other device variables, such as small variations in intrinsic loss rates, rather than yielding substantially larger linewidth-reduction factors.

Because the optimal feedback phase dispersion leads to selective locking to resonances with a favorable feedback phase $\Psi \in [-\pi, 0]$, it also facilitates a passive current tuning strategy that enhances the robustness of the SIL state against mode hops during extended operation. An unidirectional current ramp-up during the startup of a SIL laser with optimal $\Delta\Psi$ traverses the high- K locking branches asymmetrically, similarly to Fig. S2 (A). This can leave the laser in a state with a low stabilization coefficient and close to a mode hop transition, that could be initiated by noise or small drifts in system parameters during extended operation. To prevent the laser assuming continuous operation in such a state, a constant short section of opposite scan can be included as the final part of a current startup routine, on average positioning the laser closer to the stable central part of locking branch.

IV. SELF-INJECTION LOCKING RANGE

The stationary self-injection locking range at the optimal feedback phase ($\Psi = -\pi/2$ with the notation used in the main text) has been derived for example in [1] for the case of a Rayleigh-backscattering-induced optical feedback. Here, we derive the stationary self-injection locking range for the drop-port terminated mirror-induced optical feedback and for an arbitrary feedback phase.

The equation for the stationary SIL tuning curve and backreflection from a microresonator with a mirror-terminated drop port (Eqs. 1 and 2 in the main text) are

$$\omega - \omega_{LD} = \kappa_{LD} \text{Im}[\Gamma(\omega - \omega_C) e^{-i \arctan(\alpha)}], \quad (\text{S6})$$

$$\Gamma(\omega - \omega_C) = -\frac{i \kappa_{\text{bus}} \kappa_{\text{drop}} R_m e^{i\Psi}}{[\kappa_{\text{tot}}/2 - i(\omega - \omega_C)]^2}, \quad (\text{S7})$$

Equations S6 and S7 can be written together as:

$$\omega_{LD} - \omega_C = \Delta\omega - \kappa_{LD} \kappa_{\text{bus}} \kappa_{\text{drop}} R_m \frac{\kappa_{\text{tot}} \Delta\omega \sin \Psi - [\kappa_{\text{tot}}^2/4 - \Delta\omega^2] \cos \Psi}{(\kappa_{\text{tot}}^2/4 + \Delta\omega^2)^2}. \quad (\text{S8})$$

The maximum and minimum values of Equation S8 w.r.t Ψ , here denoted as the envelope $E(\Psi)$, determine the stationary locking range. The envelope cannot be analytically found, but we note that it closely follows the upper half of a Lorentzian (Fig. S7):

$$\max_{\Psi}(\omega_{LD} - \omega_C) = E(\Psi) = \left(\frac{2\pi^2}{\Psi^2 + \pi^2} - 1 \right) (\max[E(\Psi = 0)] - \min[E(\Psi = 0)]) + \min[E(\Psi = 0)], \quad (\text{S9})$$

where the extrema at $\Psi = 0$ are straightforward to solve from Equation S8:

$$\max[E(\Psi = 0)] = \frac{4\kappa_{LD} \kappa_{\text{bus}} \kappa_{\text{drop}} R_m}{\kappa_{\text{tot}}^2} \quad (\text{S10})$$

$$\min[E(\Psi = 0)] = -\frac{\sqrt{3}}{2} \kappa_{\text{tot}} - \frac{\kappa_{LD} \kappa_{\text{bus}} \kappa_{\text{drop}} R_m}{2\kappa_{\text{tot}}^2}, \quad (\text{S11})$$

Thus, the envelope $E(\Psi)$ becomes

$$E(\Psi) = \frac{\kappa_{LD} \kappa_{\text{bus}} \kappa_{\text{drop}} R_m}{\kappa_{\text{tot}}^2} \left(\frac{7\pi^2}{\Psi^2 + \pi^2} - 3 \right) - \sqrt{3} \kappa_{\text{tot}} \left(\frac{\pi^2}{\Psi^2 + \pi^2} - 1 \right), \quad (\text{S12})$$

Which agrees well with the numerically solved locking range extrema for all $\Psi \in [-\pi, \pi]$ as shown in Fig. S7 (a). The locking range $\Delta\omega_{\text{lock}}$, which is defined only for $\Psi < 0$ since the transition between the locked state and free-running state is not well defined for $\Psi > 0$ [see main text Fig. 1 (e)], thus becomes

$$\Delta\omega_{\text{lock}} = E(\Psi) + E(\Psi + \pi). \quad (\text{S13})$$

V. PHOTONIC INTEGRATED MICRORESONATOR CHARACTERIZATION

We characterize the passive optical properties of the photonic integrated microresonator used in this work with a custom scanning diode laser spectrometer, described in detail in Ref. [12]. The setup records both the bus port and drop port transmittances (see Fig. S1) as the probe laser is scanned over multiple resonances. This allows us to characterize both the bus port and drop port coupling rates κ_{bus} and κ_{drop} , respectively, by taking into account the frequency splitting in the drop port field [3]. The mirror-termination of the drop waveguide induces a frequency splitting only in the outcoupled field from the drop waveguide, while no splitting is present in the bus port transmission in the case of negligible Rayleigh backscattering, as can be observed in the experimental traces in Fig. S6. The splitting is given by

$$\Delta\omega_{\text{drop}} = \left(2[\gamma^4 + 4\gamma^3 \kappa_{\text{drop}} + 5\gamma^2 \kappa_{\text{drop}}^2 + 2(\kappa_C + \kappa_{\text{bus}})(\gamma^2 + 3\gamma \kappa_{\text{drop}} + 2\kappa_{\text{drop}}^2)(\kappa_C + \kappa_{\text{bus}} - \gamma)]^{1/2} - (\gamma + \kappa_{\text{drop}} - \kappa_C - \kappa_{\text{bus}})^2 \right)^{1/2}, \quad (\text{S14})$$

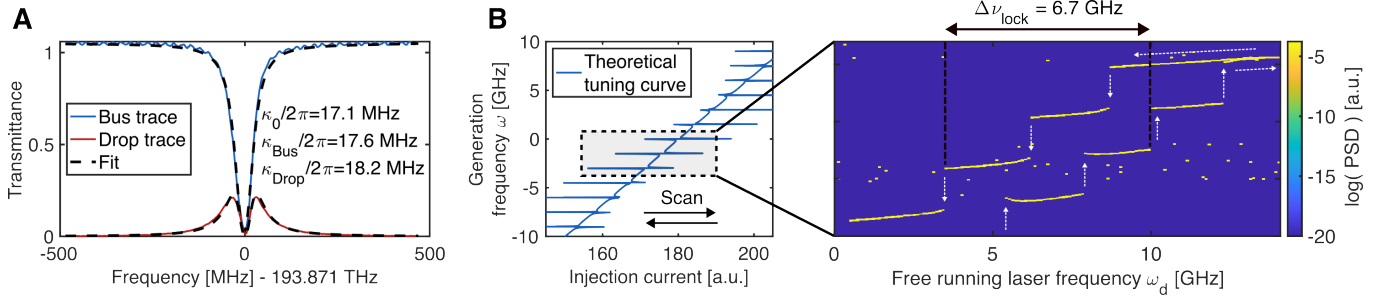


Figure S6. **Experimental characterization of resonator parameters.** (A) The intrinsic loss rate κ_0 and the external coupling rates κ_{bus} and κ_{drop} are determined by fitting split Lorentzians to the transmission traces. (B) The laser-to-PIC coupling rate κ_{LD} is calculated from the experimentally characterized locking range using Equation S13.

where γ is the Rayleigh-backscattering rate. Equation S14 is valid when the Rayleigh-backscattered counterpropagating resonator field and the drop-port-mirror induced counterpropagating field interfere constructively, and thus we choose a symmetrically split resonance with a maximized drop-port outcoupled amplitude for our further analysis. By estimating the splitting width from the drop port transmission trace to convert an underdetermined system of equations for κ_0 , κ_{bus} , κ_{drop} , and γ into a determined one, and fitting the transmission traces with split Lorentzians as shown in Fig. S6, we determine the intrinsic loss and coupler coupling rates as $\kappa_0/2\pi = 17.1$ MHz, $\kappa_{\text{bus}}/2\pi = 17.6$ MHz, and $\kappa_{\text{drop}}/2\pi = 18.2$ MHz, respectively.

To estimate the DFB-to-PIC coupling rate κ_{LD} , we first determine the locking range at the feedback phase $\Psi = -\pi/2$, which corresponds to a symmetric locking regime with highest linewidth reduction factor. We scan the DFB current with a triangular waveform through multiple resonances and record a spectrogram, from which we estimate the locking range as $\Delta\omega_{\text{lock}} = 6.7$ GHz, as shown in Fig. S6. We then use Equation S13 with $\Psi = -\pi/2$ to solve $\kappa_{\text{LD}}/2\pi = 11.195$ GHz.

VI. LIGHT-CURRENT CHARACTERISTICS OF THE ENDLESSLY SELF-INJECTION LOCKED LASER

The fiber-coupled optical output power of the self-injection-locked laser as a function of the DFB injection current is shown in Fig. S7. The discrete jumps in power occur when the laser transitions between different self-injection-locked states. These are separated by continuous regimes in which the output power follows the characteristic traces shown in Fig. 2(d), indicating regions where the DFB remains locked to a single microresonator resonance. The overall evolution of the resonance shapes follows a periodic pattern governed by the feedback-phase dispersion. Because the resonator is designed to operate close to critical coupling to maximize the linewidth-reduction factor and mode selection of locked states (see Ref. [3] and Supplementary Section II), the output power is reduced compared to the free-running laser, averaging 0.7 mW.

VII. FREQUENCY STABILITY AND ALLAN DEVIATION OF THE ENDLESSLY SELF-INJECTION LOCKED LASER

The stability of the single-mode operation of the self-injection locked laser is another crucial feature for many applications, together with turn-key operation enabled by the endless SIL design that we report. For this purpose, we assembled one laser based on the endless SIL design in a customized butterfly package and we measured the long-term frequency stability of the endless SIL laser over more than 12 hours by heterodyne beating with a phase-stabilized frequency comb (Menlo systems) for absolute reference [13]. The photonic packaging helps prevent detrimental drifts in the optical coupling into and out of the

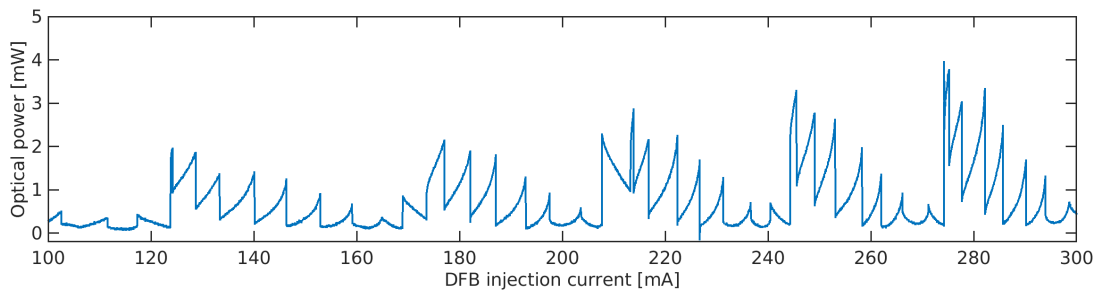


Figure S7. **LI-curve of the endlessly self-injection locked laser**

PIC throughout the duration of the measurement, as opposed to an open-air coupling setup where the photonic assembly could be inadvertently affected by air drafts or the personnel working in the lab. The measurement results presented in Fig. S8 (A) exhibit a total excursion of the laser frequency drift of around 40 MHz in over 12 hours. Notably, this continuous frequency drift over a few tens of MHz excludes any mode hops, which would necessarily span a frequency of an integer multiple of the resonator's free spectral range, i.e. around 1.5 GHz. Instead, this small frequency drift is attributable to the slow temperature drifts in the laboratory during the measurements which cause small changes in the refractive index of the cavity and hence in its resonant frequency. The quality of this long-term stability can be quantified by computing the Allan deviation of the laser output from the measured frequency profile, which is presented in Fig. S8 (B) [14]. The measured Allan deviation demonstrates a frequency stability that surpasses what is typically achievable with free-running semiconductor lasers and is even on par with the performance of the best fiber lasers over time scales above 1 s [15]. Overall, this frequency stability characteristics place the reported endless SIL laser within the performance range of state-of-the-art laboratory laser sources used for precision metrology and coherent applications.

In the experiment discussed above, the laser's photonic assembly, including the DFB diode chip, the photonic integrated circuit chip and a fiber array unit are all glued on a common substrate which sits on a Peltier thermoelectric control element: this is in order to secure the operation and durability of the electrically pumped DFB diode under drive currents, but it also provides thermal stability to the entire photonic assembly. In all the other measurements, however, the temperature of the PIC cavity was not controlled nor stabilized. Indeed, the endless SIL mechanism is insensitive to temperature: even if small changes in refractive index driven by temperature could indeed cause a small drift in the cavity resonances (as also observed in the frequency stability measurement mentioned above), this phenomenon will not significantly affect the feedback phase dispersion between consecutive modes, which is the key to the endless SIL design. This is evidenced by the fact that we reliably observed endless self-injection locking in all measurements performed across different days and laboratory environment conditions without any thermal stabilization of the PIC cavities. The measurements reported in Figure 4, as an example, consistently demonstrate self-injection locking across the entire range of the DFB drive current, corresponding to a large dynamic range in terms of laser output power, over an acquisition time of several tens of minutes.

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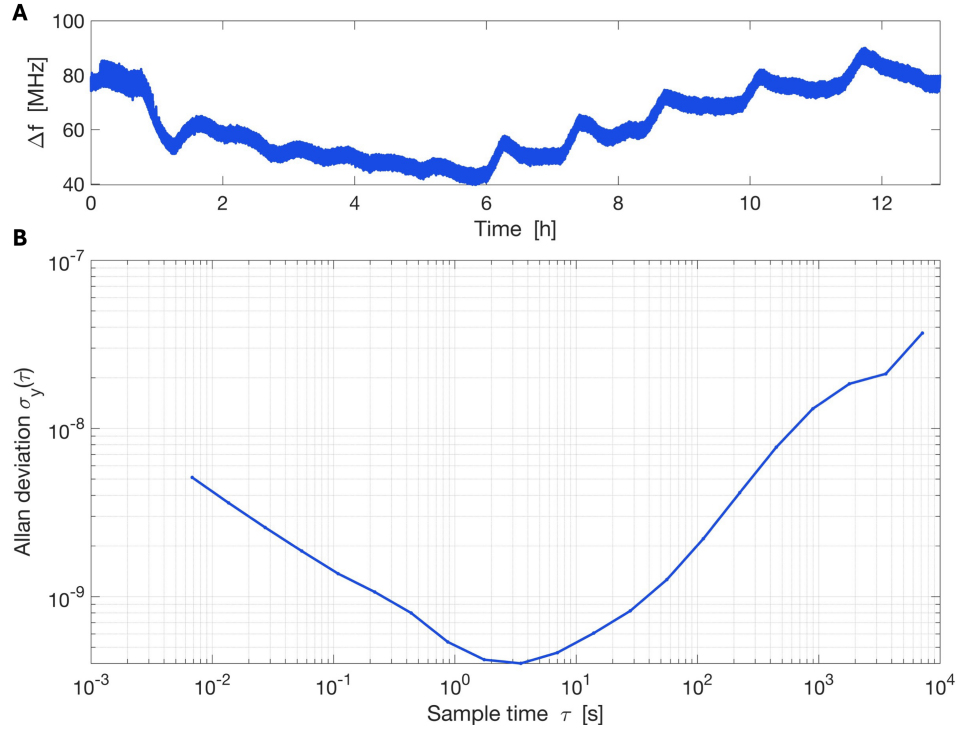


Figure S8. **Frequency stability of the endlessly self-injection locked laser.** **A** Spectrogram of the frequency drift evolution in time over an interval of over 12 hours obtained by beating the output of the packaged SIL laser with a phase-stabilized frequency comb. **B** Allan deviation plot computed from the frequency drift spectrogram in A.

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