nature physics

SUPPLEMENTARY INFORMATION
https://doi.org/10.1038/s41567-017-0011-x

In the format provided by the authors and unedited.

Coherent inflationary dynamics for Bose–Einstein condensates crossing a quantum critical point

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EXPERIMENT TECHNIQUE AND DATA ANALYSIS

Time-of-flight imaging with focusing

In order to map out the quasi-momentum states of the condensates with high resolution, we perform the time-of-flight (TOF) measurement with focusing technique[1]. Firstly, to avoid atom collisions, we reduce the scattering length to zero by switching the magnetic field right before time-of-flight. At the same time, we increase the magnetic field gradient to properly levitated the atoms into our imaging plane while we turn off the dipole traps. During time-of-flight, all the dipole traps are turned off except the one along lattice direction. By carefully tuning the intensity of the dipole beam, we are able to focus the atoms in the same momentum state into a sharp peak after a quarter of the trapping period of 60 ms. With this technique, we achieve a resolution in the momentum space to $\delta q = 0.08\ q_L$, only limited by the anharmonicity of our trap.

Seeding condensates by phase imprinting

We seed the momentum fluctuation in the quench experiment by imprinting a sinusoidal phase pattern on the condensates. Right after the seeding pulse, the wavefunction of the condensates can be written as $\psi = \psi_0 e^{i\delta\phi \sin(q'x/\hbar)}$, where ψ_0 is a constant, q' is the seeding momentum, $\delta\phi$ is the seeding amplitude and normally $\delta\phi < 1$. Thus we transfer a small fraction of the atomic population to the $\pm q'$ states. This is easy to see if we expand the wave-function using the Bessel functions, $\psi = \psi_0 \left[J_0(\delta\phi) - J_1(\delta\phi)e^{-iq'x/\hbar} - J_{-1}(\delta\phi)e^{iq'x/\hbar} + ...\right]$. After seeding, the density wave and domain structure are deterministically generated during inflation, shown in Fig.S1. The period of the density wave is $2\pi\hbar/q'$ and the domain size is exactly the half of that, $\pi\hbar/q'$.

Momentum state population and growth rate

In the ramp experiment, we want to extract the excited population ΔN in the momentum states $q \neq 0$. Firstly, we perform the focusing time-of-flight measurements on pure Bose-Einstein condensates and characterize the atom distribution in momentum space $n_0(x)$. In the following moments during the ramp, we fit the center interval, $[-\delta q, \delta q]$, of the atom distribution n_q using the characterized profile to quantify the atom population N_0 in $q = 0$ state. As a result, the excited population $\Delta N = N - N_0$ with N the total number of atoms.

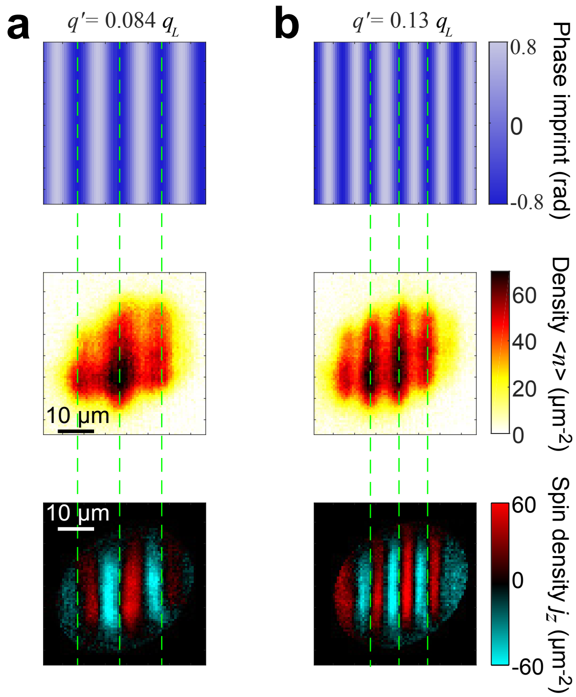

Fig. S1 Density wave and domain structure with phase imprinting. The condensates are quenched from shaking amplitude $s_c = 13.1$ nm to $s = 25$ nm after phase imprinting with the seeding momentum $q' = 0.084\ q_L$ (**a**) and $0.13\ q_L$ (**b**). The phase pattern is illustrated in the top row. The middle row shows the density wave at time $t = 12$ ms (**a**) and 8 ms (**b**). The bottom row shows the corresponding domain structure at time $t = 16$ ms (**a**) and 14 ms (**b**), which remains stationary.

In the quench experiments, to extract the atom population $N_{q'}$ in the momentum states $\pm q'$, we integrate the populations in the momentum states within the intervals $[q' - \delta q, q' + \delta q]$ and $[-q' - \delta q, -q' + \delta q]$. To extract the inflaton growth rate, we fit the fast growing interval of our data in early times where, where $N_{q'}/N \leq 0.3$, to $A \cosh(2\lambda_{q'} t)$ and both A and $\lambda_{q'}$ are simultaneously determined.

Amplitude and phase of the density wave

To accurately extract the amplitude and phase of the density wave, we have to consider the density inhomogeneity in the harmonic trap. This density inhomogenity affects little the amplitude but significantly influences the phase due to nonuniform chemical potential. To minimize this effect, we choose a rectangular subset of the center part of the atom cloud, where local density is no less than 80% of the peak density of the condensate. We integrate over the non-lattice direction and obtain density distribution along the lattice direction $\langle n(x)\rangle$, averaged over multiple shots. The density modulation $\langle \Delta n(x)\rangle$ is obtained by subtracting $\langle n(x)\rangle$ from the initial density distribution $\langle n_0(x)\rangle$ right before the shaking just starts. The amplitude and phase are then given by the Fourier transform of $\langle \Delta n(x)\rangle$.

Coherence length estimation

Here we extract a lower bound of the coherence length based on uncertainty principle in Fig. 4**b**. Given the mean square width of the atom population peaks in momentum space σ, the coherence length of the system is then $l_\phi \propto h/\sigma$. In our simulation, based on fully coherent condensates with coherence length equals to the system diameter of 30 μm, the mean square width σ' is almost half of that in the corresponding experiments. This indicates that the coherence length is $l_\phi > 15$ μm in the experiment, which is approximately 3 times larger than the domain size of 4 μm and is half of the system size, 30 μm in the direction of shaking.

THEORY OF INFLATION

Many-body Hamiltonian

Here we start with the many-body Hamlitonian in the second quantization form [2],

$$H = \sum_q \varepsilon_q \hat{a}_q^\dagger \hat{a}_q + \frac{U_0}{2V} \sum_{q,q',p} \hat{a}_{q+p}^\dagger \hat{a}_{q'-p}^\dagger \hat{a}_q \hat{a}_{q'}, \tag{1}$$

where ε_q is the single atom dispersion, U_0 is the two-body interaction energy, V is the volume of the system and $\hat{a}_{\pm q}^\dagger$ and $\hat{a}_{\pm q}$ are bosonic creation and annihilation operators of an atom with momentum $\pm q$. Shaking of the optical lattice dramatically change the single particle dispersion and in the ferromagnetic phase it is given by

$$\varepsilon_q = \epsilon \left[\left(\frac{q}{q^*} \right)^2 - 1 \right]^2 - \epsilon, \tag{2}$$

where ϵ is the kinetic energy barrier hight. Thus the dispersion is in a double-well shape in momentum space and has two minima at $q = \pm q^*$ sparated by the kinetic energy barrier

(Fig. S2**b**) [3, 4].

Hamiltonian of inflaton and solution

Under Bogliubov approximaton, we introduce a new quasi-particle field, inflaton, to rewrite the Hamiltonian of Bose-Einstein condensates. At the beginning of the phase transition, we assume that the $q = 0$ state is macroscopically occupied. Thus the condensate consists of N atoms and has a chemical potential $\mu = U_0 N/V$. The many-body Hamiltonian Eq.(1) reduces to

$$H = \frac{1}{2}N\mu + \sum_{q>0}(\mu + \varepsilon_q)(\hat{a}_q^\dagger \hat{a}_q + \hat{a}_{-q}^\dagger \hat{a}_{-q}) + \sum_{q>0}\mu(\hat{a}_q^\dagger \hat{a}_{-q}^\dagger + \hat{a}_q \hat{a}_{-q}). \tag{3}$$

Conventional Bogoliubov transformation to diagonalize the Hamiltonian using bosonic operators fails in our case because of $\varepsilon_q < 0$. Instead, we adopt a different approach by introducing the following transformation

$$\hat{\iota}_{\pm q} \ = \ u_q \hat{a}_{\pm q} + \nu_q \hat{a}_{\mp q}^\dagger, \tag{4}$$

where $\hat{\iota}_{\pm q}^\dagger$ and $\hat{\iota}_{\pm q}$ are the creation and anihilation operators of an inflaton with momentum $\pm q$ and the coefficients $u_q, \nu_q > 0$ for $\varepsilon_q < 0$ satisfy

$$u_q^2 \ = \ \frac{1}{2}(\frac{\mu}{\hbar\lambda_q} + 1) \tag{5}$$

$$\nu_q^2 \ = \ \frac{1}{2}(\frac{\mu}{\hbar\lambda_q} - 1) \tag{6}$$

$$\hbar\lambda_q \ = \ \sqrt{-\varepsilon_q(2\mu + \varepsilon_q)}. \tag{7}$$

The inflaton field operators obeys bosonic commutation relations: $[\hat{\iota}_q, \hat{\iota}_{q'}] = [\hat{\iota}_q^\dagger, \hat{\iota}_{q'}^\dagger] = 0$ and $[\hat{\iota}_q, \hat{\iota}_{q'}^\dagger] = \delta_{qq'}$.

As a result, the Hamiltonian reduces to

$$H = \sum_{q>0}\hbar\lambda_q(\hat{\iota}_q^\dagger \hat{\iota}_{-q}^\dagger + \hat{\iota}_q \hat{\iota}_{-q}) + \frac{1}{2}N\mu - \sum_{q>0}(\mu + \varepsilon_q). \tag{8}$$

Beside the two constant terms, the Hamiltonian shows that inflatons are created and annihilated in pairs with opposite momentum. Here the inflation dispersion λ_q is related to the growth rate of the inflatons (Fig. S2**a**).

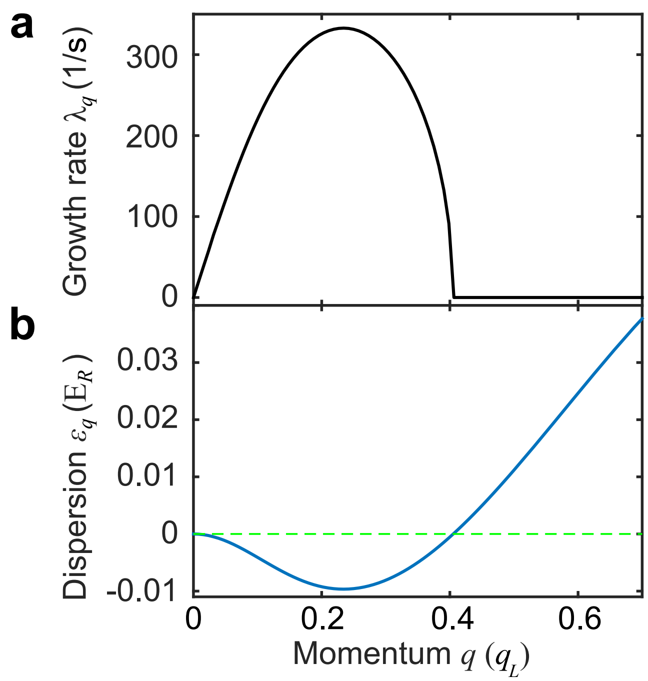

Fig. S2 Inflaton growth rate and single particle dispersion. Here the shaking amplitude $s\ = 25$ nm with atoms in an 8.9 E_R lattice. The chemical potential $\mu = h{\times}150$ Hz. We shows the inflaton growth rate λ_q (**a**) and the single particle dispersion ε_q (**b**). The green dashed line (in **b**) indicates the zero-crossing of ε_q.

To show the exponential growth, we look at the dynamics of the inflaton in the Heisenberg picture, which yields

$$\partial_t \hat{\imath}_{\pm q}(t) \ = \ \frac{i}{\hbar}[H, \hat{\imath}_{\pm q}(t)] = -i\lambda_q \hat{\imath}^{\dagger}_{\mp q}(t) \tag{9}$$

$$\partial_t \hat{\imath}^{\dagger}_{\pm q}(t) \ = \ \frac{i}{\hbar}[H, \hat{\imath}^{\dagger}_{\pm q}(t)] = i\lambda_q \hat{\imath}_{\mp q}(t). \tag{10}$$

The solutions of above equation is

$$\hat{\imath}_{\pm q}(t) \ = \ \hat{\imath}_{\pm q}(0) \cosh \lambda_q t - \hat{\imath}^{\dagger}(0)_{\mp q} i \sinh \lambda_q t \tag{11}$$

$$\hat{\imath}^{\dagger}_{\pm q}(t) \ = \ \hat{\imath}^{\dagger}_{\pm q}(0) \cosh \lambda_q t + \hat{\imath}_{\mp q}(0) i \sinh \lambda_q t. \tag{12}$$

Based on these solutions, the inflaton population $m_{\pm q}(t) \equiv < \hat{\imath}^{\dagger}_{\pm q}(t)\hat{\imath}_{\pm q}(t) >$ evolves according to

$$m_{\pm q}(t) = m_{\pm q}(0)\cosh^2 \lambda_q t + m_{\mp q}(0)\sinh^2 \lambda_q t + \sinh^2 \lambda_q t, \tag{13}$$

where the first two terms on the right-hand-side correspond to Bose stimulation and the last term originates from spontaneous emission of inflatons.

We further simplify the results by defining the total inflaton population in $\pm q$ modes $M_k = m_q + m_{-q}$ and have

$$M_q(t) + 1 = [M_q(0) + 1]\cosh 2\lambda_q t. \tag{14}$$

From this result we see that the inflaton population, including the contribution from spontaneous emission, grows exponentially with a rate of $2\lambda_q$.

Experimental observables

The exponential growth of inflaton fields is reflected from the momentum population in the time-of-flight measurements, and the structure factor of the density wave. First of all, we define the population in momentum state $\pm q$ as $N_q = \langle \hat{a}_q^\dagger \hat{a}_q \rangle + \langle \hat{a}_{-q}^\dagger \hat{a}_{-q} \rangle$. To calculate the atom population from inflatons, we engage the inverse inflaton transformation $\hat{a}_{\pm q} = u_q \hat{\iota}_{\pm q} - \nu_q \hat{\iota}_{\mp k}^\dagger$. We thus rewrite the atom population as

$$N_q + 1 = \frac{\mu}{\hbar\lambda_q}\left(M_q + 1\right) - \frac{\mu + \varepsilon_q}{\hbar\lambda_q}\left(\langle \hat{\iota}_q \hat{\iota}_{-q} + \hat{\iota}_q^\dagger \hat{\iota}_{-q}^\dagger \rangle\right). \tag{15}$$

Since $\hat{\iota}_q \hat{\iota}_{-q} + \hat{\iota}_q^\dagger \hat{\iota}_{-q}^\dagger$ commutes with the Hamiltonian and does not vary with time, the number of atoms in an inflaton is given by $\partial N_q / \partial M_q = \mu / \hbar\lambda_q$.

Secondly, the structure factor, defined as the Fourier transform of the density-density correlation function [5], can be expressed in terms of the correlations in the momentum space as

$$S_q = \frac{1}{N}\sum_{p,p'} \langle \hat{a}_{p+q}^\dagger \hat{a}_p \hat{a}_{p'-q}^\dagger \hat{a}_{p'} \rangle. \tag{16}$$

For small number of excitations $\langle \hat{a}_q^\dagger \hat{a}_q \rangle \ll \langle \hat{a}_0^\dagger \hat{a}_0 \rangle \approx N$ and $\varepsilon_q < 0$, one can rewrite S_q as

$$S_q = \frac{-\varepsilon_q}{\hbar\lambda_q}\left(M_q + 1 + \langle \hat{\iota}_q \hat{\iota}_{-q} + \hat{\iota}_q^\dagger \hat{\iota}_{-q}^\dagger \rangle\right). \tag{17}$$

Based on Eqs.(15) and (17), we futher determine the time evolution of the observables. In the begining of phase transition, we assume that there is no net source of correlated inflatons in a regular Bose-Einstein condensate

$$\langle \hat{\iota}_q \hat{\iota}_{-q} \rangle = \langle \hat{\iota}_q^\dagger \hat{\iota}_{-q}^\dagger \rangle = 0. \tag{18}$$

As a result, the initial value of inflaton population at $t = 0$ is

$$M_q(0) + 1 = \frac{\hbar \lambda_q}{\mu}[N_q(0) + 1]. \tag{19}$$

Finally, we obtain the time evolution of $N_q(t)$ and $S_q(t)$,

$$N_q(t) + 1 = [N_q(0) + 1] \cosh(2\lambda_q t) \tag{20}$$

$$S_q(t) = \frac{-\varepsilon_q}{\mu}[N_q(0) + 1] \cosh(2\lambda_q t). \tag{21}$$

NUMERICAL SIMULATION

Our numerical simulation is based on the Gross-Pitaevskii equation with an effective model in a one-dimensional lattice. In this model, space is discretized into lattice sites and kinetic motion is replaced by hopping between neighboring sites. We define $\psi_j(t)$ the value of the condensate wavefunction of site j at time t and the evolution is given by

$$i\gamma\hbar \frac{\partial}{\partial t}\psi_j(t) = -\sum_{n=1}^{n_T} t_k[\psi_{j-n}(t) + \psi_{j+n}(t)] + \left[\frac{1}{2}m\omega^2(jd)^2 + g|\psi_j(t)|^2 - \mu\right]\psi_j(t). \tag{22}$$

Here t_n is the tunneling energy between two lattice sites spatially separated by n sites, m is the atomic mass of Cs, ω is the harmonic trapping frequency, g is the coupling constant, μ is the chemical potential , d is the lattice period and n_T is the truncation number that shall be discussed later. γ is a parameter that controls the evolution in either imaginary time ($\gamma = i$) or real time ($\gamma = 1$). Particularly in imaginary time, evolution of the equation gives the ground state wavefunction of the system. In our simulation of inflation, we keep a small imaginary part by setting $\gamma = \cos\varphi + i \sin\varphi$ with $\varphi = 0.015$, which accounts for the dissipation in our system.

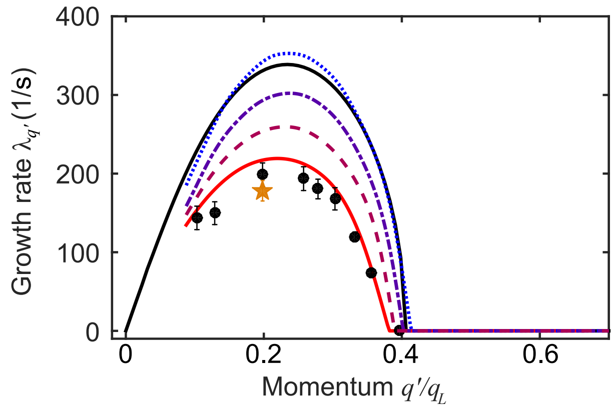

Fig. S3 Simulation of inflaton growth rate. Here we show the measured growth rate $\lambda_{q'}$ from seeded (black dot) as well as the unseeded quench experiment (orange star). Black solid line is the prediction from Bogliubov theory with effective chemical potential $\mu = h\times 125$ Hz. Lines in color from blue to red show the results from simulation with the seed amplitude $\delta\phi$: 0.1 (dot), 0.3 (dash-dot), 0.5 (dash) and 0.7 (solid) rad. In the simulation, the trapping frequency $\omega = 2\pi\times$ 12 Hz, chemical potential $\mu = h\times 150$ Hz, lattice depth is 8.9 E_R and shaking amplitude $s = 25$ nm, which are based on the corresponding experiment.

Tunneling energy t_n is determined from the Bloch band structure. In our experiment, all the atoms dominantly occupy the ground band even with lattice shaking. The tunneling energy is then the inverse Fourier transform of the ground band dispersion $t_n = \sum_q e^{-inq/\hbar}\varepsilon_q$. In a non-shaken lattice, the nearest-neighbor tunneling t_1 is the only significant term and sufficient to capture most of the physics. However, in a shaken lattice, where the ground band dispersion is significantly modified, higher order tunneling become significant as well. In our simulation, we found that $n_T = 3$ is a good truncation number and t_n with $n > 3$ are negligible.

We perform the simulation for our seeded quench experiment as the following. We first evolve equation in imaginary time to reach the ground state wavefunction in a non-shaken lattice. To implement the seed, we multiply this ground state wavefunction by a spatially-varying phase factor $e^{i\delta\theta(j)}$ with $\delta\theta(j) = \delta\phi\sin(\frac{q'}{\hbar}jd)$ as the initial wavefunction for the next step, where $\delta\phi$ is the seed amplitude and q' is the seed momentum. To simulate the quench, we now evolve the initial wavefunction with a new set of tunneling energies in a shaken lattice. Later all the observables are extracted from the wavefunction in the same way as we did in our data anlaysis.

Simulations with respect to different seed amplitudes explain the discrepancy between experiment and Bogliubov theory visible in Fig. 3**c** from the main text. Here we show the inflaton growth rate $\lambda_{q'}$ from our simulation in Fig. S3. For comparison, we also include the experimental data as well as the prediction from Bogliubov theory. First, for small initial population depletion from the $q = 0$ state where $\delta\phi \leq 0.1$, the reults from simulation agree well with that from Bogliubov theory. However, further increase of $\delta\phi$ quickly drive the system out of the perturbative regime. As a result, the growth rate $\lambda_{q'}$ decreases as the seed amplitude $\delta\phi$ increases. The measured growth rates concur with the simulation with seed amplitude $\delta\phi = 0.7$, which is estimated to be 0.6(0.1) in experiments from the initial population depletion of the $q = 0$ state.

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