nature physics

SUPPLEMENTARY INFORMATION

https://doi.org/10.1038/s41567-018-0156-2

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A null test of general relativity based on a long-term comparison of atomic transition frequencies

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Values of assumed constants

The values of all constants used in performing the calculation in the paper, and assumed to be non-varying are given in Table S1.

TABLE S1. Constants and their values

constants	value
astronomical unit, AU	149597870.691 km
speed of light in vacuum, c	299792458 $\mathrm{m\,s^{-1}}$
standard gravitational parameter of sun, GM_S	$1.327124420 \times 10^{20}$ $\mathrm{m^3s^{-2}}$
Newton's gravitational constant, G	6.67428×10^{-11} $\mathrm{m^3kg^{-1}s^{-2}}$
ratio of mass of sun and mass of Jupiter, M_S/M_J	1047.348644

Estimating β considering individual H-maser

TABLE S2. Comparing values of β from masers S2-S5

maser	# data	$\mathrm{MJD_{max}}$	$\Delta f/f\vert_{\max}$	$\mathrm{MJD_{min}}$	$\Delta f/f\vert_{\min}$	β_j
S2	380	56959	4.001×10^{-13}	53164	2.623×10^{-15}	$(1.55 \pm 4.74) \times 10^{-7}$
S3	314	56394	2.226×10^{-12}	53931	7.975×10^{-13}	$(7.24 \pm 5.57) \times 10^{-7}$
S4	363	56824	6.476×10^{-13}	51508	1.364×10^{-13}	$(3.47 \pm 4.94) \times 10^{-7}$
S5	384	54376	2.208×10^{-13}	51508	7.387×10^{-14}	$(-1.81 \pm 4.75) \times 10^{-7}$

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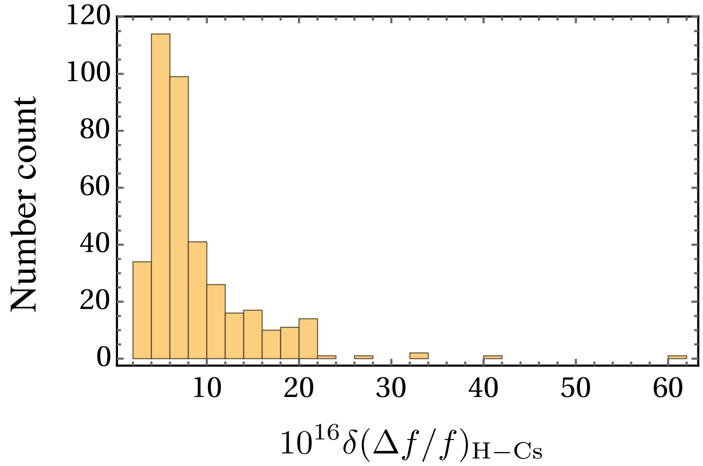

FIG. S1. Histogram of the fractional frequency difference uncertainties showing a peak population corresponding to $\approx 5\times10^{-16}$. The medians of the uncertainties before and after MJD54000 is 1.1×10^{-15} compared to 6.35×10^{-16} with 58 and 330 evaluations (number of data points) respectively. All data points before MJD54000 were included in Ashby et al.(2007)

Comparing clocks without frequency drifts

For two ideal clocks without any drifts—unlike H-masers—one could directly use the formula given below to compute the total variation of the fundamental constants. For motion of the earth in the sun's gravitational potential, the spatial gravitational potential change is

$$\Delta\Phi \equiv d\Phi = \left(GM_{\odot}/r^2\right) dr = \left(GM_{\odot}/r^3\right) \vec{r} \cdot d\vec{r}, \tag{1}$$

where r is, for example, the distance of the barycenter of the earth-moon system from the sun. Then

$$d\Phi/dt = \left(GM/r^3\right) \vec{r} \cdot \vec{v}, \tag{2}$$

where v is the velocity of earth. Eq. (2) may be used to compute the temporal variation of the potentials both due to the sun and to Jupiter, by calculating the velocity and position components of the earth with respect to sun and Jupiter using planetary ephemerides DE430. Then

$$d \ln\left(f_{\mathrm{A}}/f_{\mathrm{B}}\right)/dt = \left(\beta_{\mathrm{A}} - \beta_{\mathrm{B}}\right)\left(GM/c^2 r^3\right) \vec{r} \cdot \vec{v}, \tag{3}$$

where A and B are two different atoms that make these clocks. $(\beta_{\mathrm{A}} - \beta_{\mathrm{B}})$ is obtained as described in the main text of the paper.