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SUPPLEMENTARY INFORMATION

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Quantum stochastic resonance in an a.c.-driven single-electron quantum dot

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Supplementary Information for "Quantum stochastic resonance in an ac-driven single-electron quantum dot"

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Counting statistics without drive

Without external drive ($A = 0$) the counting statistic depends on the asymmetry $\alpha = (\Gamma_{in} - \Gamma_{out})/(\Gamma_{in} + \Gamma_{out})$. The tunneling-in rate $\Gamma_{in} = 1/\langle\tau_1\rangle$ and tunneling-out rate $\Gamma_{out} = 1/\langle\tau_0\rangle$ can be determined from the mean residence times $\langle\tau_0\rangle$, $\langle\tau_1\rangle$. Fig. S1 shows different Fano factors as a function of the asymmetry parameter α. For the total counting statistic $P(\Delta t, n_\Sigma = n_{in} + n_{out})$ the Fano factor F_Σ (red cycles) lies between $1 \leq F_\Sigma \leq 2$ and for the individual counting statistic $P(\Delta t, n_{in/out})$ the Fano factor $F_{in/out}$ (orange diamonds / green crosses) lies between $0.5 \leq F_{in/out} \leq 1$. The different Fano factors have all been determined for a counting interval Δt of 10 ms.

When the rates are symmetric ($\alpha = 0$) the Fano factor F_Σ equals one, because in this special situation the total counting statistics $P(n_{in} + n_{out})$ behaves like a Possonian process with rate $\Gamma_\Sigma = 2\Gamma_{in/out}$. With growing asymmetry the 'in' and 'out' events bunch, causing an increase of the Fano factor F_Σ. When the tunneling rates become very asymmetric ($\alpha \approx \pm 1$) two events always occur at the same time, therefore the Fano factor F_Σ must go to 2.0.

The Fano factor $F_{in/out}$ of the individual counting statistic $P(n_{in/out})$ is instead always smaller than 1.0. The black solid line shows the theoretical expected Fano factor $F_{in/out} = \frac{1}{2}(1 + \alpha^2)$ in the zero-frequency limit[S1,S2]. Due to the Coulomb-blockade two identical events ('in-in' or 'out-out') can never follow each other directly, which causes always an antibunching of the individual events. The antibunching is maximized when the rates are symmetric ($\alpha = 0$), in that situation the Fano factor becomes 0.5. When the rates are very asymmetric ($\alpha \approx \pm 1$) the counting statistic $P(n_{in/out})$ follows a Poissonian process ($F_{in/out} = 1$) with rate Γ_{in} ($\alpha = 1$) or Γ_{out} ($\alpha = -1$).

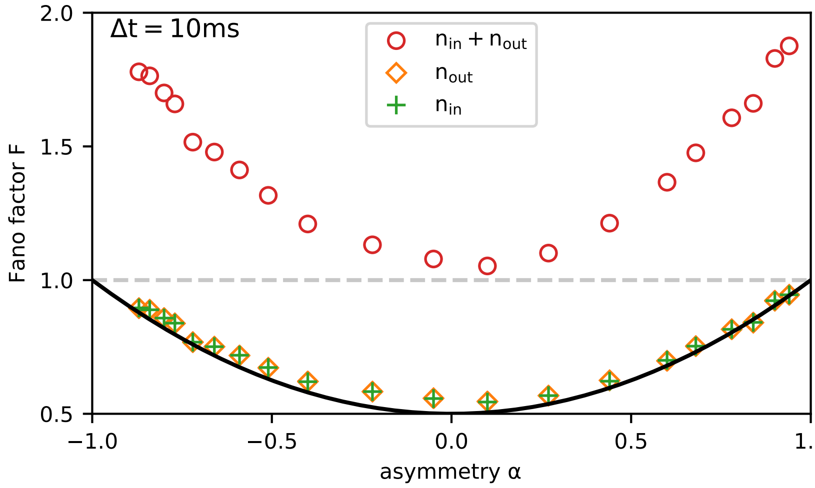

Figure S1: **Fano factor without drive**. The Fano factor without drive ($A = 0$) as a function of the asymmetry $\alpha = (\Gamma_{in} - \Gamma_{out})/(\Gamma_{in} + \Gamma_{out})$. The Fano factor F_Σ (red cycles) of the total counting statistic lies between $1 \leq F_\Sigma \leq 2$. The Fano factor F_Σ (red cycles) lies between $1 \leq F_\Sigma \leq 2$ and the Fano factor $F_{in/out}$ (orange diamonds / green crosses) lies between $0.5 \leq F_{in/out} \leq 1$. The different Fano factors have been determined for a counting interval $\Delta t = 10$ ms. The black solid line is the theoretical expected Fano factor $F_{in/out} = \frac{1}{2}(1 + \alpha^2)$.

Individual 'In' and 'Out' tunneling statistics

Fig.S2a shows the frequency dependent Fano factor $F_{in/out}$ (orange diamonds / green crosses) of the individual counting statistic $P(T_f = 1/f, n_{in/out})$ and the Fano factor F_Σ (red cycles) of the total counting statics $P(T_f = 1/f, n_\Sigma)$. Fig.S2b shows additionally the correspoding individual mean counts $\langle n_{in/out}\rangle$ as well as the total mean counts $\langle n_\Sigma\rangle := \langle n_{in} + n_{out}\rangle$. The Fano factor $F_{in/out}$ runs through a minimum, which occurs at a slightly slower driving frequency $f = 600\,\text{Hz}$ than the minimum at $f = 800\,\text{Hz}$ in the Fano factor F_Σ. The minimum in $F_{in/out}$ does not fall below 0.5, which makes a clear distinction from the non driven counting statistics (Fig. S1) difficult. This difficulty does not occur for F_Σ, which clearly falls below 1.0.

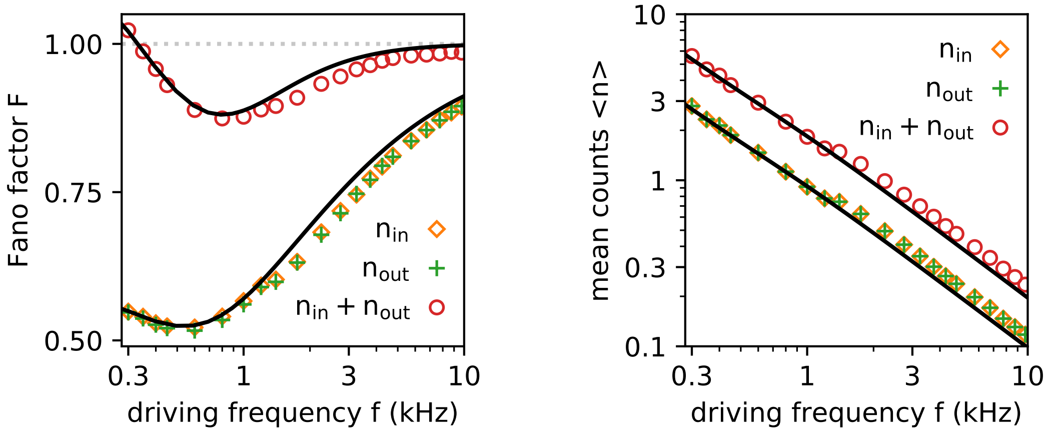

Figure S2: **Frequency dependent counting statistics**. **(a)** The Fano factor $F_{in/out}$ (orange diamonds / green crosses) of the individual counting statistics and the Fano factor F_Σ (red cycles) for the total counting statistics of a periodically driven QD with driving amplitude $A = 10\,\text{mV}$. The theoretical expected Fano factors and mean counts (black solid lines) agree nicely with the experimental values. **(b)** The corresponding individual mean counts $\langle n_{in/out}\rangle$ (orange diamonds / green crosses) and total mean counts $\langle n_\Sigma\rangle := \langle n_{in} + n_{out}\rangle$.

Frequency Locking

Here we discuss the single-electron counting statics and tunneling rates for a larger driving amplitude $A = 30\,\text{mV}$ than in the main text. Fig. S3a shows the corresponding mean counts $\langle n\rangle$ and Fig. S3b the Fano factor F as a function of the driving frequency f. For this significantly stronger drive a plateau-like feature appears in the mean counts $\langle n\rangle$, corresponding to a locking of the Rice frequency. In the same frequency range the Fano factor F is strongly suppressed below the Poissonian limit ($F_P = 1$), indicating that unwanted random tunneling events are effectively suppressed. The black solid lines in Fig. S3a,b are the theoretical counting statistics, expected from the tunneling rates in Fig. S3c. The rates have been extracted from the detector trace with $f = 1.5\,\text{kHz}$ using Eq. 4 from the main text.

The locking is a consequence of the strong modulation of the tunneling rates. In the first half of a driving period ($0 \le t < T_F/2$), the in-rate becomes so large that an electron tunnels almost with a probability of one into the QD. At the same time the out-rate is suppressed to the extent that the probability to leave the quantum dot is almost zero. In the second half of a driving period ($T_F/2 \le t < T_f$) the out-rate is increased to the extent that the ejection of the electron is almost certain. At the same time the in-rate is strongly suppressed that an electron is prevented from tunneling into the dot.

However the electron transfer is still based on the stochastic tunneling process. When the drive becomes too slow the probability of unwanted events increases and if the drive is too fast the probability to miss an tunneling event increases. Therefore the locking is only valid in a certain regime ($\min\{\Gamma_{in/out}(t)\} \ll f \ll \max\{\Gamma_{in/out}(t)\}$), which results from the same time scale matching as SR.

Although the tunneling rates are not harmonic and their forms are quite different, the double driving frequency $2f \approx 3.0\,\text{kHz}$ still is surprisingly close to the tunneling coupling $\Gamma_S = 2.5\,\text{kHz}$.

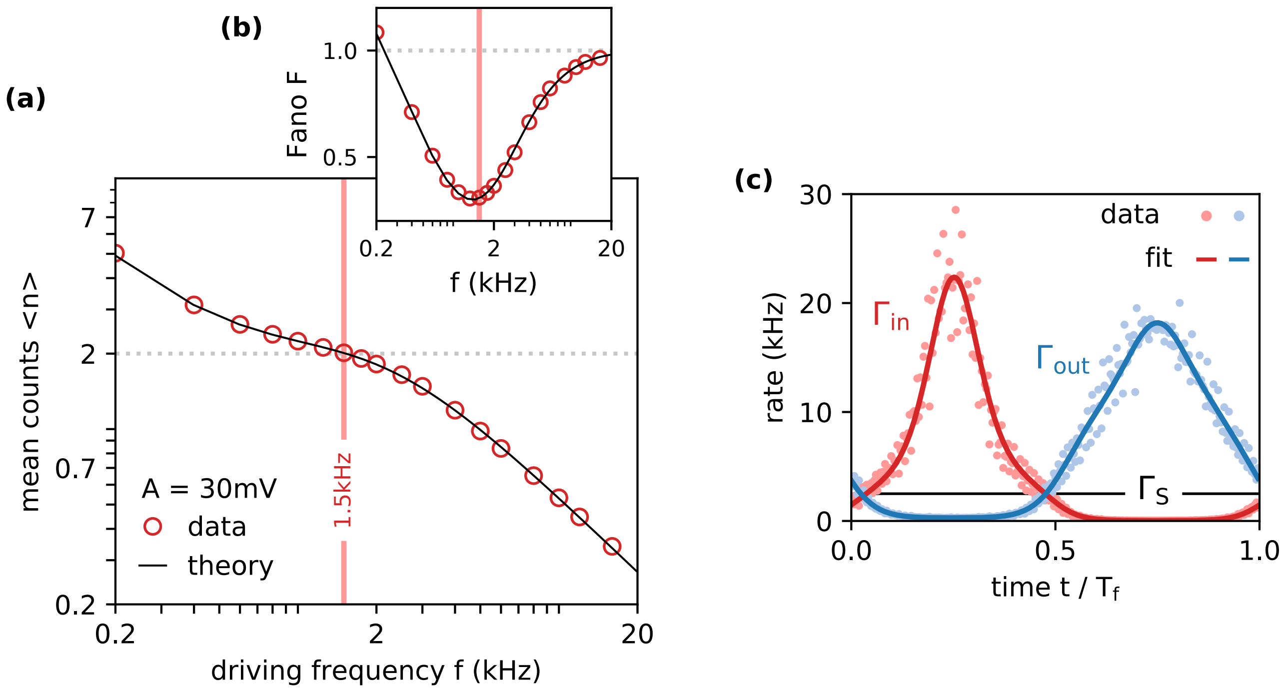

Figure S3: **Frequency locking**. **(a)** The experimental (red cycles) and theoretical (black line) number of tunneling events per period $\langle n \rangle$ as a function of the driving frequency f. **(b)** The frequency dependent Fano factor F. **(c)** Experimentally determined tunneling-in rate $\Gamma_{in}(t)$ (red dots) and tunneling-out rate $\Gamma_{in}(t)$ (blue dots). The rates have been extracted for a drive with $f = 1.5\,\mathrm{kHz}$ and $A = 30\,\mathrm{mV}$. The fits (solid lines) are based on a Fourier expansion of the logarithm of the rates. The horizontal black line marks the tunnel coupling $\Gamma_S = 2.5\,\mathrm{kHz}$ at symmetry level μ_S.

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