
Supplementary information

Directional self-locomotion of active droplets enabled by nematic environment

In the format provided by the
authors and unedited

Supplementary Information

Directional self-locomotion of active droplets enabled by nematic environment

Mojtaba Rajabi ^{1,†}, Hend Baza ^{1,†}, Taras Turiv ², Oleg D. Lavrentovich ^{1,2,*}

¹Department of Physics, Kent State University, Kent, OH 44242, USA

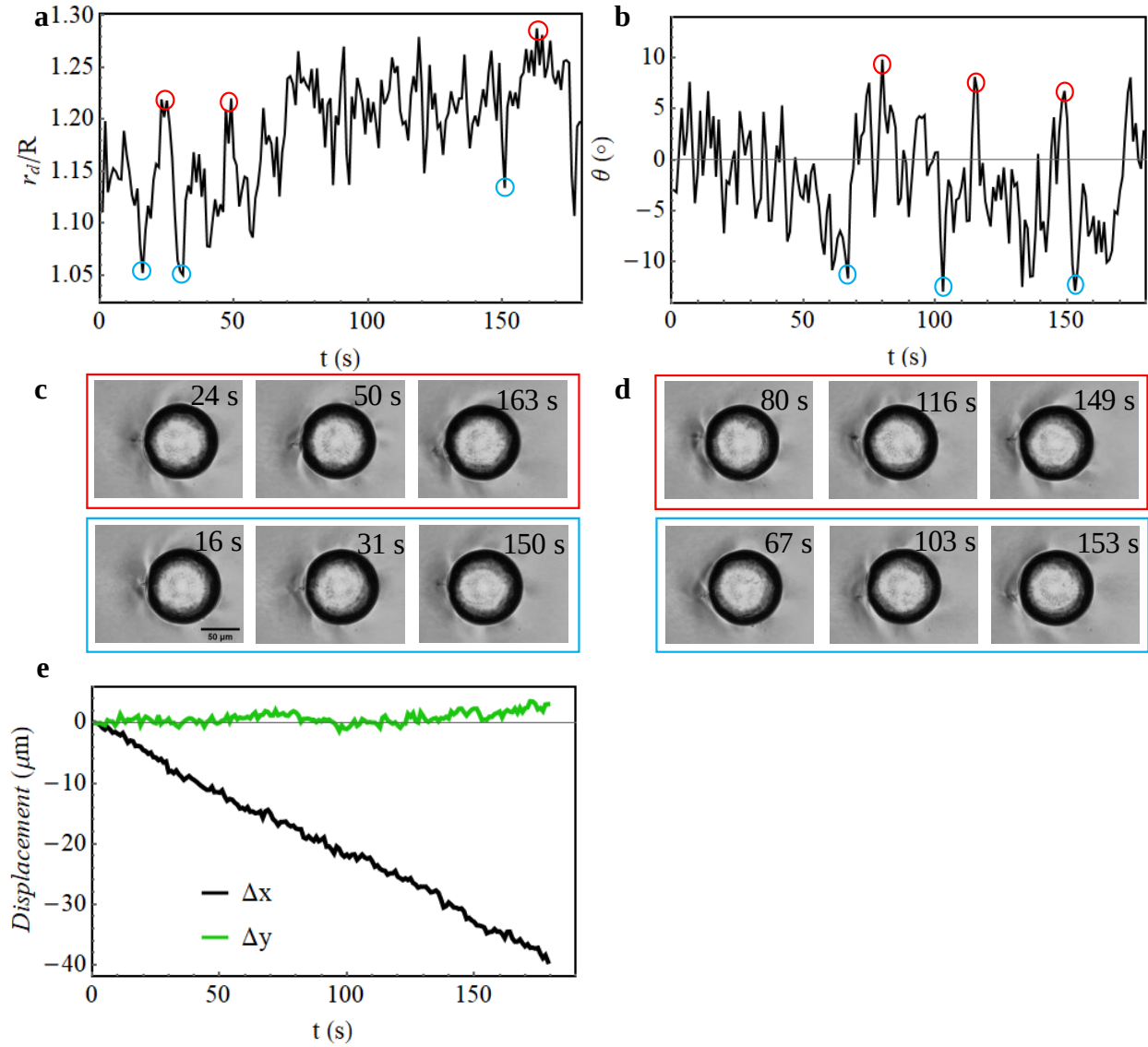
²Advanced Materials and Liquid Crystal Institute, Chemical Physics Interdisciplinary Program,
Kent State University, Kent, OH 44242, USA

* Corresponding author. Email: olavrent@kent.edu

† These authors contributed equally to this work

Torque equation for the director

The torque equation in the Ericksen-Leslie theory writes¹ $[\hat{\mathbf{n}} \times \mathbf{h}] = [\hat{\mathbf{n}} \times \gamma_1 \mathbf{N} + \gamma_2 \mathbf{A} \cdot \hat{\mathbf{n}}]$, where $\mathbf{h}$ is the molecular field that brings the director to the equilibrium state $[\hat{\mathbf{n}} \times \mathbf{h}] = 0$, as dictated by the elasticity and the possible presence of the external fields; γ_1, γ_2 are the viscous coefficients, $\mathbf{A}$ is the tensor of symmetric velocity gradients and $\mathbf{N} = \partial \hat{\mathbf{n}} / \partial t + (\mathbf{v} \cdot \nabla) \hat{\mathbf{n}} - \frac{1}{2} [\nabla \times \mathbf{v}] \times \hat{\mathbf{n}}$ describes the rate of director rotations which vanishes when the entire nematic rotates as a whole with the angular velocity $\frac{1}{2} [\nabla \times \mathbf{v}]$. This equation is not invariant with respect to the velocity reversal, as illustrated by Stark and Ventzki¹ in simulations of a Stokes drag acting on a sphere with perpendicular anchoring and a hyperbolic hedgehog.



Supplementary Figure S1. In-plane fluctuations of **a** the amplitude and **b** direction of the radius-vector $\mathbf{r}_d$ drawn from the center of the droplet to a small droplet (diameter 4 μm) trapped at the core of the hyperbolic hedgehog; θ is the angle between $\mathbf{r}_d$ and the x -axis. **c**, Snapshots of the droplet textures when r_d is maximum (top red box) and minimum (bottom blue box), as marked in **a**. **d**, Snapshots of the textures when θ is maximum (top red box) or minimum (bottom blue box), as marked in **b**. **e**, Displacements of the droplet shown in parts **a-d** vs the time of observation. The droplet diameter is $2R = 97 \mu\text{m}$, speed $v = 0.22 \mu\text{m/s}$; bacterial concentration $20c_0$; recording started 15 minutes after filling the cell. Unlike the figures in the Main text, this one shows an active droplet with a hedgehog on the left hand side; it moves to the left.

Supplementary Video 1 for Figure 1d: Self-locomotion of an active droplet with bacteria *B. subtilis* of a diameter $2R = 90\text{ }\mu\text{m}$ in a thermotropic nematic liquid crystal. The droplet distorts the director field of the nematic and produces a point defect-hedgehog on the right-hand side. Asymmetry of the director field rectifies the flows outside the droplet and enables a directional propulsion of the active drop with the hedgehog leading the way.

Supplementary Video 2 for Figure 2e,f: Self-locomotion of an active droplet with bacteria *B. subtilis* of a diameter $2R = 130\text{ }\mu\text{m}$ in a thermotropic nematic. The active flows cause large fluctuations of the director field in the surrounding nematic, so that the equatorial “Saturn ring” of a disclination shifts to the right and left. As a result, the quadrupolar symmetry of the nematic around the droplet is broken and the droplet moves in the direction in which the ring shifts. When the Saturn ring is temporarily located in the equatorial plane, the droplet shows no propulsion. At the end, the ring collapses into the point defect-hedgehog and the droplet gains the maximum speed.

Supplementary Video 3 for Figure 3: Self-locomotion of four active droplets with bacteria *B. subtilis* in a thermotropic nematic. The droplets swim along the circular trajectories set by the photoalignment of the nematic medium.

Supplementary Video 4 for Figure 4a,b,c: Fluorescent microscopy of interior flows visualized by fluorescent tracers in a droplet of diameter $2R = 125\text{ }\mu\text{m}$, $c = 20c_0$. The hyperbolic hedgehog (invisible) is located on the right-hand side of the drop. Fluorescent Suncoast Yellow spheres of diameter 200 nm, excitation wavelength 540 nm, emission wavelength 600 nm.

References

- 1 Stark, H. & Ventzki, D. Non-linear Stokes drag of spherical particles in a nematic solvent. *Europhys Lett* **57**, 60-66 (2002).