
Supplementary information

**Photonic quantum Hall effect and
multiplexed light sources of large orbital
angular momenta**

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Supplementary Information

Photonic quantum Hall effect and multiplexed light sources of large orbital angular momenta

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1. Modeling of the leaky-wave quantum Hall rings

The leaky-rings are modeled by considering the field components of the circular waveguide that constitutes the quantum Hall ring (QH-ring). The far-field radiation is calculated with a wave in the QH-ring propagating only in one direction.

(ρ, ϕ, z) represents the cylindrical coordinates and $\hat{\rho}, \hat{\phi}, \hat{z}$ are the corresponding unit vectors. $\delta(z)$ represents the Dirac delta function, a is the radius of the QH-ring, and $E_{\rho 0}, E_{\phi 0}, E_{z 0}$ are the magnitude of the complex electric field at an origin on the QH-ring. The superscript prime (') denotes a quantity at the source. $\vec{k}'$ and $\vec{l}'$ represent wave vector and propagation vector in the source and $\vec{k}' = k'_{\phi} \hat{\phi} = n_{\text{eff}} k_0 \hat{\phi}$ and $\vec{l}' = a \phi' \hat{\phi}$, where n_{eff} is the effective refractive index of the QH-ring waveguide (topological waveguide) and k_0 is the free space wave vector.

We also assumed that the source only exists inside the QH-ring at $\rho = a$ and $z = 0$, and that the wave in the source propagates in the azimuthal direction $\hat{\phi}$ (Fig. S1a), i.e., the waveguiding direction.

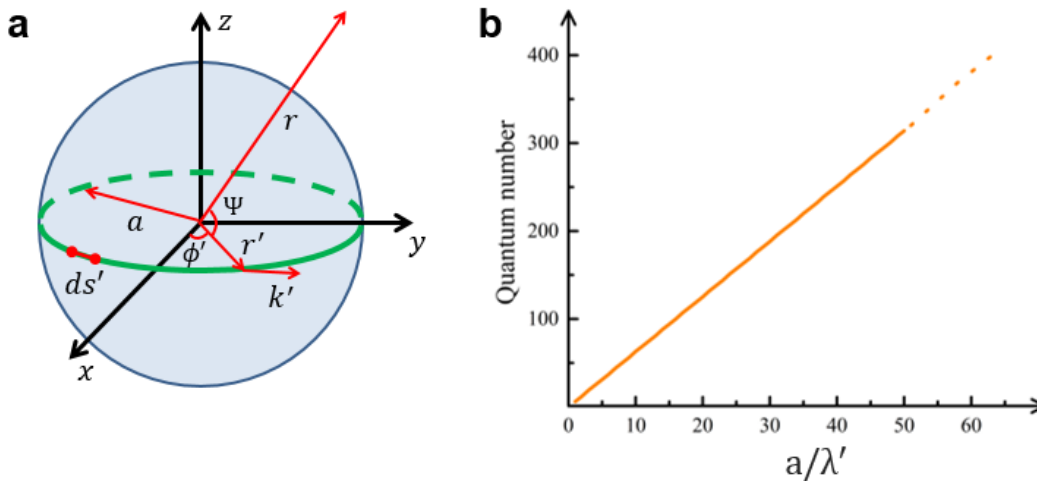


Fig. S1. a, Schematic of a QH-ring of radius a (in green). The propagating mode can be TE-like or TM-like. The cavity length is $m\lambda'$, where m is an integer representing the azimuthal order of the whispering gallery mode and λ' the guided wavelength in the QH-ring. Since the cavity is leaky, the quantum number ℓ is the same as the whispering gallery mode order m . **b**, Quantum number of OAM beam as a function of the normalized radius of the QH-ring.

In the transverse magnetic case (TM), H-field of the source is perpendicular to the propagation direction and only has the H_z component, and, the source E-field has in-plane components (E_ρ , E_ϕ). The transverse electric case (TE) can be similarly derived.

Because the edge mode is dominated by E_x , E_y , and H_z components (see Fig. S4 below), it is mostly TM polarized. For waves propagating in the QH-ring (green in Fig. S1), the field components (TM-like) in cylindrical coordinate can be approximated as:

$$\begin{aligned}\vec{E} &= (E_{\rho 0}\hat{\rho} + E_{\phi 0}\hat{\phi})\text{Exp}[-j\vec{k}' \cdot \vec{l}'] = [\hat{\rho}E_{\rho 0}\text{Exp}(-jk_\phi'a\phi') + \hat{\phi}E_{\phi 0}\text{Exp}(-jk_\phi'a\phi')]\delta(\rho - a)\delta(z) \\ &= \hat{\rho}E_\rho + \hat{\phi}E_\phi \\ \vec{H} &= \hat{z}\frac{E_{z0}}{\eta_0}\text{Exp}[-j\vec{k}' \cdot \vec{l}'] = \hat{z}\frac{E_{z0}}{\eta_0}\text{Exp}(-jk_\phi'a\phi')\delta(\rho - a)\delta(z) = \hat{z}H_z\end{aligned}$$

These fields are in the QH- ring and the evanescent waves are neglected because they do not radiate to the far-field of interest here.

By setting (for simplicity without loss of generality) $E_{\rho 0} = 1$, $E_{\phi 0} = 1$, $m = k_\phi'a$, the equivalent electric and magnetic current sources are [1]:

$$J_s = \hat{n} \times \vec{H} = \hat{z} \times \hat{z}H_z = 0$$

$$M_s = -\hat{n} \times \vec{E} = -\hat{z} \times (\hat{\rho}E_\rho + \hat{\phi}E_\phi) = (-\hat{\phi}E_\rho + \hat{\rho}E_\phi)$$

From these equivalent current sources, the far-field components are calculated to be [1]:

$$E_r \cong 0; H_r \cong 0$$

$$E_\theta \cong -\frac{jke^{-jkr}}{4\pi r}(L_\phi + \eta N_\theta); E_\phi \cong -\frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\phi)$$

$$H_\theta \cong \frac{jke^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right); H_\phi \cong -\frac{jke^{-jkr}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right)$$

E_θ , E_ϕ , H_θ , H_ϕ are the far-fields radiated from the equivalent current sources.

where

$$N_\theta = \int_{ring} [J_\rho \cos\theta \cos(\phi - \phi') + J_\phi \cos\theta \sin(\phi - \phi') - J_z] e^{jkr' \cos\psi} ds'$$

$$N_\phi = \int_{ring} [-J_\rho \sin(\phi - \phi') + J_\phi \cos(\phi - \phi')] e^{jkr' \cos\psi} ds'$$

$$L_\theta = \int_{ring} [M_\rho \cos\theta \cos(\phi - \phi') + M_\phi \cos\theta \sin(\phi - \phi') - M_z] e^{jkr' \cos \Psi} ds'$$

$$L_\phi = \int_{ring} [-M_\rho \sin(\phi - \phi') + M_\phi \cos(\phi - \phi')] e^{jkr' \cos \Psi} ds'$$

(r, θ, ϕ) represent the spherical coordinates. $\eta = \eta_0$ is wave impedance with $\eta_0 \sim 377\Omega$ being the free space wave impedance, $k = n_{eff}k_0$ is wave vector (k_0 is the free space wave vector), Ψ is the angle between the vectors r and r' , and ds' is an elementary displacement in the source. In deriving these equations, we only keep components with $1/r$ dependence, and neglected all other components with higher power of $1/r$ [1]. In the case of integer m , L_θ , L_ϕ have the following analytical solutions:

$$L_\theta = a \cos\theta \text{Exp}(-jm\phi) \left\{ -\pi [j^{m+1}J_{m+1}(x) + j^{m-1}J_{m-1}(x)] + \frac{\pi}{j} [j^{m+1}J_{m+1}(x) - j^{m-1}J_{m-1}(x)] \right\}$$

$$L_\phi = a \text{Exp}(-jm\phi) \left\{ \frac{\pi}{j} [j^{m+1}J_{m+1}(x) - j^{m-1}J_{m-1}(x)] + \pi [j^{m+1}J_{m+1}(x) + j^{m-1}J_{m-1}(x)] \right\}$$

where $x = k a \sin\theta$, j is the imaginary number with $j = \sqrt{-1}$ and $J_{m(x)}$ is Bessel functions of the first kind.

For general m (not necessarily integer), integrating E_ϕ for example, the quantum number ℓ can be calculated using the equation [2]:

$$\ell = \lim_{\rho \rightarrow \infty} \frac{1}{2\pi} \int_0^{2\pi} d\phi \frac{\partial}{\partial \phi} \arg(E_\phi)$$

In this expression, we integrate on a circular path in a plane located $10\lambda_0$ above the equivalent current source. The circular path in numerical integration was chosen to have a radius $\rho = 10a$.

Figure S1b presents the quantum number as a function of the normalized radius of cavity and it naturally obeys the relation $a/\lambda' = \ell/2\pi$. λ' is the propagating wavelength in the QH-ring.

Figure S2a shows the field intensity distribution of the OAM mode for $\ell=100$. By interfering this mode with an OAM beam with the opposite quantum number (i.e. $\ell=-100$), an interference pattern with fringes appears in which the number of fringes is $2\ell=200$ (Fig. S2b). We use this method to experimentally characterize our OAM beams of large quantum numbers.

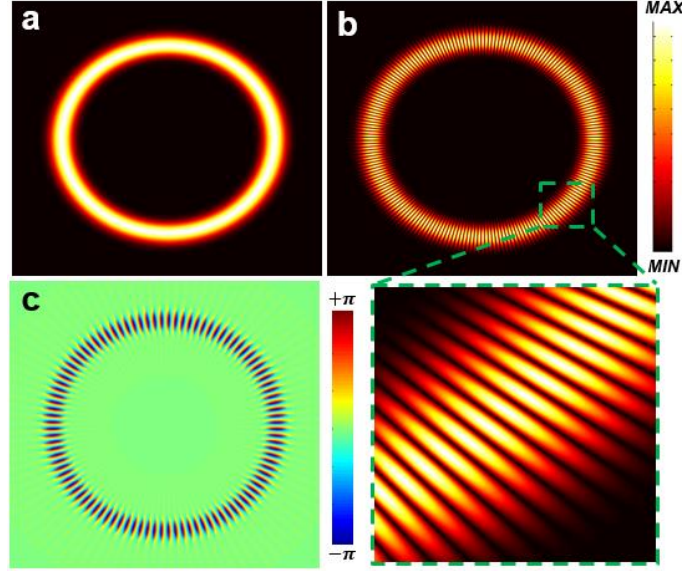


Fig. S2. **a**, Field intensity profile of an OAM mode with $\ell=100$. **b**, Interference pattern of two OAM beams with opposite ℓ numbers. The number of fringes in the interference pattern is equal to 2ℓ . **c**, Phase profile of the OAM beam with $\ell=100$.

2. Simulation of photonic crystals

The QH-ring presented in Fig. S1a is a general system. The unidirectional QH-ring can be implemented using a circular leaky-wave one-way waveguide in which photons orbit using the interface between non-trivial and trivial PhCs [3].

Band diagrams of the two PhCs presented in the manuscript are calculated using finite-element method. Figure S3 presents the band diagram of the square lattice and the triangular lattice PhCs in presence of an external Magnetic Field (MF) of $\mathbf{B}=B_0\mathbf{e}_z$ where $B_0=100$ Oe that saturates the Yttrium Iron Garnet (YIG) material, maximizing the non-diagonal components of its dielectric permittivity tensor. For a zero MF the band structure of the square lattice PhC does not exhibit any gap in the frequency range of interest. The application of $\mathbf{B}$ opens a band gap (green shaded region in Fig. S3a). The topological invariant associated with the corresponding band gap, also known as Chern number, is given for the n^{th} band by [3-4]:

$$C_n = \frac{1}{2\pi i} \int \left(\frac{\partial A_y^{nn}}{\partial k_x} - \frac{\partial A_x^{nn}}{\partial k_y} \right) d^2k,$$

where A is the Berry connection with defined as $A^{nn'}(k) = \langle E_{nk} | \nabla_k | E_{n'k} \rangle$, and the brackets denote spatial integration over a unit-cell. For each mode, n , the field components, E_{nk} , then Berry connection, $A^{nn'}$, are calculated numerically. Finally, the Chern number (C_n) is calculated using the equation above.

The band gap opened due to broken time-reversal symmetry has a non-zero sum of Chern number for all modes below the gap ($|\Delta C|=1$), and it is referred to as a non-trivial photonic gap. In the triangular lattice, there is a large band gap (with and without $\mathbf{B}$) that has a zero sum of Chern number and is thus referred to as trivial band gap.

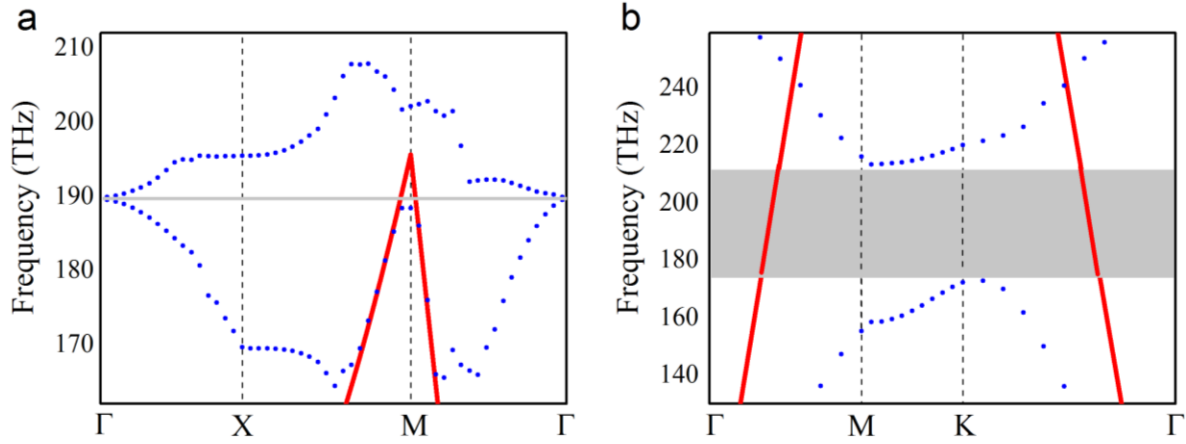


Fig. S3. a, Band diagram of a four-armed star-shaped PhC with a square lattice unit-cell. The band diagram is calculated with a MF that saturates the YIG material. The topological invariant associated to the green band gap, sum of Chern numbers below that gap, is $|\Delta C|=1$. **b**, Photonic band diagram of a triangular lattice PhC with air hole unit-cell in presence of the same MF as in Fig. S3a. Calculations indicate a zero sum of Chern number for this gap.

Figure S4 presents three-dimensional full wave simulations of the interface between the two topologically dissimilar PhCs that constitutes a one-way waveguide. An edge mode is indeed excited with a point source located at the interface, at a frequency that is within the band gap of the two PhCs, and it propagates along the interface. The field distribution inside the waveguide presented in Fig. S4 shows that the mode is predominantly in-plane with dominating components of E_x , E_y , and H_z (TM-like).

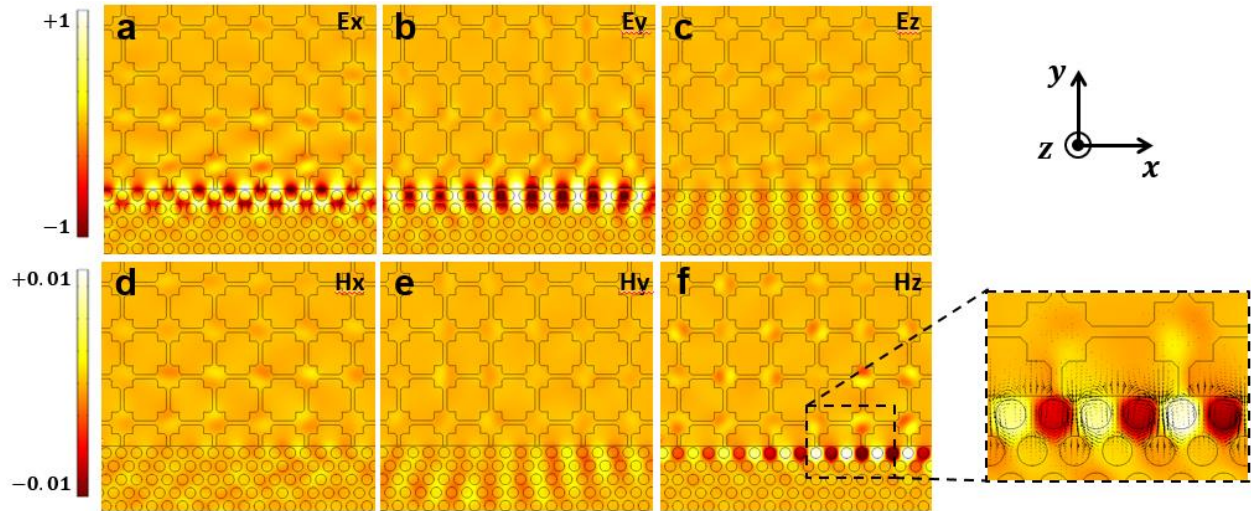


Fig. S4. Electric (**a-c**) and magnetic (**d-f**) field distribution inside the waveguide. Dominant field components are E_x , E_y , and H_z . The inset shows the total mode with H_z distribution (color plot), and E_x and E_y (vector plot). The edge state is above the light cone and thus radiates out-of-plane.

Figure S5 shows the edge mode propagating inside the topological waveguide. As the edge mode propagates, it partially scatters out. The quality factor of the edge mode is on the order of ~ 100 .

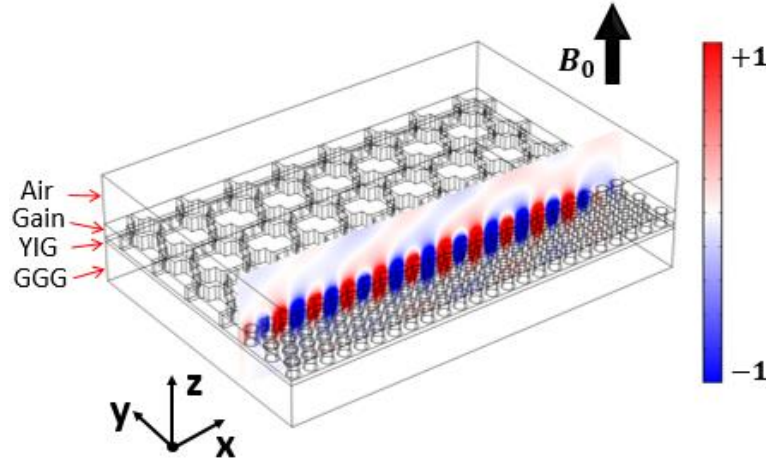


Fig. S5. Full wave simulation of the edge mode. As the edge mode propagates in forward direction, it partially leaks out. The quality factor of the edge mode is about 100.

3. Fabrication

For fabrication, we used standard nanofabrication processes consisting of electron beam lithography, reactive ion etching (RIE), and bonding. The flow chart of the fabrication used several steps schematically presented in Fig. S6. We start by removing the protective layer on top of the InGaAsP using a chemical solution of HCl:H₂O (3:1) for 60 sec (step #2). Next (step 3), the Hydrogen SilsesQuioxane (HSQ) negative tone resist is spin-coated on InGaAsP, preparing it for ebeam lithography. Using RIE the pattern is transferred onto the InGaAsP. Then, the residual HSQ layer is removed by immersing the sample into the hydrofluoric acid for about 30 sec (step #4). For successful bonding, it is crucial to thoroughly clean the substrate in order to work with a dust-free and chemically receptive surface. A thin ebeam resist (PMMA) is spin coated on the YIG substrate followed by a hard baking (step #5). Subsequently, the InGaAsP is bonded to the PMMA coated YIG substrate. The bonding process was performed with a clamp to apply uniform pressure. The sample was put into an oven for curing at 240°C for one hour under 80 kPa pressure. The curing process lasts about five hours including about one hour raising the temperature from room temperature to 240°C, one hour maintaining it at this temperature, and three hours cooling down the sample to room temperature (step #6). The last step removes the InP substrate using a chemical solution of HCl:H₂O (3:1). The whole etching process to remove the InP lasts one hour. The sample was removed from the solution and rinsed with deionized (DI) water (step #7).

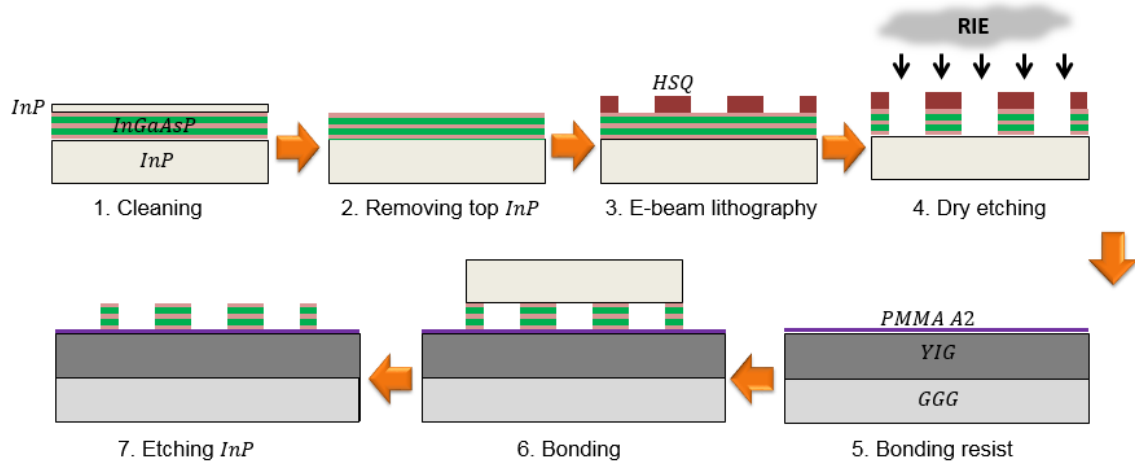


Fig. S6. Flowchart of the fabrication of the QH-rings.

Figure S7 presents scanning electron micrographs (SEM) of a fabricated and superposed QH-rings 1, 2 and 3 of the main manuscript.

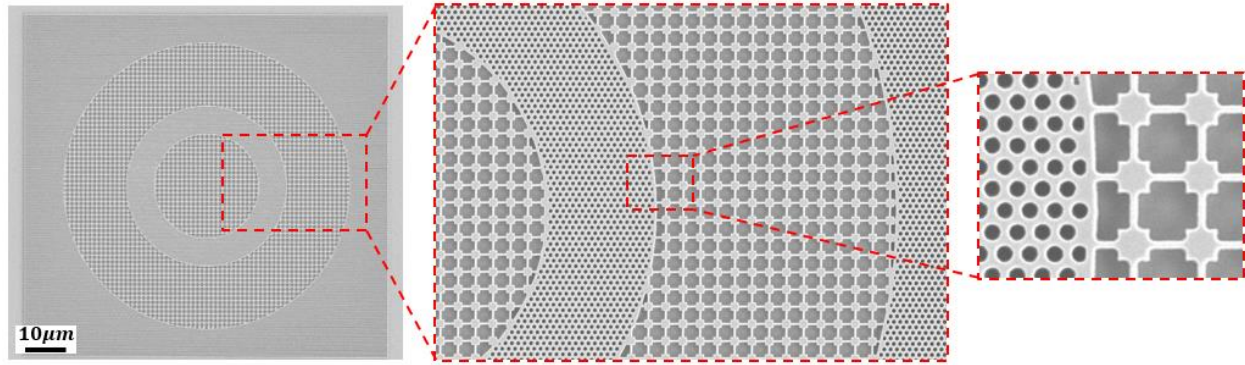


Fig. S7. Scanning electron micrograph of a superposed QH-rings 1, 2, and 3.

4. Characterization

Figure S8 depicts the photoluminescence measurement setup used to characterize the devices. To induce population inversion in the InGaAsP, the devices are optically pumped with a 1064 nm laser beam at room temperature. To finely control the power of the pump laser, we used an attenuator and a power detector. An extra-long working distance microscope objective (Mitutoyo x20) with a numerical aperture (NA) of 0.4 is used to focus the pump beam on the active area of the sample. The size of pump beam is varied between 35 μm and 60 μm depending on the size of cavity. To generate a uniform MF, a solenoid coil is used, which generates a uniform MF at the center of coil with $B_{\text{Max}}=150$ Oe. The coil is positioned in front of the objective lens and the sample is mounted on a stage, which is at the center of the coil. The emission spectra were collected using the same objective lens and emission was directed to a monochromator for spectrum analysis. The real space camera image of the emission can be directly measured from the IR camera by using a movable mirror. To perform the interference measurement, we used

two beam splitters and three mirrors at the collimated beam path as depicted in the inset of Fig. S8.

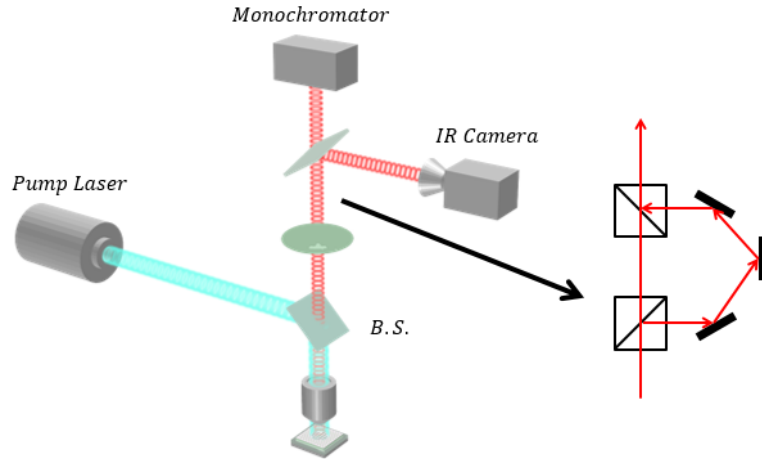


Fig. S8. Experimental setup. The setup is composed of three parts, the first part corresponds to the pumping system, the second part corresponds to the collection and spectrum analysis system using a monochromator, and the third part corresponds to the imaging system using an IR camera. The inset shows the schematic of the interference setup composed of beam splitters and mirrors to measure the quantum number of the OAM beams.

We also characterized our devices by measuring the second-order intensity correlation function ($g^2(\tau)$) with a Hanbury Brown-Twiss (HBT) interferometer, with $g^2(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2}$, where $\langle I(t) \rangle$ represents the expectation value of the intensity at time t . The HBT setup consists of two Geiger-mode avalanche photo diodes (APDs from ID Quantique-ID220) and a time-correlated single photon counting module (PicoQuant PicoHarp 300). The arrival time of each emitted photon from our device is recorded by the PicoHarp such that the time delay of two photons arriving at different APDs can be calculated to construct the correlation histogram. Figure S9 presents the experimental correlation histogram (Figs. S9a-e) and the corresponding normalized $g^2(\tau)$ (Fig. S9f-j) at five different pump power densities.

Due to nanosecond pulse pumping (290 kHz repetition rate, 6 ns pulse width), the correlation histogram inherently consists of many pulses corresponding to the optical pump pulses arriving and the device turning on. The time delay between neighboring pulses in Fig. S9a-e is approximately 3.4 μ s, which is consistent with the 290 kHz repetition rate of the pump laser. According to coherence theory, a photon bunching peak near zero-delay can be observed in $g^2(\tau)$ for a lasing device in the SpE and ASE regimes where the degree of coherence is low [5]. The width of the photon bunching peak is on the order of the coherence time, which is expected to be much smaller than the repetition period of the pump laser ($\sim 3.4 \mu$ s). Therefore, the photon bunching peak will emerge only on top of the zero-delay pulse, making it taller than the non-zero-delay pulses (Fig. S9b-d). In the lasing regime, such a photon bunching peak is suppressed since coherent emission is achieved [5]. This is clearly demonstrated in Fig. S9e.

The photon bunching peak can be more readily observed in the normalized $g^2(\tau)$, which is obtained by dividing the zero-delay pulse by the mean of the non-zero delay pulses. The mean is taken here to reduce the effect of shot noise in the experiment. Figure S9f-j show examples of the normalized $g^2(\tau)$ in the SpE (Fig. S9f), ASE (Fig. S9g-i) and StE (Fig. S9j) regimes of our devices. Evidently, the photon bunching peak visible in the ASE regime is suppressed in the StE regime. However, contrary to theoretical prediction, the photon bunching is also absent in the SpE regime (Fig. S9a,f). This experimental observation is primarily due to the timing uncertainty of our APDs, which is on the order of 100 ps. Since the coherence time in the SpE regime is much shorter than 100 ps, the averaging effect from the APDs renders the photon bunching peak invisible.

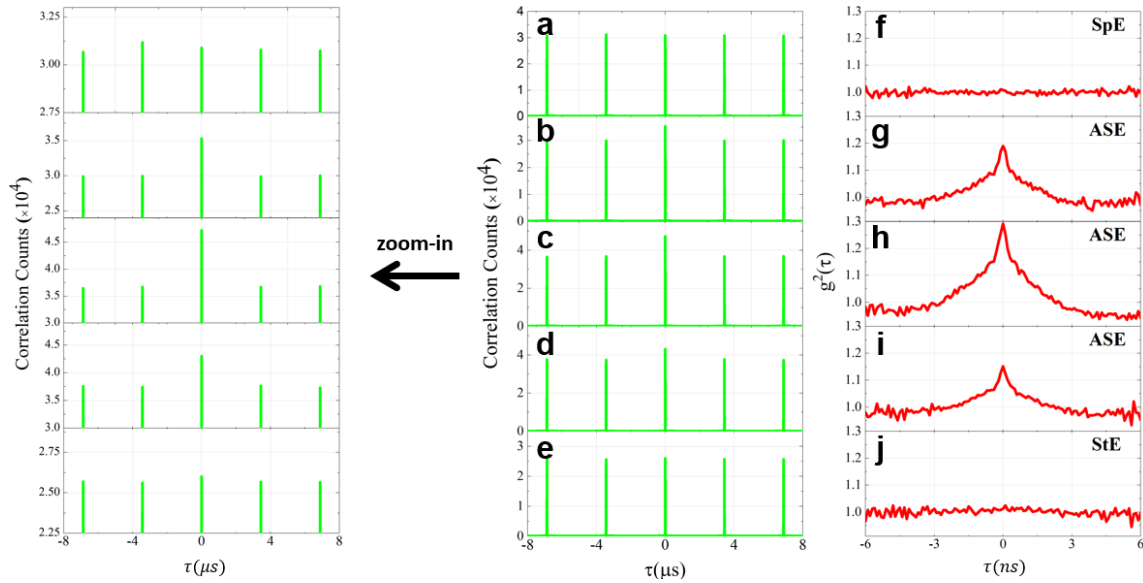


Fig. S9. Intensity correlation histograms (a-e), and the corresponding normalized $g^2(\tau)$ (f-j) in the SpE, ASE and StE regimes. The photon bunching peaks emerge in the ASE regime (b-d, g-i) signifying that stimulated emission begins to dominate in our device as it transitions into the StE regime. At the highest pump power density (e,j), the photon bunching peak is suppressed due to lasing. In the SpE regime (a,f); however, the photon bunching peak is absent because the coherence time is too short to be resolved by the APDs [6].

It is worth noticing that the correlation functions presented in Fig. S9f-j are already normalized, and we did not further normalize this function to the non-zero delay pulses. Rather, we obtained this normalized function by dividing the central (zero-delay) pulse in the raw correlation histogram (shown in Fig. S9a-e) by an average of all the side (non-zero-delay) pulses. To clarify this procedure, we have plotted the central pulse from correlation histogram Fig. S9c (see Fig. S10). For easy comparison, the average of all the side pulses from the same histogram is also presented on the same axis. Dividing the blue curve by the red curve, we obtain the normalized correlation function presented in Fig. S9h.

As shown, the g^2 correlation peak is riding on top of the central pulse only and for $|\tau| > 6\text{ns}$, the count in the central pulse (blue curve) approaches that of the side pulse (red curve). Therefore, the normalized correlation function approaches 1 for $|\tau| > 6\text{ns}$. For $|\tau| > 10\text{ns}$, the light pulse from the device has already extinguished. However, the correlation count is not zero due to dark counts from the photo detector. Upon dividing the dark counts at $|\tau| > 10\text{ns}$ in the central pulse by those of the side pulse, a near unity value is again obtained for the normalized correlation function. Therefore, the normalized correlation function will not go to zero. The tail of the correlation function in Fig. S9g,h,i goes slightly below unity. This is most likely due to lower efficiency from one of the detectors. However, it shouldn't significantly affect the zero-delay value (i.e. $g^2(0)$), from which we draw our main conclusion (see Fig. 3e in the main manuscript).

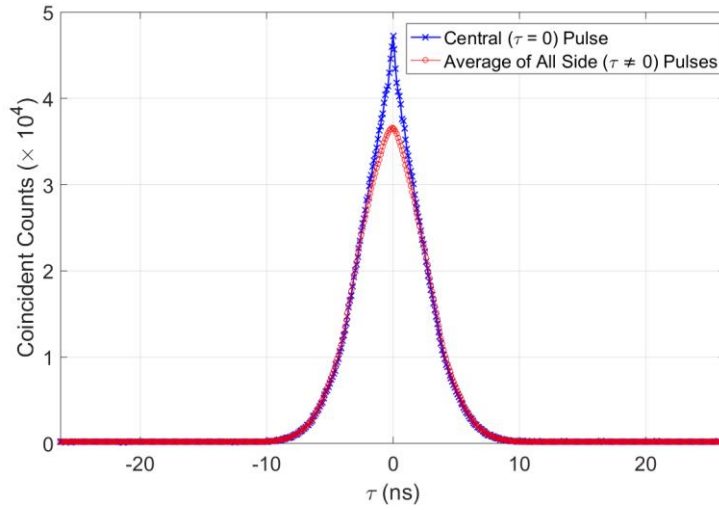


Fig. S10. A zoom-in view of Fig. S9c. The average of all side pulses (red) is plotted on top of the central pulse (blue) for comparison.

Figure 3e in the manuscript was acquired by extracting the zero-delay value, i.e., $g^2(0)$, of the normalized $g^2(\tau)$ in a wide range of pump powers. $g^2(0) \sim 1$ in both the SpE and StE regimes, confirms that our device indeed reached the lasing regime for two reasons. Firstly, the output power of our device continued to increase even at the highest pump intensities. Hence, it's impossible that the device reverses back to the SpE regime due to, for example, degradation as that should lead to a significant decline in the output power. Furthermore, we examined the full width at half maximum (FWHM) of the $g^2(\tau)$ pulses as a function of the pump power (Fig. 3f in the main manuscript). In a previous study [6], it is shown that when a laser is optically pumped with nanosecond pulses, its $g^2(\tau)$ pulse FWHM shrinks in the SpE regime, reaches a minimum in the ASE regime and broadens in the StE regime. Such variations in the $g^2(\tau)$ pulse width are related to a nonlinear effect called the delay threshold phenomenon (DTP) or dynamical hysteresis (DH), which occurs in a laser only when the peak intensity of a pump pulse is larger than the threshold intensity [6-7]. Therefore, an indirect observation of DTP via the $g^2(\tau)$ pulse width unambiguously shows lasing. The measured $g^2(\tau)$ pulse FWHM of our topological laser is

shown in Fig. S11. We further measured the $g^2(\tau)$ pulse FWHM of our pump laser at different pump powers to ensure that the shrinkage and broadening effects are not due to anomalies of the pump laser. Figure S11 clearly shows that the $g^2(\tau)$ pulse FWHM of the pump laser stays constant at all pump powers while that of the devices varies significantly. Consequently, the notable variation in the $g^2(\tau)$ pulse width results solely from the emission characteristics of our devices.

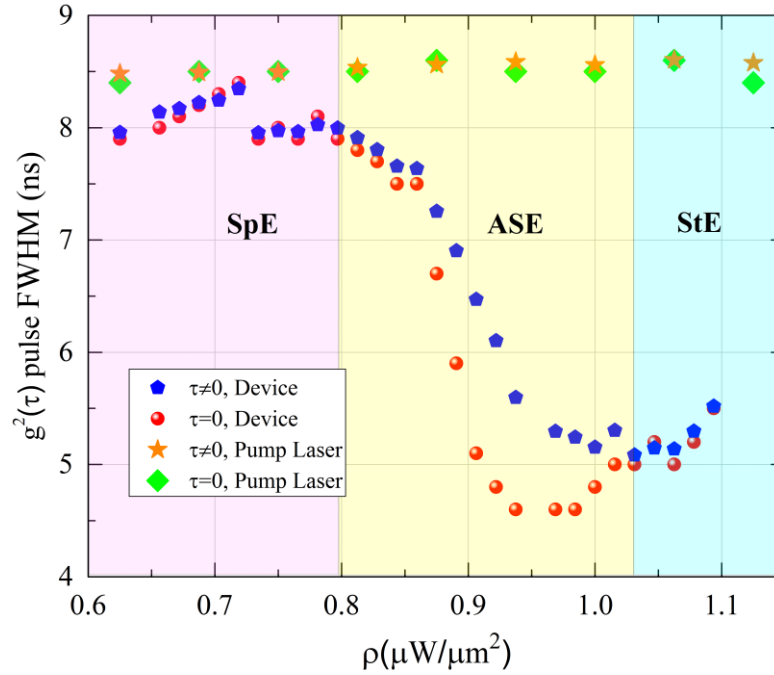


Fig. S11. The pulse FWHM of the device is plotted along with that of the pump laser. Evidently, the $g^2(\tau)$ width of the pump laser stays nearly constant while that of the device varies significantly. Therefore, the device width variations are due solely to the distinctive emission statistics in the SpE, ASE, and StE regimes, and, are not correlated with the operation of the pump laser.

Figure S12 shows the output power evolution for the other cavities in Fig. 2 of the manuscript. The measured linewidth of the laser (1.86 nm), can originate from thermal effects or fabrication imperfections including non-perfectly uniform bonding. However, second order intensity correlation measurements confirmed that QH-rings generate coherent emission.

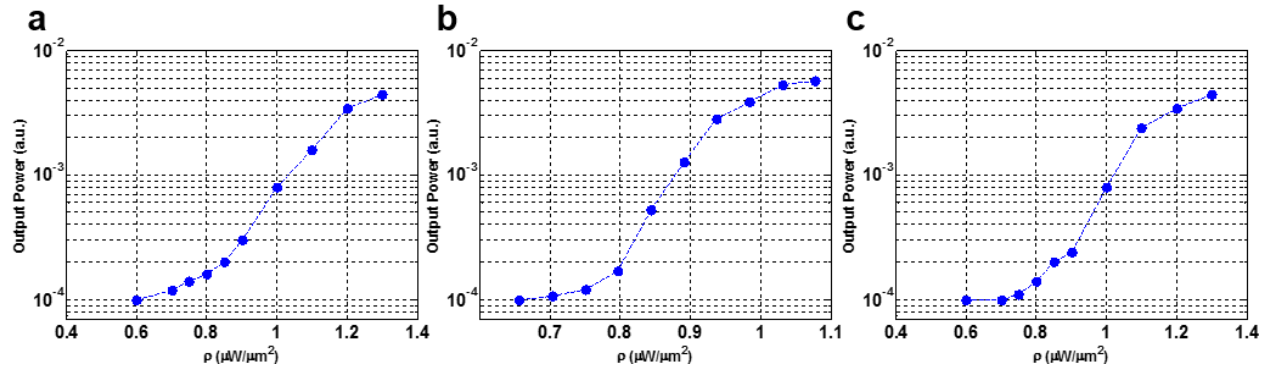


Fig. S12. Output power evolution as a function of the pump power density for cavity with (a) $\ell_1=100$, (b) $\ell_2=156$ (c) $\ell_3=276$.

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