nature portfolio

https://doi.org/10.1038/s41567-021-01463-1

Supplementary information

Direct neutrino-mass measurement with sub-electronvolt sensitivity

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Direct neutrino-mass measurement with sub-eV sensitivity

KATRIN Collaboration

1 Combined analysis of first and second neutrino mass campaign

The 1000 days of total measurement time of the KATRIN experiment will be distributed within 15-20 individual measurement campaigns. The experimental parameters will not be constant over all these campaigns, as the systematic uncertainties are reduced and the experimental performance is improved (e.g. the background rate will be lowered) with time. Nevertheless, all individual campaigns need to be combined eventually to achieve the full statistical power.

1.1 Combination of individual results

One approach is to simply multiply the m_ν^2 distributions obtained with MC-propagation of the individual measurement phases. As the uncertainties of the first and second neutrino mass campaigns are still largely statistically dominated and the main systematic uncertainties are uncorrelated, this method is well justified for the combination presented here. The best fit, obtained from the combined distribution is $m_\nu^2 = (0.11 \pm 0.34)\,\mathrm{eV}^2$. Assuming a symmetric Gaussian distribution of m_ν^2, we derive an upper limit of $m_\nu < 0.81\,\mathrm{eV}$ (90 % C.L). Adding the χ^2-profiles of the individual fits leads to statistically consistent results.

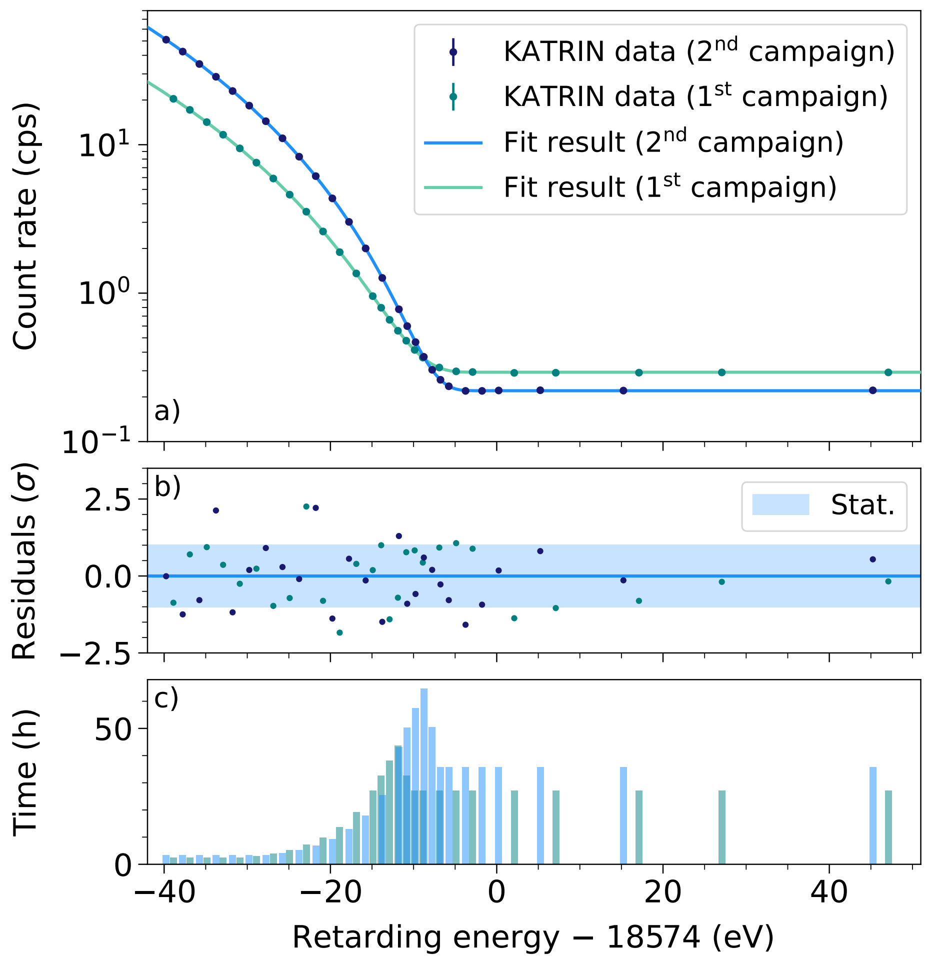
1.2 Simultaneous fit of the both neutrino mass campaigns

Another way, to combine both data sets, is by performing a simultaneous fit of both data sets with a common neutrino mass. The first and second neutrino mass campaigns were performed under different experimental conditions, e.g. the column density was increased by a factor of four and the background level was reduced by 25 % in the second campaign. Correspondingly, the systematic parameters and their uncertainties are partly different in the two campaigns. As a consequence, the data of the two campaigns cannot be described with a single effective spectrum prediction. Nevertheless, a simultaneous fit of both data sets can be performed by extending the χ^2 function to

$$\begin{aligned}
&\chi^2(m_\nu^2, \vec{\Theta}_{p1}, \vec{\Theta}_{p2}, \vec{\eta}_{p1}, \vec{\eta}_{p2}, \vec{\eta}_c) \\
&\quad = \chi^2(m_\nu^2, \vec{\Theta}_{p1}, \vec{\eta}_{p1}, \vec{\eta}_c) + \chi^2(m_\nu^2, \vec{\Theta}_{p2}, \vec{\eta}_{p2}, \vec{\eta}_c) \\
&\quad + \sum_i \left(\frac{\hat{\eta}_{p1,i} - \eta_{p1,i}}{\sigma_{\eta_{p1,i}}} \right)^2 + \sum_i \left(\frac{\hat{\eta}_{p2,i} - \eta_{p2,i}}{\sigma_{\eta_{p2,i}}} \right)^2 + \sum_i \left(\frac{\hat{\eta}_{c,i} - \eta_{c,i}}{\sigma_{\eta_{c,i}}} \right)^2 ,
\end{aligned} \tag{1}$$

where $\vec{\Theta}_{p1}$ and $\vec{\Theta}_{p2}$ depict the endpoint, background, and normalisation of the first ($p1$) and second ($p2$) measurement period. These parameters can be different in the two measurement phases, as the experimental conditions are changed. $\vec{\eta}_{p1}$ and $\vec{\eta}_{p2}$ correspond to the two sets of systematic parameters, while $\vec{\eta}_c$ illustrates the set of common systematic parameters. The neutrino mass squared m_ν^2 is a common parameter of both χ^2 functions. The last three terms are "pull terms" to constrain the systematic parameters as introduced in Eq. 8 in the main text.

For a simultaneous fit the number of free parameters and data points increases by about a factor of two compared to the fit of a single measurement campaign, which makes the minimisation computationally challenging and prone to numerical noise. To simplify the problem, we choose for this analysis to group the detector rings to one effective detector area (uniform fit), reducing the number of free parameters to $n_{\mathrm{free}} = 33$ and the number of data points to $n_{\mathrm{data}} = 55$.



Supplementary Figure 1. Measured rate at each retarding energy for the first[1,2] (turquoise points) and second (black points) KATRIN measurement campaign. The turquoise (KNM1) and blue (KNM2) lines depict a simultaneous fit to the data of both neutrino mass campaigns. For this fit the measured rates of the 12 detector rings are combined to a single detector area (uniform fit) in both campaigns. The fit description is given in Eq. 2 in the main article. The fit results are quoted in the main text of the supplementary information. b) Normalised residuals for the fit to the data of both campaigns. The blue band shows the 1-σ statistical uncertainty. c) Total measurement time at each retarding energy for the first and second measurement campaign. KNM1 and KNM2 have 27 and 28 scan steps, respectively, with slightly differing retarding energies. The most sensitive region of the spectrum where most of the measurement time is spent is shifted slightly to higher energies in the KNM2 campaign due to the improved signal to background.

The fit to both data sets is shown in Supplementary Figure 1. The $\chi^2/\text{ndof} = 51.7/48 = 1.1$ indicates excellent agreement of the model to the data. The final result of the combined fit reveals $m_\nu^2 = (0.07 \pm 0.32)\,\text{eV}^2$ and a corresponding upper limit of $m_\nu < 0.75\,\text{eV}$ (90 % CL).

A combined fit was also performed with the covariance matrix method. In this case, the dimension of the covariance matrix is given by the number of data points in the two campaigns, here $n_{\text{data}} = 55$, and the off-diagonal entries reflect the correlation between data points within and across the campaigns. We find a consistent best fit as listed in Supplementary Table 1.

In complement to the standard goodness-of-fit, we perform the Parameter-Goodness-of-Fit (PGoF) test[3] to assess the compatibility of KNM1 and KNM2 data sets. This test quantifies the penalty of combining the data sets compared to fitting them independently. We find a PGoF probability of 17 % reflecting a good agreement between the two statistically independent data sets.

It should be noted that with respect to the stand-alone first neutrino mass analysis as presented in[2], the spectrum calculation has been slightly improved. It now includes the transmission function for non-isotropic electrons, an improved parametrisation of the energy loss function, and a reduced uncertainty on the magnetic fields and column density. Finally, a possible Penning-trap induced time-dependent background, as described in the main text, is included. A re-analysis of the first neutrino mass data with these new inputs reveals a best fit of $m_\nu^2 = -1.1^{+0.9}_{-1.1}\,\text{eV}^2$ and an upper limit of $m_\nu < 1.1\,\text{eV}$ at 90 % C.L. These values are in good (10 %) agreement with respect to the published result in[1].

1.3 Bayesian combination of both neutrino mass campaigns

Another way of combining different neutrino mass measurement campaigns is by using the posterior distribution of one campaign as prior information for the other campaign. In this procedure, correlations between the data sets are neglected.

For the analysis of the first neutrino mass measurement campaign, we use the result published in[2], which includes the five major systematic effects. The posterior distribution for m_ν^2, displayed in Supplementary Figure 2 (green colour), is then used as prior distribution of m_ν^2 for the analysis of the second neutrino mass measurement campaign, which is otherwise performed with the same procedure as the stand-alone analysis.

The central value is found to be $m_\nu^2 = (0.06 \pm 0.32)\,\text{eV}^2$. The corresponding 90 % credible interval from a positive prior on m_ν^2 corresponds to a Bayesian limit of $m_\nu < 0.73\,\text{eV}$, as illustrated in Supplementary Figure 2.

References

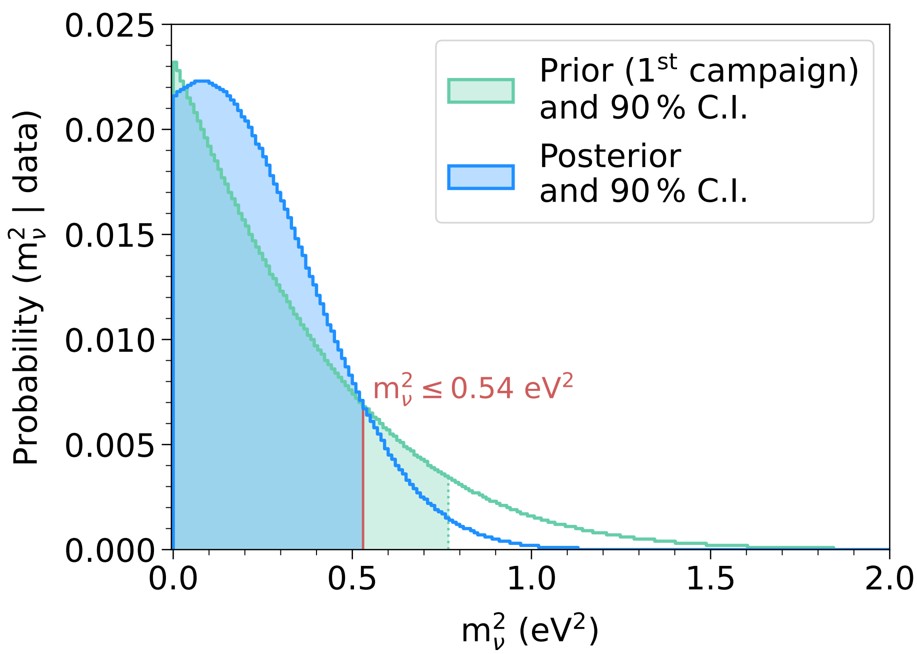
1. Aker, M. *et al.* Improved Upper Limit on the Neutrino Mass from a Direct Kinematic Method by KATRIN. *Phys. Rev. Lett.* **123**, 221802, DOI: https://doi.org/10.1103/PhysRevLett.123.221802 (2019). 1909.06048.

2. Aker, M. *et al.* Analysis methods for the first KATRIN neutrino-mass measurement. *Phys. Rev. D* **104**, 012005, DOI: https://doi.org/10.1103/PhysRevD.104.012005 (2021).

3. Maltoni, M. & Schwetz, T. Testing the statistical compatibility of independent data sets. *Phys. Rev. D* **68**, 033020, DOI: https://doi.org/10.1103/PhysRevD.68.033020 (2003). hep-ph/0304176.

Supplementary Table 1. Overview of the results of the different strategies applied to KNM2 and the KNM1 and 2 combined analysis. For simplicity we average the negative and positive 68.2 % CL uncertainties. Strategies 1 and 2 perform a full simultaneous fit to combine KNM1 and KNM2, while strategy 3 multiplies the m_ν^2 distributions from MC-propagation, and strategy 4 uses the m_ν^2 posterior of KNM1 as prior for KNM2. Furthermore, strategies 1 and 2 use the improved inputs for KNM1, while strategy 3 and 4 use the original KNM1 inputs. Limits for strategy 1-3 are based on Lokhov-Tkachov (or equivalently Feldman-Cousins) technique and strategy 4 is based on the 90% posterior quantile obtained with a flat prior for positive values of m_ν^2. These difference can explain the slight volatility in the numerical values.

	Strategies			
	1 (Pull term)	2 (Cov. matrix)	3 (MC prop.)	4 (Bayesian)
	KNM2			
m_ν^2 best fit and 68.2 % CL uncertainties (eV^2)	0.25 ± 0.35	0.26 ± 0.32	0.26 ± 0.34	0.26 ± 0.34
limit at 90 % CL (CI for Bayes) (eV)	0.87	0.90	0.91	0.85
	KNM1 and KNM2 Combined			
m_ν^2 best fit and 68.2 % CL uncertainties (eV^2)	0.07 ± 0.32	0.07 ± 0.31	0.10 ± 0.33	0.06 ± 0.32
limit at 90 % CL (CI for Bayes) (eV)	0.75	0.76	0.81	0.73



Supplementary Figure 2. Posterior distributions of the observable m_ν^2 for the Bayesian combination of the first[1,2] and second KATRIN measurement campaign. The posterior distribution of the first measurement campaign (green)[2] is used as a prior information on the neutrino mass for the second measurement campaign. The resulting posterior distribution for the second campaign is shown in blue. The 90% credible interval of the combined analysis is obtained by integrating the posterior distribution (blue) up to the 90% of the cumulative probability, as indicated by the red solid line.