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Supplementary information

Quantifying the role of antiferromagnetic fluctuations in the superconductivity of the doped Hubbard model

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Supplementary Material

I. COUPLING CONSTANT

To examine the robustness of the coupling constant fitting procedure, we repeat the same analysis for the additional doping and interaction levels shown in the phase diagram of Fig. 1 of Ref. 1

- $U = 4t$, $\mu = -0.2t$, $x \approx 0.035$.
- $U = 4t$, $\mu = -0.6t$, $x \approx 0.11$.
- $U = 5.5t$, $\mu = -0.2t$, $x \approx 0.020$, $T_c \in (t/30, t/35]$.
- $U = 5.5t$, $\mu = -0.6t$, $x \approx 0.067$, $T_c \in (t/35, t/40]$.
- $U = 6t$, $\mu = -0.8t$, $x \approx 0.076$, $T_c \in (t/30, t/35]$.
- $U = 6t$, $\mu = -0.9t$, $x \approx 0.089$, $T_c \in (t/30, t/35]$.
- $U = 6t$, $\mu = -1.1t$, $x \approx 0.12$, $T_c \in (t/40, t/45]$.
- $U = 6t$, $\mu = -1.6t$, $x \approx 0.19$.

The choice of parameters is influenced by the following considerations: for higher U, calculations become more difficult, while for lower U they are less relevant for strong correlation superconductivity. Higher dopings reduce T_c, whereas lower dopings enhance the effects of the nearby pseudogap and the effects of the AFM states near half filling, making the RPA-like one-loop spin fluctuation approximation less likely to succeed.

In general, our analysis is robust in the parameter regime presented in the paper (slightly overdoped regime away from pseudogap and AFM around half filling): We can get reasonable g^2 from our fitting procedure at $U = 4$ for $\mu = -0.6$ ($x \approx 0.11$) (see Fig. 1); at $U = 5.5t$ for $\mu = -0.6t$ ($x \approx 0.067$) (see Fig. 2); and at $U = 6t$ for $\mu = -1.1t, -1.6t$ ($x \approx 0.12, 0.19$) (see Figs. 3, 4). At $U = 6t$ for $\mu = -0.9t$ ($x \approx 0.088$) the fitting procedure is borderline (see Fig. 5). For the same interaction strength U, at slightly lower (higher) doping levels, the percentage of self-energy given by RPA-like one-loop spin fluctuation decreases (increases), consistent with expectations. Fitting with the same U and μ at different temperatures (and different momentum points) also leads to minor quantitative variations but no qualitative difference. At very high temperature, the fitting breaks down.

II. DOPING AND INTERACTION DEPENDENCE

The change of pairing fluctuations and T_c with different interactions U is shown in Refs. 1–3. In general, the change is non-linear and superconductivity is strongest around $U = 5.5 - 6$.

Below, we show calculations for the following additional parameters:

- $U = 5.5t$, $\mu = -0.6t$, $x \approx 0.067$, $T_c \in (t/35, t/40]$.
- $U = 6t$, $\mu = -1.1t$, $x \approx 0.12$, $T_c \in (t/40, t/45]$.
- $U = 6t$, $\mu = -0.9t$, $x \approx 0.089$, $T_c \in (t/30, t/35]$.

The comparison of the anomalous component given by the one-loop approximation and the DCA self-energy at these doping levels is shown in Fig. 6, Fig. 7, and Fig. 8. In general, at slightly lower (higher) doping levels the percentage of self-energy given by a one-loop spin fluctuation exchange decreases (increases).

FIGURES

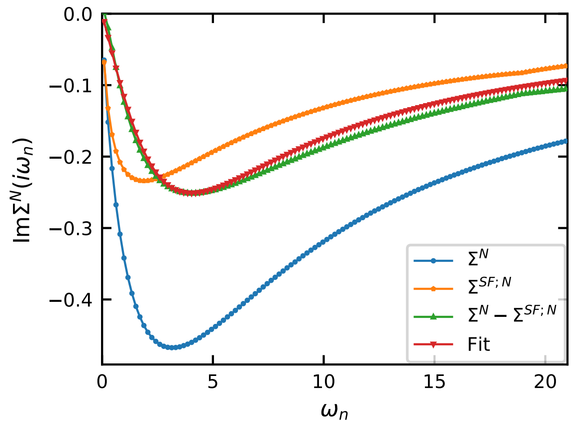

FIG. 1. Imaginary part of the normal component of the Matsubara self-energy at the antinode $(0, \pi)$ at $U = 4t$, $\beta t = 35$, $\mu = -0.6t$ ($x \approx 0.11$) compared to spin fluctuation self-energy computed with $g^2 = 2.35$.

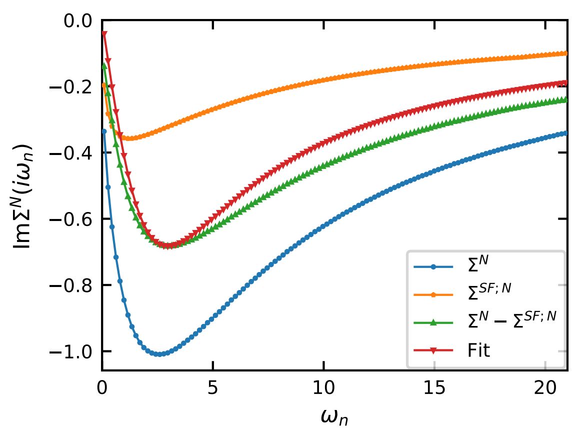

FIG. 2. Imaginary part of the normal component of the Matsubara self-energy at the antinode $(0, \pi)$ at $U = 5.5t$, $\beta t = 35$, $\mu = -0.6t$ ($x \approx 0.067$) compared to spin fluctuation self-energy computed with $g^2 = 2.90$.

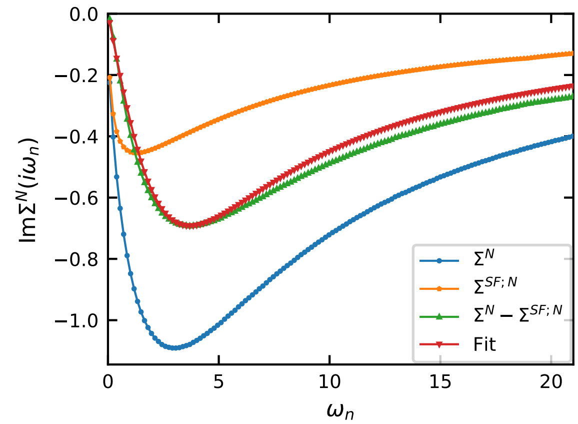

FIG. 3. Imaginary part of the normal component of the Matsubara self-energy at the antinode $(0, \pi)$ at $U = 6t$, $\beta t = 40$, $\mu = -1.1t$ ($x \approx 0.12$) compared to spin fluctuation self-energy computed with $g^2 = 3.80$.

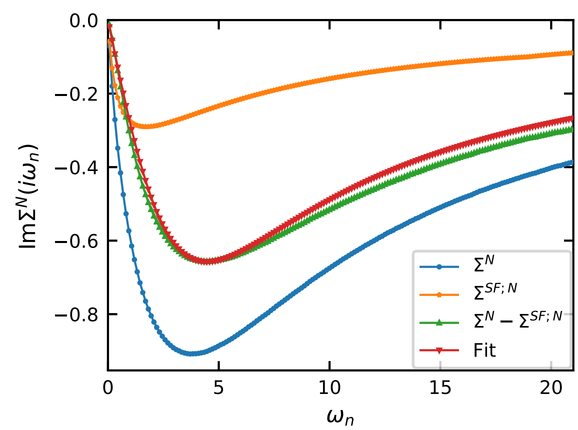

FIG. 4. Imaginary part of the normal component of the Matsubara self-energy at the antinode $(0, \pi)$ at $U = 6t$, $\beta t = 50$, $\mu = -1.6t$ ($x \approx 0.19$) compared to spin fluctuation self-energy computed with $g^2 = 2.80$.

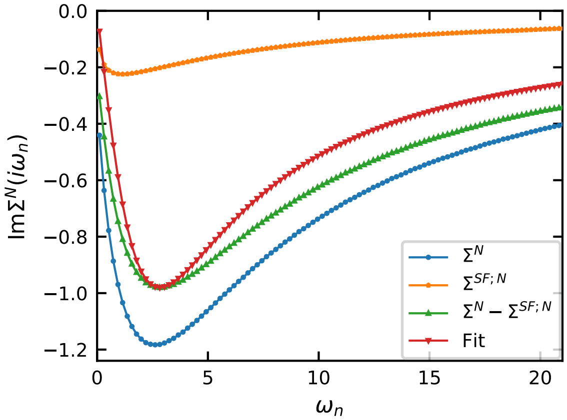

FIG. 5. Imaginary part of the normal component of the Matsubara self-energy at the antinode $(0, \pi)$ at $U = 6t$, $\beta t = 30$, $\mu = -0.9t$ ($x \approx 0.089$) compared to spin fluctuation self-energy computed with $g^2 = 1.78$.

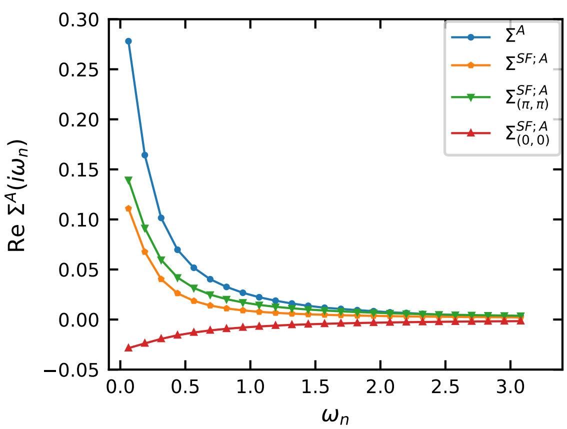

FIG. 6. Total measured anomalous self-energy Σ^A and estimated spin-fluctuation contribution $\Sigma^{SF;A}$ at $U = 5.5t$, $\beta t = 50$, $\mu = -0.6t$ ($x \approx 0.067$), with $g^2 = 2.90$. Also shown are the individual contributions to $\Sigma^{SF;A}$ from transferred momenta $Q = (\pi, \pi)$ and $Q = (0, 0)$.

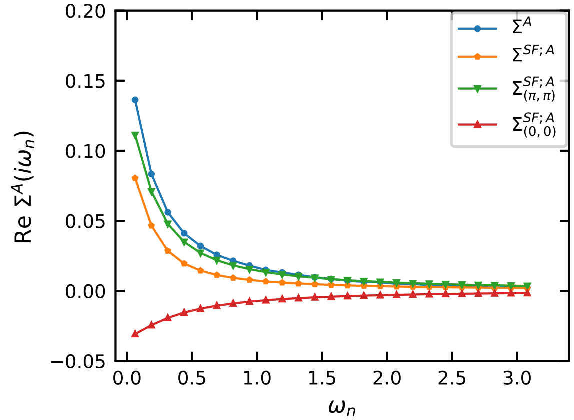

FIG. 7. Total measured anomalous self-energy Σ^A and estimated spin-fluctuation contribution $\Sigma^{SF;A}$ at $U = 6t$, $\beta t = 50$, $\mu = -1.1t$ ($x \approx 0.12$), with $g^2 = 3.80$. Also shown are the individual contributions to $\Sigma^{SF;A}$ from transferred momenta $Q = (\pi, \pi)$ and $Q = (0, 0)$.

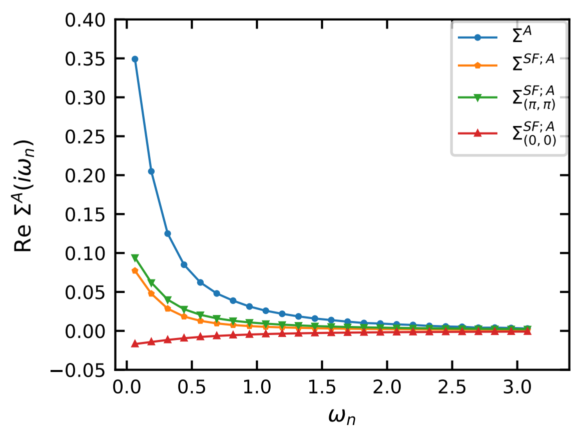

FIG. 8. Total measured anomalous self-energy Σ^A and estimated spin-fluctuation contribution $\Sigma^{SF;A}$ at $U = 6t$, $\beta t = 50$, $\mu = -0.9t$ ($x \approx 0.089$), with $g^2 = 3.80$. Also shown are the individual contributions to $\Sigma^{SF;A}$ from transferred momenta $Q = (\pi, \pi)$ and $Q = (0, 0)$.

[1] E. Gull, O. Parcollet, and A. J. Millis, Phys. Rev. Lett. **110**, 216405 (2013).
[2] X. Chen, J. P. F. LeBlanc, and E. Gull, Phys. Rev. Lett. **115**, 116402 (2015).
[3] X. Dong and E. Gull, Phys. Rev. B **101**, 195115 (2020).