

# Three-dimensional non-Abelian quantum holonomy

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Supplementary information

for

# Three-dimensional non-Abelian Quantum Holonomy

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## Remarks on the non-classicality of the measurements

To verify that the two input photons are indeed indistinguishable, we characterize the Hong-Ou-Mandel interference<sup>30</sup> of our type-I SPDC source. To this end, a variable time delay is introduced in one of the output beams before both are fed into the arms of a fiber-integrated 50:50 fiber beam splitter. The obtained delay-dependent coincidence counts are depicted in Fig. S1. A Gaussian fit shows a visibility of  $V = 99.0\%$ .

To examine the nonclassicality of the output state, a click-counting theory based on second order moments is employed<sup>S1-S3</sup>. Adhering to the procedure outlined in the supplementary material of Ref. <sup>S1</sup>, the second order matrix of moments can be directly calculated from the measured click counting statistics. Performing a linear error propagation to its eigenvalues yields the signed significance  $\Sigma_n = \bar{e}_n / \Delta e_n$ , where  $\bar{e}_n$  is the mean of the  $n$ th eigenvalue and  $\Delta e_n$  its error margin. If the probed click counting statistics are consistent with the classical bound,  $\Sigma_n \gtrsim 0$  for all eigenvalues. To clearly violate the classical bound at least one eigenvalue has to fulfil  $\Sigma_n \lesssim -3$ . Regarding the input dark state  $|D_2(z_i)\rangle$ , one eigenvalue can be found with  $\Sigma_1 \approx -22.0$ , thus clearly showing that the output state is nonclassical.

As to the input states  $|D_1(z_i)\rangle$  and  $|D_3(z_i)\rangle$ , since both photons here inhabit the same mode here, their mean photon numbers propagate classically, i.e., without quantum interference. This follows from a more general statement on linear optics: If  $N$  photons are injected into a single optical mode, the mean photon number that the detector of the  $k$ th mode measures is the multiple of the single-photon case,  $\langle \hat{n}_k \rangle^{(N)} = N \langle \hat{n}_k \rangle^{(1)}$ . This emphasizes again the relation between  $|D_1(z_i)\rangle = |0200\rangle$  and  $|d_1(z_i)\rangle = |0100\rangle$ , as well as  $|D_3(z_i)\rangle = |2000\rangle$  and  $|d_2(z_i)\rangle = |1000\rangle$  (cf. Fig. 2c).

## Detection probabilities for the measurement of $\hat{u}(\gamma)$ and $\hat{U}(\gamma)$

### a) Experiment U(2)

| Input \ Output     | $ d_1(z_i)\rangle$ | $ d_2(z_i)\rangle$ |
|--------------------|--------------------|--------------------|
| $ d_1(z_i)\rangle$ | $(87.1 \pm 0.1)\%$ | $(12.9 \pm 0.3)\%$ |
| $ d_2(z_i)\rangle$ | $(15.3 \pm 0.3)\%$ | $(84.7 \pm 0.1)\%$ |

### b) Theory U(2)

| Input \ Output     | $ d_1(z_i)\rangle$ | $ d_2(z_i)\rangle$ |
|--------------------|--------------------|--------------------|
| $ d_1(z_i)\rangle$ | 89.3%              | 10.7%              |
| $ d_2(z_i)\rangle$ | 10.7%              | 89.3%              |

### c) Simulation U(2)

| Input \ Output     | $ d_1(z_i)\rangle$ | $ d_2(z_i)\rangle$ |
|--------------------|--------------------|--------------------|
| $ d_1(z_i)\rangle$ | 90.4%              | 9.6%               |
| $ d_2(z_i)\rangle$ | 12.4%              | 87.6%              |

### d) Experiment U(3)

| Input \ Output     | $ D_1(z_i)\rangle$ | $ D_2(z_i)\rangle$ | $ D_3(z_i)\rangle$ |
|--------------------|--------------------|--------------------|--------------------|
| $ D_1(z_i)\rangle$ | $(78 \pm 1)\%$     | $(20.8 \pm 0.8)\%$ | $(2 \pm 4)\%$      |
| $ D_2(z_i)\rangle$ | $(23 \pm 2)\%$     | $(56.4 \pm 0.5)\%$ | $(20 \pm 2)\%$     |
| $ D_3(z_i)\rangle$ | $(2 \pm 4)\%$      | $(23.8 \pm 0.7)\%$ | $(74 \pm 1)\%$     |

### e) Theory U(3)

| Input \ Output     | $ D_1(z_i)\rangle$ | $ D_2(z_i)\rangle$ | $ D_3(z_i)\rangle$ |
|--------------------|--------------------|--------------------|--------------------|
| $ D_1(z_i)\rangle$ | 80%                | 19.1%              | 1%                 |
| $ D_2(z_i)\rangle$ | 19%                | 61.8%              | 19%                |
| $ D_3(z_i)\rangle$ | 1%                 | 19.1%              | 80%                |

### f) Simulation U(3)

| Input \ Output     | $ D_1(z_i)\rangle$ | $ D_2(z_i)\rangle$ | $ D_3(z_i)\rangle$ |
|--------------------|--------------------|--------------------|--------------------|
| $ D_1(z_i)\rangle$ | 81%                | 18.0%              | 1%                 |
| $ D_2(z_i)\rangle$ | 23%                | 60.0%              | 17%                |
| $ D_3(z_i)\rangle$ | 2%                 | 21.9%              | 76%                |

**Table S1:** Measurement of  $\hat{u}(\gamma)$  as depicted in Fig. 3c and  $\hat{U}(\gamma)$  as depicted in Fig. 4a. Observed, analytically calculated and numerically simulated detection probabilities for each dark state at the output facet of the chip. The numerical simulation takes into account deviations from the adiabaticity condition as well as errors that stem from the fact, that the path  $\gamma$  through coupling space is not perfectly closed. All depicted data has been projected onto the dark subspace; losses to the C and A channels are excluded.

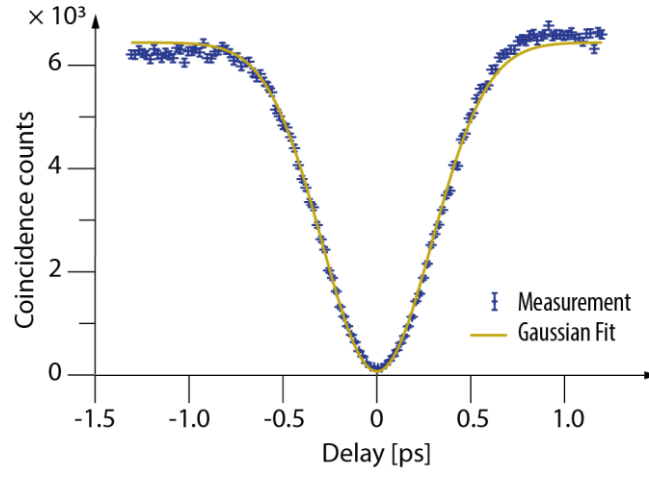


**Figure S1:** Hong-Ou-Mandel-characterization of the photon pairs. The quantum source was connected to a 50:50 fiber beam splitter and a time delay introduced in one arm. Shown are the delay-dependent coincidence counts (blue) with error bars that indicate the Poisson standard deviation. The Gaussian fit (yellow line) exhibits a visibility of  $V = 99.0\%$ , illustrating the high degree of indistinguishability of the two photons.

## Supplementary References

- S1. Sperling, J., Bohmann, M., Vogel, W., Harder, G., Brecht, B., Ansari, V. & Silberhorn, C. Uncovering Quantum Correlations with Time-Multiplexed Click Detection. *Phys. Rev. Lett.* **115**, 023601 (2015).
- S2. Bohmann, M., Kruse, R., Sperling, J., Silberhorn, C. & Vogel, W. Direct calibration of click-counting detectors. *Phys. Rev. A.* **95**, 033806 (2017).
- S3. Bohmann, M., Qi, L., Vogel, W. & Chekhova, M. Detection-device-independent verification of nonclassical light. *Phys. Rev. Res.* **1**, 033178 (2019).