

Laughlin charge pumping in a quantum anomalous Hall insulator

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Supplementary Note 1: Transport properties of the Hall-bar sample

Figures S1a-S1c respectively show the magnetic field dependence of longitudinal resistance R_{xx} , Hall resistance R_{yx} , and two-terminal resistance R_{2t} measured in the Hall bar sample made of the same heterostructure film as in the main text. The width of the Hall bar was

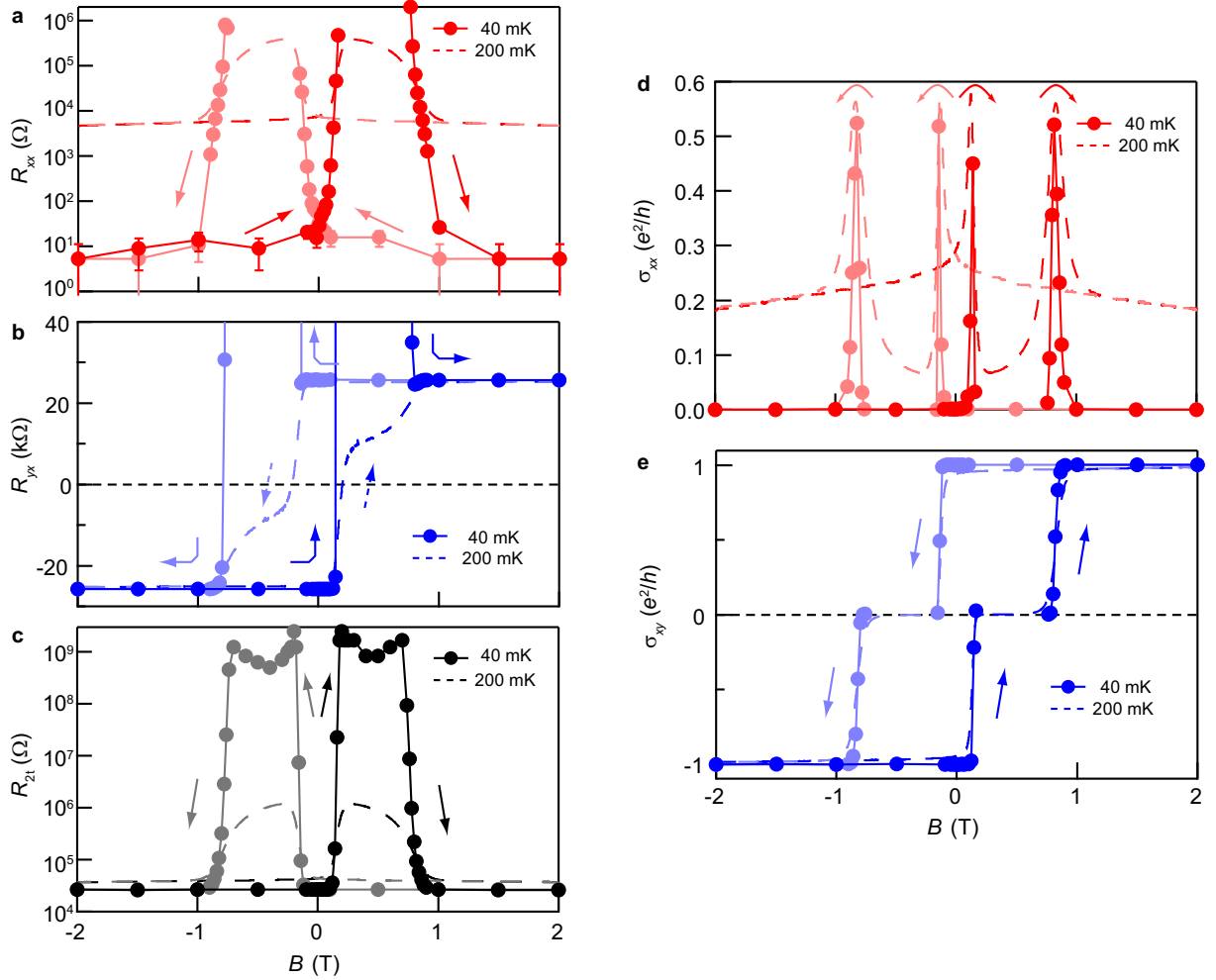


Fig. S1. Transport properties of the Hall-bar sample. **a, b, c** Magnetic field dependence of longitudinal resistance R_{xx} (**a**), Hall resistance R_{yx} (**b**), and two-terminal resistance R_{2t} (**c**). The data were taken using a Hall bar sample. Arrows indicate the magnetic field sweep directions. Error bars are statistical uncertainty from repeated runs of measurement. The excitation voltage of $50 \mu\text{V}$ at a frequency of 3 Hz was applied to the Hall bar sample and current and longitudinal and Hall voltages were measured. Data at $T = 40$ mK and 200 mK are shown. **c, d**, Magnetic field dependence of longitudinal conductivity σ_{xx} (**c**) and Hall conductivity σ_{xy} (**d**) calculated from the data in **a** and **b**.

300 μm and the distance between the voltage probes was 300 μm . The experiment was conducted in a dilution refrigerator with a base temperature of 40 mK. We measured the current using a standard lock-in technique by applying an ac voltage of 50 μV at a frequency of 3 Hz between the source and drain electrodes of the Hall bar sample. Simultaneously, we recorded the voltage drops between the longitudinal and transverse pairs of voltage electrodes and calculated R_{xx} , R_{yx} , and R_{2t} from the measured voltages and current. The quantum anomalous Hall (QAH) effect with $R_{yx} = h/e^2$ with nearly zero R_{xx} is observed clearly. With increasing B , R_{2t} becomes so large that we cannot obtain reliable data of R_{xx} and R_{yx} in the magnetic field range of $0.2 \text{ T} < B < 0.8 \text{ T}$. In this field range, R_{2t} exceeds 1 G Ω , showing consistency with the small conductance of the Corbino disk sample in Fig. 2d in the main text. The large R_{2t} is ascribed to the realization of the axion insulator (AXI) phase where magnetization directions of the two magnetic layers are anti-parallel, as discussed in the previous studies[1–4]. The non-monotonous behavior in R_{yx} in the corresponding field range can be attributed to an unintentional mixing of R_{xx} to R_{yx} . Figures S1c and S1d show the longitudinal σ_{xx} and the Hall σ_{xy} conductivity calculated from the data in Figs. S1a and S1b using the relation $\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{yx}^2}$ and $\sigma_{xy} = \frac{\rho_{yx}}{\rho_{xx}^2 + \rho_{yx}^2}$. In addition to the $\sigma_{xy} = \pm e^2/h$ plateaus of the QAH phase, the $\sigma_{xy} = 0$ plateaus of the AXI phase are clearly observed.

Supplementary Note 2: Symmetrization against the static magnetic field

Figure S2 shows the B_0 dependence of the symmetric and anti-symmetric components of the in-phase voltage amplitude V_1 calculated from the data shown in Fig. 2c. The asymmetric and symmetric components are calculated by $V_1^{\text{asym}} = (V_{1,\text{for}}(B_0) - V_{1,\text{back}}(-B_0))/2$ and $V_1^{\text{sym}} = (V_{1,\text{for}}(B_0) + V_{1,\text{back}}(-B_0))/2$, respectively. Here, $V_{1,\text{for}}(B_0)$ and $V_{1,\text{back}}(B_0)$ are the data taken by increasing (black circles in Fig. 2c) and decreasing (grey circles in Fig. 2d) B_0 , respectively. The symmetric component is almost independent of B_0 except for the two points at $B_0 = -0.05 \text{ T}$ and -0.6 T . These exceptional points correspond to the magnetization reversal where the signal is sensitive to a subtle difference in temperature or magnetic field.

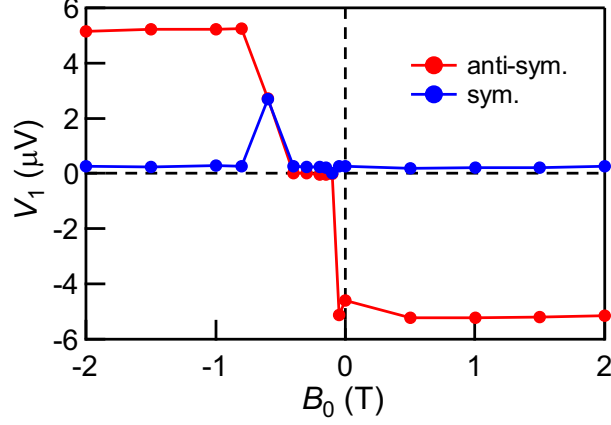


Fig. S2. Calculated symmetric and anti-symmetric components of the in-phase voltage amplitude.

Supplementary Note 3: Circuit model analysis

Here, we consider an equivalent electric circuit model for a Corbino disk sample of magnetic heterostructure of topological insulators, extending the earlier study on the Corbino disk sample of the quantum Hall effect[5]. We assume that the top and bottom surfaces have the conductivity tensors

$$\sigma^\tau = \begin{pmatrix} \sigma_{rr}^\tau & \sigma_{r\theta}^\tau \\ \sigma_{\theta r}^\tau & \sigma_{\theta\theta}^\tau \end{pmatrix} \quad (\text{S1})$$

in the cylindrical coordinate (r, θ) . The index $\tau = \text{top, bottom}$ represents the surface degree of freedom. The current density $\vec{j}^\tau$ is related to the electric field $\vec{E}$ by the Ohm's law:

$$j_r^\tau = \sigma_{rr}^\tau E_r + \sigma_{r\theta}^\tau E_\theta, \quad (\text{S2})$$

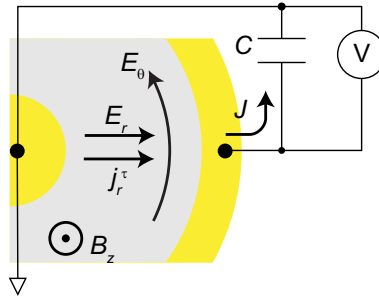


Fig. S3. Schematic of measurement circuit. Schematic top view of a part of a Corbino disk sample and the measurement circuit diagram.

$$j_\theta^\tau = \sigma_{\theta r}^\tau E_r + \sigma_{\theta\theta}^\tau E_\theta \quad (\text{S3})$$

Due to the charge conservation $\vec{\nabla} \cdot \vec{j}^\tau = 0$,

$$j_r^\tau = \frac{J^\tau}{2\pi r} \quad (\text{S4})$$

where J^τ is the electric current that flows out of each surface. By adding the current at the top and bottom surfaces, the total current $J = J^{\text{top}} + J^{\text{bottom}}$ is expressed as

$$\frac{J}{2\pi r} = (\sigma_{rr}^{\text{top}} + \sigma_{rr}^{\text{bottom}})E_r + (\sigma_{r\theta}^{\text{top}} + \sigma_{r\theta}^{\text{bottom}})E_\theta \quad (\text{S5})$$

$$= \sigma_{rr}^{\text{tot}} E_r + \sigma_{r\theta}^{\text{tot}} E_\theta, \quad (\text{S6})$$

where $\sigma_{rr}^{\text{tot}} = \sigma_{rr}^{\text{top}} + \sigma_{rr}^{\text{bottom}}$ and $\sigma_{r\theta}^{\text{tot}} = \sigma_{r\theta}^{\text{top}} + \sigma_{r\theta}^{\text{bottom}}$. This current J is sent to the capacitor C of the outer electric circuit. Then, a voltage V_r appears between the capacitor as

$$V_r = Z_C J, \quad (\text{S7})$$

where Z_C is the impedance of the capacitor. When a time-dependent homogeneous magnetic field B_z is applied in the z direction, an electric field is produced in the azimuth direction as

$$E_\theta = \frac{1}{2}r \left(-\frac{\partial B_z}{\partial t} \right), \quad (\text{S8})$$

according to Farady's law. In our analysis, we neglect the contribution of the self inductance L because its impedance is negligibly small in the frequency range of our experiment. By plugging in eqs. (7) and (8) into eq. (6),

$$E_r = \frac{1}{\sigma_{rr}^{\text{tot}}} \left(\frac{V_r}{2\pi r Z_C} - \sigma_{r\theta}^{\text{tot}} \frac{1}{2}r \left(-\frac{\partial B_z}{\partial t} \right) \right). \quad (\text{S9})$$

Because V_r is related to E_r by

$$-V_r = \int_{r_1}^{r_2} dr E_r, \quad (\text{S10})$$

the integration of both sides of eq. (9) yields

$$-V_r = \int_{r_1}^{r_2} dr \frac{1}{\sigma_{rr}^{\text{tot}}} \left(\frac{V_r}{2\pi r Z_C} - \sigma_{r\theta}^{\text{tot}} \frac{1}{2}r \left(-\frac{\partial B_z}{\partial t} \right) \right). \quad (\text{S11})$$

$$(\text{S12})$$

Then, V_r is obtained as

$$V_r = \frac{\frac{1}{4}(r_2^2 - r_1^2) \frac{\sigma_{r\theta}^{\text{tot}}}{\sigma_{rr}^{\text{tot}}} \left(-\frac{\partial B_z}{\partial t} \right)}{1 + \frac{\ln(r_2/r_1)}{2\pi \sigma_{xx}^{\text{tot}} Z_C}} = \frac{-\frac{i}{4}(r_2^2 - r_1^2) \frac{\sigma_{r\theta}}{\sigma_{rr}} \omega B_1}{1 + R_0/Z_C} e^{i\omega t}. \quad (\text{S13})$$

Here, $R_0 = \frac{\ln(r_2/r_1)}{2\pi\sigma_{xx}^{\text{tot}}}$ is the resistance of the Corbino disk sample. We also assumed $B_z(t) = B_1 e^{i\omega t}$. When σ_{rr} is small enough to satisfy $R_0 \gg |Z_C|$,

$$V_r \sim \frac{-\frac{\pi}{2}(r_2^2 - r_1^2)\sigma_{r\theta}^{\text{tot}} B_1}{C \ln(r_2/r_1)} e^{i\omega t} \quad (\text{S14})$$

or

$$Q = CV_r \sim \frac{-\frac{\pi}{2}(r_2^2 - r_1^2)\sigma_{r\theta}^{\text{tot}} B_1}{\ln(r_2/r_1)} e^{i\omega t}, \quad (\text{S15})$$

where Q is the amount of charge accumulated in the capacitor C . The voltage V_r and Q oscillate in-phase to the alternating magnetic field. The in-phase component amplitude V_1 is independent of the frequency ω . Using the amplitude V_1 , the Hall conductivity $\sigma_{r\theta}^{\text{tot}}$ can be expressed as

$$\sigma_{r\theta}^{\text{tot}} = -\frac{2 \ln(r_2/r_1) CV_1}{\pi(r_2^2 - r_1^2) B_1}. \quad (\text{S16})$$

Because there are two capacitances, an intentionally inserted capacitance C_0 and a parasitic capacitance C_p in the actual experimental setup, $C = C_0 + C_p$ is used in the main text. Here, we neglect the capacitance of Corbino disk sample itself because it is much smaller than the other capacitance. In the limit of $r_1 \rightarrow r_2$, the formula recovers the geometry-independent expression as

$$\sigma_{r\theta}^{\text{tot}} \rightarrow -\frac{CV_1}{\pi r_2^2 B_1} = -\frac{Q}{\Phi}, \quad (\text{S17})$$

where Q is the pumped charge and Φ is the magnetic flux passing through the center hole.

Supplementary Note 4: Deviation from the frequency-independent plateau

In the present experiment, the frequency-independent plateau of $-V_1^{\text{asym}}/B_1$ starts to deviate at a relative low frequency of $f \sim 10$ Hz with increasing f , as shown in Fig. 3a in the main text. We assign the deviation to the increase in G_{rr} by elevated electron temperature. We measured the conductance G_{rr} at various temperatures without applying the ac magnetic field B_{ac} and at the base temperature under the applications of B_{ac} , as shown in Figs. S4a and S4b. Assuming that G_{rr} is dominated by the electron temperature, a comparison of the values in Fig. 4b to the G_{rr} - $1/T$ curve can give an estimation of the electron temperature. The estimated electron temperature is about 70 mK at $B_1 = 0.560$ mT and $f = 10$ Hz. At this electron temperature, the QAH state is not completely broken (The Hall resistance is still close to h/e^2). But $G_{rr} \sim 2.3 \times 10^{-8} \Omega^{-1}$ at $B_1 = 0.560$ mT and

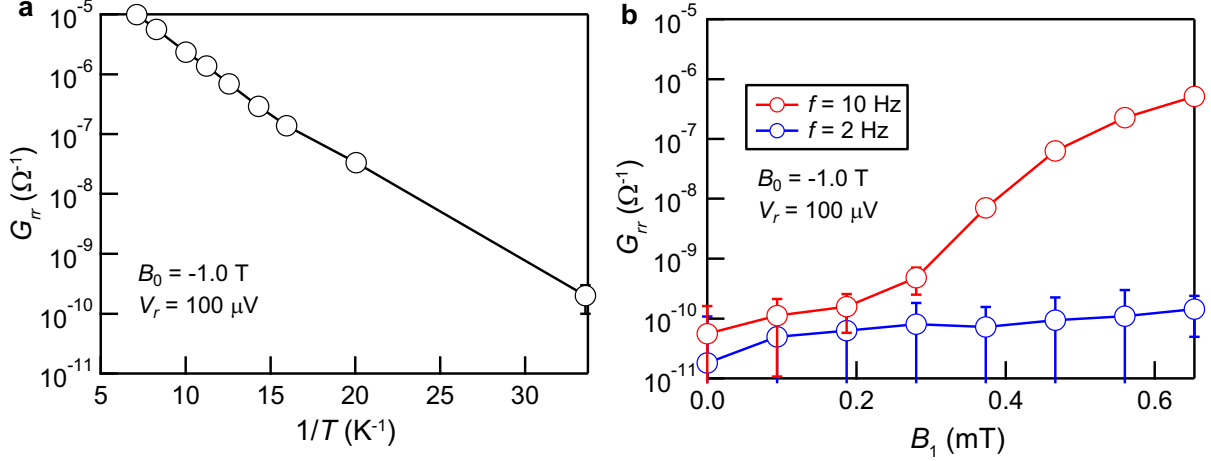


Fig. S4. Increase in G_{rr} by increasing temperature and by applying B_{ac} . **a**, Temperature dependence of G_{rr} plotted against reciprocal of temperature $1/T$. **b**, Dependence of G_{rr} on the B_{ac} amplitude B_1 . The data corresponding to the B_{ac} frequency $f = 2$ Hz and 10 Hz are shown. Error bars are statistical uncertainty from repeated runs of measurement.

$f = 10$ Hz is so large that the condition $|Z_C| \ll R_0 = 1/G_{rr}$ for observing the f -independent plateau does not hold anymore. Thus, the deviation starts at a relatively low frequency. For the mechanism of the increase in G_{rr} , the Joule heating by the eddy current is likely.

Supplementary Note 5: Relevant electronic states to the charge pumping

The charge pumping originates from the shifts of the occupied electronic states extending over the system size. In this sense, the present charge pumping shares the same mechanism of the charge pumping with the quantum Hall (QH) states. Therefore, we believe it is appropriate to use the term “Laughlin’s charge pumping” in our study. At the same time, the electronic structure of the delocalized states responsible for the charge pumping is different between the QAH and QH states; the gapped two-dimensional Dirac fermion system for the QAH state and the Landau levels for the QH state.

In a two-dimensional Dirac fermion system under time-reversal symmetry without an energy gap, all the electronic states are delocalized even in the presence of disorder. When the energy gap opens via the exchange interaction, the electronic states near the band gap become localized due to the disorder. The effect of disorder on the gapped Dirac system was theoretically studied in Ref. [6]. We consider that the occupied electronic states extending

over the system size contribute to the charge pumping.

In our sample, the Fermi energy is located in the energy gap, and the electronic states are localized at the Fermi energy, resulting in nearly zero longitudinal conductance. This localization at the Fermi energy is essential to measure the charge pumping. Otherwise, the pumped charge can relax internally in the sample via the dissipative transport at the Fermi energy.

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