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Observation of temporal reflection and broadband frequency translation at photonic time interfaces

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Supplementary Information

S1. Theoretical Design and Optimization of the Loaded Transmission Line Metamaterial

Each unit cell of our transmission line, once the switches have been activated, can be approximately modelled by the T-network shown in Fig. S1a, where d is the physical length of the cell (i.e. one turn of the meandered microstrip on our printed circuit board), and $jb = j\omega C Z_o$ is the susceptance of the capacitive load, normalized against Z_o, the characteristic impedance of the unloaded TLM. The voltage and currents at the input port of such a network can be related to those at the output port by its ABCD matrix, which reads[S1],

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\cos\theta - \frac{b}{2}\sin\theta\right) & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \left(\cos\theta - \frac{b}{2}\sin\theta\right) \end{bmatrix}. \tag{S1}$$

Here, $\theta = k_o d$ is the total propagation phase through the unloaded transmission-line section. Applying Bloch boundary condition yields the dispersion relation for an infinitely long chain of this unit cell as

$$\cos(\beta d) = \cos\theta - \frac{b}{2}\sin\theta. \tag{S2}$$

Furthermore, the Bloch wave impedance of the periodic structure is given by

$$Z_B = \pm\frac{B Z_o}{\sqrt{A^2 - 1}}. \tag{S3}$$

In Fig. S1b and Fig. S1c, we plot the normalized dispersion relation and the Bloch wave impedance of our TLM, for $\theta = 72°$ and $b = 2.753$. where $\omega_o = 2\pi \times 100$ MHz is the designated maximum operating frequency. In this case, the loaded TLM to exhibit a Bloch impedance is exactly $Z_o/2$ at $\omega = \omega_o/2 = 2\pi \times 50$ MHz, and retains approximately the same value for lower frequencies.

Having determined the ideal frequency translation ratio f_2/f_1, as well as the wave impedances before (Z_1) and after (Z_2) switching, one can calculate the theoretical temporal reflection and refraction coefficient of the TLM. For generality, we present a set of parametric studies that were conducted to identify the best design parameters for the TLM, which can be scaled to accommodate different specifications (i.e., different temporal reflection magnitude and frequency translation ratios).

At the design stage, two key parameters need to be determined: the length of the unit cell, which is directly proportional to the phase accumulation; and the appropriate shunt load b. Both parameters will have significant impact on various aspects of the time-interface. Crucially, together, they determine the band gaps of our TLM. Since we wish to operate in a regime in which the TLM is almost translationally homogeneous and dispersionless, we must pay special attention to avoid the first band gap. To this end, we perform parametric studies on R and f_2/f_1 by sweeping θ and b (which is equivalent to sweeping the cutoff frequency of the first band gap, ω_c); the results for R and f_2/f_1 are plotted in Fig. S1d and Fig. S1e respectively. In Fig. S1d, we use a red line to denote the combinations of (θ, ω_c) that gives the maximum temporal reflection magnitude.

S2. Practical Implementation

The practical circuit layout is shown in Fig. S2a and Fig. S2b, while the choices of all circuit components are listed in Table S1. A simplified schematic of our experimental set up for time domain measurements is shown in Fig. S2c. The input signal is launched from an arbitrary waveform generator (Keysight M8195A) into a symmetric T-connector. Identical copies of the input (both with 3.5 dB attenuation) are fed to Port 1 of the transmission line as well as channel 1 of an oscilloscope (Tektronix DPO4104), which records $V_1(t)$. Port 2 of the transmission line is connected to channel 2 of the oscilloscope, which records $V_2(t)$. Both oscilloscope channels have an input impedance of 50 Ω, and record in the averaged mode with 512 samples.

S3. Compensation of the T-connectors and Impedance Mismatch

We measured the signal via an oscilloscope at the Port 1 at $x = 0$, denoted as $V_{o1}(t)$, which is a linear superposition of input and time-reflected signals. Considering the effect of the T-connector that connects the source (denoted by subscript "s"), transmission-line (subscript "t"), and the oscilloscope (subscript "o"), we can estimate $V_{o1}(t)$ for the case of closing switches as

$$V_{o1}(t) \approx \mathcal{F}^{-1}\left\{S_{os}(\omega)\frac{\mathcal{F}[V_{inc}(0,t)]}{S_{ts}(\omega)} + S_{ot}(\omega)(1+\Gamma)\mathcal{F}\left[V_{ref}(0,t)\right]\right\}, \tag{S4}$$

where $\mathcal{F}$ denotes the Fourier transform. Then S-parameters measurement of the T-connector gives $S_{os} \approx S_{ts} \approx S_{ot} \approx 0.665$ from $10 \sim 100$ MHz. Γ is the spatial reflection coefficient considering the impedance mismatch between the switched TLM and the T-connector, which reads,

$$\Gamma = \frac{Z_0 - Z_{TL}}{Z_0 + Z_{TL}}, \tag{S5}$$

where $Z_{TL} = Z_1(\approx 50\ \Omega = Z_0)$ with all switches open, while $Z_{TL} = Z_2$ if all switches are closed. The effective impedance of the TLM can be retrieved from its measured S-parameters. Similar analysis can be done for other quantities, e.g., that are needed in the main text.

S4. Retrieval of the Time-Reflection and Time-Refraction Coefficients

The reflection coefficient at a time-interface at $t = t_s$ is defined as

$$R(k) = \frac{\tilde{V}_{ref}(k, t_s^+)}{\tilde{V}_{inc}(k, t_s^-)} = \frac{\int_{-\infty}^{+\infty} V_{ref}(x, t_s^+)e^{jkx}dx}{\int_{-\infty}^{+\infty} V_{inc}(x, t_s^-)e^{jkx}dx}, \tag{S6}$$

where k is the real part of the wavenumber conserved across the time-interface. $\tilde{V}_{inc,ref}(k,t)$ refer to the incident and reflected signals in the momentum space, while $V_{inc,ref}(x,t)$ are the voltages expressed in the real space. We assume that the dispersion and the loss of the transmission line are small enough: $\omega \approx vk$ and then $V_{inc}(x,t) \approx V_{inc}\left(0, t-\frac{x}{v_1}\right)e^{-\alpha_1 x}$, $V_{ref}(x,t) \approx V_{ref}\left(0, t+\frac{x}{v_2}\right)e^{\alpha_2 x}$. The loss rates α_1 and α_2 can be retrieved by measuring the *S*-parameters of the unloaded and loaded transmission lines, respectively. By taking Fourier transform of the incident and reflected signals recorded at the input port, one can calculate the reflection coefficient measured at the input port for a given wavenumber k and corresponding frequencies $\omega_{1,2}$:

$$\begin{aligned}\tilde{R}(k) = \frac{\mathcal{F}\left[V_{ref}(0,t)\right]\big|_{\omega_2}}{\mathcal{F}\left[V_{inc}(0,t)\right]\big|_{\omega_1}} &\approx \frac{e^{-jkx}\int_{-\infty}^{+\infty} V_{ref}\left(0, t+\frac{x}{v_2}\right)e^{-j\omega_2 t}dt}{e^{jkx}\int_{-\infty}^{+\infty} V_{inc}\left(0, t-\frac{x}{v_1}\right)e^{-j\omega_1 t}dt} \\ &\approx e^{-j2kx}e^{-(\alpha_1+\alpha_2)x}\frac{\int_{-\infty}^{+\infty} V_{ref}(x,t)e^{-j\omega_2 t}dt}{\int_{-\infty}^{+\infty} V_{inc}(x,t)e^{-j\omega_1 t}dt}.\end{aligned} \tag{S7}$$

To obtain the exact temporal reflection coefficient, we need to compensate both the phase and loss during the round trip that a wave travels. Without loss of generality, we assume $t_s = 0$. Then, by changing the integration variable $t = -x_r/v_1$ for the denominator and $t = -x_r/v_2$ for the numerator in Eq. (S7), we obtain the relation between the corrected reflection coefficient and that measured at the input port:

$$R(k) \approx \frac{\omega_2}{\omega_1}\tilde{R}(k)e^{j2kx_r}e^{(\alpha_1+\alpha_2)x_r}, \tag{S8}$$

Here, x_r entails the physical distance that a wave travels from the input port until it gets reflected by the time-interface, which can be retrieved via $x_r \approx -\frac{1}{2}\partial_k[\arg(\tilde{R})]$.

Therefore, the procedure for retrieving the time-reflection coefficients for a given wavenumber k can be summarized as follows:

a) Measure the *S*-parameters and the dispersion diagrams of unloaded and loaded transmission lines.

b) Read the corresponding frequencies $\omega_{1,2}$; Retrieve the loss rate $\alpha_{1,2}$ from the *S*-parameters.

c) Calculate the reflection coefficient at the input port $\tilde{R}(k)$ using Eq. (S7).

d) Compensate the loss and accumulated phase using Eq. (S8).

Similar procedure also applies to retrieving the time-refraction coefficient, where the relation between the time-refraction coefficient $\tilde{T}(k)$ referring to the two ports and the actual temporal refraction coefficient $T(k)$ reads:

$$T(k) \approx \frac{\omega_2}{\omega_1}\tilde{T}(k,x)e^{jkL}e^{(\alpha_1 x_r+\alpha_2 x_t)}, \tag{S9}$$

where $L = x_r + x_t$ is approximately the total length of our transmission-line, and x_t denotes the retrieved distance that the wave travels after the time-interface until it exits the transmission-line.

S5. Derivation of the Temporal Scattering Coefficients

Consider a homogeneous transmission-line (TL) whose distributed capacitance can be changed instantaneously at $t = 0$, by closing or opening a switch connected to a shunt capacitor C_p, as shown in Fig. S3. The values of the capacitance are chosen such that the effective distributed capacitance of the TL stays at C_1 for $t < 0$ while it jumps to C_2 for $t > 0$. Applying a Laplace transform $\mathcal{L}\{v(t)\} = \int_0^{+\infty} v(t)e^{-st}dt = V(s)$ on both voltage $v(z,t)$ and current $i(v,t)$, one can write the wave equation for $t > 0$ as:

$$(\partial_z^2 - s^2 L_1 C_2)V = -L_1 C_1 (s + j\omega_1) v(z, 0^-). \tag{S10}$$

Assuming a harmonic variation e^{-jkz} of the fields in space, we can solve for the voltage in *s*-domain. For connecting a parallel capacitor as shown in Fig. S3a,

$$V(z,s) = \frac{C_1}{C_2}\frac{s + j\omega_1}{s^2 + \omega_2^2} v(z, 0^-), \tag{S11}$$

while for the case of opening the switch as shown in Fig. S3b,

$$V(z,s) = \frac{s + j\frac{C_1}{C_2}\omega_1}{s^2 + \omega_2^2} v(z, 0^-). \tag{S12}$$

In both cases, ω_2 is not only the eigenfrequency of the TL after switching, but also relates to the poles of the fields in *s*-domain. It reads:

$$\omega_2 = \omega_1\sqrt{C_1/C_2} = \omega_1 Z_2/Z_1, \tag{S13}$$

which predicts the theoretical frequency translation ratio.

More interestingly, by applying the initial-value theorem, Eqs. (S11) and (S12) will entail different temporal boundary conditions: when closing the switch (Fig. S3a) the voltage is discontinuous and the charge stored in capacitors is conserved:

$$C_2 v(z, 0^+) = C_1 v(z, 0^-), \tag{S14}$$

while in the case where the switch is opened (Fig. S3b), the voltage remains continuous, yet the charges are not because C_p is suddenly disconnected from the TL system:

$$v(z, 0^+) = v(z, 0^-). \tag{S15}$$

Numerical examples of these two cases are demonstrated in Fig. S3c and S3d: when connecting C_p such that $C_2 = 4C_1$, the voltage on the TL is discontinuous at $t = 0$, while the total charge in the system is conserved, though part of the charge stored in C_1 is transferred to C_p

instantaneously. The frequency is halved as expected. By contrast, when disconnecting C_p from the TL such that $C_2 = C_1/4$. The voltage is continuous across the time-interface, while the total charge decreases as we exclude C_p from the system. The frequency doubles as expected in this case.

Having derived these boundary conditions, we can readily find the temporal reflection and transmission coefficients for switching on a parallel capacitor:

$$\begin{aligned} T_{on} &= \frac{1}{2}\left(\frac{C_1}{C_2} + \sqrt{\frac{C_1}{C_2}}\right) = \frac{Z_{on}\left(Z_{on} + Z_{off}\right)}{2Z_{off}^2}, \\ R_{on} &= \frac{1}{2}\left(\frac{C_1}{C_2} - \sqrt{\frac{C_1}{C_2}}\right) = \frac{Z_{on}\left(Z_{on} - Z_{off}\right)}{2Z_{off}^2}; \end{aligned} \tag{S16}$$

and for switching off a parallel capacitor:

$$\begin{aligned} T_{off} &= \frac{1}{2}\left(1 + \sqrt{\frac{C_1}{C_2}}\right) = \frac{Z_{on} + Z_{off}}{2Z_{on}}, \\ R_{off} &= \frac{1}{2}\left(1 - \sqrt{\frac{C_1}{C_2}}\right) = \frac{Z_{on} - Z_{off}}{2Z_{on}}. \end{aligned} \tag{S17}$$

In our experiments with temporal slabs, we observed two time-reflected signals, and both are out of phase with respect to the incident signal. This can only be explained via Eqs. (S16) and (S17): in fact, in our setup the effective impedance is decreased when the loaded capacitors are switched on, i.e., $Z_{on} < Z_{off}$, and vice versa. Both temporal reflection coefficients $R_{on,off}$ will be negative, hence, both time-reflected waves ($R_{on}T_{off}$ and $T_{on}R_{off}$) will be out of phase to the incidence, while the two time-refracted waves ($T_{on}T_{off}$ and $R_{on}R_{off}$) will be in phase.

S6. Measurement of Frequency Conversion with Wave Packets

While determining the broadband frequency translation properties of our TLM, we inject wave packets with different center frequencies, and observe the converted center frequency of the output. Fig. S4 shows a sample of the time-domain measurement used to quantify the frequency conversion of our time-interface when the wave impedance is abruptly reduced via the closing of the switches. The top left plot of Fig. S4 shows the incident (black) and the reflected (red) signals. Note that the effects of the T-connectors and impedance mismatch have not been compensated as they are not crucial for this study. The bottom plot shows the transmitted (blue) signal. Visually, it is clear that the frequencies of the time-scattered waves are lower than that of the incident wave, as the period appears to be longer. We perform Fourier transform on all three signal components. As revealed by the spectra on the right, the original signal has a center frequency of 60 MHz, while the time-reflected and the time-transmitted signals have center frequencies of approximately 35 MHz, corresponding to a down conversion ratio of about 0.55.

S7. Temporal Slab Acting on a Wave Packet

An ideal temporal slab, formed with two sequential time-interfaces must be unbounded in space. We cannot emulate such a system effectively with our fabricated TLM unless the injected wave packets have very narrow durations (i.e. wide bandwidth). Otherwise, the time-reflections and refractions cannot be completely contained inside the TLM as the temporal slab is being actuated.

In order to study the effect of a temporal slab on a true narrow-band wave packet, we numerically simulate a circuit constructed with three cascaded copies of our TLM. For some arbitrary value of "ON" duration (τ =15ns), we sweep the center frequency (f_1) of the injected pulse (17.5MHz spectral FWHM), and observe the total time reflection (which consists of two interfering wave packets) in both the time domain and the frequency domain. The results for $f_1 \in [30, 35, 38, 41, 46]$ MHz are plotted in Fig. S5. In the left column, we plot the input voltage wave form. The total time reflection induced by the temporal slab for each case is plotted in the middle column. Here, the interference between the two TR pulses are clearly visible. A Fourier transform of the reflected output yields the frequency spectra in the right column. Here, a reflection null persists at approximately 38MHz. The frequency of this null, inversely proportional to the slab duration[S2], is consistent with the experimentally measured value in Fig. 3d of the main text.

The study done in this section also hints at the scalability of our printed circuit board TLM platform: connecting multiple modular samples opens the opportunity to implement more complex forms of time metamaterials.

S8. Spatial Homogeneity of the Time-Interface

In our experiment, the control signal for the switches were carried by a pair of transmission lines with very short but nonetheless finite electrical length. Hence, strictly speaking, the switches are not perfectly synchronized. This may introduce some degree of spatial inhomogeneity and serve to detune the wave momentum. However, the control TL is much shorter than the main TL (by more than 40 times). This means that in the perspectives of a pulse propagating on the main TL, the change in the distributed capacitance appears effectively homogeneous. To further enhance the spatial homogeneity, we feed the control signal from the center of the sample, instead of the end. This serves to reduce the total transit time of the control signal across the entire sample by another factor of two, compared to the transit time of the signal on the main TL.

To prove the above assertion, we perform circuit simulations with different degrees of switch desynchronization; the results are shown in Fig. S6. Here, we inject an asymmetric signal in the same form as that used in Fig. 2e of the main text. The grey curves depict the voltages at port 1 and 2 of the TLM if we turn on all of the switches in a perfectly synchronized fashion. In other words, there is no time delay between the activation of adjacent switches. The dashed red curve corresponds to the simulated results when the electrical lengths between adjacent switches (L_c) match our experiment exactly (<1cm). It can be seen that the red curves overlap the ideal grey curves almost exactly, showing that our experimental platform can realize an effectively homogeneous time-interface that can almost conserve momentum for sufficiently low frequencies.

S9. Characterization of the Circuit Switching Speed

As pointed out by the main text as well as previous works in the literature[S3,S4], in order to induce strong temporal reflections, the time-interface must be sufficiently fast. Ideally, the duration of the interface should be much shorter than the period of the highest frequency wave component under consideration. In our transmission line metamaterial, the duration of the time-interface is primarily dictated by the rise and fall times of the switches. Hence, we experimentally characterize the switching speed of our circuit, and numerically investigate its implications on our implemented time-interface.

To characterize the circuit switching speed, we fabricate a single unit cell of the TLM. We feed a 100MHz sinusoidal signal into the input port, and examine the envelope of the transmitted waveform. Changing the state of the switch with a control signal will result in a transition in the output envelope, whose rise/fall time can be used to quantify the "sharpness" of our time-interface. In this work, we define the interface duration as the time it takes for the output envelope to make a 10%-to-90% or a 90%-to-10% transition. In our measurement, the interface duration is measured to be approximately 3ns, which agrees with the specifications supplied by the manufacturer of the switches.

We also utilize different forms of control signal such as a square wave or a triangular wave. As seen from the measured output envelope in Fig. S7a (black curves), the slew rate of the control signal has no appreciable influence on the duration of the time-interface. As long as V_{sw} can pass the required activation threshold, the switches will have the same transient response. This is an important property since in a realistic operating scenario, the slew rate of the control signal will be limited by the input reactance of the switches. For example, Fig. S7b depicts the actual form of the control signal (V_{TTL}) used in obtaining Fig. 2e of the main text.

The slow rise and fall times of the switch input voltage also makes it difficult to precisely define the duration of a temporal slab. Hence, in the main text, we use τ to refer to the "ON"-state duration of the control signal supplied by the function generator, which is a well-defined rectangular pulse train.

S10. Effect of Finite Switching Time

In this section, we numerically investigate the influence of finite circuit switching time on the scattering properties of our time-interfaces. First, we compare the time reflections generated by an ideal time-interface (instantaneous switching) and a realistic interface with 3ns 10%-to-90% switch rise time (which corresponds to our experiment). We first inject an asymmetric pulse into the circuit, and probe the time-reflected and time-refracted waveforms at the input and output ports in the time domain. The results are plotted in Fig. S7c. Here, the black curve corresponds to the simulated voltages when the switches have almost instantaneous rise times, while the red curves correspond to the voltages when the rise time is 3ns. It can be seen that the TR and TT waves are almost the same for both cases, which suggests that our platform behaves almost the same as a true time-interface for the frequency components contained in the asymmetric pulse.

In Fig. S7d, we plot the peak amplitude of the time reflection as a function of the rise time, normalized against the peak value when $\tau_{rise} \approx 0$. It can be seen that the 3ns 10%-to-90% rise time produced a time reflection with amplitude that is approximately 90% of the ideal output. The match would be even better when operating at lower frequencies.

S11. Measurement of Inverted Temporal Slab

In this section, we report measurements of inverted temporal slabs formed by an "ON-OFF-ON" switching sequence, with different "OFF"-durations τ. This provides the complementary control experiment for the "OFF-ON-OFF" temporal slabs discussed in the main text (Fig. 3c, 3d and 3e). It should be reiterated that τ corresponds to the "OFF" duration of the control signal (an ideal rectangular pulse train). It does not equal to the actual duration of the realized temporal slab, due to the input reactance of the switches. However, as discussed in the section "Characterization of the Circuit Switching Speed", this does not degrade the sharpness of the "facets" of our temporal slab.

In the experiment, we launch broadband pulses into the TLM, and observe the TR output produced by the inverted temporal slab. The time domain measurements for three different values of τ are plotted in Fig. S8a. In each case, two TR pulses with inverted polarity compared to the input can be observed. In contrast with the "OFF-ON-OFF" temporal slab examined in the main text, which produced two TR pulses with almost equal magnitudes, here we see that the second pulse is much more attenuated than the first. The amplitude difference is exacerbated as τ is increased. This can be explained by the loss of the TLM in its "ON" state. As we increase τ, the second pulse will have travelled further in the TLM before the ending of the temporal slab. Therefore, it must traverse a longer distance inside the lossy TL before returning to port 1, meaning it will experience more attenuation.

We carry out the same Fourier analysis as is done in the main text, to obtain the total reflection spectrum of each inverted temporal slab (Fig. S8b). As expected, we observe reflection minima for certain frequencies. The location of the dips can be tuned by adjusting τ. Due to the strong dissipative loss in the "ON"-state of the TLM, we see that the reflections never reach exactly zero. This is reminiscent of the properties a lossy Fabry-Perot etalon.

S12. Video Leakage of the Switches

Video leakage refers to the generation of spurious signals at the RF ports of an integrated circuit switch, typically caused by the switch driver when a large voltage spike (such as that caused by our control signal) occurs at the logic input. Such signals can travel through our transmission line and contaminate the voltage measurements at the input and output ports, especially when it overlaps with our signal pulses and/or their temporal reflections, as seen by the upper plot in Fig. S9.

Although video leakage is unavoidable during measurement, we can remove its effect through post processing. We use a logic control signal with higher frequency than the signal pulse train. This will produce multiple identical copies of the video leakage signal, with period T. Some copies will inevitably overlap with the signal pulses (which have period T_s). However, most of the copies of the video leakage will appear in between the signal pulses. Since the leakage is almost identical during each switching cycle, we can simply subtract the measured waveforms by its copy, shifted by T. This will remove all video leakage, while leaving intact the desired signal.

Supplemental Figure Captions

Fig. S1: Design and analysis of the transmission-line metamaterial. (**a**) Equivalent T-network for one unit cell of our TLM when the switches are closed. Here, d is the physical length of the unit cell (0.2080m) and $jb = j2.753$ is the normalized load susceptance at 100 MHz. (**b**) Normalized dispersion relation and (**c**) normalized Bloch impedance of our transmission line, assuming a unit-cell electrical length of $\theta = kd = 72°$, and an normalized load susceptance of $b = 2.753$, both evaluated at 100 MHz. The blue lines and the red lines correspond to the unloaded and the loaded lines respectively. In order to identify the optimal unit-cell electrical length θ and cutoff frequency of the first band gap ω_c, which is related to the load susceptance b, two dimensional parametric studies were done to generate maps of (**d**) the temporal reflection coefficient and (**e**) the frequency translation ratio. The red line in panel (a) corresponds to the combination of (θ, ω_c) that yields the largest reflection coefficient. The white boxes with dotted borders in both panels denote regions in which the load susceptance become inductive (i.e., negative).

Fig. S2: Practical circuit layout and measurement setup. (**a**) Layout for the unit cell. The descriptions of the components can be found in Table S1. (**b**) Schematic for the complete TLM, showing how the unit cells are connected. (**c**) Simplified schematic of the experimental set up for time domain measurements. Power supply for the switches and a separate function generator for switch control are not shown.

Fig. S3: Circuit model of time-switched transmission-line (TL). Panels (**a**) and (**b**) correspond to the loading and removal of a shunt capacitor respectively. $C_1 (C_2)$ denotes the value of effective distributed capacitance of the TL before (after) closing or opening the switch. Panel (**c**) demonstrates the time variation of voltages and charges on each capacitor, as well as their total values sensed by the TL, corresponding to a numerical example of panel (a) with $C_2 = 4C_1$. Panel (**d**) shows the same quantities for a numerical example of Panel B with $C_2 = C_1/4$.

Fig. S4: A sample of the measurement used to characterize the frequency conversion of our time-interface when the wave impedance reduced. The left column shows the incident (black), time-reflected (red) and time-refracted (blue) signals. The right column shows their corresponding Fourier transform. A frequency down conversion by about 0.55 is clearly visible through the peak of the frequency spectra.

Fig. S5: Circuit simulations for a temporal slab with a fixed "ON" time (τ =15ns). The injected pulses are in the form of Gaussian wave packets with different carrier frequencies and 17.5MHz spectral FWHM. The simulated circuit consists of 3 cascaded copies of our TLM sample. The left column depicts the time domain waveform of the input pulse, while the middle column shows the total time-reflected output. The right column shows the frequency spectra of the output. All of the spectra exhibit reflection nulls at approximately 38MHz, which agrees with measured results in Fig. 3d of the main text.

Fig. S6: Circuit simulations demonstrating the effect of switch desynchronization. The black curve corresponds to the simulated voltages at ports 1 and 2 assuming perfect synchronization. The red curve ("Practical"), which matches the ideal one almost exactly, depicts the simulation of our measurement set up. This shows that despite the finite time delay between adjacent unit cells, our set up can emulate a homogeneous time-interface with very high fidelity.

Fig. S7: Investigation on the effect of finite switching time. (**a**) Experimental characterization of the circuit switching speed using different types of control signal. Here, a 100MHz sinusoidal signal is transmitted through a single unit cell sample of the TLM, and the envelope of the output waveform is recorded (black curve). We activate the switch inside the unit cell using different types of control signals V_{sw}, with drastically different slopes. Both a rectangular control signal (top) and a triangular control signal (bottom) produced identical output envelopes, demonstrating that the rise time of V_{sw} has very little influence on the switching speed of the circuit. (**b**) A sample measurement of the control signal used to induce the time-interface. The slew rate of the signal is very low, due to the input reactance of the switches. However as shown in panel (a), simply crossing the activation threshold of the switch (2.3V according to the datasheet) is sufficient to induce a sharp time-interface. This particular control signal was used to induce the time-interface examined in Fig 2e of the main text. To further verify the fidelity of the time interface, we perform circuit simulation of the TLM with different 10%-to-90% rise time for the switches, and plot the (**c**) time domain voltage measured at the input port. The input is a Gaussian wave packet with 50MHz center frequency and 40MHz spectral FWHM. The blue curve represents the simulation results when the circuit has the same rise time (3ns) as our selected RF switches; it is seen to closely match the black curve, which corresponds to the case when the rise time is near zero. As we further increase the rise time to 8ns (green) and 12ns (yellow), the amplitude of the time reflection become significantly weaker. (**d**) Peak amplitude (normalized against the ideal case) of the time reflected wave packet as a function of the switch rise time. The amplitude corresponding to the case of 3ns rise time was approximately 90% of the ideal amplitude.

Fig. S8: Measurement of temporal interference induced by the inverted temporal slab. (**a**) Time domain waveform recorded at port 1 for various values of “OFF” duration τ. As τ is increased, so does the separation between the two TR pulses. The second TR pulse is heavily attenuated for larger values of τ, since it experiences increased attenuation. (**b**) Reflection spectra of inverted temporal slab in the wavenumber (bottom axis) and frequency (top axis) domain. We see distinct reflection minima whose locations are tunable via adjustment of τ.

Fig. S9: Compensation of video leakage from the switches. Top plot shows the uncompensated waveform measured at the input port of the TL, with multiple spurious spikes caused by the rising and falling edges of our control signal. The bottom plot shows the compensated waveform, obtained through subtracting the top plot by a shifted copy of itself. The compensated waveform has much more clearly observable input and TR pulses. A spurious inverted copy of the main signal remains, but can be discarded during analysis.

Table S1: List of components used to fabricate our transmission line, referred to the schematic shown in Fig. S2.

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