
Directed percolation and puff jamming near the transition to pipe turbulence

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SUPPLEMENTARY INFORMATION

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This Supplementary Information contains details of the experiments as well as theoretical and numerical calculations of models of puff interactions near the laminar-turbulent transition in pipe flow.

CONTENTS

I. Multi-scale stochastic models of transitional turbulence and puff dynamics	1
II. Experimental Setup	2
III. Puff sizes	4
IV. Puff-splitting model	5
A. Stochastic hydrodynamics for puff-splitting model	6
B. Master equation for the puff-splitting model	7
C. Universality class of the puff-splitting model	8
D. Effects of puff suppression and pushing	9
V. Numerical simulations of the puff-splitting model	9
A. Lattice model of puff dynamics	10
1. Detailed description of lattice model	10
2. Puff-pushing and suppression	10
3. Comparison to experimental values	10
4. Finite-size analysis in the lattice model	11
B. Continuum model of puff dynamics	13
1. Puff diffusion coefficient	13
2. Life and splitting times	13
3. Significance and details on errors estimation	15
VI. Data and code availability statement	17
VII. Acknowledgments	17
References	17

I. MULTI-SCALE STOCHASTIC MODELS OF TRANSITIONAL TURBULENCE AND PUFF DYNAMICS

In the model considered in the main text, the unit of turbulence is considered to be a single puff, and the model describes the interactions between puffs treated as point particles. The dynamics of these puffs is modeled by rules that encapsulate the actual hydrodynamic interactions arising from the way in which one puff influences the mean velocity profile across the pipe.

The justification for the point-puff approximation comes from experiment, direct numerical simulation (DNS) and theoretical consideration. In the main text,

we show experiments and DNS results that indicate how the puff dimensions do not change significantly as one varies the Reynolds number and passes through the transition to turbulence. This shows that the singular behavior that we report is a result of collective puff interactions over system-wide scales, reflected in diverging correlation lengths in space and time that emerge from the directed percolation (DP) picture.

This conclusion also emerges from theoretical analysis. One can start by asking what is the dynamics of turbulence in general, and in particular on scales smaller than a single puff? How can effective stochastic models on multiple scales be obtained from the Navier-Stokes equations, which capture the essential features of the laminar-turbulent transition at scales ranging from within a puff to the macroscopic scale considered in the point-puff models presented here? What, if any, is the relationship between these models and the point-puff models? Do they also exhibit the characteristics of directed percolation, as originally implied by Pomeau [1]?

These questions are distinct from the model considered here, and can be addressed by attempting to derive the effective theory for turbulence from the Navier-Stokes equation, as was done in Ref. [2]. There, numerical simulations suggested that the most singular contribution to the behavior of a single puff near the onset of turbulence arose from the interplay between two weak-modes that necessarily vanish near the transition: a collective large-scale zonal flow and small-scale turbulent anisotropy. The former mode is an emergent mean flow created (activated) by the small-scale turbulence; on the other hand, the emergent mean flow suppresses (inhibits) the small-scale turbulence. The resulting effective theory has been termed a “predator-prey” or “ecological” model of the laminar-turbulent transition [2], and is a non-equilibrium counterpart to what would be Landau theory in an equilibrium transition.

Remarkably, this ecological model turns out also to be in the universality class of directed percolation, at least asymptotically close to the transition. In order to further elucidate this finding, a mean field derivation of this single puff model from the Navier-Stokes equations has been presented [3].

The final bridge between the small-scale structure of the laminar-turbulent transition and the point-puff models presented here was the extension of the ecological model for a single puff to include streamwise shear interactions between patches of turbulence [4]. This work showed that it is possible to recapitulate the entire phase diagram of transitional pipe and quasi-one-dimensional

Taylor-Couette flow, including the transitions from puffs to weak then strong slugs as Re is increased through the transition [5, 6].

Although the methodological origin of directed percolation from this multi-scale perspective is very different from what happens in the puff-splitting model of the present paper, the puff-splitting model in the present work describes decay, merging and splitting of point particles and these dynamics are sufficient for the directed percolation universality class to emerge.

In summary, we conclude from experiment RG theory and DNS that even with strong puff-puff interactions of the suppression-pushing type, the universality class for the transition to pipe turbulence is consistent with directed percolation, and that the point-puff model used in the main text is fully justified as a description of the critical regime.

II. EXPERIMENTAL SETUP

In this section, we provide complete details of the experimental setup and measurement techniques. Setup A in Fig. S1 shows a schematic of the experiments used to measure the microscopic rules governing puff dynamics. The measurements were carried out in a $D = 4 \pm 0.01$ mm pipe of length $L/D = 1500$, made of 6 segments of precision glass tubes joined together with acrylic connectors. A few of these connectors had 0.8 mm holes (perpendicular to the pipe axis) that acted as ports for pressure measurement or to perturb the flow. Puffs can be easily identified by a sharp increase in pressure drop across the pipe and by monitoring the number of puffs at different positions along the pipe, we can monitor puff decays and splittings. To avoid any consequent changes in the flow-rate and hence Re , the flow was driven in a constant mass flux mode using a metering pump from Fuji Techno Industries Corporation (model HYSA 12). This pump features three pistons operating 180° out of phase, driven by a precision servo motor ensuring a pulsation free and accurate flow rate (better than 0.1%) over the entire operating range. The temperature of the fluid, distilled water, was measured by platinum RTD probes at the inlet and outlet of the pipe. The fluid viscosity was calculated from a fit to standard temperature-viscosity data. The fluid temperature was maintained constant and equal to $22 \pm 0.05^\circ C$ by means of a heat exchanger resulting in an estimated variation of ± 5 in terms of Re over several days. The flow is perturbed $150D$ downstream of the inlet and the laminar profile can be considered fully developed. The laminar flow is then disturbed by simultaneously injecting and withdrawing fluid by a push-pull perturbation via two 0.8 mm diametrically opposed holes in one of the acrylic connectors. As illustrated by Fig. S1, the perturbation mechanism consists of a coil to move a small magnetic cylinder, which injects water from one of the ports, while simultaneously withdrawing it from the other port, thus ensuring a disturbance with zero net

flux. It is known (see, for example, [7]) that provided a perturbation is successful in creating turbulence, the resulting puff is independent of the perturbation details and all puffs are statistically identical, with a fixed average length at a given Re . The state of the flow is monitored by measuring the differential pressure drop across a distance of $3D$, using variable-reluctance differential pressure sensors from Validyne (model DP-45). As a puff passes the pressure ports, the increased skin friction results in an increased differential pressure across the pair of pressure ports. Thus, the passage of a puff across the measurement point shows up as a peak in the pressure signal at that time instant as seen in Fig. S2a, which shows the signal corresponding to a train of puffs.

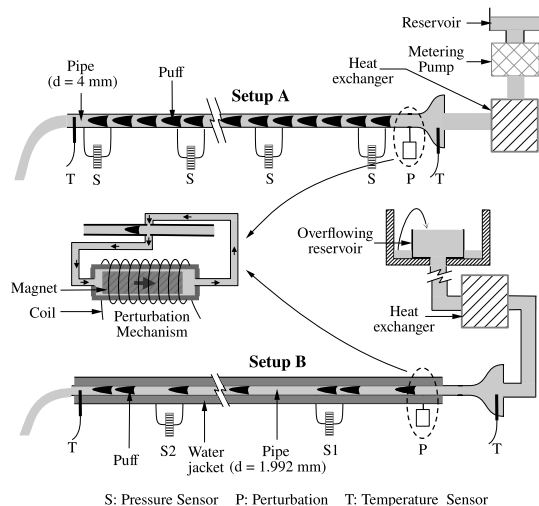


FIG. S1 A schematic of the experimental set-ups and the perturbation mechanism used to trigger puffs. Setup A was used to measure the microscopic rules governing puff motion and interactions. Setup B, is the quasi-periodic pipe used to obtain the steady state characteristics for different Re .

To quantify the inter-puff repulsion, two kinds of measurements were carried out, with the pressure signals being monitored $75D$ and $425D$ downstream of the perturbation. In the first series of measurements, two puffs were generated close to one another, at various separations. The pressure traces were used to determine the time at which the puffs passed each of the measurement points. This could be used to determine the mean speed of each puff as well as their relative speed. These measurements reveal that the speed of the downstream puff is affected by the upstream one and depends on their separation. However the speed of the puff that is upstream is unaffected and it has the same speed as an isolated puff, with its Re dependence matching the results of [5]. In the second type of measurement, a train of 100 equally spaced puffs is generated and measuring the time delays between each of the peaks in the pressure signal (corresponding to puffs) yields the speed of each of the puffs. Due to the repulsion, the first few puffs do spread out by the time they reach the second sensor and are ignored for

further processing, but the inter-puff distance of all the other puffs is maintained. The experiment is repeated 10 times for the same inter-puff spacing and the velocity is averaged over all the puffs to obtain a mean speed. As each puff in the train has a puff upstream, their speed is different from that of an isolated puff. The speed of a puff as a function of distance from an upstream one can be obtained by repeating this experiment for different inter-puff spacings.

As a further check, this information can be used to calculate the increase in relative separation between puffs that are initially closely spaced, and the results are in good agreement with the earlier measurements involving only 2 puffs.

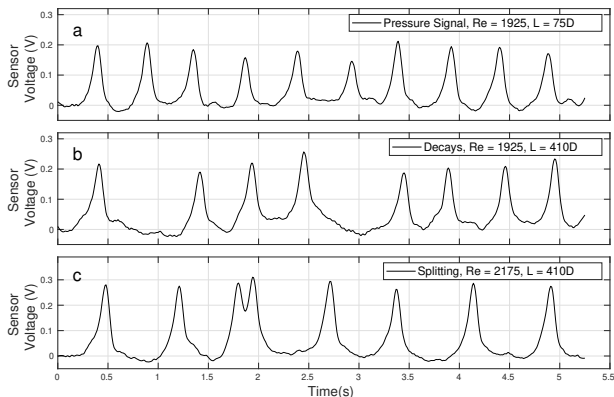


FIG. S2 (a) A typical pressure trace from the sensor at $75D$ for a puff train with an inter-puff spacing of $60D$, at $Re = 1925$. (b). The same puff train at $410D$ showing two decays, identifiable by the larger spacing at two locations, as compared to the signal at $60D$ above. (c) A puff train with an inter-puff spacing of $100D$ at $Re = 2175$ showing a smaller than typical spacing at one location, indicating a puff splitting.

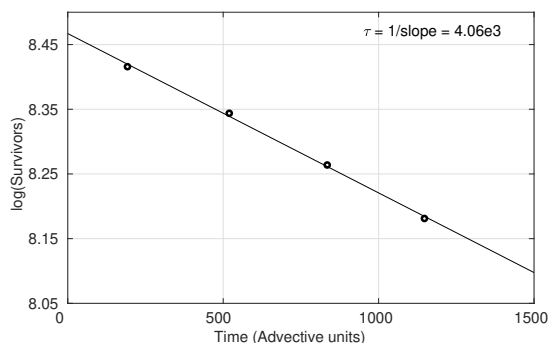


FIG. S3 A straight line fit of the logarithm of survivors (puffs that have survived without decaying) at different times (obtained from the 4 measurement locations), showing an exponential decay of survivors. The inverse of the slope is the time-scale of decay. The figure is obtained from measurements at $Re = 1925$ and an inter-puff spacing of $60D$, some pressure traces of which are shown in Fig. S2(a) and (b).

To determine the dependence of inter-puff spacing on

decay and splitting lifetimes, the same set-up was used, but with 4 sensors at $75D$, $410D$, $73D$ and $1055D$ downstream of the perturbation point. A train of equally spaced puffs with a specific spacing was generated and the pressure traces recorded at each sensor. As mentioned previously, except for the first few puffs, the train travelled downstream while maintaining the inter-puff spacing, unless a splitting or decay occurred. Indeed, decays and splittings were identified by significantly larger or smaller gaps between adjacent puffs than the initially chosen spacing. Examples of typical pressure traces are shown in Fig. S2. Fig. S2(a) shows a portion of the puff train consisting of 10 puffs with an inter-puff spacing of $60D$ at $Re = 1925$, recorded at the first pressure sensor, located just downstream ($75D$) of the perturbation. Fig. S2(b) shows the same portion of the train at the next sensor, located $410D$ downstream of the perturbation point. Two decays can be identified due to the larger spacing as compared to the original train. Similarly, Fig. S2c shows the pressure trace at the same sensor, but at $Re = 2175$, for an initial inter-puff spacing of $100D$, where one splitting event can be identified due to a smaller inter-puff spacing than originally chosen. Thus, for a given puff train, the number of decays or splittings at each sensor can be determined. In practice, due to the faster than exponential decrease (increase) in decay (splitting rates), there are practically only decays for $Re < 2030$ and only splittings for $Re > 2050$, and hence it suffices to count the number of puffs at each sensor location to determine the number of decays (or splittings) at each sensor.

With this information, the fraction that survive without a decay or split can be plotted as a function of time (equivalent to downstream location as the puffs advect downstream at a well defined average speed). This reveals an exponentially decaying or memoryless distribution of the survivors allowing us to extract a time-scale from a fit. Note that this is a slight modification of a technique to obtain lifetimes that has become standard for pipe flow; see, for example, [8]. Fig. S3 shows the survivor distribution for $Re = 1925$ and an inter-puff spacing of 60 , corresponding to the measurements whose pressure traces are shown in Fig. S2a and b. These measurements are repeated for different Re and inter-puff spacings to yield characteristic decay and splitting times as a function of both puff spacing and Re .

Finally, by looking at the distribution of arrival times of 1000 isolated puffs at these 4 locations, the diffusive nature of the puff propagation was shown and the diffusion coefficient calculated as a function of Re .

The long time-scale experiments to experimentally obtain the steady-state turbulent fraction as a function of Re require a much bigger system size and a tighter control on Re for long periods of time. As measurements were carried out closer to critical as compared to [7], the setup here is a modified version of the one used in that study, resulting in a tighter control of Re and temperature. A schematic of the set-up is shown as setup B

in Fig. S1. The pipe had a nominal inner diameter of $D = 2\text{mm}$ and a total length of 18 meters, or $9000D$. However a determination of the diameter by measuring the pressure drop across a section of the pipe using a precision, piezo-resistive, differential pressure sensor (model PN1M from Sensors) gave a value of 1.992 mm and this value is what is used in calculating Re . As before, this consisted of glass pipe-sections joined together with custom made acrylic connectors, some of which had ports for perturbing the flow or measurement of pressure. The pipe is gravity-driven, with the working fluid, de-ionized water, being supplied from reservoir that is more than 20 meters above the pipe. The reservoir is continuously overflowing, which ensures a precise pressure-head that drives the flow. The reservoir is mounted on motorized, vertical guide-rails, permitting fine adjustments to the height of the reservoir, and hence the flow rate and Re . The convergent section through which the water enters the pipe and the precise machining of the connectors ensures that the pipe remains laminar up to $Re \approx 5000$, which is far above the Re range studied.

The flow-rate is continuously measured to an accuracy of better than 0.05% by collecting the water exiting the pipe for a fixed amount of time and weighing it with a precision electronic scale. RTD platinum probes were used to measure the temperature at the inlet and exit of the pipe. A little before entering the pipe, the water passes through a heat exchanger, with an associated temperature controller. As shown in the schematic, there is an acrylic pipe with an diameter of 50 mm, that is co-axial to the 2 mm measurement pipe and surrounds it. Water, whose temperature is regulated by a temperature-controller is circulated in the annular space between these pipes which acts as a water jacket to regulate temperature variations of the measurement pipe. To allow a better control, the outer pipe is heat-insulated by covering it with standard pipe insulation. The two temperature controllers (one for the working fluid and the other for the water jacket) are used in a feedback loop with a target of maintaining a zero temperature difference between inlet and outlet of the the measurement pipe and a desired value (usually 20° at the inlet. Both targets are achieved to better than $\pm 0.01^\circ$ for as long as desired. The temperature of the working fluid is taken to be the mean of the temperatures at the two ends of the pipe. As with the earlier constant flux set-up, the viscosity of the water is determined by a fit to standard data. With this viscosity and the measured flow-rate, the Re is calculated. Experiments to measure the turbulent fraction close to the critical point can take anywhere between several hours to several weeks and it is important to maintain the Re for such extended periods of time. In the course of the experiment, the Re can change due to two reasons. One is the temperature associated viscosity change, with a change of 0.01° resulting in a change of around 1 in Re . Another is the change in frictional drag due a change in the number of puffs in the pipe due to splitting or decay, with one extra puff causing a decrease of around 1 in Re .

A feedback loop counters any such deviation from the set Re by an adjustment of the vertical position of the reservoir and hence the flow-rate, in order to bring back the Re to the set value. With this method, a long term stability of the Re to better than ± 1 can be achieved.

The flow is perturbed $150D$ from entrance using the same push-pull perturbation described earlier for the pump driven set-up and shown in the schematic of the perturbation mechanism in Fig.S1. The puffs are monitored near the pipe entrance and exit using DP-45 variable-reluctance differential pressure sensors from Validyne, which measure the pressure across $8D$. Specifically, the locations are $245D$ and $8495D$ downstream of the perturbation point, and are indicated in the schematic by points S1 and S2 respectively

The procedure to determine the steady-state turbulent fraction is as follows. A number of puffs, typically uniformly spaced, are generated using the perturbation and serve as the initial condition. These puffs advect downstream. As each of them passes the pressure measurement location S2, a peak is detected in the pressure signal. This peak detection automatically triggers the perturbation to generate a new puff near the inlet. Shortly thereafter, the newly generated puff passes point P1, generating a peak in the pressure signal which confirms that it was successfully generated. Thus, the pattern of puffs passing the point P2 and exiting the pipe is generated at the pipe inlet and verified at point S1. As puffs are statistically identical, this procedure should not change the statistical evolution of the flow. In this way, the evolution of turbulence can be followed for long periods of time till it reaches a steady-state. In this paper, we use the puff density (number of puffs per unit length) as a proxy for the turbulent fraction as the former quantity is well defined while the latter one depends on the cut-off chosen for differentiating laminar from turbulent flow. The mean puff density at a given Re is calculated by simply counting the number of puffs in one period and then averaging over long times in the steady state.

III. PUFF SIZES

In the main paper, we point out that, as is typical of a phase transition, the microscopic rules such as puff interactions or the diffusion coefficient but vary smoothly as we pass the critical point and do not exhibit any sudden change there. Here, we present additional evidence to strengthen this claim by looking at typical puff widths at different Re . Well separated (non-interacting) puffs were triggered and the pressure signal recorded further downstream. The pressure signals for 10 puffs were phase averaged (by using the peak of the pressure signal as a reference point), yielding an ensemble-averaged pressure signal. This was repeated at several Re below and above critical and the results are shown in Fig. S4. The width of the pressure peak is a measure of the puff width, and

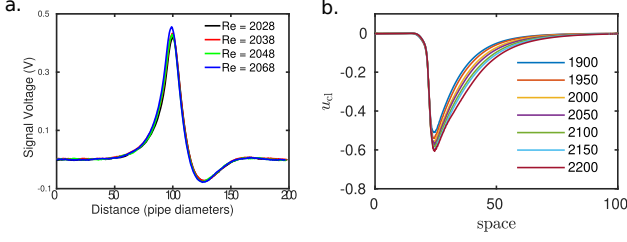


FIG. S4 Profile of a single puff as a function of Re , showing puff widths at various Re straddling the critical point, showing that the puff sizes change slowly and smoothly with Re . (a) Experimental voltage signal from a pressure sensor (b) Centreline velocities obtained from DNS

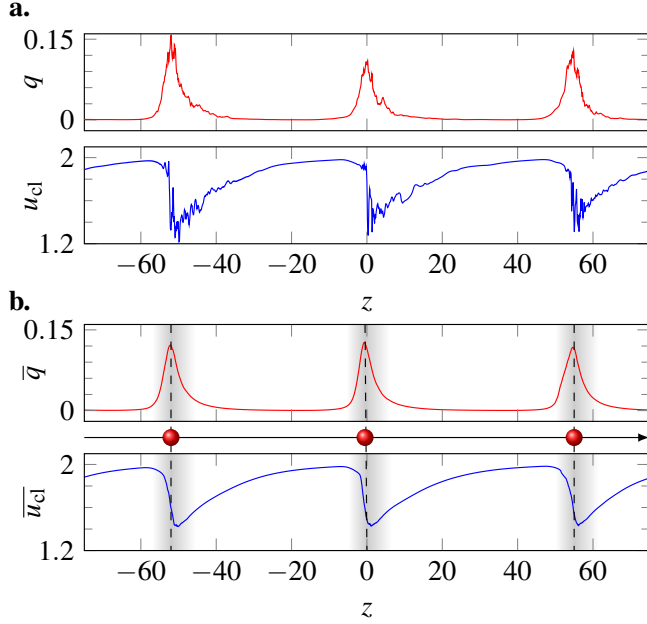


FIG. S5 a. Instantaneous turbulent kinetic energy of the perturbation averaged over cross-section of the pipe $q(z, t) = \int (\mathbf{u} - \mathbf{u}_{lam})^2 d^2\mathbf{r}$ and the streamwise centerline velocity u_{cl} from a direct numerical simulation showing three interacting turbulent puffs. $\mathbf{u}$ is the velocity field, $\mathbf{u}_{lam}$ is the laminar velocity field. **b.** Time averaged, in a co-moving frame, q and u_{cl} for the same simulation. While each puff is unique due to a fully 3d and temporal dynamics (a), their time-averaged counterpart are statistically equivalent (b). The spatially localized turbulent kinetic energy justifies the point-like particle approach and the long tail of the centerline velocity perturbation provides an insight into the origin of their interactions.

as is clear, this does not change appreciably with Re as we pass through the critical point.

IV. PUFF-SPLITTING MODEL

In this chapter of the Supplementary Information, we develop a theoretical description of the statistical me-

chanics of puffs near the critical Reynolds number. We start with a description of turbulent puffs in a pipe that is coarse-grained so that each puff is considered to be a single particle that can move on a line or a one-dimensional (1D) lattice. The turbulent puffs can spontaneously relaminarize (i.e. “die”), diffuse and split (i.e. “birth”). A general configuration will be a gas of puffs that is very dilute near the laminar-turbulence transition, and becomes increasingly dense as the Reynolds number is increased. We wish to understand the functional form of this mean density field, which we will sometimes call the turbulent fraction. In addition to their intrinsic dynamics, puffs are being advected downstream by the mean flow, which we will take to be moving from left to right.

Systems of particles hopping predominantly in one direction along a line, with strong repulsive interactions, arise in many fields of physics, and are generally described by a class of models known as asymmetric exclusion processes (ASEP) [9, 10]. The variant of these models that is appropriate here as a minimal model is based upon the totally asymmetric exclusion process (TASEP), in which only unidirectional hopping is allowed. The question that is usually asked of ASEP models is how is the bulk dynamics of the particles affected by the boundary conditions? In these models, particles are injected at a given rate on the left and removed with a given rate on the right of the system. In our case we are primarily interested in the case of periodic boundary conditions. Particles are, however, not conserved in our case: they are born through splitting, and they die through decay. Thus the appropriate model for puffs combines TASEP with the additional birth and death processes of DP.

Let n_i be the occupation number of the site i in a 1D periodic lattice of N sites. Then the puff-splitting model (ignoring for now puff-puff interactions) is given by the following stochastic processes:

1. Puff at site i is removed with rate ω_d (*decay*)
2. Puff at site i moves to site $i+1$, if that site is empty, with rate p (*propagation*)
3. Puff at site i creates a puff at site $i+1$, if that site is empty, with rate ω_s (*splitting*)

To implement this model, random asynchronous updating should be used, in which the three rate processes are carried out on randomly chosen particles in random order, with probabilities per unit time given by the coefficients p , ω_d and ω_s .

The mean field equation for this process is given by

$$\partial_t \langle n_i \rangle = -p \langle n_i (1 - n_{i+1}) \rangle + p \langle n_{i-1} (1 - n_i) \rangle - \omega_d \langle n_i \rangle + \omega_s \langle n_{i-1} (1 - n_i) \rangle \quad (\text{S1})$$

The first term describes the process in which a puff hops away from site i onto the neighbouring downstream site $i+1$, but only if the destination site is empty. This

process happens with a rate p . If the puff does not hop, it is effectively moving backwards (i.e. upstream) compared to the mean flow of the puffs. The second process is similar but is a flow onto site i from the upstream site $i - 1$. Together these terms represent single puff hopping unless blocked by a downstream puff. The third term represents decay, and the fourth term puff-splitting. Note that this term resembles the second half of the puff hopping process, but its effect is different because there is no term which effectively removes the puff at site i , as does the first term in equation (S1).

Living in one dimension, puff-puff interactions are very strong, and there are two additional microscopic interactions that need to be considered in addition to the fact the puffs cannot pass through one another: a puff that is upstream of another occupying its nearest-neighbour site can neither split nor hop to the right, until the downstream puff has itself hopped downstream or decayed. First, the lifetime of the puff that is downstream of its nearest-neighbour will have a drastically shortened lifetime (by perhaps 5 orders of magnitude) compared to a free puff. We will call this “suppression”; it arises from the way in which a puff distorts the mean velocity profile. Second, the puffs have a strong short-range repulsion, which we will call “pushing”: a puff that is created by splitting will for a short distance experience a faster downstream velocity that has the effect of separating it quickly from its mother puff. If the daughter does not get pushed then it will have a high likelihood of decaying rapidly.

These interactions can be added to the simple puff-splitting model by two additional rules. *Suppression* can be modeled by assuming that the decay rate is much greater if a puff is immediately downstream of another puff, i.e. $\omega_d \rightarrow \tilde{\omega}$. *Pushing* can be modeled by introducing an increased hopping rate $\tilde{p}$ if a puff is immediately downstream of another puff, i.e. $p \rightarrow r$. For now, we will not include these terms.

If we set $p = 0$, then this model describes purely splitting and decay. Since splitting followed by decay represents hopping (i.e. anisotropic diffusion), and there is a limit of one puff per site, this model includes the four basic processes of directed percolation (DP): diffusion, de-coagulation, coagulation and annihilation [11]. For ω_d large enough, the ultimate fate of the system is to be empty. As ω_d becomes smaller, there will be a continuous transition in the DP universality class to a state with persistent mean number of particles $\rho \equiv \langle n_i \rangle \sim (\omega_c - \omega_d)^\beta$, with $\beta = 0.276$ in 1D.

The question that we now address is how the additional terms in the puff-splitting model modify the critical behavior from DP. We will answer the question by writing down a stochastic hydrodynamic model for the puff-splitting model, and then analyze its critical behaviour using renormalization group arguments.

A. Stochastic hydrodynamics for puff-splitting model

The discrete stochastic puff-splitting model can be expressed as a field theory using standard techniques that

use the Doi formalism [12], followed by a Martin-Siggia-Rose [13–15] transformation to a coherent state path integral [16]. We will not use that method here, although we will need to use the result later. An alternative, shown in the next Section, is to derive the stochastic equation by writing down the master equation, and then performing a Kramers-Moyal expansion to second order to derive the corresponding Fokker-Planck equation, and thence the Langevin equation [17]. In this coarse-grained formulation that neglects higher order derivatives, the lattice becomes a continuum ($i \rightarrow x$). This amounts to writing $n_i \rightarrow \rho(x)$ and

$$n_{i+1} \approx \rho(x) + \partial_x \rho(x) + \frac{1}{2} \partial_x^2 \rho(x) \quad (\text{S2})$$

and reading off the noise, to obtain

$$\begin{aligned} \partial_t \rho = & -\omega_d \rho + \frac{1}{2} (p + \omega_s) \partial_x^2 \rho - (p + \omega_s) \partial_x \rho + \omega_s \rho (1 - \rho) \\ & + (2p + \omega_s) \rho \partial_x \rho - \frac{\omega_s}{2} \rho \partial_x^2 \rho + \sqrt{\rho(1 - \rho)} \eta \end{aligned} \quad (\text{S3})$$

where $\eta(x, t)$ is a white noise delta-correlated in space and time. The dynamics is controlled by multiplicative noise, which has the effect that the empty state is absorbing. This form of the noise arises because of the rule that there can only be one particle per site. If this were enforced by an explicit coagulation term of the form $2n_i \rightarrow n_i + \emptyset$, the equations would be the same but the minus sign in the noise term would be replaced by a plus sign.

Equation (S3) is similar to the Langevin equation for DP, but with the addition of three terms: the term proportional to $\partial_x \rho$ and the last two terms of second order in ρ . The linear term arises because the puff dynamics is diffusive (the second order term, linear in ρ) but in a frame moving with the net velocity of the puffs. This anisotropic diffusion can be removed by a Galilean transformation into the comoving frame. The second term is proportional to $\rho \partial_x \rho$ and has the functional form of the advective nonlinearity in Burgers equation. Such terms naturally arise in TASEP models, and loosely speaking lead to shocks that model the bunching up of particles, as occurs in traffic jams, transcription by ribosomes moving on mRNA etc. These shocks are regularized by the higher order derivative terms, including the third extra term and the linear diffusive term. We refer to the terms $\rho \partial_x \rho$ and $\rho \partial_x^2 \rho$ as Burgers-like terms. We will refer to the terms $\rho \partial_x \rho$ and $\rho \partial_x^2 \rho$ as Burgers-like terms, because they are of the form density multiplying a derivative of the density.

B. Master equation for the puff-splitting model

In this Section, we derive equation (S3). The puff splitting model is based on the reactions of decay, hopping and splitting process in one dimension:

$$A_i \xrightarrow{\omega_d} E_i, \quad (S4)$$

$$A_i + E_{i+1} \xrightarrow{p} E_i + A_{i+1}, \quad (S5)$$

$$A_i + E_{i+1} \xrightarrow{\omega_s} A_i + A_{i+1}, \quad (S6)$$

where A_i denotes a particle at site i , E_i represents the vacancy at site i , and $i = 1, 2, \dots, L$. We introduce the coarse-grained particle density centered at site i over a length scale of K lattice sites with a uniform kernel as n_i , specifically,

$$n_i = \frac{1}{K} \sum_{j=-K/2}^{K/2} N_{i+j}, \quad (S7)$$

where $N_i = 0, 1$ is the particle number at each site i . We denote the state vector by $\mathbf{n} = (n_1, n_2, \dots, n_L)^T$ and define the lattice density operator $\mathcal{E}_i^\pm$ as

$$\mathcal{E}_i^\pm f(n_i) = f(n_i \pm \Delta), \quad (S8)$$

and the master equation can be represented as

$$\partial_t P(\mathbf{n}, t) = \frac{1}{\Delta} \sum_i \left[(\mathcal{E}_i - 1) \omega_d n_i + (\mathcal{E}_{i-1} \mathcal{E}_i^{-1} - 1) \omega_s n_{i-1} (1 - n_i) + (\mathcal{E}_{i-1} \mathcal{E}_i^{-1} - 1) p n_{i-1} (1 - n_i) \right] P(\mathbf{n}, t), \quad (S9)$$

where $\Delta = 1/K$. By applying Kramers-Moyal expansion with truncation to the second order,

$$\mathcal{E}_i^\pm \approx 1 \pm \Delta \partial_{n_i} + \frac{1}{2} \Delta^2 \partial_{n_i}^2 + \dots, \quad (S10)$$

equation (S9) can be rewritten into a Fokker-Planck equation:

$$\partial_t P(\mathbf{n}, t) = - \sum_i \frac{\partial (A_i P)}{\partial n_i} + \frac{1}{2} \Delta \sum_{i,j} \frac{\partial^2 (B_{ij} P)}{\partial n_i \partial n_j}, \quad (S11)$$

where

$$A_i = -\omega_d n_i - p n_i (1 - n_{i+1}) + (p + \omega_s) n_{i-1} (1 - n_i), \quad (S12)$$

$$B_{ij} = [\omega_d n_i + \omega_s n_i (1 - n_{i+1}) + p n_i (1 - n_{i+1}) + p n_{i-1} (1 - n_i)] \delta_{ij} + 2p n_i (1 - n_j) \delta_{j,i+1}. \quad (S13)$$

The corresponding Langevin equation in Ito's sense is

$$\partial_t \mathbf{n} = \mathbf{A}(\mathbf{n}) + \boldsymbol{\xi}(t), \quad (S14)$$

$$\langle \xi_i(t) \xi_{i'}(t') \rangle = \Delta B_{ii'} \delta(t - t'). \quad (S15)$$

In the continuum limit in space, the density operator is defined as

$$\tilde{\mathcal{E}}_x^\pm f[\rho(y)] = f[\rho(y) \pm \Delta \delta(y - x)], \quad (S16)$$

and equation (S9) becomes

$$\begin{aligned} \partial_t P(\rho, t) = \frac{1}{\Delta} \left\{ \int (\tilde{\mathcal{E}}_x - 1) \omega_d \rho(x) P dx \right. \\ + \iint \left(\tilde{\mathcal{E}}_y \tilde{\mathcal{E}}_x^{-1} - 1 \right) p \rho(y) [1 - \rho(x)] \delta(x - y - \epsilon) P dx dy \\ \left. + \iint \left(\tilde{\mathcal{E}}_x^{-1} - 1 \right) \omega_s \rho(y) [1 - \rho(x)] \delta(x - y - \epsilon) P dx dy \right\}, \end{aligned} \quad (S17)$$

where ϵ is the lattice spacing.

The Kramers-Moyal expansion gives

$$\tilde{\mathcal{E}}_x^\pm \approx 1 \pm \Delta \frac{\delta}{\delta \rho(x)} + \frac{1}{2} \Delta^2 \frac{\delta^2}{\delta \rho(x)^2} + \dots, \quad (\text{S18})$$

and with truncation to the second order, equation (S17) becomes

$$\begin{aligned} \partial_t P(\rho, t) \approx & - \int \frac{\delta}{\delta \rho(x)} \{ -\omega_d \rho(x - \epsilon) + (p + \omega_s) \rho(x - \epsilon) [1 - \rho(x)] - p \rho(x) [1 - \rho(x + \epsilon)] \} dx \\ & + \Delta \int \frac{\delta^2}{\delta \rho(x)^2} \{ \omega_d \rho(x) + (p + \omega_s) \rho(x - \epsilon) [1 - \rho(x)] + p \rho(x) [1 - \rho(x + \epsilon)] \} dx \\ & + \Delta \iint \frac{\delta^2}{\delta \rho(x) \delta \rho(y)} \{ -2p \rho(y) [1 - \rho(x)] \delta(x - y + \epsilon) \} dx dy. \end{aligned} \quad (\text{S19})$$

By using

$$\rho(x \pm \epsilon) \approx \rho(x) \pm \epsilon \partial_x \rho(x) + \frac{1}{2} \epsilon^2 \partial_x^2 \rho(x) + \dots, \quad (\text{S20})$$

equation (S19) can be written into a Fokker-Planck equation:

$$\partial_t P(\rho, t) \approx - \int \frac{\delta}{\delta \rho(x)} [\mathcal{A}(\rho, x) P] dx + \frac{1}{2} \Delta \iint \frac{\delta^2}{\delta \rho(x) \delta \rho(y)} [\mathcal{B}(\rho, x, y) P] dx dy, \quad (\text{S21})$$

where

$$\begin{aligned} \mathcal{A}(\rho, x) = & -\omega_d \rho(x) + \omega_s \rho(x) [1 - \rho(x)] + \frac{1}{2} \epsilon^2 (p + \omega_s) \partial_x^2 \rho(x) \\ & - \epsilon (p + \omega_s) \partial_x \rho(x) + \epsilon (2p + \omega_s) \rho(x) \partial_x \rho(x) - \frac{1}{2} \epsilon^2 \omega_s \rho(x) \partial_x^2 \rho(x), \end{aligned} \quad (\text{S22})$$

$$\begin{aligned} \mathcal{B}(\rho, x, y) = & \left\{ \omega_d \rho(x) + (2p + \omega_s) \rho(x) [1 - \rho(x)] + \frac{1}{2} \epsilon^2 (p + \omega_s) \partial_x^2 \rho(x) \right. \\ & \left. - \epsilon (p + \omega_s) \partial_x \rho(x) + \epsilon \omega_s \rho(x) \partial_x \rho(x) - \frac{1}{2} \epsilon^2 (2p + \omega_s) \rho(x) \partial_x^2 \rho(x) \right\} \delta(x - y) \\ & - 2p \left\{ \rho(x) [1 - \rho(x)] + [1 - \rho(x)] \left[-\epsilon \partial_x \rho(x) + \frac{1}{2} \epsilon^2 \partial_x^2 \rho(x) \right] \right\} \delta(x - y + \epsilon). \end{aligned} \quad (\text{S23})$$

The corresponding Langevin equation in Ito's sense is

$$\partial_t \rho(x) = \mathcal{A}(\rho, x) + \xi(x, t), \quad (\text{S24})$$

$$\langle \xi(x, t) \xi(y, t') \rangle = \Delta \mathcal{B}(\rho, x, y) \delta(t - t'). \quad (\text{S25})$$

By setting $\epsilon = 1$, near transition $\rho \ll 1$, one obtains

$$\begin{aligned} \partial_t \rho \approx & -\omega_d \rho + \omega_s \rho (1 - \rho) + \frac{1}{2} (p + \omega_s) \partial_x^2 \rho - (p + \omega_s) \partial_x \rho \\ & + (2p + \omega_s) \rho \partial_x \rho - \frac{1}{2} \omega_s \rho \partial_x^2 \rho + \sqrt{a\rho - b\rho^2} \eta, \end{aligned} \quad (\text{S26})$$

where $\eta(x, t)$ is a Gaussian white noise and a and b are functions of ω_d, ω_s and p . The nonlinear term in the square root that multiplies the white noise η can be replaced by $\sqrt{\rho}$ near the transition where $\rho \rightarrow 0$. This equation is of the form of equation (S3).

C. Universality class of the puff-splitting model

The puff-splitting model without the Burgers-like terms is in the universality class of DP. We now examine whether or not the Burgers-like terms change the universality class, by calculating whether or not they are relevant or irrelevant in the renormalization group sense at the fixed point that controls DP. To do that, we will

use the field theoretic RG, in which the critical dynamics of DP is generated by a Martin-Siggia-Rose action of the form [16]

$$\begin{aligned} A(\rho, \tilde{\rho}) = & \int d^d x dt \left[\tilde{\rho} (\partial_t + D(r - \partial_x^2)) \rho \right. \\ & \left. - u_3 \tilde{\rho} (\rho - \tilde{\rho}) \rho + u_4 \tilde{\rho}^2 \rho^2 + O(\rho^5) \right] \end{aligned} \quad (\text{S27})$$

where $\tilde{\rho}$ is a so-called response field that arises during the representation of the problem as a field theory, u_3 and u_4 are coupling constants of cubic and quartic terms, D is an effective diffusion coefficient, and r is a control parameter that vanishes at the critical point (for an excellent review, see Ref. [16], pp. 173-174). The requirement that the action be dimensionless implies that the dimensions of the fields can be expressed in terms of the momentum cut-off μ as follows: $[x] = \mu^{-1}$, $[t] = \mu^{-2}$, $[\rho] = \mu^{d/2}$, $[D] = \mu^0$, $[r] = \mu^2$, $[u_3] = \mu^{2-d/2}$, $[u_4] = \mu^{2-d}$.

The lowest order nonlinearity beyond free field theory is the cubic term, whose scaling dimension is $2 - d/2$, implying that the critical behavior deviates from mean field theory for $d < 4$ (for an introduction to the renormalization group for DP, see Ref. [16], p. 404 et seq.). The cubic nonlinearity is the only contributor to the non-analytic scaling behavior below four dimensions, and the critical behavior can be calculated using (e.g. an expansion in $4 - \epsilon$ dimensions) (see (e.g.) [18]). The quartic coupling constant u_4 has scaling dimension $2 - d$, so as the momentum cut-off $\mu \rightarrow \infty$, the quartic coupling gets weaker and weaker, and is irrelevant in the RG sense at the four dimensional fixed point that controls the behavior for $d < 4$. Thus, even as low as $d = 1$, DP is controlled by the cubic nonlinearity only. We now apply the same reasoning to the Burgers-like terms in equation (S3), but work directly with the stochastic differential equation rather than the field theoretic action.

The stochastic differential equation corresponding to the DP action of equation (S27) is [16]:

$$\partial_t \rho = -D(r - \partial_x^2)\rho - u_3 \rho^2 + \xi \quad (\text{S28})$$

$$\langle \xi(x, t) \xi(x', t') \rangle = 2u_3 \rho(x, t) \delta(x - x') \delta(t - t') \quad (\text{S29})$$

Note that the dimension of all the terms in the equation is $\mu^{2+d/2}$. Thus, to see how the Burgers-like terms scale under renormalization, we calculate the coupling constant dimensions using the known scaling dimensions of space and the field ρ . Denoting the coupling constant of the Burgers term as λ_1 , we equate

$$[\lambda_1 \rho \partial_x \rho] = \mu^{2+d/2} \quad (\text{S30})$$

to obtain

$$[\lambda_1] = \mu^{1-d/2} \quad (\text{S31})$$

At the fixed point in $d = 4$, this has negative scaling dimensions and flows to zero as $\mu \rightarrow \infty$. We conclude that this Burgers-like term is irrelevant for DP. Similarly, the coupling constant λ_2 for the Burgers-like term $\rho \partial_x^2 \rho$ scales as $\mu^{-d/2}$ and thus is also irrelevant at the DP fixed point. These results are consistent with earlier studies briefly mentioned in Ref. ([19]) (p. 471 and 477).

D. Effects of puff suppression and pushing

The experiments suggest that puffs that are close and downstream of another puff, experience an additional

rate of decay, $\tilde{\omega}$ and a higher velocity to the right. We can model suppression in the stochastic hydrodynamic model by a term that in mean field theory follows:

$$\partial_t \langle n_i \rangle = -\tilde{\omega} \langle n_{i-1} n_i \rangle \quad (\text{S32})$$

and model pushing by an enhanced hopping at a rate r for nearest neighbour downstream puffs:

$$\partial_t \langle n_i \rangle = -r \langle n_i n_{i-1} (1 - n_{i+1}) \rangle + r \langle (1 - n_i) n_{i-1} n_{i-2} \rangle. \quad (\text{S33})$$

Going to the continuum stochastic hydrodynamical description, these terms will all be represented by high order operators of the form

$$I_{\alpha\beta\gamma} \equiv \lambda_{\alpha\beta\gamma} \rho^\alpha (\partial_x^\beta \rho)^\gamma \quad (\text{S34})$$

where the coupling constants $\lambda_{\alpha\beta\gamma}$ scale as $\mu^{y_{\alpha\beta\gamma}}$ with

$$y_{\alpha\beta\gamma} = 2(2 - \alpha - \gamma) - \beta\gamma. \quad (\text{S35})$$

For $\alpha, \beta, \gamma \geq 1$, $y_{\alpha\beta\gamma} < 0$, so the effect of puff suppression and pushing, at least at large scales is irrelevant and should not change the universality class of the transition from DP.

V. NUMERICAL SIMULATIONS OF THE PUFF-SPLITTING MODEL

To verify these predictions, we have performed numerical simulations. The RG calculation reveals the asymptotic critical scaling, but in practice there might be slow crossovers to the true critical behavior, due to the finite length of a pipe and the finite time for sampling puff dynamics. These effects will be acute near any putative critical point, and so it is important to be able to predict how these effects will be manifested in experiments and simulations. In addition, the divergence of the relaxation time near the critical point means that simulations may be biased by long-lived transients, and these need to be taken into account also.

To rule out sensitive dependence on the numerical implementation, two different computer codes with different update rules and parameter sets were used to check the results. In the first, a minimal discrete model was used to check numerically the conclusions based on perturbative RG arguments given above. In the second, a continuum stochastic dynamics simulation was performed, with a greater level of realism regarding the interactions between the puffs, as determined experimentally. In both simulations, we observe the emergence of a jammed or crystalline phase above the turbulence transition due to the ‘‘traffic jams’’ experienced by puffs. As the transition is approached from higher Reynolds numbers, the crystalline phase effectively melts due to strong fluctuations through the appearance of interspersed disordered regions. Both simulations, when analysed properly, show that the asymptotic universality class is directed percolation, but we identify strong artifacts that complicate the direct observation in experiments of the asymptotic critical scaling.

A. Lattice model of puff dynamics

In order to understand in detail how the phase diagram of interacting puffs depends on individual puff splitting and decay rates, we constructed an additional minimal model that we call the lattice model to differentiate it from the continuum minimal described in the main text.

This minimal model is a computationally-efficient lattice model of the puff interaction dynamics uncovered in our experiments, closely related to the statistical hydrodynamics model. This model permits even greater scale-up than the continuous puff model, and allows us to readily scan through the respective strengths of each interaction and quantify its effects on the DP scaling. In the coarse grained model, we simulate the puff motion by right hopping particles, which push and suppress neighbors and split and decay similarly to the rules described above and explained in the SI. Near the critical point, the puff density is low and τ_d and τ_s are both much larger than the time it takes for a puff to diffuse a distance similar its spatial extent. Thus we can ignore the puff motion below 12D and use this scale as our fundamental lattice unit and the step size for the puff hopping.

1. Detailed description of lattice model

The discrete model consists of a lattice of length L . The dynamics is controlled by a set of parameters

ω_s	the puff splitting rate
ω_d	the puff decay rate
p	the puff hopping rate
λ_{push}	pushing length-scale
q_{sup}	suppression parameter for puffs within a distance λ_{push}
q_{push}	pushing parameter for puffs within a distance λ_{push}

In the basic version of the model, particles hop to the right with a probability p in each time step. We permit puff suppression by allowing a right hopping particle to invade an occupied neighboring site and thereby eliminate a downstream puff/particle. Note that we have also considered a variant of the model where we do not permit puff suppression, meaning that particles do not hop if there would be a collision with an already occupied site. The lifetime $\tau_{d,\infty}$ and time between splitting events, $\tau_{s,\infty}$, are given by a phenomenological fit to experimental data

$$\tau_d(\text{Re}) = \exp[\exp(0.005898 \text{ Re} - 9.1871)],$$

The corresponding expression for the splitting rate is given by

$$\tau_s(\text{Re}) = \exp[\exp(-0.003165 \text{ Re} + 9.2871)].$$

The simulation of the model is performed using one of two update rules, an upstream update rule, where the puffs

in each time step are updated sequentially from the left to the right and a stochastic update rule (producing similar results (in almost all circumstances)) where puffs are picked randomly one-by-one and updated accordingly.

The dimensionless diffusion constant D of the right hopping process is

$$\tilde{D} = \frac{p(1-p)}{2}$$

2. Puff-pushing and suppression

The puff-pushing alters the hopping probability p of a puff x_i depending on the distance $|x_i - x_{i-1}|$ to an upstream puff x_{i-1} . If that distance is smaller than some pushing length λ_{push} , the hopping probability is set to one. Thus, the hopping probability of a puff at x_i is p if $|x_i - x_{i-1}| > \lambda_{push}$ and $q_{push} = 1$ otherwise. Similarly, we use the following expression for the probability of puff-suppression: when a puff is within a distance $d < d_{sup}$ from an upstream puff, the suppression probability at x_i is zero if $|x_i - x_{i-1}| > \lambda_{push}$ and q_{sup} otherwise. For the phase diagram of the puff density in the main paper, $q_{sup}=2\text{e-}5$.

3. Comparison to experimental values

In this section, we show how to relate the parameters in the discrete model to those in the experiments and continuous model. The experiments have the following characteristic parameters

D^*	4e-3 m
U^*	0.4995 ms ⁻¹
L^*	6.3e-5 m ² s ⁻¹
T^*	8e-3s = $(D^*)/(U^*)$

In the experiments, a puff has a spatial extent of $\lambda = 12D^*$. For a pipe of length 7800D and with a pushing zone reaching out to $\lambda_{push} = 4\lambda$, a system being in the perfect puff-crystal state would contain around $7800/48 = 162$ puffs. If we convert these numbers to units of the discrete model, the corresponding system size would be $162\lambda_{push}/\lambda$ lattice units. We can perform meaningful simulations down to values of $\lambda_{push} = 2\lambda$ i.e. a discrete system representing the experimental system would consist of around 350 lattice units.

Regardless of the model, the scaling of the dimensionless variables should result in the same variables with dimensions, e.g.

$$L^* = \frac{p(1-p)}{2} \frac{\lambda^2}{\tau} \quad (\text{S36})$$

Here λ is a characteristic length scale and τ a characteristic time scale for the model.

All the relevant timescales in the model are scaled with this length and time, where e.g. dimensionless reaction rates become

$$\omega_{\text{decay}}(\text{Re}) = \frac{\tau}{\tau_{\text{decay}}(\text{Re})}$$

$$\omega_{\text{split}}(\text{Re}) = \frac{\tau}{\tau_{\text{split}}(\text{Re})}$$

The scale parameters are determined by matching the model to the characteristics of the experiment. First we choose our length scale to be as large as possible without losing the microscopic details, e.g. the length scale should be smaller than the pushing distance such that puff-pushing is resolved. Here we choose

$$\lambda = 12 D^*,$$

i.e. the spatial extent of a puff. We choose $p = 0.5$ and therefore our timescale follows from equation (S36),

$$\tau = \frac{p(1-p)}{2} \frac{\lambda^2}{L^*}. \quad (\text{S37})$$

Note that it is computationally advantageous to use as large as possible a diffusion constant, which would mean $p = 0.5$.

With this choice of parameters, a puff would at $\text{Re} = 2039$ diffuse a length ℓ_d between splitting or decaying events,

$$\ell_d = \lambda \sqrt{p \frac{\tau_d(\text{Re}_c)}{\tau}} = 4900\lambda = 19m \quad (\text{S38})$$

or $\Delta t = 5 \cdot 10^7 \tau = 1.2 \cdot 10^8 s$. At $\text{Re} = 2050$, the scales separating the splitting events are $\Delta t = 2.4 \cdot 10^7 \tau$ and $\ell_d = 1744\lambda$.

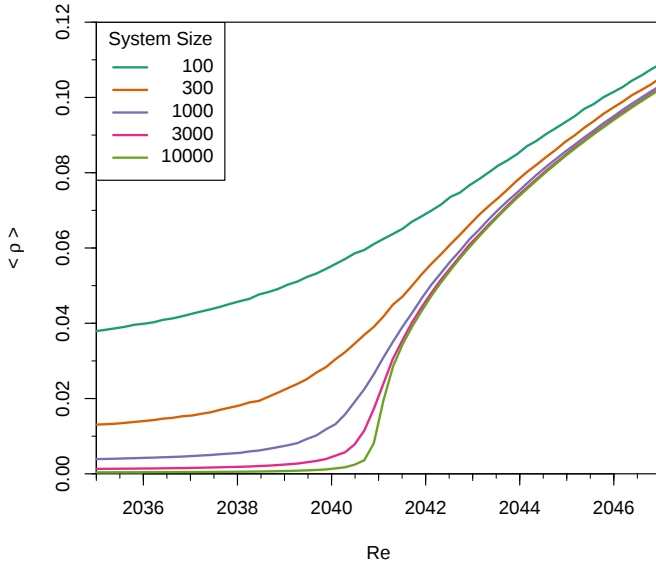


FIG. S6 Numerical simulations of the average density in the lattice model for different values of the system size L .

4. Finite-size analysis in the lattice model

It is well established that to overcome the difficulty of identifying the critical point and measuring critical exponents in simulations one should use finite-size scaling [18, 20]. In this section, we perform finite-size scaling analysis on the lattice model by adopting the procedure in the contact process model by Tomé and de Oliveira [21]. The idea is to remove the absorbing state by forbidding the last particle to decay, and therefore the behavior in the subcritical regime becomes continuous and scalable due to the finite-time effect. The density calculated in the numerical simulations as a function of Re is shown in Fig. S6. The scaling relation of the first and the second moment of the density can be written as

$$\langle \rho \rangle = L^{-\beta/\nu_\perp} F(\epsilon L^{1/\nu_\perp}), \quad (\text{S39})$$

$$\langle \rho^2 \rangle = L^{-2\beta/\nu_\perp} \tilde{F}(\epsilon L^{1/\nu_\perp}), \quad (\text{S40})$$

where $\epsilon = \text{Re} - \text{Re}_c$ and F and $\tilde{F}$ are universal functions. Therefore the reduced second-order cumulant

$$U = \frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle^2} \quad (\text{S41})$$

is independent of the system size L at the critical point which can be read off in Fig. S7. In the figure, we see that the reduced second-order cumulant starts to intersect at a unique Re for large enough values of Re , as expected by theory.

The stationary probability in the scaling form becomes

$$P(\rho, \epsilon, L) d\rho = F_P(\rho L^{\beta/\nu_\perp}, \epsilon L^{1/\nu_\perp}) L^{\beta/\nu_\perp} d\rho, \quad (\text{S42})$$

where the universal function F_P shows data collapse in the numerical simulations (Fig. S8). Therefore from the identified critical point, the critical exponent for the density in the lattice model is confirmed to be same as the $1 + 1$ DP as shown in Fig. S9.

We performed the numerical simulations on a periodic lattice of length L (see Fig. S10). In addition to the nearest-neighbor rules *propagation*, *decay* and *splitting* described above, additional rules are implemented modeling *suppression* and *pushing*. Specifically, in each time step all puffs on the lattice are submitted randomly to one of three actions, move, split and decay. After we have picked a given action, we execute it with a probability computed from predefined rates. The move shifts a puff on site i one lattice unit ahead at a rate p provided that the site $i + 1$ is vacant. A splitting event of a puff on site i introduces a new puff either at site $i + 1$ or site $i + 2$ with pre-defined rates ω_s and ω_{push} , respectively. The push is only possible if the sites $i + 1$ and $i + 2$ are vacant. Please note that we could instead have introduced the new puff on either of the sites $i - 1$ or $i - 2$ without changing quantitatively the dynamics. A puff on a site i decays with a rate ω_d if the site $i + 1$ is vacant. Otherwise it decays at a higher rate $\tilde{\omega}$. We then performed

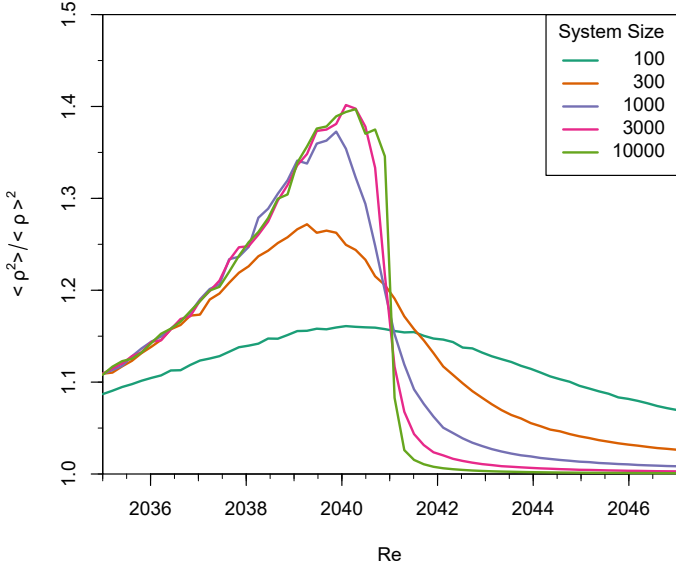


FIG. S7 The reduced second-order cumulant calculated in the numerical simulations of the lattice model as a function of Re and system size L . For sufficiently large L , the curves all intersect at a unique value of Re , thus identifying the actual critical Re that would occur in an infinite system. which is independent of the system size at the critical point. The first and second order moments were computed by averaging over 10^7 time steps after an initial transient of $5 \cdot 10^6$ time steps.

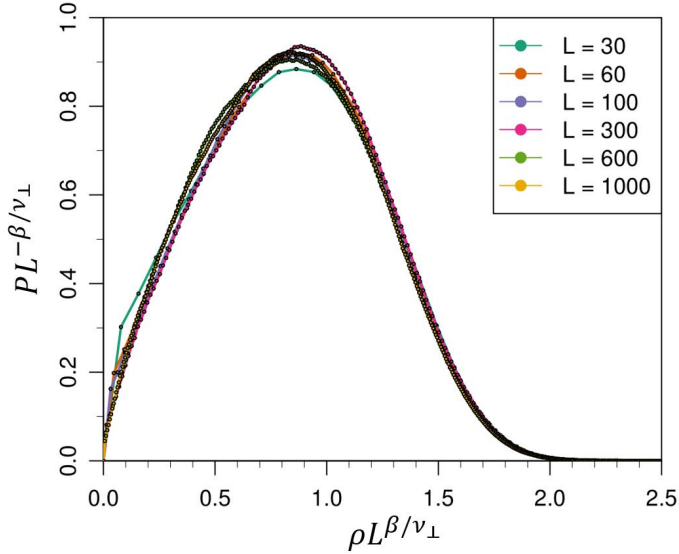


FIG. S8 Data collapse of rescaled probability density in the numerical simulations of the lattice model. The probability was estimated by sampling over 10^7 time steps and after an initial transient of $5 \cdot 10^6$ time steps

the simulations for different systems sizes and for a given number of time steps. We output the density of puffs on the lattice at regular time steps and check whether the system has collapsed into the absorbing state. We also take care to remove long-lived transients, and only take data once the transients have decayed. This required us

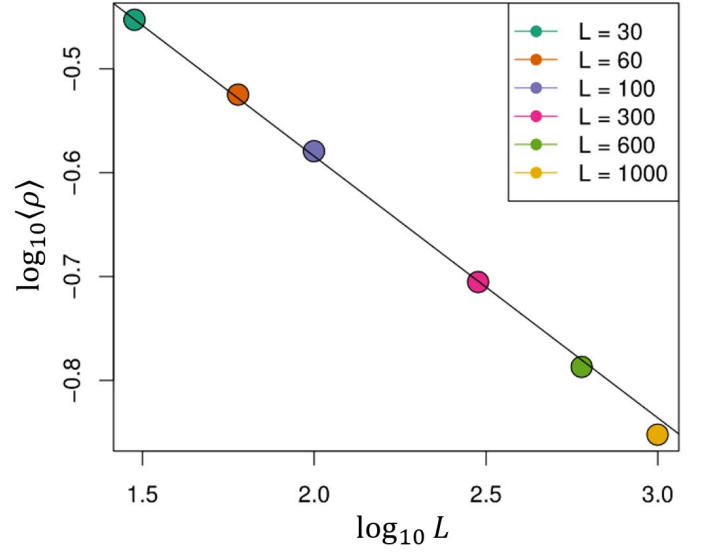


FIG. S9 The average density at the critical point in the numerical simulations of the lattice model. The fitted solid line has a slope of $-\beta/\nu_\perp$ with $\nu_\perp \sim 0.109$ which is consistent with the known critical exponents of $1+1$ DP. The average density was computed from the simulations presented in Fig. S8.

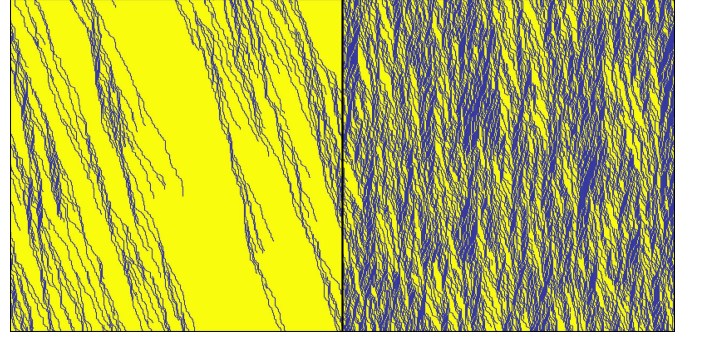


FIG. S10 Numerical simulations of the lattice model for systems respectively below (left panel) and above (right panel) the critical point. Clearly visible above the critical point are traffic jams of puffs, forming a crystalline phase, interspersed with regions of disordered puffs. As the transition is approached from above, the disordered eventually dominate, leading to the pure DP behavior at the critical point. In these simulations, $\omega_d = \tilde{\omega} = 0.01$, $p = \tilde{p} = 0.5$, and $\omega_{push} = \omega_s$ with $\omega_s = 0.0175$ in the left panel and $\omega_s = 0.0375$ in the right panel.

to run as much as 10^7 time steps before equilibration, when close to the transition.

The results of our simulations are summarized in Fig. S11. The turbulent fraction ρ varies with the splitting rate $(\omega_s - \omega_c)^\beta$ with $0.27 < \beta < 0.285$, consistent with the expectation from DP. We have verified that the puff dynamics is in a range where there are strong pushing and “traffic jam” effects.

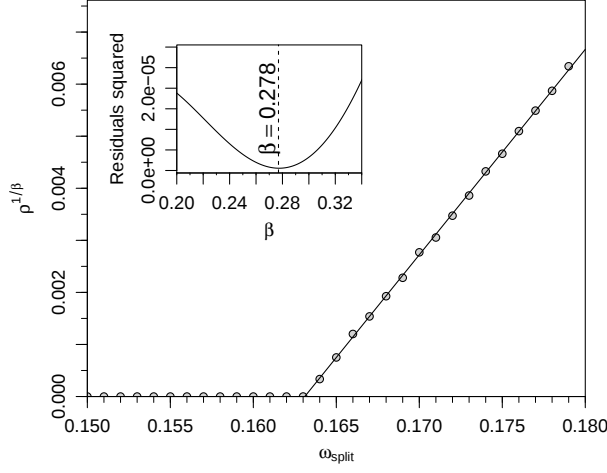


FIG. S11 Turbulent fraction ρ as a function of splitting rate, for a system size $L = 3200$ and averaged over 400 simulations and after a transient of $T = 10^7$ time steps. The vertical axis is the turbulent fraction $\rho^{1/\beta}$ where β has been estimated by a linear model whose residual is shown as function of β in the inset. The value of $\beta = 0.278$ used in the vertical axis varies slightly but has a sample variation in the range 0.270 to 0.285, which is consistent with the known value $\beta \approx 0.276$ for DP. In these simulations, $\omega_d = 0.04$, $\tilde{\omega} = 0.2$, $p = r = 0.5$, and $\omega_{push} = \omega_s$ as on the abscissa.

B. Continuum model of puff dynamics

In our *continuum model*, each puff is considered a point particle interacting with its closest neighbors, using the measured effective potential between puffs. We solve the equation of motion for each puff in the frame co-moving at v_∞

$$\frac{dx_i}{dt} = v_{\text{puff}}(x_i - x_{i-1}) + \sqrt{2D_{\text{puff}}}\xi_i(t) \quad (\text{S43})$$

where x_i is the streamwise coordinate of the i^{th} puff, $v_{\text{puff}}(x_i - x_{i-1})$ is the pushing speed of puff i as a result of puff $i - 1$ as measured in (S44), and $\xi_i(t)$ is a Gaussian white noise with mean zero and standard deviation 1, thus modeling the random walk of an isolated puff with an effective diffusion coefficient D_{puff} . In addition to equation (S43), each puff experiences decay and splitting with rates $\tau_{d,i} = \tau_d(\text{Re}, x_i - x_{i-1})$ and $\tau_{s,i} = \tau_s(\text{Re}, x_{i+1} - x_i)$ respectively, accounting for the presence of an upstream/downstream puff as measured in the experiment.

The pushing speed, v_{puff} , is well-described by an exponential decay:

$$v_{\text{puff}}(l) - v_\infty = A \exp(-l/l_c), \quad (\text{S44})$$

with $A = 0.22$, $l_c = 12$ and v_∞ is the travelling speed of an isolated puff.

1. Puff diffusion coefficient

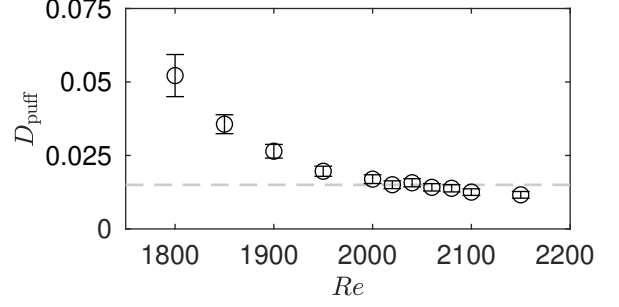


FIG. S12 Evolution of the effective diffusion coefficient versus Re . The dashed line indicates the choice made for the simulation of the continuous model. The diffusion coefficient is estimated from the evolution of the variance of the time of arrival distribution of 1000 puffs. Data are represented by their best fit $\pm$ their 95% confidence interval.

Measuring the evolution of the arrival time distribution at various location, we have been able to confirm that the puff propagation is purely diffusive. We could therefore measure an effective diffusion coefficient D_{puff} for various Re (Fig. S12). This diffusive coefficient is decreasing with Re before reaching a plateau for $\text{Re} \gtrsim 2000$. The higher value for low Re is understood in terms of puff stability. For $\text{Re} = 1800$, a puff will experience many near-relaminarization events before eventually relaminarizing. During such events, the puff energy will decrease substantially and its speed will increase accordingly. This translates into an increased diffusion coefficient. In the transitional range ($2000 < \text{Re} < 2100$), the puff diffusion coefficient is nearly constant around $D_{\text{puff}} = 0.015$ (dashed line in Fig. S12) and therefore, for simplicity, we have chosen not to use a Re -dependent diffusion coefficient in the model and used the value which is approximately constant for the range of Re of interest here.

2. Life and splitting times

A key feature of turbulent puff in pipe flow is their memoryless nature, ie. their life and splitting time statistics follow an exponential distribution. It is therefore possible for each Re to define a characteristic life and splitting time. Experiment [22] indicates that these times for the decay and splitting of an isolated puff depend on Re super exponentially, a result in agreement with theoretical expectations [23] (for a summary of recent work, see [24]). Here, we determine how the presence of a neighboring puff changes these statistics.

We measured puff lifetime and splitting time distributions for puff trains with different spacing and checked that each distribution remains exponential. Fig. S14.a shows the evolution of the life time (circle) and splitting time (square) with Re for different puff to puff dis-

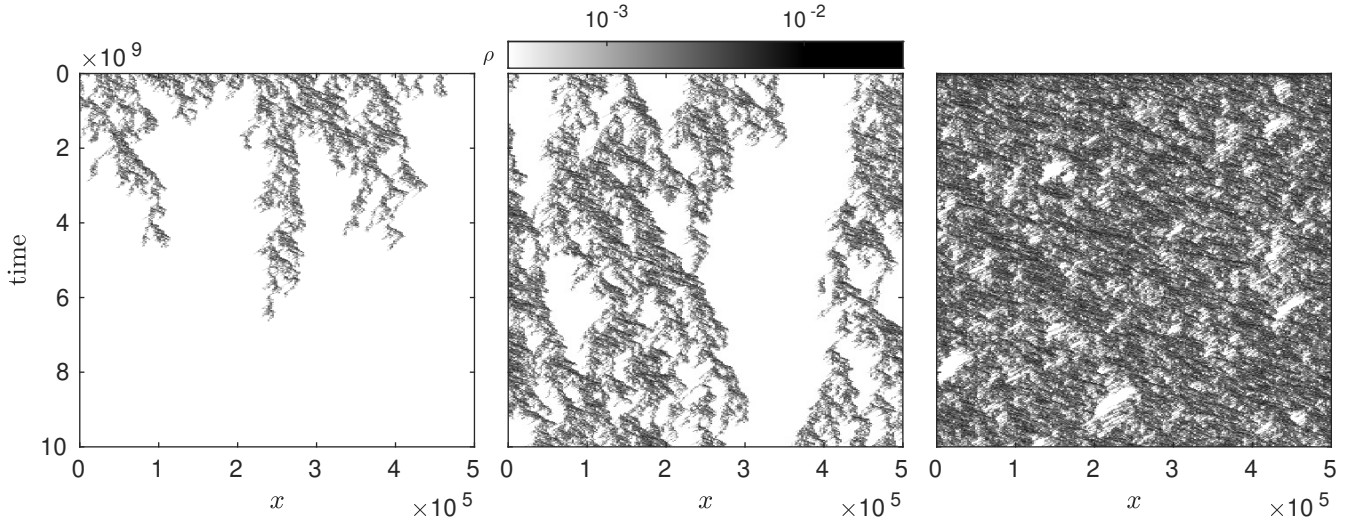


FIG. S13 Spacetime plots of the local puff density ρ at (from left to right) increasing Reynolds numbers for the continuous numerical model. Left panel shows subcritical behaviour ($\text{Re} = 2040.0$), while central panel is at $\text{Re} = 2040.22 \sim \text{Re}_c$ and right panel is above the critical point ($\text{Re} = 2040.6$).

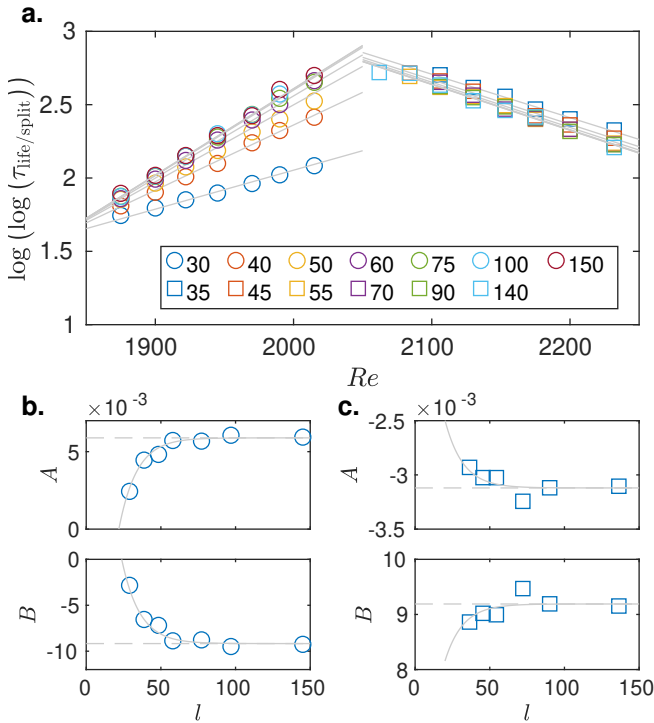


FIG. S14 a. Lifetime (circles) and splitting time (square) for various puff-to-puff distances l . Solid lines indicates a super-exponential fit to the data. **b.** Coefficient A and B of the super exponential fit $\log(\log(\tau)) = A \text{Re} + B$ for life times. **c.** Coefficient A and B of the same super exponential fit for splitting times.

tances, l . We note that for each l the Re dependence remains super exponential. For each spacing we fit the Re dependence as $\log(\log(\tau)) = A \text{Re} + B$. Fig. S14.b

and S14.c show the dependence of A and B with l for decay and splitting respectively. In both cases we decided to fit it with an exponentially decaying function $A - A_\infty \propto \exp(-l/l_c)$ where $l_c = 12$ was chosen to correspond to the characteristic distance identified in the pushing interaction, in order to limit the number of free parameters.

Eventually, the fit was done using a global non-linear fit on all data using the fitting function

$$\log(\log(\tau)) = (A_\infty + A \exp(-l/l_c)) \text{Re} + (B_\infty + B \exp(-l/l_c))$$

where $l_c = 12$ was imposed. The best fit gave the following values where superscript l , and s respectively stand for lifetime and splitting time.

$$\begin{aligned} A_{\infty,l} &= 5.898 \cdot 10^{-3} & A_{\infty,s} &= -3.165 \cdot 10^{-3} \\ B_{\infty,l} &= -9.187 & B_{\infty,s} &= 9.287 \\ A_l &= -3.687 \cdot 10^{-2} & A_s &= 4.714 \cdot 10^{-3} \\ B_l &= 67.45 & B_s &= -8.583 \end{aligned}$$

Representative spacetime plots of the puff configurations for increasing Re are shown in Fig. S13. The extreme sensitivity of the puff density to Re is evident, reflecting the fact that the critical region for the laminar-turbulent transition is very small.

The A_∞ and B_∞ coefficients are to be compared with those found in Avila *et al.* [22]. Although slightly different, they remain within the error bars and were the ones we used for building the model. This anecdotal and seemingly minor difference has actually far reaching consequences as it changes the Reynolds number $\text{Re}_\times$ where life and splitting times are equal. As a consequence, it dictates the value of the DP critical Reynolds number Re_c . This makes a direct comparison with experimental data extremely sensitive to the values of $\text{Re}_\times$ and explains the shift we had to impose in Fig. 3.a of

the main paper. Here, we should emphasize the difference between *accuracy* and *precision*. While it is easy to get reproducible experimental conditions when controlling temperature within a fraction of a degree and using the same experimental set-up, it is difficult to ensure absolute accuracy across different experiments. This has various causes such as absolute calibration of temperature sensors and accuracy of glass tubing. Once acknowledged, the agreement between the continuous model and the experimental turbulent fractions (measured in a different set-up) is excellent.

Finally, the choice of the fitting function is not unique and has no physical support other than being short range and therefore decaying exponentially. We have tested other fitting functions such as error functions but did not find any qualitative differences. Therefore the main outcome from the continuous model, the DP phase transition, is robust with respect to the functional form we choose to fit our experimental data.

3. Significance and details on errors estimation

Life time and splitting time for a given (Re, l) pair is estimated by fitting a Poisson distribution to the survivability/splitting distribution. The number of puffs used to build the distribution is given in tables I and II. The error is estimated by calculating the 95% confidence interval of the fit.

Re	l	τ_{life}	$\Delta\tau_{\text{life}}$	N_{puff}
1875	30	3.06e+02	1.91e+02	3900
1875	39	4.50e+02	9.12e+01	4800
1875	50	5.93e+02	5.40e+01	4500
1875	59	6.12e+02	4.87e+01	4500
1875	79	6.80e+02	5.00e+01	4600
1875	99	6.71e+02	1.44e+01	4600
1875	149	7.77e+02	2.68e+01	4600
1900	29	4.13e+02	4.65e+02	3300
1900	39	8.17e+02	2.19e+02	3600
1900	49	1.26e+03	8.91e+01	3500
1900	59	1.53e+03	1.24e+02	3400
1900	79	1.76e+03	1.22e+02	3200
1900	98	1.79e+03	1.72e+02	3000
1900	148	1.85e+03	9.98e+01	4400
1922	29	5.83e+02	7.70e+02	4400
1922	39	1.72e+03	2.23e+02	5100
1922	49	2.90e+03	5.45e+02	5100
1922	59	4.14e+03	4.15e+02	4800
1922	78	5.30e+03	4.03e+02	4600
1922	97	5.30e+03	4.48e+02	4900
1922	147	5.51e+03	3.19e+02	3900
1945	29	7.88e+02	8.83e+02	5000
1945	39	3.50e+03	4.55e+02	4200
1945	48	7.43e+03	2.09e+03	5100
1945	58	1.43e+04	3.55e+03	6000
1945	77	1.75e+04	2.18e+03	5000
1945	97	2.19e+04	2.97e+03	5000
1945	145	1.93e+04	9.59e+03	5600
1970	29	1.23e+03	8.95e+02	8300
1970	39	1.20e+04	3.66e+03	7200
1970	48	2.48e+04	7.09e+03	7000
1970	57	5.83e+04	1.27e+03	7000
1970	76	8.90e+04	6.55e+03	7000
1970	95	8.58e+04	9.98e+03	5000
1970	144	8.08e+04	7.94e+04	4900
1990	29	1.90e+03	8.38e+02	14600
1990	38	2.73e+04	8.20e+03	14300
1990	48	6.30e+04	3.33e+04	11000
1990	57	1.98e+05	5.81e+04	17000
1990	76	3.35e+05	1.52e+05	17000
1990	95	4.89e+05	1.64e+05	18000
1990	142	7.40e+05	6.20e+05	12200
2015	29	3.09e+03	7.87e+02	12600
2015	38	7.17e+04	2.18e+04	12100
2015	47	2.64e+05	2.11e+05	12000
2015	57	1.70e+06	5.36e+05	12000
2015	75	1.45e+06	1.20e+06	12000
2015	141	2.85e+06	2.39e+06	13000

TABLE I Details of the statistical analysis for measuring the life time of a puff. N_{puff} corresponds to the number of puffs generated to build the survivability distribution.

Re	l	τ_{split}	$\Delta\tau_{\text{split}}$	N_{puff}
2084	46	3.63e+06	1.33e+06	47000
2084	56	2.61e+06	1.10e+06	44000
2084	139	3.58e+06	3.01e+06	32000
2106	37	3.05e+06	1.49e+06	41000
2106	46	1.65e+06	1.11e+06	41000
2106	55	9.00e+05	1.73e+05	38000
2106	73	1.45e+06	9.93e+05	21000
2106	92	9.76e+05	3.41e+05	20000
2106	138	1.11e+06	2.33e+05	28000
2130	37	8.84e+05	2.66e+05	31000
2130	46	6.26e+05	2.94e+05	30000
2130	55	3.99e+05	2.90e+05	26000
2130	73	4.41e+05	1.68e+05	11000
2130	91	3.56e+05	1.57e+05	11000
2130	137	2.72e+05	1.31e+05	8000
2153	37	3.97e+05	1.62e+05	15000
2153	45	1.75e+05	6.11e+04	15000
2153	54	1.80e+05	5.82e+04	15000
2153	72	1.36e+05	2.36e+04	5000
2153	90	1.63e+05	1.56e+04	5000
2153	136	1.22e+05	6.73e+03	5000
2175	36	1.36e+05	5.56e+04	5000
2175	45	8.01e+04	3.52e+04	5000
2175	54	6.12e+04	6.58e+03	5000
2175	72	6.49e+04	2.03e+04	5000
2175	90	6.93e+04	1.16e+04	5000
2175	135	7.34e+04	2.50e+04	5000
2200	36	6.34e+04	9.86e+03	5000
2200	45	4.06e+04	6.26e+03	5000
2200	54	2.94e+04	5.03e+03	5000
2200	72	2.99e+04	9.44e+02	5000
2200	89	2.57e+04	4.12e+03	5000
2232	36	2.82e+04	1.33e+04	3000
2232	45	1.62e+04	4.34e+03	3000
2232	54	1.18e+04	1.57e+03	4900
2232	71	1.09e+04	1.12e+03	4900
2232	89	1.06e+04	1.02e+03	4900
2232	134	8.93e+03	3.68e+03	4900
2261	36	1.26e+04	4.08e+03	1000
2261	45	7.68e+03	4.16e+02	1000
2261	54	6.09e+03	1.59e+03	4900
2261	71	5.14e+03	5.12e+02	4800
2261	89	5.28e+03	9.39e+02	4900
2261	133	4.90e+03	8.57e+02	4900

TABLE II Details of the statistical analysis for measuring the splitting time of a puff. N_{puff} corresponds to the number of puffs generated to build the splitting distribution.

Re	ρ	ρ_{min}	ρ_{max}	Δ_t	N
2040.21	1.51e-04	9.72e-05	2.41e-04	5.0e+11	499998
2040.22	5.61e-04	4.94e-04	6.34e-04	5.0e+11	499998
2040.23	7.10e-04	6.32e-04	7.94e-04	3.9e+11	386701
2040.24	8.54e-04	6.39e-04	1.11e-03	2.5e+11	249999
2040.26	9.93e-04	8.21e-04	1.19e-03	2.0e+11	199998
2040.28	1.16e-03	9.70e-04	1.34e-03	2.0e+11	199998
2040.30	1.25e-03	1.09e-03	1.41e-03	5.0e+10	49999
2040.40	1.64e-03	1.52e-03	1.79e-03	3.5e+10	34877
2040.50	1.92e-03	1.81e-03	2.05e-03	1.5e+10	15316
2040.60	2.15e-03	2.05e-03	2.28e-03	9.3e+09	9324
2040.70	2.39e-03	2.29e-03	2.50e-03	7.4e+09	7422
2040.80	2.61e-03	2.51e-03	2.71e-03	5.0e+09	5018
2040.90	2.79e-03	2.68e-03	2.88e-03	3.4e+09	3414
2041.00	3.00e-03	2.94e-03	3.07e-03	1.7e+09	1692
2042.00	4.49e-03	4.41e-03	4.58e-03	1.3e+09	1279
2043.00	5.67e-03	5.60e-03	5.73e-03	1.2e+09	1165
2044.00	6.61e-03	6.53e-03	6.67e-03	1.0e+09	1031
2045.00	7.43e-03	7.36e-03	7.50e-03	9.3e+08	931
2046.00	8.12e-03	8.05e-03	8.17e-03	7.5e+08	752
2047.00	8.73e-03	8.68e-03	8.79e-03	6.7e+08	669
2048.00	9.26e-03	9.21e-03	9.33e-03	6.4e+08	644
2049.00	9.76e-03	9.72e-03	9.82e-03	6.1e+08	606
2050.00	1.02e-02	1.01e-02	1.03e-02	5.1e+08	514
2052.00	1.10e-02	1.09e-02	1.10e-02	4.6e+08	464
2055.00	1.20e-02	1.19e-02	1.20e-02	4.2e+08	420
2060.00	1.32e-02	1.32e-02	1.33e-02	3.7e+08	371
2065.00	1.42e-02	1.42e-02	1.42e-02	3.4e+08	340
2070.00	1.50e-02	1.50e-02	1.51e-02	3.5e+08	349
2080.00	1.63e-02	1.63e-02	1.64e-02	3.3e+08	328
2090.00	1.74e-02	1.74e-02	1.74e-02	2.8e+08	282
2100.00	1.82e-02	1.82e-02	1.83e-02	3.4e+08	342

TABLE III Details of the statistical analysis for measuring the equilibrium puff density ρ . The puff density is time averaged with Δ_t the duration of the interval and N the number of samples.

VI. DATA AND CODE AVAILABILITY STATEMENT

All experimental and computational data presented in this paper, the codes used to generate those data and the codes and scripts used to generate the figures are available online in the Zenodo repository 10.5281/zenodo.10308791 at <https://zenodo.org/doi/10.5281/zenodo.10308791>

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Correspondence to Björn Hof (bhof@ist.ac.at) and Nigel Goldenfeld (nigelg@ucsd.edu).

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