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Direct excitation of Kelvin waves on quantized vortices

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Effect of Particles Decorating Quantized Vortices on the Dispersion Relation of Kelvin Waves

Here, we discuss the dispersion relation of Kelvin waves when particles decorate the quantized vortex within the framework of the vortex filament model. Note that we do not address potential changes in the internal structure of quantized vortices caused by the decorating particles, as this discussion requires a thorough study of the particle-vortex interaction and is beyond the scope of the current manuscript. Under the vortex filament model, the dispersion relation of Kelvin waves can be derived from the equation of motion (Eq. (6)). Utilizing the local induction approximation, we obtain the following equation:

$$\frac{d\boldsymbol{s}}{dt} = \beta \boldsymbol{s}' \times \boldsymbol{s}'' - \alpha \boldsymbol{s}' \times (\beta \boldsymbol{s}' \times \boldsymbol{s}'') + \alpha' \boldsymbol{s}' \times [\boldsymbol{s}' \times (\beta \boldsymbol{s}' \times \boldsymbol{s}'')]\,,$$

where $\beta = \frac{\kappa k^2}{4\pi} \log(1/ka)$, and we assume $\boldsymbol{v}_n = 0$ for simplicity. Assuming a helical wave with a small amplitude $\epsilon(t)$ along the z-axis, we can write:

$$\boldsymbol{s} = (\epsilon(t)\cos\phi, \epsilon(t)\sin\phi, z),$$

where $\phi = kz - \omega t$. By neglecting higher order infinitesimals, we derive:

$$\begin{aligned}
\frac{d\boldsymbol{s}}{dt} &= \beta \boldsymbol{s}' \times \boldsymbol{s}'' + \alpha\beta \boldsymbol{s}'' - \alpha'\beta \boldsymbol{s}' \times \boldsymbol{s}'' \\
&= \beta(1-\alpha')\boldsymbol{s}' \times \boldsymbol{s}'' + \alpha\beta \boldsymbol{s}'' \\
&= (1-\alpha')\beta \begin{pmatrix} k^2\epsilon\sin\phi \\ -k^2\epsilon\cos\phi \\ 0 \end{pmatrix} + \alpha\beta \begin{pmatrix} -k^2\epsilon\cos\phi \\ -k^2\epsilon\sin\phi \\ 0 \end{pmatrix}.
\end{aligned}$$

We thus obtain the dispersion relation of the helical Kelvin wave as:

$$\omega = (1-\alpha')\beta k^2.$$

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The friction coefficient α' is known to be negligibly small (see Table 1 in [22]). We will now estimate the effective change in the friction coefficient due to the decorating nanoparticles, which are localized along the vortex core with an inter-particle distance of d. The force on the particles is described as Stokes drag in the Methods section of the main manuscript:

$$\boldsymbol{F}_S = -6\pi R\eta \boldsymbol{v}_L,$$

where the particle is assumed to travel with the velocity $\boldsymbol{v}_L$ of the vortex segment. Assuming d is sufficiently small, we consider the equation of motion for a vortex segment of length d. Then, the drag force is:

$$\boldsymbol{F}_D = (\gamma_0 \boldsymbol{s}' \times \boldsymbol{s}' \times \boldsymbol{v}_L - \gamma_0' \boldsymbol{s}' \times \boldsymbol{v}_L)\, d.$$

By neglecting the force along $\boldsymbol{s}'$, the first term becomes $-\gamma_0 \boldsymbol{v}_L d$, which can effectively absorb the Stokes drag:

$$-\gamma_0 \boldsymbol{v}_L d - 6\pi R\eta \boldsymbol{v}_L = -\gamma_{0,\mathrm{eff}} \boldsymbol{v}_L d = -(\gamma_0 + 6\pi R\eta/d)\boldsymbol{v}_L d.$$

We can thus calculate the effective friction coefficient α'_{eff} from $\gamma_{0,\mathrm{eff}}$ and γ_0' using Eq. (8) and (9). Although the exact value of d is unknown in the experiment, we can safely assume that it is similar to or larger than the spatial resolution of the experiment, approximately 20 µm, since we observed separated silicon nanoparticles in the experiment. Therefore, we estimate the upper bound of α'_{eff} to be approximately 0.02, and the dispersion relation remains virtually unchanged by the presence of decorating particles.

Discrepancy between experimental and theoretical dispersion relation of Kelvin waves

Here, we discuss possible origins for the discrepancy between the experimental and theoretical prefactor C in the dispersion relation. One plausible explanation is the observation angle relative to the quantized vortex. Given that our observation volume has a depth of 5 mm, a slightly inclined vortex remains observable. However, this inclination can lead to an underestimation of the wavelength of the excited Kelvin waves, corresponding to an overestimation of the wavenumber and resulting in a smaller prefactor than the theoretical one. Our three-dimensional reconstruction method could address this issue by appropriately modeling the deformed helical shape of the Kelvin waves. Another possible explanation involves

the alteration of the effective thickness of the quantized vortex cores due to the decorating nanoparticles. Assuming this effect primarily describes the discrepancy, we can estimate the effective thickness a_{eff} by solving the equation $0.56 = \log(1/ka_{\text{eff}})/4\pi$. The estimated value of $a_{\text{eff}} = 88$ nm is consistent with the size of the silicon nanoparticles used in the experiment. A further detailed discussion is necessary to explore the mechanisms behind the change in the effective thickness of the vortex core.