
Deterministic remote entanglement using a chiral quantum interconnect

In the format provided by the
authors and unedited

SUPPLEMENTARY INFORMATION

A. Device and Experimental Setup

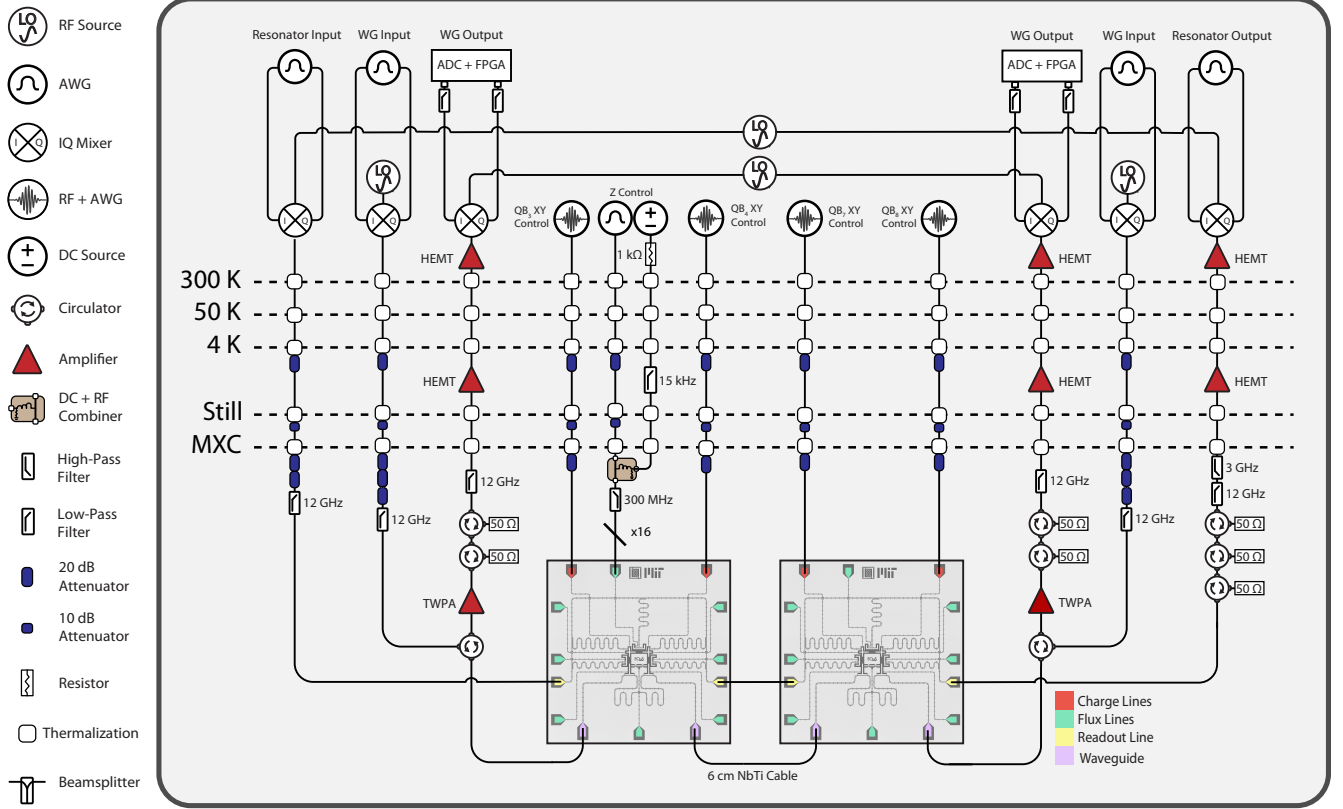


FIG. S1. **Experimental setup.** Wiring schematic of the device and all electronics used to perform the experiment. Note that only one flux line configuration is shown (green), but each qubit and coupler is coupled to a flux line with separate, but identical, control electronics.

This experiment was conducted in a Bluefors XLD1000 dilution refrigerator, which operates at base temperature of around 15 mK throughout the experiment. The experimental setup is shown in Fig. S1. The device is protected from ambient magnetic fields by superconducting and Cryoperm-10 shields below the mixing chamber (MXC) stage. Each end of the waveguide is connected to a microwave circulator for dual input-output operation. To minimize thermal noise from higher temperature stages, the inputs are attenuated by 20 dB at the 4K stage, 10 dB at the Still stage, and 60 dB (40 dB for resonator readout input) at the MXC stage. The output signals are filtered with 3 GHz high-pass and 12 GHz low-pass filters. Two additional isolators are placed after the circulator at the MXC stage to prevent noise from higher-temperature stages travelling back into the sample. Traveling wave parametric amplifiers (TWPA) are used at the MXC stage and high electron mobility transistor (HEMT) amplifiers are used at 4 K and room-temperature stages of the measurement chain to amplify the outputs from the device. The signals are then downconverted to an intermediate frequency using an IQ mixer, after which they are filtered, digitized, and demodulated. All qubits and tunable couplers are also equipped with their own flux bias lines. A DC + RF combiner is used for all flux lines to provide both static and dynamic control of the qubit/coupler frequencies. The DC and RF inputs are joined by a RF choke at the MXC stage before passing through a 300 MHz low-pass filter. The RF flux control lines are attenuated by 20 dB at the 4K stage, and by 10 dB at the 1K stage. The data qubits are equipped with local charge lines for independent single-qubit XY gates. The specific control and measurement equipment used throughout the experiment is summarized in Table S1. The relevant parameters of the device used in the experiment are summarized in Table S2.

Component	Manufacturer	Model
Dilution Refrigerator	Bluefors	XLD1000
RF Source	Rohde & Schwarz	SGS100A
DC Source	QDevil	QDAC
Control Chassis	Keysight	M9019A
AWG	Keysight	M3202A
ADC	Keysight	M3102A

TABLE S1. **Summary of control equipment.** The manufacturers and model numbers of experimental control equipment.

Parameter	Q ₁	Q ₂	Q ₃	Q ₄	Q ₅	Q ₆	Q ₇	Q ₈
Frequency (GHz)	5.0	5.0	4.9	4.67	5.0	5.0	4.9	4.67
$\gamma/2\pi$ (MHz)	17.7	17.3	-	-	17.9	17.1	-	-
T_1 (μ s)	-	-	7.9	4.4	-	-	8.1	6.0
T_2^* (μ s)	-	-	3.2	2.0	-	-	4.5	5.4

TABLE S2. **Summary of device parameters.** The operational qubit frequencies, qubit-waveguide coupling strengths γ , and T_1 and T_2^* of the data qubits are given for the waveguide (Q_{1,2,5,6}) and data qubits (Q_{3,4,7,8}) on both modules used throughout the experiment.

B. Error Budget Analysis

Loss Mechanisms	Measurement Type	Leftward Protocol	Rightward Protocol
Directionality Error	Field Amplitude	1.3%	1.4%
Missed Absorption	Field Amplitude	2.9%	5.7%
Total Coherent Loss		4.2%	7.1%
Propagation Loss	Scattering	18%	18%
Data-Qubit Decoherence	M.E. Simulation	6.8%	6.8%
Sideband Loss	Field Amplitude	1.5%	1.5%
Total Incoherent Loss		26.3%	26.3%
Total Photon Loss		30.5%	33.4%

TABLE S3. **Summary of losses in the directional emission and absorption protocol.**

Loss Mechanisms	Measurement Type	Leftward Protocol	Rightward Protocol
Propagation Loss	Field Amplitude	20%	23%
Q ₃ and Q ₇ decay	Field Measurement	2.6%	1.4%
Q ₃ and Q ₇ decay	M.E. Simulation	2.9%	2.9%

TABLE S4. **Alternate incoherent loss assessments of the directional emission and absorption protocol.**

We quantify coherent protocol losses with the measured photon-field amplitudes at both ends of the waveguide following separate emission and absorption experiments. For example, in the case of leftward emission (module B $\rightarrow$ module A), to estimate photon loss caused by imperfect directionality, we emit photons from module B in the rightward direction and calculate the integrated coherent power $P_{R,E} = \int_W |\langle \hat{a}_{R,E}(\omega) \rangle|^2 d\omega$ to calibrate the gain of the rightward detector in the interval $W \in [\omega_{ph} - \delta\omega, \omega_{ph} + \delta\omega]$, where $\omega_{ph}/2\pi = 5.0$ GHz and $\delta\omega/2\pi = 40$ MHz. We compare this to the integrated power of the photon field on the same detector following the absorption experiment $P_{R,A} = \int_W |\langle \hat{a}_{R,A}(\omega) \rangle|^2 d\omega$. We calculate an upper bound for coherent photon loss due to directionality error of $P_{R,A}/P_{R,E} = 1.3\%$ for the leftward protocol. The directionality error of the rightward protocol (module A $\rightarrow$ module B) is 1.4%.

Similarly, the protocol is subject to another form of coherent loss, which we call missed absorption. For leftward emission, this error is characterized by the integrated power measured by the leftward detector after absorption, $P_{L,A} = \int_W |\langle \hat{a}_{L,A}(\omega) \rangle|^2 d\omega$. To calculate the missed absorption loss, we emit photons in the leftward direction and detune the absorber module, placing it in transparency mode. We measure the integrated power on the leftward detector $P_{L,E} = \int_W |\langle \hat{a}_{L,E}(\omega) \rangle|^2 d\omega$ to calculate the missed absorption loss $P_{L,A}/P_{L,E} = 2.9\%$. Likewise, the missed absorption loss resulting from the rightward protocol is 5.7%. In summary, the total coherent loss to the waveguide

is 4.2% in the leftward direction and 7.1% in the rightward direction.

With photon-field measurements, we also estimate incoherent photon loss $\zeta_{L/R}$ along the waveguide in between the modules. Though we use a superconducting NbTi coaxial cable between microwave packages, the printed-circuit-board traces are made of gold-plated copper, with additional losses introduced by the ceramic dielectric. Similarly, the K connectors on the packages are made of gold-plated beryllium copper. To measure ζ_L , we emit photons from both modules individually in the leftward direction and calculate the ratios of their integrated powers measured at the leftward detector. We measure $\zeta_L = 20\%$ and $\zeta_R = 18\%$, noting that we are not accounting for directionality error. Similarly, assuming reciprocity $\zeta = \zeta_L = \zeta_R$, we estimate the loss between the waveguide $\zeta = 18\%$ (-0.86 dB) with single-qubit resonant scattering measurements as a function of probe power.

Using master-equation simulations, we estimate data-qubit dephasing and lifetime contribute roughly 6.8% to incoherent photon loss. We estimate that the data-qubit lifetimes are limited by direct decay to the waveguide. To illustrate this, in the field amplitude measurement following the absorption experiment, we observe a signal component at 4.9 GHz, which is the frequency of the data qubits (Q_3 and Q_7) closest to the photon frequency of $\omega_{ph}/2\pi = 5.0$ GHz. From the integrated power at the data qubit frequency, we calculate a photon loss of 2.6% (1.4%) during the leftward (rightward) emission experiment. For comparison, we find the lifetimes of these two data qubits exclusively contribute 2.9% in photon loss in simulation.

In addition, the parametric pulses used to release the photon to the waveguide drive higher-order sideband transitions, which also appear in the field-amplitude signals. The integrated power loss at the frequency of the visible sideband at 5.1 GHz is 1.5% in both directions. We estimate a total incoherent photon loss of 26.3%, including contributions from propagation loss, data-qubit decoherence, and sideband loss.

In total, we account for 30.5% (33.4%) photon loss in the leftward (rightward) direction. These known sources of loss are summarized in Table S3. We expect sideband transitions at frequencies outside our digitization bandwidth cause additional photon loss at the single-percent level. The remaining error may be due to modulation-induced error; the scaling of microwave-induced errors in superconducting circuits with drive amplitude, such as leakage to higher levels, is not completely understood [60, 61]. We estimate that by improving the circuit design for optimal data qubit coherence, improving the 50- Ω match of the waveguide, and mitigating photon loss, we can achieve absorption efficiencies greater than 90%.

C. Single-Qubit Scattering – Waveguide Loss Measurement

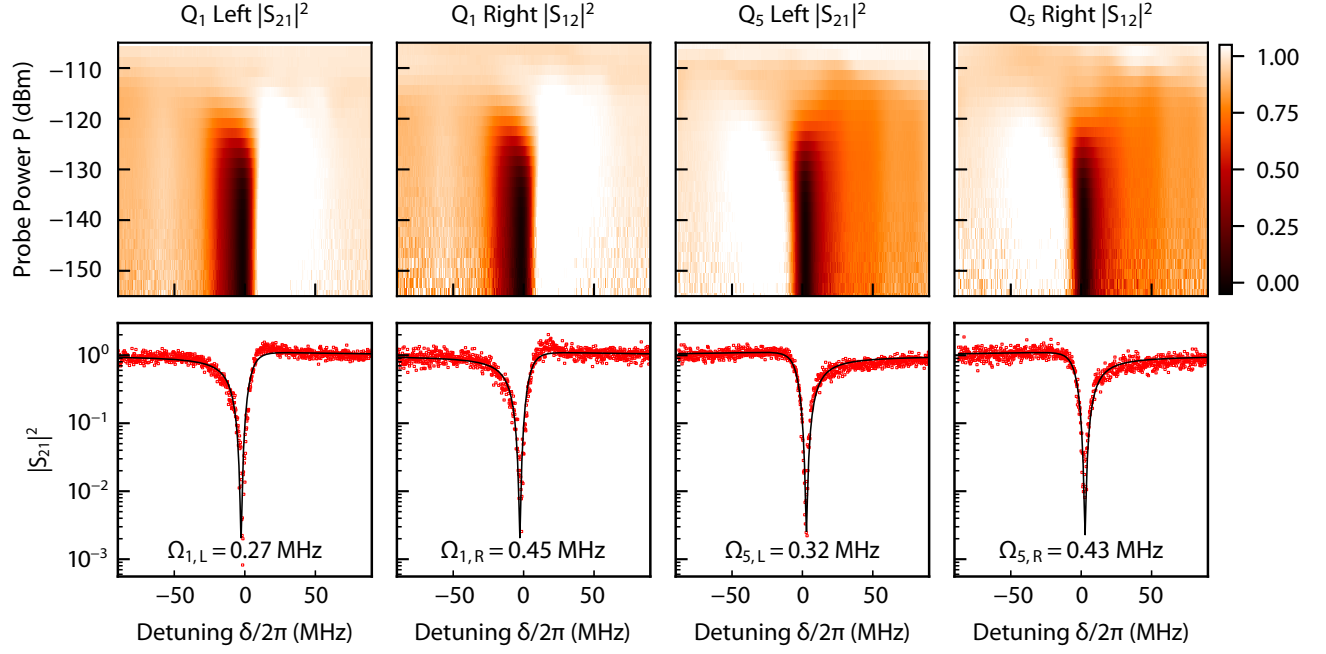


FIG. S2. **Waveguide-qubit spectroscopy.** Magnitude of the transmission spectrum of a coherent probe incident on a qubit through the waveguide as a function of the qubit-probe detuning $\delta/2\pi$ and the probe power P . The measurement is repeated separately on Q₁ and Q₅ individually, which belong to different modules, with a probe propagating in the leftward or rightward direction. We fit the 2D scans to Eq. S3 to extract the qubit-waveguide coupling rate $\gamma/2\pi \approx 17$ MHz and the drive amplitudes $\Omega_{1,R/L}$ and $\Omega_{5,R/L}$ which we define at the lowest input powers in both directions. We show line cuts of the 2D scan with the Lorentzian fits at the lowest probe powers. We measure the propagation loss $\zeta \approx 18\%$ between the modules along the cable.

To fully characterize the emission and absorption protocol, we must determine the loss between the modules housed in separate microwave packaging. One method relies on elastic scattering measurements, which enable absolute power calibration as well as the extraction of qubit-waveguide coupling and decoherence rates.

We send a coherent input tone through the waveguide which couples to a single waveguide qubit with strength γ . For low probe powers with average photon numbers $\ll 1$, the qubit acts as a mirror to a single photon in the waveguide at a time [62–64]. The master equation for driven qubit-waveguide system is [65]

$$\partial_t \hat{\rho} = -i[\hat{H}, \hat{\rho}] + \gamma D[\hat{\sigma}^-] \hat{\rho} + \frac{\gamma_\phi}{2} D[\hat{\sigma}_z] \hat{\rho}. \quad (\text{S1})$$

The single-qubit Hamiltonian is $\hat{H} = \frac{1}{2}\delta\hat{\sigma}_z + \frac{1}{2}\Omega_p\hat{\sigma}_x$, γ_ϕ is the pure dephasing rate, $\delta = \omega - \omega_p$ is the qubit-probe detuning, and $\Omega_p = \sqrt{2\gamma P/\hbar\omega_p}$ is the drive strength of the probe with power P . With a rightward-propagating probe input, the output from the right end of the waveguide is

$$\hat{a}_R = \hat{a}_R^{\text{in}} + \sqrt{\frac{\gamma}{2}}\hat{\sigma}^-. \quad (\text{S2})$$

Therefore, the transmission amplitude $S_{21} = \langle \hat{a}_R \rangle / \langle \hat{a}_R^{\text{in}} \rangle$ is [65]

$$S_{21}(\delta, \Omega_p) = 1 - \frac{\gamma(1 - i\frac{\delta}{\gamma_2})}{2\gamma_2 \left(1 + \left(\frac{\delta}{\gamma_2} \right)^2 + \frac{\Omega_p^2}{\gamma_2} \right)}, \quad (\text{S3})$$

where $\gamma_2 = \gamma/2 + \gamma_\phi$ is the total decoherence rate of the qubit. Transmission measurements as a function of probe power P and detuning δ , as shown in Fig. S2 give $\gamma/2\pi \approx 17$ MHz. Given that we use highly asymmetric

junctions through a $2-J_c$ fabrication process, the pure dephasing rates γ_ϕ are negligible. We also extract the absolute power of microwave tones incident on the waveguide qubits.

To characterize the attenuation η^2 between the modules—the probability a photon travels from one module to another—we compare the power calibration for two corresponding waveguide qubits Q_1 and Q_5 in different packages. We measure transmission as a function of probe power P for each qubit in both propagation directions for a total of 4 scans to extract $\Omega_{1,R/L}$, $\Omega_{5,R/L}$, $P_{1,R/L}$, and $P_{5,R/L}$. Assuming $P_{1,R} = \eta^2 P_{5,R}$, and $P_{1,L} = \eta^2 P_{5,L}$,

$$\frac{\Omega_{1,R}^2}{\Omega_{5,R}^2} = \frac{\gamma_1 P_{1,R} \omega_5}{\gamma_5 P_{5,R} \omega_1} = \frac{\gamma_1 \omega_5}{\eta^2 \gamma_5 \omega_1}, \quad (\text{S4})$$

$$\frac{\Omega_{5,L}^2}{\Omega_{1,L}^2} = \frac{\gamma_5 P_{5,L} \omega_1}{\gamma_1 P_{1,L} \omega_5} = \frac{\gamma_5 \omega_1}{\eta^2 \gamma_1 \omega_5}. \quad (\text{S5})$$

Combining these expressions, we find an analytical expression for attenuation between qubits that depends only upon the drive strength of the probe from four independent measurements,

$$\eta^2 = \frac{\Omega_{1,L} \Omega_{5,R}}{\Omega_{1,R} \Omega_{5,L}}. \quad (\text{S6})$$

In our system, the classical propagation and scattering loss between the qubits on different modules is $\zeta = 1 - \eta^2 = 18\%$. This is the most significant contribution to overall photon loss during the emission and absorption protocol.

D. System Model

1. Chiral Waveguide Quantum Electrodynamics (wQED)

To formulate a master equation model for wQED, we assume that the waveguide constitutes a Markovian environment—i.e. the waveguide hosts a 1D photonic continuum of time-independent modes. This approximation holds in the regime where the bandwidth of the continuum is much larger than the bandwidth of the emitted photons, which allows us to assume instantaneous interactions between atoms and the continuum of modes. This is valid for our system, since the inter-module separation is much smaller than the emitted photon bandwidth ($d \approx 10 \text{ cm} \ll 2\pi\nu/\gamma_{\text{ph}} \approx 17 \text{ m}$, where ν is the speed of light in the waveguide and $\gamma_{\text{ph}}/2\pi = 7 \text{ MHz}$ is the photon linewidth). The photon travel time between modules ($\Delta t = d/\nu = 0.84 \text{ ns}$) is negligible compared to the temporal extent of the photon wavepacket ($2\pi/\gamma_{\text{ph}} = 90 \text{ ns}$). Therefore, the waveguide qubits on different modules interact through the waveguide modes simultaneously.

By taking the Markov approximation, the master equation for a general system of resonant waveguide qubits, each coupled bidirectionally to a common waveguide at a total rate γ , can be written in the waveguide-qubit frame as [66]

$$\dot{\hat{\rho}} = -i \left[\frac{\gamma}{4} \sum_{j,l} \sin(k|x_j - x_l|) (\hat{\sigma}_j^\dagger \hat{\sigma}_l + \hat{\sigma}_j \hat{\sigma}_l^\dagger), \hat{\rho} \right] + \gamma \sum_{j,l} \cos(k|x_j - x_l|) \mathcal{D}[\hat{\sigma}_j, \hat{\sigma}_l] \hat{\rho}. \quad (\text{S7})$$

where x indicates the position of the atom along the waveguide. The sinusoidal Hamiltonian term governs the waveguide-mediated coherent exchange interactions while the cosinusoidal Lindbladian term (when $j \neq l$) describes the correlated decay effects.

Alternatively, the chiral wQED formalism considers the left- and right-propagating modes in the waveguide independently. The chiral master equation is written as

$$\dot{\hat{\rho}} = -\frac{i}{\hbar} [\hat{H}_L + \hat{H}_R, \hat{\rho}] + \gamma_L \mathcal{D}[\hat{c}_L] \hat{\rho} + \gamma_R \mathcal{D}[\hat{c}_R] \hat{\rho}, \quad (\text{S8})$$

where γ_L (γ_R) describes the rate of emission of the atoms into the leftwards (rightwards) propagating modes of the waveguide. $\hat{c}_L = \sum_j e^{ikx_j} \hat{\sigma}_j$ and $\hat{c}_R = \sum_j e^{-ikx_j} \hat{\sigma}_j$ are the leftwards and rightwards collective jump operators for the system. The coherent waveguide-mediated interactions are described by the terms

$$\hat{H}_L = -\frac{i\hbar\gamma_L}{2} \sum_{j < l} (e^{ik|x_l - x_j|} \hat{\sigma}_j^\dagger \hat{\sigma}_l - \text{h.c.}) \quad (\text{S9})$$

$$\hat{H}_R = -\frac{i\hbar\gamma_R}{2} \sum_{j < l} (e^{ik|x_l - x_j|} \hat{\sigma}_l^\dagger \hat{\sigma}_j - \text{h.c.}). \quad (\text{S10})$$

In the bidirectional limit, where $\gamma_L = \gamma_R$, the leftwards and rightwards phase factors interact to give rise to the sinusoidal distance-dependent waveguide-mediated interaction and correlated decay terms, reducing the chiral master equation reduces to S7. In the opposite limit, where each atom is perfectly chiral, either γ_L or γ_R is equal to zero. When $\gamma_R = 0$, the master equation reduces to

$$\dot{\hat{\rho}} = -\frac{i}{\hbar} [\hat{H}_L, \hat{\rho}] + \gamma_L \mathcal{D}[\hat{c}_L] \hat{\rho}. \quad (\text{S11})$$

In this regime, the interactions of an atom with the waveguide modes and the interactions between atoms are unidirectional; this system is often referred to as *cascaded*. Since only the leftwards phase factor (e^{+ikx}) remains, there are no distance-dependent dynamics and the phase is trivial. While in the bidirectional case, only the distance between the atoms, and not the ordering, matters; in the cascaded case, the ordering is critical, since the output of each atom can only drive other atoms downstream without back-action. This non-Hermitian effect can only be obtained from an open system.

In our system, each qubit is coupled bidirectionally to the waveguide. However, we engineer the interactions within a pair of waveguide qubits so that each module behaves as a perfectly chiral atom.

2. Chiral Scattering

To better understand how our modules function as perfectly chiral atoms, we can investigate the scattering response of the two-waveguide-qubit system. This system is made up of two resonant qubits coupled bidirectionally to a common waveguide at a total rate γ ($\gamma_R = \gamma_L = \gamma/2$). The waveguide qubits are positioned a distance of $\lambda/4$ apart, where λ is the wavelength corresponding to the qubits' frequency. The propagation phase between the two waveguide qubits is $\phi = k\Delta x = \frac{\pi}{2}$. At this distance, the correlated decay term is equal to zero and the waveguide-mediated interaction is equal to $\gamma/2$, which we cancel with a tunable coupler.

We define leftwards and rightwards collective decay operators for this system, $\hat{c}_L = \hat{\sigma}_1^- + e^{i\frac{\pi}{2}}\hat{\sigma}_2^-$ and $\hat{c}_R = \hat{\sigma}_1^- + e^{-i\frac{\pi}{2}}\hat{\sigma}_2^-$, and write the leftwards-driven system Hamiltonian,

$$\hat{H} = -\frac{\delta}{2}(\hat{\sigma}_z^1 + \hat{\sigma}_z^2) + ia_{\text{in}}\sqrt{\frac{\gamma}{2}}[(\hat{\sigma}_1^- - \hat{\sigma}_1^+) + (e^{i\frac{\pi}{2}}\hat{\sigma}_2^- - e^{-i\frac{\pi}{2}}\hat{\sigma}_2^+)] \quad (\text{S12})$$

$$= -\frac{\delta}{2}(\hat{\sigma}_z^1 + \hat{\sigma}_z^2) + ia_{\text{in}}\sqrt{\frac{\gamma}{2}}(\hat{c}_L - \hat{c}_L^\dagger), \quad (\text{S13})$$

and the input-output equations as

$$\begin{aligned} \hat{a}_L^{\text{out}} &= \hat{a}_L^{\text{in}} + \sqrt{\frac{\gamma}{2}}\hat{c}_L, \\ \hat{a}_R^{\text{out}} &= \sqrt{\frac{\gamma}{2}}\hat{c}_R, \end{aligned} \quad (\text{S14})$$

where $\delta = \omega_q - \omega_p$ and $\langle \hat{a}_L^{\text{in}} \rangle = i\sqrt{\frac{P}{\hbar\omega}}$ is the drive amplitude, which must be in the low power regime with $\langle \hat{a}_L^{\text{in}} \rangle \ll 1$ to avoid saturating the qubit. Using the Heisenberg equations of motion $\dot{\hat{\sigma}}_j^- = -i[\hat{\sigma}_j^-, \hat{H}] - (\gamma + \Gamma_{\text{nr}})\hat{\sigma}_j^-$, where Γ_{nr} is the non-radiative decay rate, we take the steady state approximation $\dot{\hat{\sigma}}_j^- = 0$ to find

$$\langle \hat{\sigma}_1^- \rangle = \frac{-\langle \hat{a}_L^{\text{in}} \rangle \sqrt{\frac{\gamma}{2}}}{i\delta + (\gamma + \Gamma_{\text{nr}})/2}, \quad (\text{S15})$$

$$\langle \hat{\sigma}_2^- \rangle = \frac{i\langle \hat{a}_L^{\text{in}} \rangle \sqrt{\frac{\gamma}{2}}}{i\delta + (\gamma + \Gamma_{\text{nr}})/2}. \quad (\text{S16})$$

Plugging these results into S14, we obtain

$$t = \frac{\langle \hat{a}_L^{\text{out}} \rangle}{\langle \hat{a}_L^{\text{in}} \rangle} = 1 - \frac{\gamma}{i\delta + (\gamma + \Gamma_{\text{nr}})/2}, \quad (\text{S17})$$

$$r = \frac{\langle \hat{a}_R^{\text{out}} \rangle}{\langle \hat{a}_L^{\text{in}} \rangle} = 0. \quad (\text{S18})$$

Comparing these results to the scattering response of a leftwards driven general chiral atom [67],

$$t = \frac{\langle \hat{a}_L^{\text{out}} \rangle}{\langle \hat{a}_L^{\text{in}} \rangle} = 1 - \frac{\gamma_L}{i\delta + \Gamma_{\text{tot}}/2}, \quad (\text{S19})$$

$$r = \frac{\langle \hat{a}_R^{\text{out}} \rangle}{\langle \hat{a}_L^{\text{in}} \rangle} = \frac{-\sqrt{\gamma_R\gamma_L}}{i\delta + \Gamma_{\text{tot}}/2}, \quad (\text{S20})$$

we see that our system has a scattering response identical to that of a perfectly cascaded atom—the effective γ_L of the module is equal to γ , while the effective γ_R is equal to zero. The cascaded scattering response for the lossless system is characterized by unity transmission with a phase change of 2π and zero reflection. If we were to drive the system to the right instead, we would find the opposite response; the effective γ_L would be zero, while the effective γ_R would be equal to γ .

Furthermore, note that the rightwards emitting state, $|\psi_R\rangle = \frac{1}{\sqrt{2}}(|10\rangle + e^{i\frac{\pi}{2}}|01\rangle)$, is in the null space of the leftwards collective decay operator, $\hat{c}_L|\psi_R\rangle = 0$. Therefore, this state can be considered to be subradiant in the leftwards direction, since destructive interference inhibits decay into the ground state via the waveguide. Similarly, the leftwards emitting state, $|\psi_L\rangle = \frac{1}{\sqrt{2}}(|10\rangle + e^{-i\frac{\pi}{2}}|01\rangle)$, is in the null space of the rightwards collective decay operator, $\hat{c}_R|\psi_L\rangle = 0$.

We can understand the two-waveguide-qubit system as *selectively perfectly chiral*: if we drive to the left (right) or prepare the system in the state $|\psi_L\rangle$ ($|\psi_R\rangle$), the system behaves as a single atom with perfect chiral coupling to the leftward (rightward) modes in the waveguide. However, this behavior is only valid in the single-photon regime, unless we engineer a large cross-Kerr interaction between the two waveguide qubits in order to enforce a single-photon limit on the module.

3. Cascaded Emission and Absorption Model

To simulate emission and absorption, we construct a master equation model for the entire eight qubit system. We first consider the four waveguide qubits. Assuming that all qubits are resonant, the master equation of this sub-system in the qubit frame is

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}_J, \hat{\rho}] + \gamma \left[\sum_{j \in \{1,2,5,6\}} \mathcal{D}[\hat{\sigma}_j^-] \hat{\rho} + \cos(kd)(\mathcal{D}[\hat{\sigma}_1^-, \hat{\sigma}_5^-] \hat{\rho} + \mathcal{D}[\hat{\sigma}_2^-, \hat{\sigma}_6^-] \hat{\rho} + \text{h.c.}) \right. \\ \left. + \cos(kd + \frac{\pi}{2})(\mathcal{D}[\hat{\sigma}_1^-, \hat{\sigma}_6^-] \hat{\rho} + \text{h.c.}) + \cos(kd - \frac{\pi}{2})(\mathcal{D}[\hat{\sigma}_2^-, \hat{\sigma}_5^-] \hat{\rho} + \text{h.c.}) \right]. \quad (\text{S21})$$

The waveguide-mediated interaction term $\hat{H}_J$ is

$$\hat{H}_J = \frac{\gamma}{2} \left(\sin(kd)(\hat{\sigma}_1^+ \hat{\sigma}_5^- + \hat{\sigma}_2^+ \hat{\sigma}_6^- + \text{h.c.}) + \sin(kd + \frac{\pi}{2})(\hat{\sigma}_1^+ \hat{\sigma}_6^- + \text{h.c.}) + \sin(kd - \frac{\pi}{2})(\hat{\sigma}_2^+ \hat{\sigma}_5^- + \text{h.c.}) \right). \quad (\text{S22})$$

Waveguide-mediated interactions and correlated decay only occur between qubits on separate modules, since we cancel intra-module qubit coupling and there is zero correlated decay between qubits spaced $\lambda/4$ apart on the same module. To better understand how the four-waveguide-qubit system leads to cascaded interactions, we can instead treat each module as a unit, defining leftwards and rightwards collective jump operators for modules A and B

$$\begin{aligned} \hat{c}_{L,A}^- &= \hat{\sigma}_1^- + e^{i\frac{\pi}{2}} \hat{\sigma}_2^- & \hat{c}_{R,A}^- &= \hat{\sigma}_1^- + e^{-i\frac{\pi}{2}} \hat{\sigma}_2^-, \\ \hat{c}_{L,B}^- &= \hat{\sigma}_5^- + e^{i\frac{\pi}{2}} \hat{\sigma}_6^- & \hat{c}_{R,B}^- &= \hat{\sigma}_5^- + e^{-i\frac{\pi}{2}} \hat{\sigma}_6^-. \end{aligned} \quad (\text{S23})$$

This allows us to rewrite S21 in a manner that mimics the chiral wQED formalism

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}_L + \hat{H}_R, \hat{\rho}] + \frac{\gamma}{2} \left(\mathcal{D}[\hat{c}_{L,A}^- + e^{ikd} \hat{c}_{L,B}^-] \hat{\rho} + \mathcal{D}[\hat{c}_{R,A}^- + e^{-ikd} \hat{c}_{R,B}^-] \hat{\rho} \right), \quad (\text{S24})$$

where

$$\begin{aligned} \hat{H}_L &= \frac{-i\gamma}{4} (e^{ikd} \hat{c}_{L,A}^+ \hat{c}_{L,B}^- - e^{-ikd} \hat{c}_{L,A}^- \hat{c}_{L,B}^+) \\ \hat{H}_R &= \frac{-i\gamma}{4} (e^{ikd} \hat{c}_{R,A}^- \hat{c}_{R,B}^+ - e^{-ikd} \hat{c}_{R,A}^+ \hat{c}_{R,B}^-). \end{aligned} \quad (\text{S25})$$

Because $\hat{c}_{L,A}|\psi_{R,A}\rangle = 0$, if we prepare module A in the rightwards-emitting state, the leftwards contributions to the system dynamics are nullified, and the system behaves in a rightwards-cascaded manner. A photon is emitted from module A and absorbed by module B. In this cascaded process, the distance between the two modules is irrelevant to the dynamics and only manifests as a phase in the absorbed state. The reverse process occurs if we instead prepare module B in the leftwards-emitting state.

To account for propagation loss between the modules, we follow the approach in [68, 69] by introducing a fictitious beamsplitter between the modules that allows a photon to propagate in either direction with probability η^2 . We

define SLH triplets [70, 71]

$$G_{R(L),A(B)} = \left(I, \sqrt{\frac{\gamma}{2}} \begin{pmatrix} \hat{c}_{R(L),A(B)}^- \\ 0 \end{pmatrix}, 0 \right), \quad (S26)$$

$$G_{BS} = \left(\begin{pmatrix} \frac{\eta}{\sqrt{1-\eta^2}} & -\sqrt{1-\eta^2} \\ \sqrt{1-\eta^2} & \eta \end{pmatrix}, 0, 0 \right), \quad (S27)$$

$$G_{\phi_{WG}} = \left(e^{ikd}, 0, 0 \right), \quad (S28)$$

where $G_{R(L),A(B)}$ is the rightwards (leftwards) triplet for module A (B), G_{BS} is the beamsplitter triplet, and $G_{\phi_{WG}}$ is the triplet describing the acquired propagation phase between the modules. We solve the rightwards cascaded system using the series product [70]

$$G_1 = G_{R,B} \triangleleft G_{\phi_{WG}} \triangleleft G_{BS} \triangleleft G_{R,A} = \left(e^{ikd} \begin{pmatrix} \frac{\eta}{\sqrt{1-\eta^2}} & -\sqrt{1-\eta^2} \\ \sqrt{1-\eta^2} & \eta \end{pmatrix}, \begin{pmatrix} \sqrt{\frac{\gamma}{2}}(\eta \hat{c}_{R,A}^- + e^{-ikd} \hat{c}_{R,B}^-) \\ \sqrt{\frac{\gamma}{2}}\sqrt{1-\eta^2} \hat{c}_{R,A}^- \end{pmatrix}, \frac{-i\gamma\eta}{4} (e^{ikd} \hat{c}_{R,A}^- \hat{c}_{R,B}^+ - e^{-ikd} \hat{c}_{R,A}^+ \hat{c}_{R,B}^-) \right). \quad (S29)$$

Similarly, the leftwards cascaded system gives

$$G_2 = G_{L,A} \triangleleft G_{\phi_{WG}} \triangleleft G_{BS} \triangleleft G_{L,B} = \left(e^{ikd} \begin{pmatrix} \frac{\eta}{\sqrt{1-\eta^2}} & -\sqrt{1-\eta^2} \\ \sqrt{1-\eta^2} & \eta \end{pmatrix}, \begin{pmatrix} \sqrt{\frac{\gamma}{2}}(\hat{c}_{L,A}^- + \eta e^{ikd} \hat{c}_{L,B}^-) \\ \sqrt{\frac{\gamma}{2}}\sqrt{1-\eta^2} \hat{c}_{L,B}^- \end{pmatrix}, \frac{-i\gamma\eta}{4} (e^{ikd} \hat{c}_{L,A}^+ \hat{c}_{L,B}^- - e^{-ikd} \hat{c}_{L,A}^- \hat{c}_{L,B}^+) \right). \quad (S30)$$

Combining these two systems using the concatenation product, $G_1 \boxplus G_2$ [70], we find that waveguide mediated interaction Hamiltonian now has a multiplicative factor of η , $\hat{H} = \eta(\hat{H}_L + \hat{H}_R)$, and the Lindbladian terms are modified as such:

$$\begin{aligned} \hat{c}_1 &= \eta \hat{c}_{R,A}^- + e^{-ikd} \hat{c}_{R,B}^- \\ \hat{c}_2 &= \sqrt{1-\eta^2} \hat{c}_{R,A}^- \\ \hat{c}_3 &= \hat{c}_{L,A}^- + \eta e^{ikd} \hat{c}_{L,B}^- \\ \hat{c}_4 &= \sqrt{1-\eta^2} \hat{c}_{L,B}^- \end{aligned} \quad (S31)$$

$\hat{c}_1$ and $\hat{c}_2$ describe photon transfer from modules A to B, while $\hat{c}_3$ and $\hat{c}_4$ describe photon transfer from modules B to A. Therefore, a non-zero η generates photon loss by creating an asymmetry in the collective loss operators $\hat{c}_1$ and $\hat{c}_3$ and by introducing operators $\hat{c}_2$ and $\hat{c}_4$, which describe on-site loss on the upstream qubits.

We incorporate the data-waveguide qubit interactions by adding Hamiltonian terms describing the parametric coupling between each data-waveguide qubit pair. We modulate the tunable coupler at the data-waveguide qubit detuning with shaped fast-flux pulses to generate a time-dependent coupling $g(t)$ to emit a symmetric photon. We use the reverse pulses $g(-t)$ to absorb the photon. In the detuning frame and under the rotating wave approximation, these terms are expressed as

$$H = g_1(t)(\sigma_1^+ \sigma_3^- + \sigma_3^+ \sigma_1^-) + g_2(t)(\sigma_2^+ \sigma_4^- + \sigma_4^+ \sigma_2^-) + g_3(-t)(\sigma_5^+ \sigma_7^- + \sigma_7^+ \sigma_5^-) + g_4(-t)(\sigma_6^+ \sigma_8^- + \sigma_8^+ \sigma_6^-). \quad (S32)$$

For simplicity, the initial physics-based experimental calibration starts with pulse envelopes that generate a symmetric, hyperbolic secant photon envelope $f(t)$ with bandwidth γ_{ph} ,

$$f(t) = \frac{\sqrt{\gamma_{ph}}}{2} \text{sech}\left(\frac{\gamma_{ph} t}{2}\right). \quad (S33)$$

To create this photon wavepacket, we must apply coupling pulses with the envelope $g(t)$

$$g(t) = \frac{\gamma_{ph}}{4 \cosh\left(\frac{\gamma_{ph} t}{2}\right)} \frac{(1 + e^{\gamma_{ph} t})\gamma/\gamma_{ph} + 1 - e^{\gamma_{ph} t}}{\sqrt{(1 + e^{\gamma_{ph} t})\gamma/\gamma_{ph} - e^{\gamma_{ph} t}}}, \quad (S34)$$

where γ is the waveguide-qubit coupling rate. The speed of the emission protocol is limited by γ ; $\gamma_{\text{ph}} \leq \gamma$. When $\gamma_{\text{ph}} = \gamma$, $g(t)$ reduces to a hyperbolic secant function.

Finally, in order to add the effects of data qubit non-radiative decay and pure dephasing to the model, we add the following Lindbladian terms for each data qubit to the master equation

$$\sum_{i \in \{3,4,7,8\}} \Gamma_{\text{nr},i} \mathcal{D}[\sigma_i^-] \hat{\rho} + \frac{\gamma_{\phi,i}}{2} \mathcal{D}[\sigma_z^i] \hat{\rho}. \quad (\text{S35})$$

By initiating the data qubits in module A (B) in $|\psi_{\text{R},\text{A}}\rangle$ ($|\psi_{\text{L},\text{B}}\rangle$) and using appropriately shaped coupling pulses $g(t)$, we observe single-photon transfer between the modules in simulation. We use the full eight-qubit master equation model to fit to the results in Fig. 2 and extract the contribution of decoherence to photon loss.

E. Two-module Scattering Model

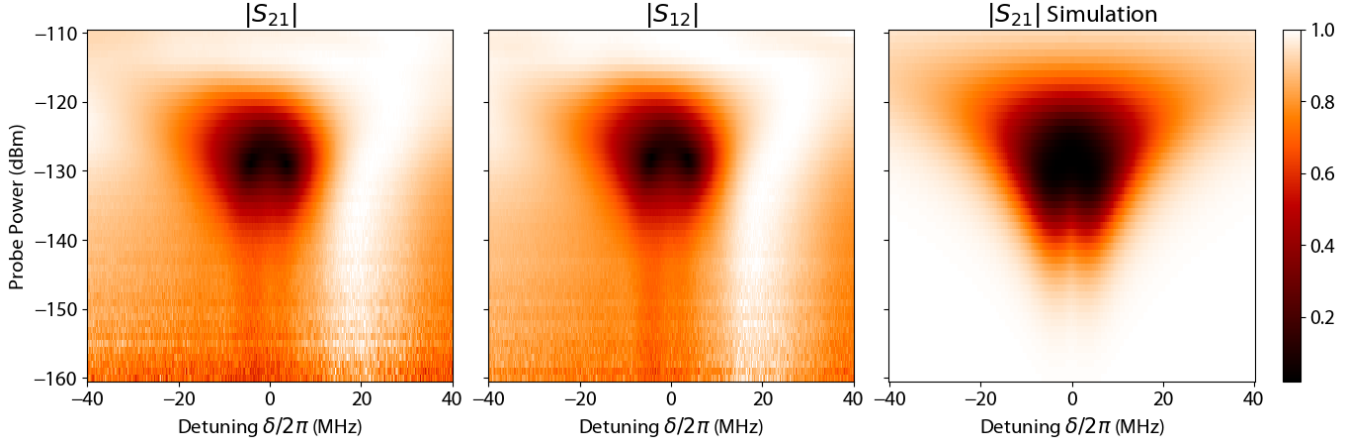


FIG. S3. **Scattering response of four-waveguide-qubit system.** Transmission spectrum of a coherent probe through the waveguide incident on the four waveguide qubits across both modules, as a function of the qubit-probe detuning $\delta/2\pi$ and the probe power P , in both propagation directions (left, center) and in simulation (right). Waveguide-mediated interactions between waveguide qubits on the same module are cancelled using tunable couplers. Each module functions as a cascaded atom, resulting in approximately unity transmission. At higher powers, the cascaded atom approximation breaks as waveguide-qubit pairs are doubly populated, resulting in transmission dips. In this regime the spectrum exhibits a subtle distance-dependence, which allows us to estimate that our inter-module distance is near a multiple of $\lambda/8$ by matching to simulation. At very high powers, all waveguide qubits are saturated, and the spectrum returns to unity transmission.

We also use a master equation model to simulate the four-emitter system when driven through the waveguide. We can derive the master equation using an SLH approach similar to that of the previous section. The module triplets now include an extra Hamiltonian term in the probe frame

$$G_{R(L),A(B)} = \left(I, \sqrt{\frac{\gamma}{2}} \begin{pmatrix} \hat{c}_{R(L),A(B)}^- \\ 0 \end{pmatrix}, \frac{\delta}{2} (\hat{\sigma}_{1(5)}^z + \hat{\sigma}_{2(6)}^z) \right), \quad (S36)$$

where δ is the emitter-drive detuning. The drive triplet is defined as

$$G_{\text{drive}} = \left(I, \begin{pmatrix} \alpha \\ 0 \end{pmatrix}, 0 \right), \quad (S37)$$

where $\alpha = \langle \hat{a}_R^{\text{in}} \rangle$ is the amplitude of the coherent rightward drive. For the rightwards-driven cascaded system, we solve $G_3 = G_{R,B} \triangleleft G_{\phi_{\text{WG}}} \triangleleft G_{\text{BS}} \triangleleft G_{R,A} \triangleleft G_{\text{drive}}$ to find

$$\hat{L}_3 = \begin{pmatrix} \eta\alpha + \sqrt{\frac{\gamma}{2}}(\eta\hat{c}_{R,A}^- + e^{-ikd}\hat{c}_{R,B}^-) \\ \sqrt{\frac{\gamma}{2}}\sqrt{1-\eta^2}\hat{c}_{R,A}^- \end{pmatrix} \quad (S38)$$

$$\hat{H}_3 = \sum_{i \in \{1,2,5,6\}} \frac{\delta}{2} \hat{\sigma}_z^i + \eta \hat{H}_R + \frac{1}{2}(\Omega \hat{c}_{R,A}^+ + \Omega^* \hat{c}_{R,A}^-) + \frac{1}{2}\eta(e^{ikd}\Omega \hat{c}_{R,B}^+ + e^{-ikd}\Omega^* \hat{c}_{R,B}^-). \quad (S39)$$

To complete the system, we concatenate G_3 with the leftwards (un-driven) cascaded system (S30), $G_4 = G_3 \boxplus G_2$, and calculate

$$\hat{L}_4 = \begin{pmatrix} \eta\alpha + \sqrt{\frac{\gamma}{2}}(\eta\hat{c}_{R,A}^- + e^{-ikd}\hat{c}_{R,B}^-) \\ \sqrt{\frac{\gamma}{2}}\sqrt{1-\eta^2}\hat{c}_{R,A}^- \\ \sqrt{\frac{\gamma}{2}}(\hat{c}_{L,A}^- + \eta e^{ikd}\hat{c}_{L,B}^-) \\ \sqrt{\frac{\gamma}{2}}\sqrt{1-\eta^2}\hat{c}_{L,B}^- \end{pmatrix} \quad (S40)$$

$$\hat{H}_4 = \sum_{i \in \{1,2,5,6\}} \frac{\delta}{2} \hat{\sigma}_z^i + \eta(\hat{H}_R + \hat{H}_L) + (\Omega \hat{c}_{R,A}^+ + \Omega^* \hat{c}_{R,A}^-) + \eta(e^{ikd}\Omega \hat{c}_{R,B}^+ + e^{-ikd}\Omega^* \hat{c}_{R,B}^-), \quad (S41)$$

where $\Omega = -i\sqrt{\frac{\gamma}{2}}\alpha/2$. We see that the propagation loss in the interconnect also affects the drive—the Hamiltonian drive term for the downstream module B has a multiplicative factor of η , as does the drive amplitude α in $\hat{L}_4$. We can simulate the system by solving the master equation with $\hat{H}_4$ and the Lindbladian terms given by $\hat{L}_4$, excluding the drive term (equivalent to S31). The steady-state solutions for $\langle \hat{c}_{R,A}^- \rangle$ and $\langle \hat{c}_{R,B}^- \rangle$ are then plugged into the input-output equation

$$\hat{a}_R^{\text{out}} = \eta \hat{a}_R^{\text{in}} + \sqrt{\frac{\gamma}{2}}(\eta \hat{c}_{R,A}^- + e^{-ikd} \hat{c}_{R,B}^-) \quad (\text{S42})$$

to solve for $S_{21} = \langle \hat{a}_R^{\text{out}} \rangle / \langle \hat{a}_R^{\text{in}} \rangle$. The simulation for our specific system is shown in Fig. S3 for a range of drive powers. In the low power regime, we observe unity transmission—which is the expected behaviour of two cascaded atoms. We use this fact to calibrate our system such that the waveguide qubits are all resonant and spaced at a $\lambda/4$ distance with zero coupling within each module. At higher powers, a single module can be populated with two or more excitations, causing the cascaded model to break down and resulting in dips in the transmission spectrum.

F. Time-domain reflectometry with single, microwave photons

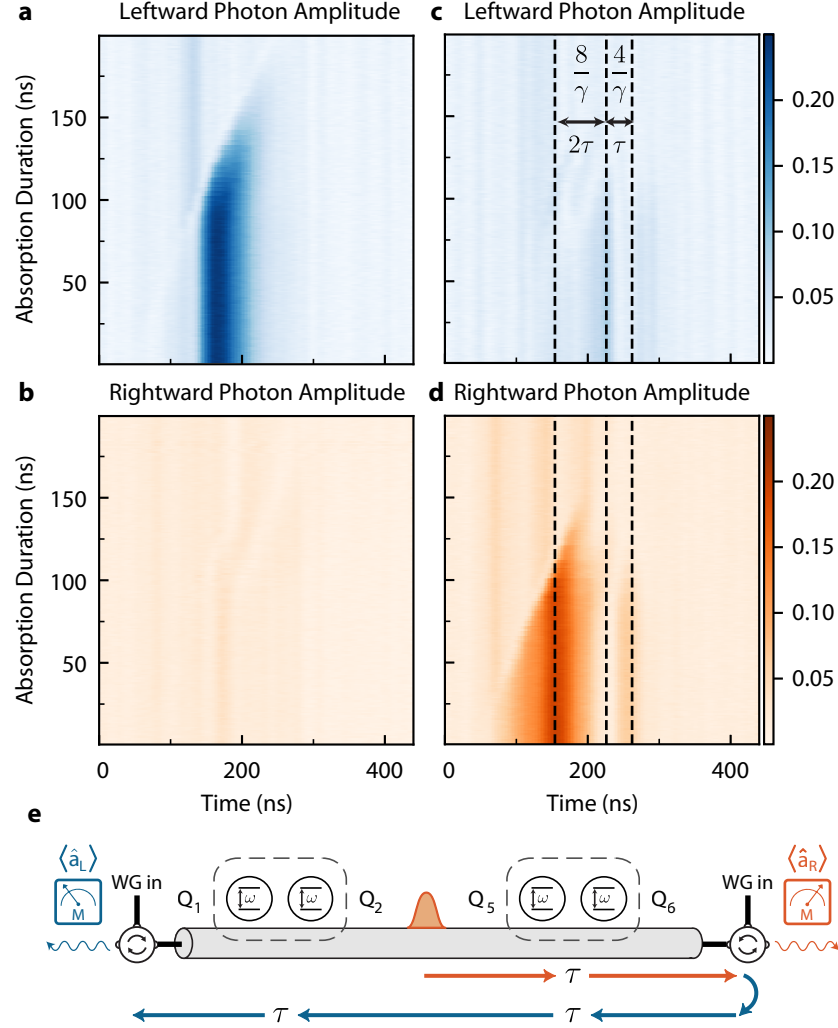


FIG. S4. **Photon amplitude as a function of absorption pulse duration.** a,b) Photon field amplitudes at the left and right ends of the waveguide during the leftward (c,d rightward) emission and absorption experiment. We sweep the time duration of the flux pulses that absorb the photon from 0 to 200 ns. We observe the gradual absorption of the leftward photon. As indicated by the dashed lines, the unabsorbed photon sees delays that are integer multiples of $\tau = 4/\gamma$ that result from the chiral interaction with waveguide qubits on both modules. This suggests the presence of impedance mismatches in between the modules and beyond the interconnect. When the absorption pulses reach their full duration, these signals are suppressed. e) Example rightward photon signal path in response to impedance mismatches downstream. Each time the photon encounters a “transparent” module it picks up a time delay of τ . In the case of rightward emission in part c,d, the photon signal sees a significant impedance mismatch, the circulators for example, beyond the modules because this signal is diminished following photon absorption.

We operate the modules in “transparency” mode in two different ways, either by detuning the waveguide qubits away from the directional photon frequency as done in Fig. 2d/e, or by keeping the waveguide qubits resonant and turning off the absorbing flux pulses as done in Fig. S4. In the latter case, the destructive interference effect guarantees unity transmission of the photon past the resonant waveguide qubits with a time delay of $4/\gamma$ and slight wavepacket

distortion [72, 73]. Starting from the input-output equations of one module,

$$\begin{aligned}\langle \hat{a}_L \rangle &= \langle \hat{a}_L^{\text{in}} \rangle + \sqrt{\frac{\gamma}{2}} (\langle \hat{\sigma}_1^- \rangle + i \langle \hat{\sigma}_2^- \rangle), \\ \langle \hat{a}_R \rangle &= \langle \hat{a}_R^{\text{in}} \rangle + \sqrt{\frac{\gamma}{2}} (\langle \hat{\sigma}_1^- \rangle - i \langle \hat{\sigma}_2^- \rangle),\end{aligned}\tag{S43}$$

with the condition that an input signal with detuning δ from the qubits comes from only one direction, $\langle \hat{a}_R^{\text{in}} \rangle = 0$ or $\langle \hat{a}_L^{\text{in}} \rangle = 0$, we calculate the transmission coefficient in the frequency domain,

$$S_{LR} = S_{RL} = \frac{\langle \hat{a}_{L/R} \rangle}{\langle \hat{a}_{L/R}^{\text{in}} \rangle} = \frac{\gamma/2 + i\delta}{\gamma/2 - i\delta}.\tag{S44}$$

For a small photon linewidth $\gamma_{\text{ph}} \leq 0.5\gamma$, which is the case in our experiment, the distortions on the wavepacket are negligible [72]. We obtain the phase imparted on the directional photon by the transparent module

$$\theta(\delta) = \arctan\left(\frac{\gamma\delta}{\gamma^2/4 - \delta^2}\right),\tag{S45}$$

which gives the group delay of the photon wavepacket,

$$\tau = \left. \frac{\partial \theta(\delta)}{\partial \delta} \right|_{\delta=0} = -\frac{4}{\gamma}.\tag{S46}$$

The experiments shown in Fig. S4 showcase several of these delays. When the absorbing pulse duration is zero, the center of the photon wavepacket that lands at the expected detector is already $4/\gamma = 37.4$ ns delayed compared to the experiment where the waveguide qubits on the absorber module are detuned in Fig. 2d/e. The first dotted line already includes a first delay imparted by the resonant waveguide qubits.

These are not the only signal delays shown in the bottom half of these plots. An illustrative example is shown by the rightward emission experiment in Fig. S4c-e. After the initial photon emission, there is another signal at the leftward end of the waveguide that is delayed by $8/\gamma$, which means the photon sometimes interacts with resonant modules twice more. This implies that an impedance mismatch after the absorber module, likely a circulator, reflects the photon backwards in the leftward direction sometimes, which results in interactions with both modules on its way to the leftward waveguide detector. The longer absorption pulses represented by the top half of the plots suppress the reflection signals. In other words, the module absorbs the photon before it has a chance to reflect off impedance mismatches outside the interconnect. The time delay imposed by the transparent module provides a means to slow down the photon pulse and observe the effect of impedance mismatches near and along the interconnect—a measurement similar to time-domain reflectometry.

G. Reinforcement Learning Optimization

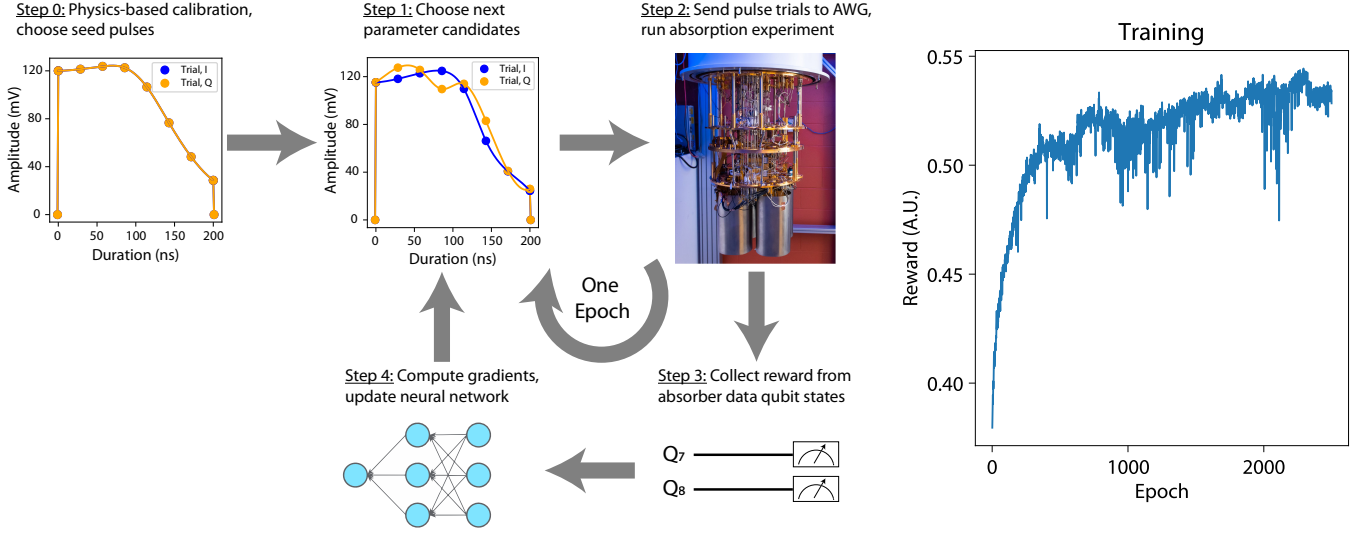


FIG. S5. **Reinforcement learning optimization for maximal photon absorption.** Following a manual physics-based calibration of the shaped emission and absorption, we provide seed pulses to the reinforcement learning algorithm for all 6 protocol pulses. To allow freedom in the potential pulse shapes, we segment emission and absorption seed pulses into 8 time steps, each with an in-phase (I) and quadrature (Q) component. We optimize a total of 73 parameters, including pulse amplitudes, frequencies, and phases. We use proximal policy optimization (PPO), which then samples the next set of parameters from Gaussian distributions centered around the seed pulse parameters to form a batch of 150 trial pulse sets. We run the absorption experiment with the batch of pulse sets and collect a reward, which we define with the relation between the data qubits on the absorber module, Q_7 excited XOR Q_8 excited, after each set of trial pulses. The PPO algorithm then computes the gradients, updates neural network parameters, and then calculates a new Gaussian probability distribution for each parameter. We repeat this cycle for around 1000 epochs until the reward converges to a value, which takes around 2.5 hours.

PPO Parameter	Value
Learning rate	0.005
Number of policy updates	20
Importance ratio clipping	0.05
Batch size	150
Number of averages	1000
Value prediction loss coefficient	0.5
Gradient clipping	1.0
Log probability clipping	0.0
Neural network layers	4
Neural network nodes per layer	10

TABLE S5. **Reinforcement learning parameters.** Hyper-parameters of the proximal policy optimization (PPO) algorithm used to optimize the emission and absorption protocol.

We use model-free reinforcement learning to maximize absorption efficiency for both photon propagation directions. There are six pulses involved in the protocol, each with interdependent frequencies, phases, and amplitudes. Four of the pulses emit and absorb the photon, and the shapes of their envelopes in relation to each other is the key to maximal absorption. We start with ideal pulse shapes that theoretically produce the photon shape $\propto \text{sech}(\gamma t/2)$, which we calibrate manually. We parameterize these four pulses with eight time steps, each with in-phase and quadrature components, and optimize a total of 73 parameters.

We use an algorithm known as proximal policy optimization (PPO), following the approach described and implemented in [60, 74, 75]. We adapt the code from [76], which is built on TF-agents (Tensor Flow). For each epoch, we send a batch of 150 trial pulses (1k averages for each trial) and run the emission and absorption experiment and simultaneously measure the populations of the data qubits on the absorber module with single-shot qubit state read-

out. At the end of the protocol, the data qubits are entangled in the single-photon subspace. We seek to maximize the degree of entanglement, so we define the reward as the sum number of counts of the two-qubit states $|01\rangle$ or $|10\rangle$.

Given the rewards, the PPO algorithm computes gradients and updates neural network parameters for the next epoch. The neural network produces a Gaussian probability distribution for each parameter, from which the next batch of pulse trials is selected. We run the optimization for roughly 1000 epochs and see an absorption efficiency improvement of up to 10% compared to the physics-based calibration.

The simplest manual calibration produces a symmetric photon wavepacket, such that the absorption pulses are the time-reverse of the emission pulses. Remarkably, the reinforcement learning agent produces photon shapes, as shown in Fig. 2d/e that are less symmetric than what we can calibrate by hand. This suggests that practice, a symmetric wavepacket is not necessarily optimal for maximal absorption efficiency. By distorting the control pulses and thus, the photon wavepacket, the agent apparently incorporates experimental imperfections such as control line distortions and impedance mismatches along the waveguide between the module when optimizing the wavepacket.

There could be additional distortions along the photon readout/amplification chain beyond the modules that impact the observed photon shape without impacting the emission and absorption protocol. The photon shape is important only once it arrives at the absorber module, so it could be the case that the optimizer distorts the photon to account for impedance mismatches between the modules.

Since we have control of both the absorption rate and the emission rate of the modules, we can in principle emit asymmetric wavepackets through temporally tuning the coupling strength on the emitter, and then absorb that asymmetric wavepacket by accordingly tuning the coupling strength absorption side. The optimizer takes this into account, apparently, to realize a higher efficiency.

H. Quantum Network Considerations

Experimental Interconnect	Almanakly (2024)	Kurpiers (2018) [77]	Zhong (2019) [78]	Zhong (2021) [79]	Wu (2024) [80]
Physical Mechanism	Directional photon	Itinerant photon	Itinerant photon	Resonant mode	Router circuit
Number of nodes per interconnect	Arbitrary number N	Two	Two	Two	Single digits
Network Connectivity	All-to-all on waveguide	All-to-all chain of circulator-node pairs	Point-to-point	Point-to-point	All-to-all for a few nodes
Communication flow	Bidirectional	Unidirectional	Bidirectional	Bidirectional	Bidirectional
Distance-dependence	None	None	Yes	Yes	Yes
On-chip implementation	Available	Not available	Available	Available	Available
Additional notes	Enables long-distance coupling without the need for sequential iteration.	Requires lossy circulator for impedance matching. Strictly unidirectional communication.	Emission speed must be faster than travel time between nodes, constraining possible node distances and protocol speeds.	Protocol operation and speed depends on the free-spectral range of the resonator.	Only several nodes, they must be qubit-scale distance apart. Larger distances unavailable.

TABLE S6. **State-of-the-art superconducting quantum interconnects.** Network architecture evaluation of several select representative demonstrations of quantum interconnects.

In general, superconducting interconnects to date have not been compatible with all-to-all connectivity in both long-distance and on-chip implementations. Point-to-point communication, where an interconnect only connects two computational nodes, is not feasible for the large-scale system sizes required to realize fault-tolerant quantum computation. We compare the features of several experimental superconducting interconnects in Table S6.

In addition to the advantage created by all-to-all connectivity, a larger chiral quantum network supports parallelized communication. For a given waveguide, photons can travel in two directions, doubling the channel throughput. Communication can also be multiplexed over different photon frequency bands spaced on the order of a few qubit-waveguide coupling rates ($\gamma/2\pi = 17$ MHz in this work). In a large quantum network, with the protocol described in the main text, we can simultaneously remotely entangle many pairs of modules by exploiting the time, frequency, and direction dimensions of photon propagation.

We could also generate remote, multi-partite W states over an arbitrary number of modules N by extending the remote entanglement protocol presented in the main text. By adjusting the shapes or durations of the emission pulses, we could emit a photon to the left and/or right with probability $(N-1)/N$. Similarly, the absorber pulses can be shaped such that each absorber module receives the photon with probability $1/N$.

We also note that long-distance coupling is advantageous also for logical qubit operations of surface code logical qubits. It would enable non-nearest-neighbor coupling of logical qubits, rather than sequential nearest-neighbor operations. Additionally, many low-density parity check (LDPC) codes have encodings that require non-nearest-neighbor couplings, and these interconnects would be a means to realize such couplings efficiently. In both cases, the fact that we need not have a unique (dedicated) interconnect between every pair of physical qubits, or an interconnect limited to only a few (more than two, but fundamentally limited) physical qubits, is an advantage of our approach. It would be unwieldy to have unique interconnects between every pair of qubits we wish to connect; this clearly does not scale well.

-
- [60] L. Ding, M. Hays, Y. Sung, B. Kannan, J. An, A. Di Paolo, A. H. Karamlou, T. M. Hazard, K. Azar, D. K. Kim, B. M. Niedzielski, A. Melville, M. E. Schwartz, J. L. Yoder, T. P. Orlando, S. Gustavsson, J. A. Grover, K. Serniak, and W. D. Oliver, *Phys. Rev. X* **13**, 031035 (2023).
- [61] M. F. Dumas, B. Groleau-Paré, A. McDonald, M. H. Muñoz Arias, C. Lledó, B. D’Anjou, and A. Blais, *Phys. Rev. X* **14**, 041023 (2024).
- [62] I.-C. Hoi, C. M. Wilson, G. Johansson, T. Palomaki, B. Peropadre, and P. Delsing, *Phys. Rev. Lett.* **107**, 073601 (2011).

- [63] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Y. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, *Science* **327**, 840 (2010).
- [64] I.-C. Hoi, C. M. Wilson, G. Johansson, J. Lindkvist, B. Peropadre, T. Palomaki, and P. Delsing, *New Journal of Physics* **15**, 025011 (2013).
- [65] M. Mirhosseini, E. Kim, X. Zhang, A. Sipahigil, P. B. Dieterle, A. J. Keller, A. Asenjo-Garcia, D. E. Chang, and O. Painter, *Nature* **569**, 692 (2019).
- [66] H. Pichler, T. Ramos, A. J. Daley, and P. Zoller, *Phys. Rev. A* **91**, 042116 (2015).
- [67] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, *Nature* **541**, 473 (2017).
- [68] A. Irfan, M. Yao, A. Lingenfelter, X. Cao, A. A. Clerk, and W. Pfaff, *Phys. Rev. Res.* **6**, 033212 (2024).
- [69] K. Reuer, J.-C. Besse, L. Wernli, P. Magnard, P. Kurpiers, G. J. Norris, A. Wallraff, and C. Eichler, *Phys. Rev. X* **12**, 011008 (2022).
- [70] A. F. Kockum, G. Johansson, and F. Nori, *Phys. Rev. Lett.* **120**, 140404 (2018).
- [71] J. Combes, J. Kerckhoff, and M. Sarovar, *Advances in Physics: X* **2**, 784 (2017).
- [72] N. Gheeraert, S. Kono, and Y. Nakamura, *Phys. Rev. A* **102**, 053720 (2020).
- [73] D. Weiss, S. Puri, and S. Girvin, *PRX Quantum* **5**, 020312 (2024).
- [74] V. V. Sivak, A. Eickbusch, H. Liu, B. Royer, I. Tsioutsios, and M. H. Devoret, *Phys. Rev. X* **12**, 011059 (2022).
- [75] V. V. Sivak, A. Eickbusch, B. Royer, S. Singh, I. Tsioutsios, S. Ganjam, A. Miano, B. L. Brock, A. Z. Ding, L. Frunzio, S. M. Girvin, R. J. Schoelkopf, and M. H. Devoret, *Nature* **616**, 50 (2023).
- [76] V. V. Sivak, “quantum-control-rl,” <https://github.com/v-sivak/quantum-control-rl>.
- [77] P. Kurpiers, P. Magnard, T. Walter, B. Royer, M. Pechal, J. Heinsoo, Y. Salathé, A. Akin, S. Storz, J.-C. Besse, S. Gasparinetti, A. Blais, and A. Wallraff, *Nature* **558**, 264 (2018).
- [78] Y. P. Zhong, H.-S. Chang, K. J. Satzinger, M.-H. Chou, A. Bienfait, C. R. Conner, É. Dumur, J. Grebel, G. A. Peairs, R. G. Povey, D. I. Schuster, and A. N. Cleland, *Nature Physics* **15**, 741 (2019).
- [79] Y. Zhong, H.-S. Chang, A. Bienfait, É. Dumur, M.-H. Chou, C. R. Conner, J. Grebel, R. G. Povey, H. Yan, D. I. Schuster, and A. N. Cleland, *Nature* **590**, 571 (2021).
- [80] X. Wu, H. Yan, G. Andersson, A. Anferov, M.-H. Chou, C. R. Conner, J. Grebel, Y. J. Joshi, S. Li, J. M. Miller, R. G. Povey, H. Qiao, and A. N. Cleland, *Phys. Rev. X* **14**, 041030 (2024).