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Supplementary information

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Experimentally probing Landauer's principle in the quantum many-body regime

In the format provided by the authors and unedited

S1 Unitarity of dynamics

We demonstrate the global conservation of both von Neumann entropy and energy throughout the time evolution in our experiment, as shown in Fig. S1. This observation provides direct evidence that the underlying dynamics are unitary, a critical assumption for the validity of using Landauer's principle for characterising the out-of-equilibrium dynamics in our experiment.

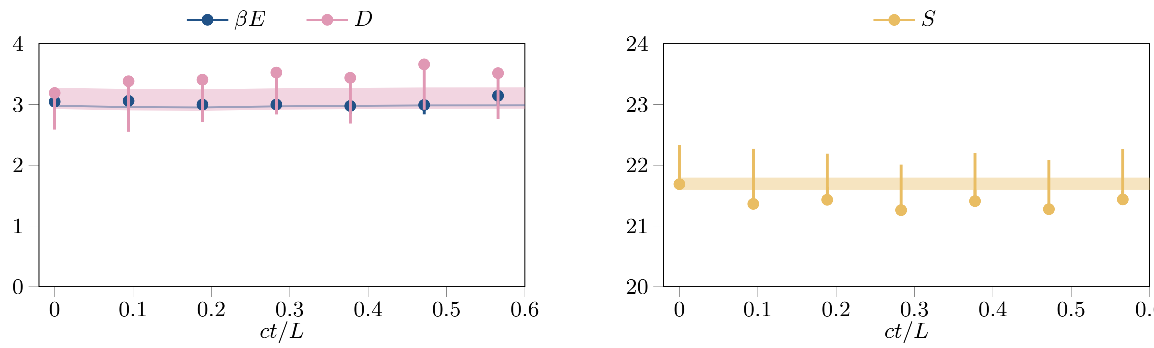

Fig. S1 Unitarity of time evolution. The energy, relative entropy and von Neumann entropy remain globally conserved over time, providing evidence of the unitarity of the dynamics. See Fig. 2 in the main text for more details on the error bars and shaded areas.

Under the assumption of global unitary dynamics for the composite system's quantum state ϱ_{SE}, Landauer's principle (Eq. (5) in the main text) can be derived. Let us define the two equivalent decompositions (or *definitions*) of generalised entropy production, $\Delta\Sigma$, as

$$\Delta\Sigma^{(\mathrm{left})} := \beta_{\mathrm{E}}\Delta E_{\mathrm{E}} + \Delta S, \qquad \Delta\Sigma^{(\mathrm{right})} := \Delta I + \Delta D. \tag{S1}$$

In our theoretical simulations, the equality $\Delta\Sigma^{(\mathrm{left})} = \Delta\Sigma^{(\mathrm{right})}$ is satisfied by construction, as it follows directly follows from the assumptions of unitary dynamics. In the experiment, however, this equality is far from guaranteed. Each of the quantities $\beta_{\mathrm{E}}\Delta E_{\mathrm{E}}, \Delta S, \Delta I$ and ΔD is computed independently from tomographically reconstructed covariance matrices obtained at different times t, where at each time step different time intervals are used for the reconstruction (Fig. 1**c** in the main text). Nevertheless, Fig. S2 shows that the two different decompositions $\Delta\Sigma^{(\mathrm{left})}$ and $\Delta\Sigma^{(\mathrm{right})}$ match well. This agreement thus further validates the assumptions underpinning Landauer's principle, namely, unitarity and valid density matrices (or covariance matrices, here) throughout the time evolution.

S2 Scaling of absolute quantities

We include a plot of the absolute quantities of those in Eq. (5) in the main text in Fig. S3 for the experiment.

S3 Continuum limit and role of boundary conditions

In this section we compare the effect of Neumann boundary conditions ($\partial_z\varphi(z)|_{z=0,L} = 0$) and Dirichlet boundary conditions ($\varphi(z)|_{z=0,L} = 0$) on the information-theoretic quantities within Landauer's principle

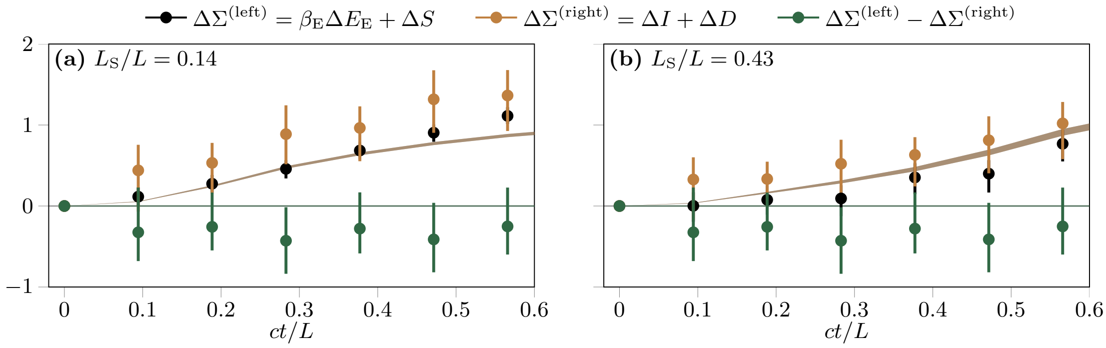

Fig. S2 Comparing the two different decompositions of generalised entropy production. We observe good agreement between $\Delta\Sigma^{(\mathrm{left})}$ and $\Delta\Sigma^{(\mathrm{right})}$, as shown here as a function of time for subregion size ratios **a,** $L_{\mathrm{S}}/L = 0.14$ and **b,** $L_{\mathrm{S}}/L = 0.43$. This provides further evidence that the assumptions behind Landauer's principle are well satisfied in our experiment.

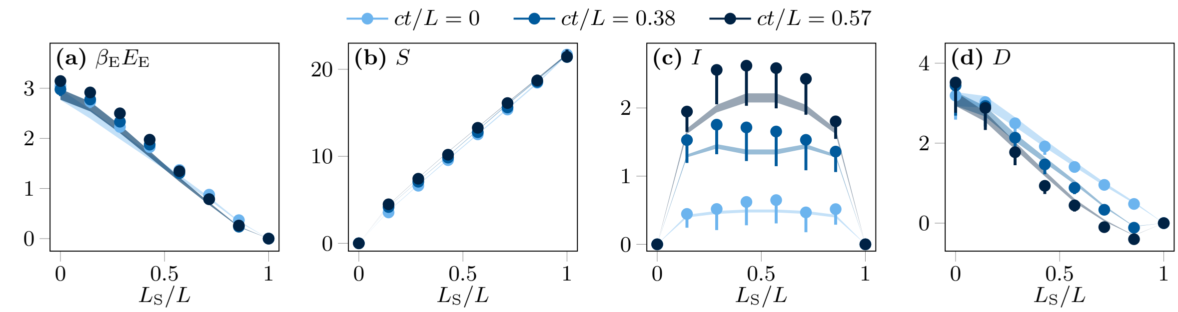

Fig. S3 Scaling of different absolute quantities in Landauer's principle. The quantities are shown as a function of subregion size for various times $ct/L = 0.19, 0.38, 0.57$. The experimental data is in agreement with the quantum field theory simulation results using Neumann boundary conditions.

(Eq. 5 in the main text) in the continuum limit. The quantum field is discretised on a sufficiently fine grid to ensure convergence, achieved when higher momentum modes no longer affect the results significantly. The simulation parameters closely match experimental values, assuming a homogeneous density profile $n_{1\mathrm{D}}$.

The plots showing the scaling with subregion size at various times are presented in Fig. S4. The scaling of $\beta_{\mathrm{E}}\Delta E_{\mathrm{E}}$ can be explained by the initial symmetric energy density profile, which is higher at the edges than in the bulk for Neumann boundary conditions and lower for Dirichlet boundary conditions (Fig. 4**a**,-**b**) in the main text). The dominant contribution for the considered quench, ΔI, shows scaling symmetrically about $L_{\mathrm{S}}/L = 0.5$ by construction. For Dirichlet boundary conditions, small systems (or environments) do not increase in correlations with its environment for $ct/L_{\mathrm{S}} > 1$ as correlations reach a plateau phase in time (Fig. 4**e**). Neumann boundary conditions, however, actually yield ΔI to be higher for the case of a small system than for the case of system and environment being of equal size. As discussed in the main text, the zero mode causes the results for Dirichlet boundary conditions, which are well described by the quasiparticle picture, to diverge from those for Neumann boundary conditions after $ct/L_{\mathrm{S}} > 1$.

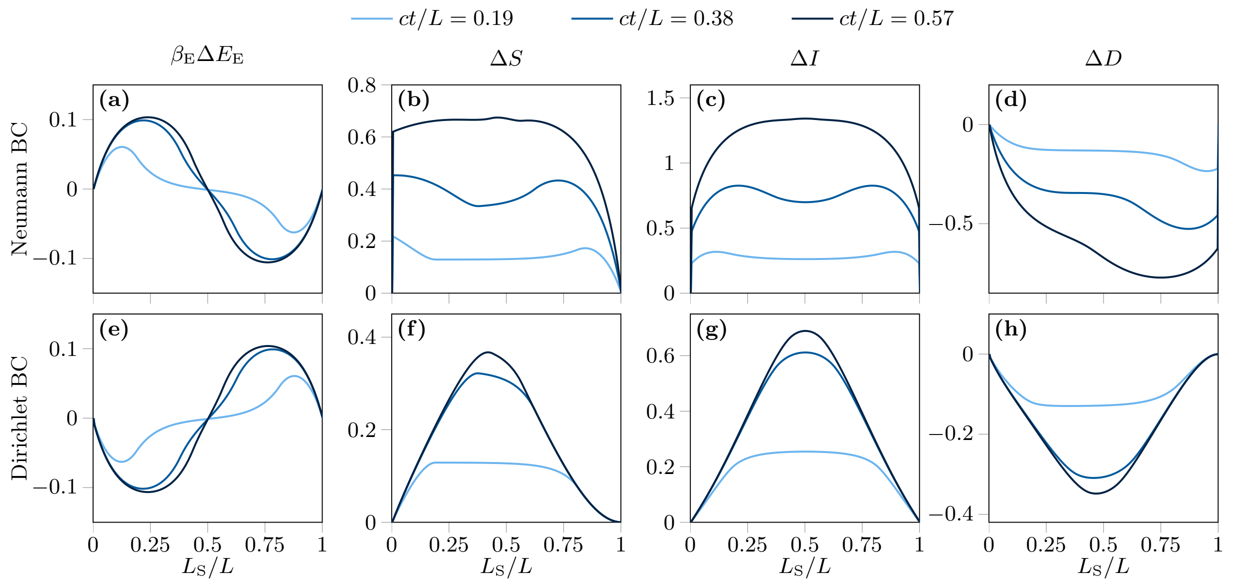

Fig. S4 Scaling of different quantities in Landauer's principle in the continuum limit for Neumann and Dirichlet boundary conditions. Quantum field theory simulations are used in the continuum limit, similar to Fig. 3 in the main text, but without accounting for experimental inhomogeneities. **a)-d)** Neumann boundary conditions and **e)-h)** Dirichlet boundary conditions.

S4 Exploiting the extremality of Gaussian states

The validity of the Gaussian tomography scheme has been confirmed by measuring the normalised averaged connected fourth-order correlation functions, as demonstrated in previous works [1–3]. However, leveraging properties of Gaussian quantum information, we argue that the information-theoretic quantities in Landauer's principle can also be estimated using our second-moment tomography scheme, even for time evolution under a non-Gaussian model with significant higher-order correlation functions. This justifies the potential use of our experimental protocol under the lens of Landauer's principle for more complex protocols and interacting models.

To begin, we define G as the Gaussian state that shares the same second moments as the *true* state ϱ that may be non-Gaussian, reflecting a preparation involving interactions. Central to the argument are several extremality properties of Gaussian states. Firstly, Gaussian states G have the largest von Neumann entropy $S(G) \geq S(\varrho)$ among all states ϱ with a given covariance matrix. Secondly, we can make use of the property

$$\mathrm{Tr}[\varrho(t) \log G(0)] = \mathrm{Tr}[G(t) \log G(0)] \,, \tag{S2}$$

as any Gaussian state can be written as a matrix exponent of a quadratic polynomial, thereby causing the matrix logarithm of the relative entropy to cancel out. Thirdly, Ref. [4] has demonstrated that the quantum conditional entropy is also maximised for Gaussian states, and the inequality

$$S(\varrho_S(t)) - S(\varrho_{SE}(t)) \geq S(G_S(t)) - S(G_{SE}(t)) \tag{S3}$$

is true. This is less obvious, as the expression under consideration is a difference of von Neumann entropies. The generalised entropy production of the true dynamics $\Delta\Sigma := \beta_{\rm E}\Delta E_{\rm E} + \Delta S = \Delta I + \Delta D$ can be written as

$$\begin{aligned} \Delta\Sigma &:= \Delta I + \Delta D \\ &= S(\varrho_{\rm S}(t)) + S(\varrho_{\rm E}(t)) - S(\varrho_{\rm SE}(t)) - S(\varrho_{\rm S}(0)) - S(\varrho_{\rm E}(0)) + S(\varrho_{\rm SE}(0)) \\ &\quad - S(\varrho_{\rm E}(t)) - \mathrm{Tr}[\varrho_{\rm E}(t)\log\gamma_{\rm E}^{\beta_{\rm E}}] + S(\varrho_{\rm E}(0)) + \mathrm{Tr}[\varrho_{\rm E}(0)\log\gamma_{\rm E}^{\beta_{\rm E}}] \\ &= S(\varrho_{\rm S}(t)) - S(\varrho_{\rm S}(0)) - \mathrm{Tr}[\varrho_{\rm E}(t)\log\gamma_{\rm E}^{\beta_{\rm E}}] + \mathrm{Tr}[\varrho_{\rm E}(0)\log\gamma_{\rm E}^{\beta_{\rm E}}] \\ &= \Delta S - \mathrm{Tr}[\varrho_{\rm E}(t)\log\gamma_{\rm E}^{\beta_{\rm E}}] + \mathrm{Tr}[\varrho_{\rm E}(0)\log\gamma_{\rm E}^{\beta_{\rm E}}]\,, \end{aligned} \tag{S4}$$

where we have used the unitary invariance of the von Neumann entropy. Clearly, the generalised entropy production $\Delta\Sigma^G := \beta_{\rm E}\Delta E_{\rm E}^G + \Delta S^G = \Delta I^G + \Delta D^G$ assuming an initially Gaussian state and assuming Gaussian dynamics is given by

$$\begin{aligned} \Delta\Sigma^G &:= \Delta I^G + \Delta D^G \\ &= \Delta S^G - \mathrm{Tr}[G_{\rm E}(t)\log\gamma_{\rm E}^{\beta_{\rm E}}] + \mathrm{Tr}[G_{\rm E}(0)\log\gamma_{\rm E}^{\beta_{\rm E}}], \end{aligned} \tag{S5}$$

where $\Delta E_{\rm E}^G = \mathrm{Tr}[(G_{\rm E}(t) - G_{\rm E}(0))H_{\rm E}]$, $\Delta S^G = S(G_{\rm S}(t)) - S(G_{\rm S}(0))$, $\Delta I^G = I_{\rm SE}^G(t) - I_{\rm SE}^G(0)$ and $\Delta D^G = D(G_{\rm E}(t)||\gamma_{\rm E}^{\beta_{\rm E}}) - D(G_{\rm E}(0)||\gamma_{\rm E}^{\beta_{\rm E}})$. By utilising the property in Eq. (S2) and considering that $\gamma_{\rm E}^{\beta_{\rm E}}$ is Gaussian we can write

$$\Delta\Sigma = \Delta S - \mathrm{Tr}[G_{\rm E}(t)\log\gamma_{\rm E}^{\beta_{\rm E}}] + \mathrm{Tr}[G_{\rm E}(0)\log\gamma_{\rm E}^{\beta_{\rm E}}] = \Delta S - \Delta S^G + \Sigma^G\,. \tag{S6}$$

This also implies that

$$\Delta E_{\rm E} = \Delta E_{\rm E}^G \tag{S7}$$

demonstrating that covariance matrices are entirely sufficient for capturing the energetic dynamics, even when the true dynamics are not Gaussian. This is important, as in this way, one can measure second moments experimentally and still get bounds for the true state that may show signatures of interactions.

To proceed further, we must assume that the initial state post-quench is also Gaussian, i.e., $\varrho_{SE,S,E} = G_{SE,S,E}$. At this step, we have not made the additional assumption of Gaussian dynamics to bound the quantities in Landauer's principle. By the maximality of entropy for Gaussian states, we have $\Delta S \leq \Delta S^G$, we quickly deduce that the generalised entropy production is upper-bounded by the Gaussian value, i.e., $\Delta\Sigma \leq \Delta\Sigma^G$. Similarly, we infer that the change in mutual information is upper-bounded by the Gaussian value [4], i.e., $\Delta I \leq \Delta I^G$. Utilising Eq. (S2), we find that the change in relative entropy is lower-bounded by the Gaussian value, i.e.,

$$\Delta D \geq \Delta D^G\,. \tag{S8}$$

However, in both cases, these inequalities alone do not provide quantitative information on the extent to which these quantities dominate each other in the true state. To gain a more precise understanding of the relative contributions of mutual information and relative entropy in this context, a more comprehensive theoretical model and advanced experimental methods are required for an experiment modelled by an arbitrary interacting theory.

An alternative approach to probing Landauer's principle in interacting models may involve calculating classical entropies instead of their quantum counterparts. Recent work [5, 6] shows that classical entropies, which can be determined without full knowledge of the composite system's state and are more accessible experimentally, can still capture key features of their quantum analogues.

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