

Observation of the nonlinear chiral thermoelectric Hall effect in tellurium

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Supplementary Note 1: Symmetry constraints on the NCTE Hall effect

In this section, we explain the conditions under which the NCTE Hall effect is observed based on symmetry analysis. The electric current $\mathbf{J}$ driven by both an electric field and a temperature gradient is denoted as follows:

$$J_i = \sigma_{ijk} E_j \left(-\frac{\partial_k T}{T} \right), \quad (\text{S1})$$

where σ_{ijk} is the third-rank nonlinear thermoelectric conductivity tensor with $i, j, k \in \{1, 2, 3\}$. The indices 1, 2, and 3 are associated with the Cartesian coordinates x , y , and z , respectively. The component tables of the third-rank conductivity tensor χ for two different external fields in chiral crystals are provided in Table S1. Since tellurium (Te) belongs to the point group 32, the nonlinear thermoelectric conductivity tensor in Te is expressed as follows¹:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xxx} & 0 & 0 & 0 & \sigma_{xyy} & \sigma_{xzy} & 0 & \sigma_{xyz} & 0 \\ 0 & \sigma_{yyx} & \sigma_{yzx} & \sigma_{yxy} & 0 & 0 & \sigma_{yxz} & 0 & 0 \\ 0 & 0 & \sigma_{zyx} & 0 & \sigma_{zxy} & 0 & 0 & 0 & 0 \end{pmatrix} \quad (\text{S2})$$

where $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yyx} = -\sigma_{yxy}$, $\sigma_{xzy} = -\sigma_{yzx}$, $\sigma_{xyz} = -\sigma_{yxz}$, and $\sigma_{zxy} = -\sigma_{zyx}$. The coordinates x and z correspond to the a -axis and c -axis, respectively. Here, the NCTE Hall current $\mathbf{J} = (J_x, J_y, J_z)$ is given by the following:

$$J_x = \frac{\sigma_{xzy} - \sigma_{xyz}}{2} \left(E_z \left(-\frac{\partial_y T}{T} \right) - E_y \left(-\frac{\partial_z T}{T} \right) \right), \quad (\text{S3})$$

$$J_y = \frac{\sigma_{yzx} - \sigma_{yxz}}{2} \left(E_z \left(-\frac{\partial_x T}{T} \right) - E_x \left(-\frac{\partial_z T}{T} \right) \right), \quad (\text{S4})$$

$$J_z = \frac{\sigma_{zyx} - \sigma_{zxy}}{2} \left(E_y \left(-\frac{\partial_x T}{T} \right) - E_x \left(-\frac{\partial_y T}{T} \right) \right). \quad (\text{S5})$$

Therefore, the NCTE Hall effects in the x , y , and z directions in Te are finite. Similarly, for other chiral point groups, the NCTE Hall effect can have a finite value based on symmetry analysis. Note that when the contribution from the Berry curvature dipole (BCD) in the NCTE Hall effect is considered, an additional constraint is added: the BCD tensor needs to be traceless². In the case of Te, the BCD contribution is not forbidden because of the crystal symmetry^{3, 4}.

Considering the second-order nonlinear current response due to electric fields, $J'_i = \sigma'_{ijk} E_j E_k$, the currents generated in the x , y , and z directions in the systems with point group 32 are as follows:

$$J'_x = \sigma'_{xxx} (E_x^2 - E_y^2) + (\sigma'_{xzy} + \sigma'_{xyz}) E_y E_z, \quad (\text{S6})$$

$$J'_y = 2\sigma'_{yxy} E_x E_y + (\sigma'_{yxz} + \sigma'_{yzx}) E_x E_z, \quad (\text{S7})$$

$$J'_z = 0. \quad (\text{S8})$$

In the c -axis direction, the second-order nonlinear electrical response is forbidden by symmetry³. This symmetry constraint holds regardless of whether the second-order nonlinear response is driven by the intrinsic BCD mechanism or by extrinsic mechanisms such as side jump and skew scattering. This

symmetry constraint also applies to the second-order response of the temperature gradient $(\nabla T)^2$. Therefore, the NCTE Hall currents in the c -axis direction of Te are unaffected by both the second-order nonlinear responses and the linear response.

As shown in Table S1, all chiral systems can show finite NCTE Hall conductivity. On the other hand, as shown in Table S2, the NCTE Hall effect is forbidden for most noncentrosymmetric and nonchiral systems due to symmetry constraints. The exceptions are point groups $\bar{4}2m$ and $\bar{4}$, which are nonchiral but allow the NCTE Hall response.

Table S1 | Components of the third-rank conductivity tensor χ_{ijk} for the chiral point groups. The red cells in the tables represent finite components.

1										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	χ_{111}	χ_{121}	χ_{131}	χ_{112}	χ_{122}	χ_{132}	χ_{113}	χ_{123}	χ_{133}
	2	χ_{211}	χ_{221}	χ_{231}	χ_{212}	χ_{222}	χ_{232}	χ_{213}	χ_{223}	χ_{233}
	3	χ_{311}	χ_{321}	χ_{331}	χ_{312}	χ_{322}	χ_{332}	χ_{313}	χ_{323}	χ_{333}

Number of independent coefficients: 27

2										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	χ_{121}	0	χ_{112}	0	χ_{132}	0	χ_{123}	0
	2	χ_{211}	0	χ_{231}	0	χ_{222}	0	χ_{213}	0	χ_{233}
	3	0	χ_{321}	0	χ_{312}	0	χ_{332}	0	χ_{323}	0
Number of independent coefficients: 13										

222										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 6										

4										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	χ_{132}	χ_{113}	χ_{123}	0
	2	0	0	χ_{231}	0	0	χ_{232}	χ_{213}	χ_{223}	0
	3	χ_{311}	χ_{321}	0	χ_{312}	χ_{322}	0	0	0	χ_{333}
Number of independent coefficients: 7										
$\chi_{131} = -\chi_{232}, \chi_{132} = -\chi_{231}, \chi_{113} = \chi_{223}, \chi_{123} = -\chi_{213}, \chi_{311} = \chi_{322}, \chi_{312} = -\chi_{321}$										

422										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0

Number of independent coefficients: 3

$\chi_{132} = -\chi_{231}$, $\chi_{123} = -\chi_{213}$, $\chi_{312} = -\chi_{321}$

Table S1 (continued)

3										
X_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	X_{111}	X_{121}	X_{131}	X_{112}	X_{122}	X_{132}	X_{113}	X_{123}	0
	2	X_{211}	X_{221}	X_{231}	X_{212}	X_{222}	X_{232}	X_{213}	X_{223}	0
	3	X_{311}	X_{321}	0	X_{312}	X_{322}	0	0	0	X_{333}
Number of independent coefficients: 9 $X_{111} = -X_{122} = -X_{221} = -X_{212}, \quad X_{112} = X_{121} = X_{211} = -X_{222}, \quad X_{131} = X_{232}, \quad X_{132} = -X_{231}$ $X_{113} = X_{223}, \quad X_{123} = -X_{213}, \quad X_{311} = X_{322}, \quad X_{312} = -X_{321}$										

32										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	χ_{111}	0	0	0	χ_{122}	χ_{132}	0	χ_{123}	0
	2	0	χ_{221}	χ_{231}	χ_{212}	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 4										
$\chi_{111} = -\chi_{122} = -\chi_{221} = -\chi_{212}, \chi_{132} = -\chi_{231}, \chi_{123} = -\chi_{213}, \chi_{312} = -\chi_{321}$										

6										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	χ_{132}	χ_{113}	χ_{123}	0
	2	0	0	χ_{231}	0	0	χ_{232}	χ_{213}	χ_{223}	0
	3	χ_{311}	χ_{321}	0	χ_{312}	χ_{322}	0	0	0	χ_{333}
Number of independent coefficients: 7										
$\chi_{131} = \chi_{232}, \chi_{132} = -\chi_{231}, \chi_{113} = \chi_{223}, \chi_{123} = -\chi_{213}, \chi_{311} = \chi_{322}, \chi_{312} = -\chi_{321}$										

622										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 3										
$\chi_{132} = -\chi_{231}, \chi_{123} = -\chi_{213}, \chi_{312} = -\chi_{321}$										

23										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 2 $\chi_{132} = \chi_{213} = \chi_{321}, \chi_{123} = \chi_{231} = \chi_{312}$										

Table S1 (continued)

432										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 1 $\chi_{123} = -\chi_{132} = \chi_{231} = -\chi_{213} = -\chi_{321} = \chi_{312}$										

Table S2 | Components of the third-rank conductivity tensor χ_{ijk} for the noncentrosymmetric and nonchiral point groups. The red cells in the tables represent finite components. For centrosymmetric point groups, all tensor components are zero.

m										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	χ_{111}	0	χ_{131}	0	χ_{122}	0	χ_{113}	0	χ_{133}
	2	0	χ_{221}	0	χ_{212}	0	χ_{232}	0	χ_{223}	0
	3	χ_{311}	0	χ_{331}	0	χ_{322}	0	χ_{313}	0	χ_{333}
Number of independent coefficients: 14										

mm2										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	0	χ_{113}	0	0
	2	0	0	0	0	0	χ_{232}	0	χ_{223}	0
	3	χ_{311}	0	0	0	χ_{322}	0	0	0	χ_{333}

Number of independent coefficients: 7

$\bar{4}$										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	χ_{132}	χ_{113}	χ_{123}	0
	2	0	0	χ_{231}	0	0	χ_{232}	χ_{213}	χ_{223}	0
	3	χ_{311}	χ_{321}	0	χ_{312}	χ_{322}	0	0	0	0
Number of independent coefficients: 6										
$\chi_{132} = \chi_{231}, \chi_{131} = -\chi_{232}, \chi_{113} = -\chi_{223}, \chi_{123} = \chi_{213}, \chi_{311} = -\chi_{322}, \chi_{312} = \chi_{321}$										

4mm										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	0	χ_{113}	0	0
	2	0	0	0	0	0	χ_{232}	0	χ_{223}	0
	3	χ_{311}	0	0	0	χ_{322}	0	0	0	χ_{333}

Number of independent coefficients: 4

$\chi_{131} = \chi_{232}, \chi_{113} = \chi_{223}, \chi_{311} = \chi_{322}$

$\bar{4}2m$										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 3										
$\chi_{132} = \chi_{231}, \chi_{123} = \chi_{213}, \chi_{312} = \chi_{321}$										

Table S2 (continued)

3m										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	χ_{121}	χ_{131}	χ_{112}	0	0	χ_{113}	0	0
	2	χ_{211}	0	0	0	χ_{222}	χ_{232}	0	χ_{223}	0
	3	χ_{311}	0	0	0	χ_{322}	0	0	0	χ_{333}
Number of independent coefficients: 5										
$\chi_{112} = \chi_{221} = \chi_{211} = -\chi_{222}, \chi_{131} = \chi_{232}, \chi_{113} = \chi_{223}, \chi_{311} = \chi_{322}$										

$\bar{6}$										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	χ_{111}	χ_{121}	0	χ_{112}	χ_{122}	0	0	0	0
	2	χ_{211}	χ_{221}	0	χ_{212}	χ_{222}	0	0	0	0
	3	0	0	0	0	0	0	0	0	0

Number of independent coefficients: 2

$\chi_{111} = -\chi_{122} = -\chi_{221} = -\chi_{212}, \chi_{112} = \chi_{121} = \chi_{221} = -\chi_{222}$

6mm										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	χ_{131}	0	0	0	χ_{113}	0	0
	2	0	0	0	0	0	χ_{232}	0	χ_{223}	0
	3	χ_{311}	0	0	0	χ_{322}	0	0	0	χ_{333}
Number of independent coefficients: 4 $\chi_{131} = \chi_{232}, \quad \chi_{113} = \chi_{223}, \quad \chi_{311} = \chi_{322}$										

$\bar{6}m2$										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	χ_{121}	0	χ_{112}	0	0	0	0	0
	2	χ_{211}	0	0	0	χ_{222}	0	0	0	0
	3	0	0	0	0	0	0	0	0	0
Number of independent coefficients: 1										
$\chi_{112} = \chi_{121} = \chi_{211} = -\chi_{222}$										

$\bar{4}3m$										
χ_{ijk}	jk									
i		11	21	31	12	22	32	13	23	33
	1	0	0	0	0	0	χ_{132}	0	χ_{123}	0
	2	0	0	χ_{231}	0	0	0	χ_{213}	0	0
	3	0	χ_{321}	0	χ_{312}	0	0	0	0	0
Number of independent coefficients: 1										
$\chi_{123} = \chi_{132} = \chi_{231} = \chi_{213} = \chi_{321} = \chi_{312}$										

Supplementary Note 2: Electrical properties of the tellurium samples and the control materials

Supplementary Figure S1 shows the temperature dependence of the electrical resistivity for samples A and B with different chiralities. As shown in the figure, the temperature dependences of the resistivity for samples A and B are very similar, and their absolute values are close. Therefore, the possibility of a difference in the temperature dependence of the electrical resistivity as a reason for the sign reversal of the NCTE Hall effect is ruled out. The behaviour at high temperatures ($T > 200$ K) is quintessentially semiconducting, whereas the medium temperature region ($50 \text{ K} < T < 200 \text{ K}$) shows a flat temperature dependence due to the contribution of the impurity carriers. At low temperatures ($T < 50 \text{ K}$), the resistivity increases due to the freezing of the carriers and shows a typical behaviour of variable range hopping.

Figure S2a and S2b show the ordinary Hall effects for samples A and B measured at 300 K. The Hall coefficient is obtained by linear fitting of the experimental data up to 1 T, with $R_H = -6.88 \times 10^{-5} \text{ m}^3 \text{ C}^{-1}$ for sample A and $R_H = -1.30 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$ for sample B. Consequently, the carrier density is $n = 9.07 \times 10^{22} \text{ m}^{-3}$ for sample A and $n = 4.78 \times 10^{22} \text{ m}^{-3}$ for sample B. These values are nearly the same as the typical values reported for the Te samples⁵. Since both samples exhibit a negative Hall coefficient, we can conclude that electrons are the dominant carriers near room temperature. The deviation from linearity at high magnetic fields is likely caused by the effect of mixed contributions from multiple carriers.

In the main text, we demonstrate that the NCTE Hall effect is not observed in polycrystalline Bi. For additional information, the temperature dependence of the electrical resistivity of Bi and the ordinary Hall effect are shown in Supplementary Figures S1 and S3, respectively. Both the resistivity and the ordinary Hall response of Bi are rather small compared to Te. As a comparison when measuring samples of similar resistivity, we additionally measure the ordinary Hall and NCTE Hall effects for polycrystalline germanium (Ge) as shown in Supplementary Figure S4. In Ge, as in the case of Bi, we find no NCTE Hall response that varies with temperature gradient, in sharp contrast to the NCTE Hall response in Te. This result further confirms that the NCTE Hall effect is a phenomenon that requires inversion symmetry breaking.

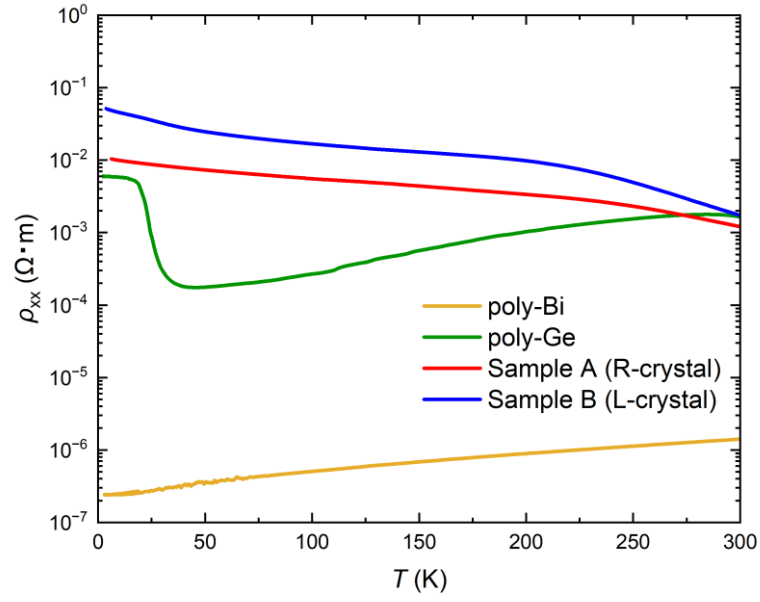


Figure S1 | Temperature dependence of the resistivity of Te, Bi, and Ge. The red and blue solid lines correspond to samples A (right-handed crystal) and B (left-handed crystal), respectively.

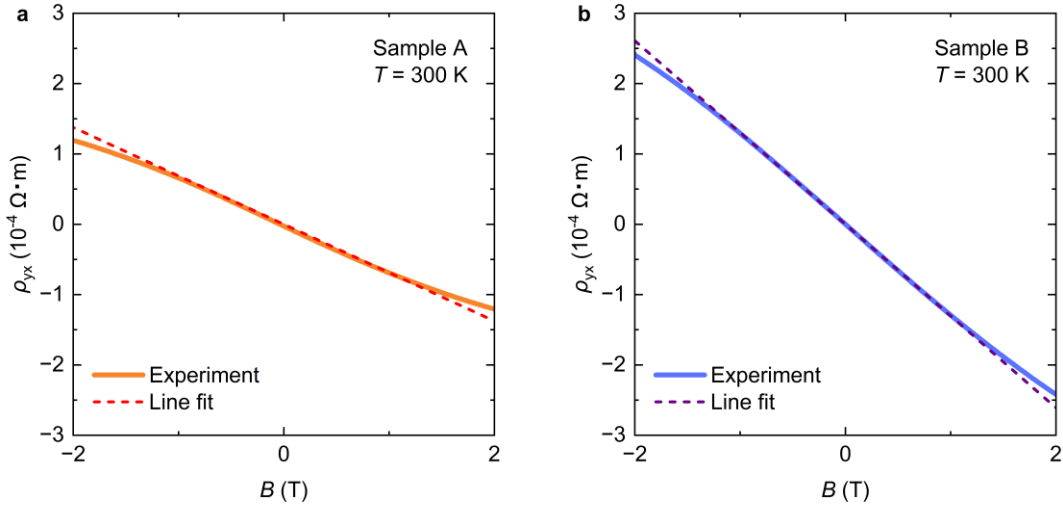


Figure S2 | Ordinary Hall resistivity at 300 K. **a**, Hall resistivity of sample A. The orange solid line and red dashed line represent the experimental data and the linear fitting curve for ± 1 T data, respectively. **b**, Hall resistivity of sample B. The blue solid line and purple dashed line represent the experimental data and the linear fitting curve for ± 1 T data, respectively.

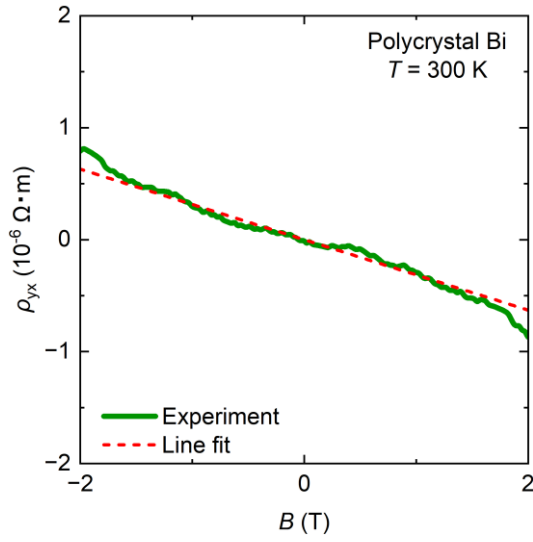


Figure S3 | The ordinary Hall resistivity of polycrystalline Bi. Red dashed line represents a linear fitting curve.

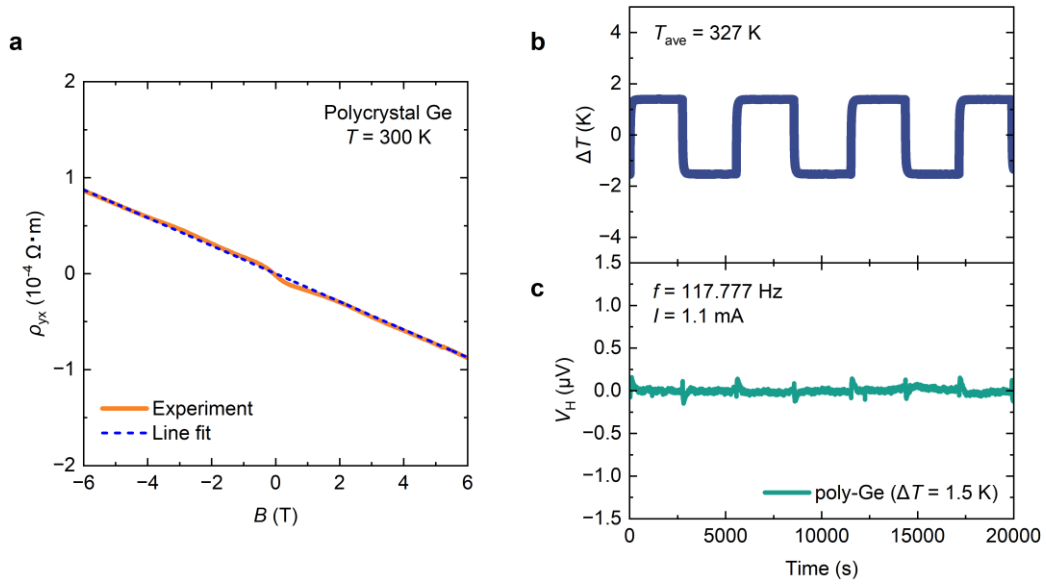


Figure S4 | Ordinary Hall and NCTE Hall effect measurement for Ge. **a**, The ordinary Hall resistivity of Ge. The orange solid line and blue dashed line represent the experimental data and the linear fitting curve for ± 5 T data, respectively. **b**, Time profile of the temperature difference (ΔT) in the sample. **c**, The NCTE Hall measurement for Ge polycrystalline. No clear signal oscillation is observed in Ge in contrast to Te.

Supplementary Note 3: Additional information about the experimental setup

(i) Elimination of the contamination of other nonlinear responses

As described in the Methods, in the present experimental setup, an AC electric field of frequency ω is given and the fundamental frequency component of the Hall signal is detected using a lock-in amplifier. Since second-order electric field responses would appear as frequency-doubled (or higher-harmonic) components, the second-order contributions are excluded in our setup. In principle, a third-order or higher response could appear as a contribution to the fundamental frequency component, e.g., $(I\sin(\omega t))^3 = I^3(3/4\sin(\omega t) - 1/4\sin(3\omega t))$. However, in such cases, the signal behaves nonlinearly with respect to the magnitude of the current. The signal observed in this study is clearly linear in both current and temperature gradient, and therefore the influence of higher-order response is considered negligible.

(ii) Ruling out longitudinal voltage contribution to the observed Hall voltage

Here, we discuss the possibility of false Hall signals being observed in Te due to contamination with longitudinal voltage signals.

(ii-a) Effect of the average temperature change

In our measurement, offset tuning is performed using the five-terminal method with $\Delta T = 0$. However, if the average temperature deviates during temperature modulation, the offset changes due to changes in the resistance distribution inside the sample, and a finite value may be mixed in as a false Hall signal. Supplementary Figure S5 shows the temperature dependence of the Hall voltage at $\Delta T = 0$ by using a Te sample

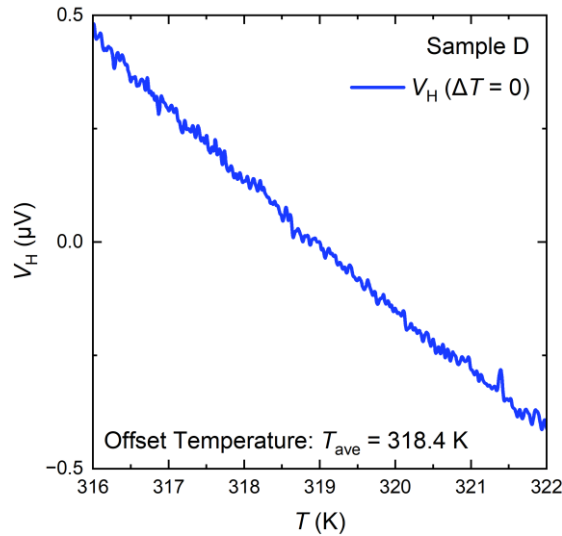


Figure S5 | Temperature dependence of the Hall signal at $\Delta T = 0$ condition. Zero-offset is set at $T = 318.4$ K.

(sample D) where the zero offset tuning is done at $T_{\text{ave}} = 318.4$ K. When the sample average temperature is shifted, the Hall voltage shows a linear change around the offset temperature. Estimating from Supplementary Figure S5, an offset change of ≈ 0.16 μV per 1 K deviation in T_{ave} would occur. Since the average temperature shift during temperature gradient inversion is controlled to about 0.1 K, the offset shift caused by this effect is estimated to be about 0.02 μV at most. This is clearly smaller than the observed NCTE Hall signal of about 1 μV . Therefore, it is difficult to explain the observed Hall signal by this offset shift due to the mismatch in T_{ave} .

(ii-b) Comparison with the longitudinal voltage during measurement

Even without a quantitative discussion, it is qualitatively expected that if the leakage of the longitudinal voltage component is the major contribution to the observed Hall signal, the waveforms of longitudinal voltage (V_L) and Hall voltage (V_H) are expected to be linked. To examine this possibility, the time profiles of simultaneous measurement of longitudinal and transverse voltages are shown in Supplementary Figure S6. As shown in Supplementary Figure S6b, the longitudinal voltage remains almost constant and uncorrelated with the temperature gradient change shown in Supplementary Figure S6a. This indicates that the average temperature of the sample remains nearly constant during the measurement, validating our assumption. The signal

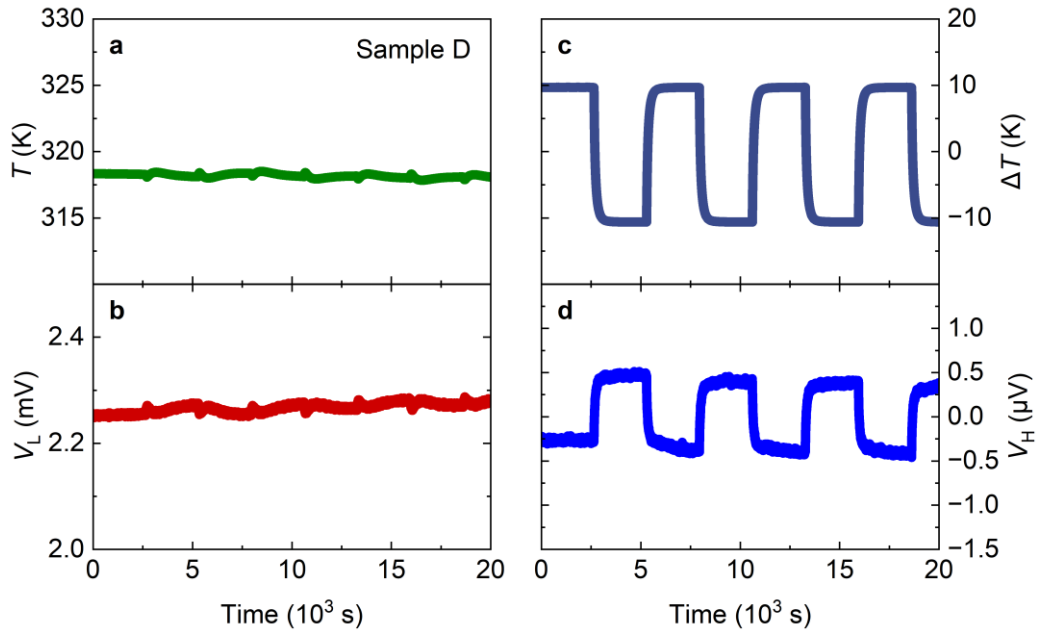


Figure S6 | Simultaneous longitudinal voltage and NCTE Hall measurements. **a**, Time profile of the average temperature of the sample D. **b**, longitudinal voltage (V_L) change. **c**, temperature difference (ΔT) change during measurement. **d**, observed Hall signal (V_H).

change in V_L is linked to the change in average temperature that occurs during the temperature reversal shown in Supplementary Figure S6c. On the other hand, the time profile of V_H has a distinctly different behaviour from that of V_L , suggesting that V_L contamination plays a negligible role in the observed V_H .

(iii) Reproducibility of the sign change upon reversing the crystal chirality

We additionally performed the NCTE Hall measurement using another enantiomeric pair of Te, sample C (R-handed) and sample D (left-handed), to check the reproducibility of the sign change upon reversing the crystal chirality. Supplementary Figure S7 shows the time profile of the NCTE Hall measurement on samples C and D. As shown in Supplementary Figure S7, measurements with different samples reproduced the relationship between the chirality and the sign of the Hall effect, confirming a definite correlation between the chirality and the sign of the Hall signal

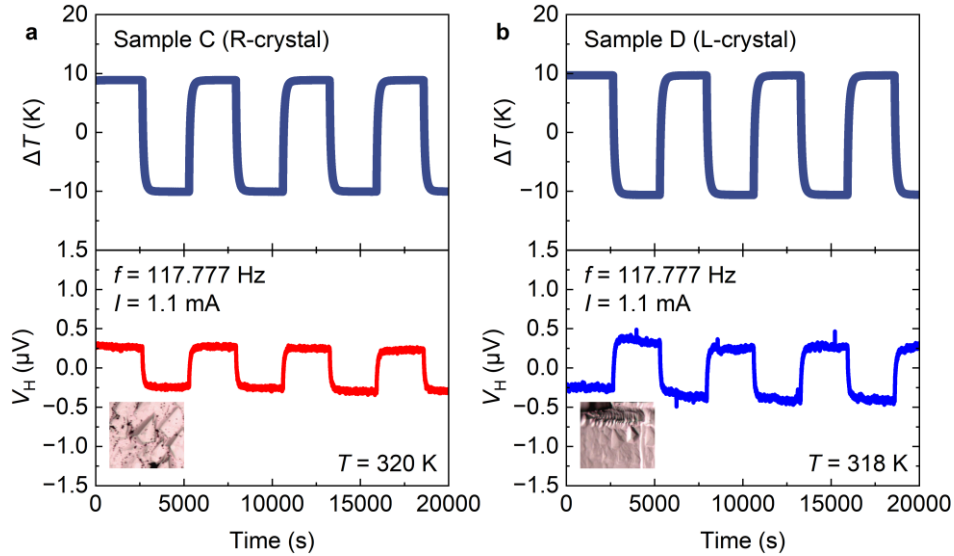


Figure S7 | Time profile of the NCTE Hall response of the other enantiomer pair. **a**, Right-handed crystal (sample C). **b**, Left-handed crystal (sample D). The insets are the pictures of the etch pit.

Supplementary Note 4: Microscopic theory of the NCTE Hall effect

According to microscopic theory⁶, the total NCTE Hall conductivity σ_{NCTE} can be described as the sum of contributions from the BCD, σ_z^{BC} , and orbital magnetisation, σ_z^{OM} ; these are defined as follows:

$$\sigma_z^{\text{BC}} = e^2 \tau \sum_{n,k} (\varepsilon_{nk} - \mu) \left(-\frac{\partial f}{\partial \varepsilon} \right)_{\varepsilon=\varepsilon_{nk}} \times \left[(\partial_{k_z} \varepsilon_{nk}) \Omega_n^z - \frac{1}{2} \{ (\partial_{k_x} \varepsilon_{nk}) \Omega_n^x + (\partial_{k_y} \varepsilon_{nk}) \Omega_n^y \} \right], \quad (\text{S9})$$

$$\sigma_z^{\text{OM}} = -e\tau \sum_{n,k} (\varepsilon_{nk} - \mu) \left(-\frac{\partial f}{\partial \varepsilon} \right)_{\varepsilon=\varepsilon_{nk}} \mathbf{\nabla}_k \cdot \mathbf{m}_{nk}^{\perp}, \quad (\text{S10})$$

where f and $\mathbf{\Omega}_n = \mathbf{\nabla}_k \times \mathbf{A}_n(\mathbf{k})$ are the Fermi distribution function and the Berry curvature with $\mathbf{A}_n(\mathbf{k}) = -i\langle n(\mathbf{k}) | \mathbf{\nabla}_k n(\mathbf{k}) \rangle$ as the Berry connection, respectively; and ε_{nk} and $|n(\mathbf{k})\rangle$ are the eigenenergy and eigenvector of the Hamiltonian. In addition, $\mathbf{m}_{nk}^{\perp} = \mathbf{m}_{nk} - m_{nk}^z \hat{\mathbf{e}}_z = (m_{nk}^x, m_{nk}^y, 0)$ is the orbital magnetic moment written by the interband Berry connection,

$$\mathbf{m}_{nk} = \frac{e}{2} \sum_m (\varepsilon_{mk} - \varepsilon_{nk}) \text{Im}[\mathbf{A}_{nm}(\mathbf{k}) \times \mathbf{A}_{mn}(\mathbf{k})], \quad (\text{S11})$$

$$\mathbf{A}_{mn}(\mathbf{k}) = -i\langle n(\mathbf{k}) | \mathbf{\nabla}_k m(\mathbf{k}) \rangle. \quad (\text{S12})$$

An important feature is that both contributions are proportional to the square of the electron charge, e^2 . Thus, the sign of the NCTE Hall effect is independent of the type of carrier. The band structure near the top of the valence band is approximately isotropic in Te, which is expected to suppress the contribution of the Berry curvature dipole σ_z^{BC} . Therefore, the contribution from the orbital magnetic moment σ_z^{OM} , which is predicted from microscopic theory, is very important^{3,6}. σ_z^{OM} can be interpreted as the Nernst effect due to orbital magnetisation caused by the applied current or the Hall effect due to orbital magnetisation caused by the temperature gradients.

Supplementary Note 5: Sample preparation

We provide the details of the single crystal growth used in our study. Single crystals of Te were produced via the physical vapour transport (PVT) method⁷. A mass of 0.36 g of bulk Te was vacuum-sealed in a quartz tube and heated in a three-zone electric furnace. The temperatures at the source side, T_1 , and on the crystal growth side, T_2 and T_3 , were adjusted following the time profile shown in Supplementary Figure S8a. A picture of the obtained crystal is shown in Supplementary Figure S8b. The typical size was ≈ 1 cm in the direction of the c -axis, with a thickness of 2–4 mm.

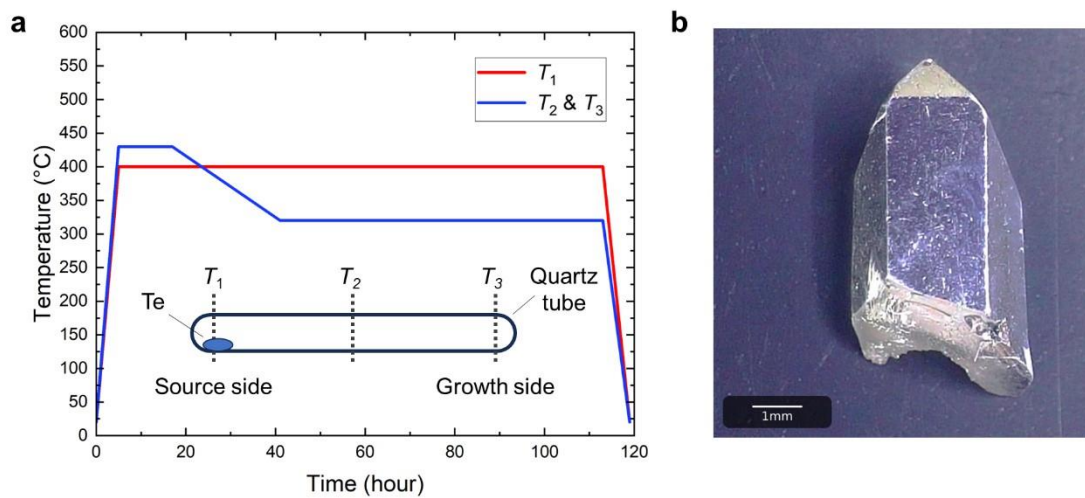


Figure S8 | Synthesis of the Te crystals. **a**, Time profile of Te synthesis via the PVT method. **b**, Obtained crystal of Te.

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