
Strong coupling of a microwave photon to an electron on helium

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Contents

S1 Electron-photon coupling energy	2
S2 Electron cluster properties up to $N = 4$	2
S3 Electron frequency calculations and FEM parameters	3
S4 Modeling resonator transmission spectra	4
S5 Estimating electron motional state frequencies from resonator transmission spectra	6
S6 Analytical model of electron motional state spectrum	7
S7 Purcell decay	9
S8 Classical helium vibrations	10
S9 Temperature-dependent observation of vacuum Rabi splitting	10
S10 Prospects for spin readout	10

S1 Electron-photon coupling energy

The coupling energy between the single electron motional state and the resonator photon is given by $\hbar g = |\vec{d}| V_{\text{zpf}}/\ell$, where $\vec{d} = -el_y\vec{y}$ is the dipole moment of the electron, $l_y = \sqrt{\hbar/2m_e\omega_e} \approx 36$ nm is the zero-point motion amplitude of electron in the $\vec{y}$ -direction (on resonance $\omega_e = \omega_r$), V_{zpf} is the zero point voltage fluctuation of the resonator and ℓ parametrizes the effective size of the region over which the microwave field is concentrated within the dot. We design a compact quantum dot, resulting in $\ell \approx 2.2$ μm at its center, as determined from FEM calculations. This value is slightly larger than the physical size of the dot due to electric field screening effects from surrounding electrodes (primarily the Bottom electrode), which partially shield the field generated by the resonator electrodes.

Leveraging the high kinetic-inductance of TiN, we design a high-impedance resonator composed of two arms formed by coplanar waveguide structures. Each arm has a center nanowire with a width of 175 nm and a gap of 5 μm to the adjacent ground plane. Each arm of the resonator has a length of 135 μm . At the open end, the resonator arms are terminated at the quantum dot, which is modeled as a capacitive coupler between two arms with capacitance $C_{\text{dot}} = 0.028$ fF, as determined from FEM. At the opposite end, the two arms are galvanically connected to each other and to an additional nanowire (“tail”), which allows a DC bias to be applied to the nanowire. We model the nanowire resonator as two symmetrically coupled lumped LC-circuits [1, 2], where each half has capacitance $C_r = 5.8$ fF and inductance $L_r = 85$ nH. This value is within 10% of the value calculated from a conformal mapping calculation. Our resonator supports two modes distinguished by their spatial voltage distributions: the *differential* mode, characterized by out-of-phase voltage fluctuations at the open ends of the arms, and the *common* mode, which has in-phase voltage fluctuations [2]. Due to the symmetry of the quantum dot geometry, only the differential mode produces an electric field capable of driving electron motion. The estimated resonator impedance for the differential mode is $Z = \sqrt{L_r/C_r} \approx 3.8$ k Ω , corresponding to a zero-point voltage fluctuation amplitude of $V_{\text{zpf}} = \sqrt{2\hbar\omega_r/C_r} \approx 40$ μV . This yields a designed single-electron coupling strength of $g/2\pi \approx 110$ MHz.

In the experiments the voltages are tuned to align the electron motion with the electric field of the resonators differential mode along the $\vec{y}$ -direction, thereby maximizing the coupling between the electron and resonator photons.

In our numerical calculations described below, we set the resonator tail inductance to $L_{\text{tail}} = 109$ nH to match the common mode frequency with the experimentally observed value of 3.8 GHz.

S2 Electron cluster properties up to $N = 4$

In this section, we present the details for modeling resonance frequency shifts $\Delta\omega_r$ shown in Fig.1 e in main text. The results provide an explanation for the smaller $\Delta\omega_r$ observed for $N = 1, \dots, 4$ electrons shown in Fig.1 e in main text compared to the single electron frequency shifts presented in Fig.2 in main text. This behavior arises because the motional frequencies of the electron clusters are far higher than the resonator frequency (>20 GHz) for the voltages in Fig.1 e in main text.

To model the resonance frequency shift due to N trapped electrons, we begin by extracting the electrostatic potential from FEM simulations and numerically solve for the electron configuration that minimizes the total potential energy, taking into account the repulsive Coulomb interaction between electrons (see Fig.1 d in main text) [3]. This is followed by formulating the coupled equations of motion in matrix form, which includes the resonator and two-dimensional electron motion, including charge-photon coupling (details in Ref. [1]). We numerically solve

for the coupled eigenmodes and eigenfrequencies using the `quantum_electron` python package [4]. Finally, the resonator frequency shift is calculated by subtracting the bare differential mode frequency,

$$\omega_r = \frac{1}{\sqrt{L_r C_r}}, \quad (\text{S1})$$

from the frequency of the eigenmode with the strongest cavity participation. The resulting shifts are shown as black horizontal lines in Fig.1 e in main text.

Fig. S4 b shows good agreement between the modeled and measured resonance frequency shift, especially for $V_r < 0.12$ V. For more positive V_r (gray shaded areas in Fig. S4), the total energy of the electron clusters exceeds the depth of the trapping potential (Fig. S5), potentially explaining the kink in the measured value of $\Delta\omega_r$ for $N = 1, \dots, 4$. Furthermore, Fig. S4 a shows that the eigenfrequency of the electron modes remains above 20 GHz for all values of V_r , which results in small measured frequency shifts $\Delta\omega_r/2\pi \leq 2$ MHz presented in Fig.1 e in main text. These results show that the classical equations of motion accurately model the electron dynamics in the experiment.

S3 Electron frequency calculations and FEM parameters

Finite element modeling (FEM) of the trapping potential is carried out using the technique outlined in the Methods section. Electron frequencies are extracted by solving the single electron 2D Schrödinger equation using the electrostatic trapping potential from a given dot voltage configuration. In several experiments, achieving the electron resonance condition was only possible when voltage asymmetries were applied to the Guard electrodes. This suggests the presence of a stray in-plane electric field at the location of the electron in the dot. In simulations, we account for this asymmetry by adding two terms in the FEM model to represent static compensating electric fields E_y and E_x . In addition to introducing compensating fields, adjustments are made to the other electrode voltages from their experimental values until the electron frequencies calculated from FEM simulations align well with those observed in experiments (Fig. S6). Notably, a Bottom electrode offset of $\sim +0.1$ V is needed in all adjusted voltage configurations to find good agreement between experimental and predicted FEM values. The $+0.1$ V adjustment is consistent with the difference in the metal work function of the niobium Bottom electrode and the top titanium nitride electrodes [5, 6]. Other deviations in the FEM parameters from the voltages used in the experiments likely arise due to fabrication abnormalities that slightly alter the dot electrode geometry from the simulated geometry, or due to screening from nearby charges. The FEM parameters used to generate the electron frequencies in Fig. S6a and Fig. S6b are listed in Table S1 and Table S2, respectively.

The disappearance of strong avoided crossing features (i.e. large frequency shifts $\Delta\omega_r$) in the upper left region of the $\omega_r/2\pi$ map shown in Fig.2 a in main text and Fig. S6a indicate the deviation of the electron susceptibility from a two-level model and may be attributed to excitations into higher-lying excited states. This behavior is qualitatively supported by our FEM simulation results. In Fig. S7 we show the transition frequency between the ground and first excited states ω_{01} (a), along with transition frequency between the first and second excited states ω_{12} (b) and the electron anharmonicity $\alpha_e = (\omega_{12} - \omega_{01})/2\pi$ (c) calculated using the voltages in Table S2. We find that for the voltage configurations $V_u < -4.45$ V and -2.04 V $< V_r < -2.03$ V both $\omega_{01} = \omega_r$ and $\omega_{12} = \omega_r$ contour lines exhibit complex shapes. In combination with observation that $\alpha_e = 0$ in the same region, this might result in a nontrivial and potentially complex electron response.

For the voltage configurations at which we perform the measurements in Fig.2 b, Fig.2 d, Fig.3, and Fig.4 in main text the predicted anharmonicities are large, and allow us to treat

our electron as a two level system.

S4 Modeling resonator transmission spectra

To model the experimental transmission spectrum (including effects of coupling to the electron), we start with the following Hamiltonian in the frame rotating with a probe tone at frequency $\omega_p/2\pi$ [7]:

$$\begin{aligned} \mathcal{H}/\hbar = & (\omega_r - \omega_p)a^\dagger a + (\omega_e - \omega_p)\frac{\sigma_z}{2} \\ & + g(a^\dagger\sigma^- + a\sigma^+) + i\sqrt{\kappa_1}(a_{\text{in},1}a^\dagger - a_{\text{in},1}^*a). \end{aligned} \quad (\text{S2})$$

Here, $\omega_r/2\pi$ is the bare resonator frequency, ω_e is the bare electron frequency, $a^\dagger, a$ are the raising and lowering operators of the resonator, $\sigma^\pm = \sigma_x \pm i\sigma_y$ are the Pauli matrices raising and lowering a electron excitation respectively. Further, $a_{\text{in},1}$ is the photon flux of the input field (applied at port 1 of the resonator) and $\kappa_{1,2}/2\pi$ are the input and output coupling rates for the resonator. Note that this Hamiltonian approximates the electron as a two level system.

To calculate transmission spectra from this Hamiltonian we use the Heisenberg equations of motion for a and σ^- , which read

$$\dot{a} = i(\omega_r - \omega_p)[a^\dagger a, a] + ig\sigma^- [a^\dagger, a] - a_{\text{in},1}\sqrt{\kappa_1} [a^\dagger, a] \quad (\text{S3})$$

$$\dot{\sigma}^- = \frac{i}{2}(\omega_e - \omega_p) [\sigma_z, \sigma^-] + ig [\sigma^+, \sigma^-] a \quad (\text{S4})$$

For a compact notation we will now use the notation $\Delta_{ab} = \omega_a - \omega_b$. With the commutators $[\sigma_z, \sigma^-] = -2\sigma^-$, $[\sigma^+, \sigma^-] = \sigma_z$, $[a^\dagger a, a] = -a$ and $[a, a^\dagger] = 1$, and adding in dissipative terms, we can simplify

$$\dot{a} = -\left(\frac{\kappa_{\text{tot}}}{2} + i\Delta_{rp}\right)a - ig\sigma^- + a_{\text{in},1}\sqrt{\kappa_1} \quad (\text{S5})$$

$$\dot{\sigma}^- = -(\Gamma_2 + i\Delta_{ep})\sigma^- + ig\sigma_z a \quad (\text{S6})$$

Here $\Gamma_2 = \gamma_1/2 + \Gamma_\varphi$ is the electron decoherence rate. For transmission spectra, we are only interested in the steady-state values of a and σ^- , so we may set $\dot{a} = 0$ and $\dot{\sigma}^- = 0$. Furthermore, we will neglect resonator-qubit correlations by approximating $\langle\sigma_z a\rangle \approx \langle\sigma_z\rangle\langle a\rangle$, valid for the weak probe conditions [8]. Under these approximations we have

$$-\left(\frac{\kappa_{\text{tot}}}{2} + i\Delta_{rp}\right)\langle a\rangle - ig\langle\sigma^- \rangle + a_{\text{in},1}\sqrt{\kappa_1} = 0 \quad (\text{S7})$$

$$-(\Gamma_2 + i\Delta_{ep})\langle\sigma^- \rangle + ig\langle\sigma_z\rangle\langle a\rangle = 0 \quad (\text{S8})$$

Solving this set of equations for $\langle a\rangle$ and $\langle\sigma^- \rangle$ gives

$$\langle\sigma^- \rangle = \frac{g\langle\sigma_z\rangle}{\Delta_{ep} - i\Gamma_2}\langle a\rangle \quad (\text{S9})$$

$$\langle a\rangle = \frac{a_{\text{in},1}\sqrt{\kappa_1}}{\kappa_{\text{tot}}/2 + i\Delta_{rp} + i\chi(\omega_e)\langle\sigma_z\rangle} \quad (\text{S10})$$

Eq. (S10) describes the intracavity field including coupling to the electron via the susceptibility $\chi(\omega_e)$:

$$\chi(\omega_e) = \frac{g^2}{\Delta_{ep} - i\Gamma_2}. \quad (\text{S11})$$

To translate this into a transmission spectrum we use the standard input-output theory of a two-sided cavity (see Fig. S8):

$$a_{\text{in},1} + a_{\text{out},1} = \sqrt{\kappa_1} \langle a \rangle \quad (\text{S12})$$

$$a_{\text{in},2} + a_{\text{out},2} = \sqrt{\kappa_2} \langle a \rangle \quad (\text{S13})$$

Since experimentally, we do not apply a probe tone at port 2 ($a_{\text{in},2} = 0$) these equations simplify to

$$a_{\text{out},1} = \sqrt{\kappa_1} \langle a \rangle - a_{\text{in},1} \quad (\text{S14})$$

$$a_{\text{out},2} = \sqrt{\kappa_2} \langle a \rangle. \quad (\text{S15})$$

These equations combined with Eq. (S10) can be used to describe the reflection and transmission spectrum:

$$S_{11} = -1 + \frac{\kappa_1}{\kappa_{\text{tot}}/2 + i\Delta_{rp} + i\chi\langle\sigma_z\rangle} \quad (\text{S16})$$

$$S_{21} = \frac{a_{\text{out},2}}{a_{\text{in},1}} = \frac{\sqrt{\kappa_1\kappa_2}}{\kappa_{\text{tot}}/2 + i\Delta_{rp} + i\chi\langle\sigma_z\rangle}. \quad (\text{S17})$$

These results are consistent with Refs. [9, 7].

Origin of the asymmetric transmission spectrum

The experimentally observed transmission spectrum is asymmetric (see Fig. S9), which generally originates from impedance mismatches [10], coupling to nearby electromagnetic modes [11] or direct input-output cross-talk [12]. Regardless of the exact origin of the asymmetry, we follow the theoretical description of Ref. [13] to incorporate crosstalk that bypasses the resonator, see Fig. S8. This background crosstalk is given by a scattering matrix

$$S = e^{i\zeta} \begin{pmatrix} \sqrt{1-T}e^{i\theta} & -i\sqrt{T} \\ -i\sqrt{T} & \sqrt{1-T}e^{-i\theta} \end{pmatrix}, \quad (\text{S18})$$

where the diagonal elements are reflections from each port, and off-diagonal elements are the transmission crosstalk. We now modify S_{21} by adding the transmission crosstalk to Eq. (S15):

$$a_{\text{out},2} = -i\sqrt{T}e^{i\zeta}a_{\text{in},1} + \sqrt{\kappa_2}\langle a \rangle. \quad (\text{S19})$$

This results in an additional frequency-independent term in S_{21} ,

$$\begin{aligned} S_{21} &= S_{21}^{\text{parasitic}} + S_{21}^{\text{res}} \\ &= -i\sqrt{T}e^{i\zeta} + \frac{\sqrt{\kappa_1\kappa_2}}{\kappa_{\text{tot}}/2 + i\Delta_{rp} + i\chi\langle\sigma_z\rangle}, \end{aligned} \quad (\text{S20})$$

and it is the interference of these two terms that explains the asymmetric transmission spectrum [12].

Correcting for interference from background microwave transmission

While Eq. (S20) captures the essence of the transmission spectrum, there is a substantial deviation between fit and experimental data at higher probe frequencies (see Fig. S9 a), which complicates fitting to electron parameters g and Γ_2 when $\omega_e \approx \omega_r$. This suggests that the experimental data consists of three components: S_{21}^{res} , $S_{21}^{\text{parasitic}}$ and additional S_{21}^{other} , where the latter is due to additional transmission not captured by T and ζ . Crucially, S_{21}^{other} does not depend on voltage. To facilitate fitting of nearly resonant transmission spectra, we present a method to eliminate these additional transmission effects from our experimental data is similar to the background subtraction method of Ref. [14]:

1. For a far detuned ($\omega_e \gg \omega_r$) experimental transmission spectrum S_{21}^{exp} , fit to Eq. (S20) to determine S_{21}^{res} and $S_{21}^{\text{parasitic}}$ by fitting only to the data near the peak frequency.
2. Determine the additional parasitic component S_{21}^{other} from the fit residuals

$$S_{21}^{\text{other}} = S_{21}^{\text{exp}} - S_{21}^{\text{res}} - S_{21}^{\text{parasitic}} \quad (\text{S21})$$

3. Because we assume S_{21}^{other} and $S_{21}^{\text{parasitic}}$ do not depend on voltage, we may subtract $S_{21}^{\text{parasitic}} + S_{21}^{\text{other}}$ from measured transmission spectra at arbitrary voltage, which leaves only the information of the resonator-electron subsystem:

$$S_{21}^{\text{res}} = S_{21}^{\text{exp}} - S_{21}^{\text{parasitic}} - S_{21}^{\text{other}}. \quad (\text{S22})$$

Fig. S9 c demonstrates this compensation method for an off-resonant spectrum ($\omega_e \gg \omega_r$, blue) and nearly resonant spectrum ($\omega_e \approx \omega_r$, orange). The nearly resonant spectrum shows the characteristic double peak vacuum Rabi splitting with peaks of approximately equal height, suggesting that the parasitic transmission was effectively removed and is indeed voltage independent. The raw data (Fig. S9 b) and compensated data both fit well to a model with parameters $g/2\pi = 118$ MHz and $\Gamma_2/2\pi = 75$ MHz, which further suggests that the compensation method does not introduce a bias when determining g and Γ_2 .

S5 Estimating electron motional state frequencies from resonator transmission spectra

Besides direct measurement with two-tone spectroscopy, the electron motional state frequency can also be estimated from the measured resonator frequency ω_r . Here, we explain this method, which is used for Fig.3 b in the main text.

The resonator frequency shift in the off-resonant regime is given by the real part of the electron susceptibility [9], as defined in Eq. (S11):

$$\Delta\omega_r = \text{Re}(\chi\langle\sigma_z\rangle) = \frac{g^2\Delta_{er}\langle\sigma_z\rangle}{\Delta_{er}^2 + \Gamma_2^2}. \quad (\text{S23})$$

When the electron-resonator detuning Δ_{er} is large relative to the electron linewidth, and if the electron remains in its ground state, the measured resonator frequency shift $\Delta\omega_r$ is approximated by $\Delta\omega_r = g^2/\Delta_{er}$. Thus, the inferred electron frequency is given by

$$\omega_e = \omega_r + g^2/\Delta\omega_r, \quad (\text{S24})$$

where $\omega_r/2\pi$ is the bare resonator frequency uncoupled from the electron. The electron frequency calculated from the resonator spectroscopy measurement is displayed in Fig.3 b in the

main text. For this calculation, we find the best agreement between the two-tone spectroscopy data and the extracted electron frequency from resonator spectroscopy for $g/2\pi = 110\text{MHz}$. The bare resonator frequency ω_r is determined with an empty dot and the model further assumes that only two electron states couple to the resonator. For most voltage configurations, the anharmonicity of the system is high (Fig. S10) and justifies this use of a two-state model for the electron used to produce the fit from the spectroscopy data in Fig.3 b in the main text.

In Fig.3 b in the main text we compare the estimate using Eq. (S24) with two-tone spectroscopy, which show good agreement. This lends further credibility to our use of a two-level approximation method for Fig.3 b in the main text.

S6 Analytical model of electron motional state spectrum

The previous section described how to accurately model the measured electron frequency curve shown in Fig.3 b in the main text using FEM. To gain further understanding about the experimental electrostatic potential and the trapped electron, we study a simple model with three ingredients: (a) a harmonic confinement (b) a constant electric field (c) an anharmonic term. We show that the combination of these three ingredients qualitatively modifies the voltage dependence from $\omega_e \propto \sqrt{V}$ to what is observed in the experiment. It also reproduces the minimum frequency observed in the experiment.

The simple one dimensional electrostatic potential that reproduces the experimental data is given by

$$U = -eV(y) = a_1 y^2 + a_2 y^4 - eE_y y, \quad (\text{S25})$$

where E_y is the in-plane electric field, a_1 is the harmonic term, and a_2 contains the anharmonicity. From the FEM model we have observed that when all other electrodes are biased appropriately, the Unload electrode sweeps the harmonic component $a_1 \propto V_u$ through 0. To calculate the effect of all terms on the effective motional frequency, we differentiate U twice with respect to y :

$$\tilde{\omega}_e^2 = \frac{e}{m_e} \frac{\partial^2 U}{\partial y^2} = \frac{2a_1}{m_e} + \frac{12a_2}{m_e} y_0^2, \quad (\text{S26})$$

where y_0 is the coordinate of the potential minimum.

In the absence of anharmonicity ($a_2 = 0$), we retrieve $\tilde{\omega}_e^2 = 2a_1/m_e$ and since a_1 varies linearly with the applied electrode voltage, $\tilde{\omega}_e \propto \sqrt{V}$ as shown by the blue curve in Fig. S11.

If the anharmonicity is non-zero, we must calculate y_0 by differentiating the electrostatic potential (S25) once with respect to y :

$$\left. \frac{\partial U}{\partial y} \right|_{y=y_0} = 2a_1 y_0 + 4a_2 y_0^3 - eE_y = 0. \quad (\text{S27})$$

The resulting equation for y_0 is

$$y_0^3 + \frac{a_1}{2a_2} y_0 - \frac{eE_y}{4a_2} = 0, \quad (\text{S28})$$

which is of the form $x^3 + px + q = 0$, and is also known as a depressed cubic equation. From inspection, we see that

$$p = \frac{a_1}{2a_2} \quad (\text{S29})$$

$$q = -\frac{eE_y}{4a_2} \quad (\text{S30})$$

$$\frac{p}{q} = -\frac{2a_1}{eE_y}. \quad (\text{S31})$$

The exact solution to this equation can be classified according to the sign of the determinant D , which is defined as

$$D = -(4p^3 + 27q^2) \quad (\text{S32})$$

- If $D > 0$ the cubic has three distinct real solutions. This happens if the potential has a clear double well shape (the third solution is the bump in between the wells), for example. In practice, this happens for negative anharmonicity $a_2 < 0$ and for small E_y .
- If $D < 0$, the cubic has one real solution and two complex conjugate pair solutions. This happens if the potential has one clear minimum, either because $a_2 > 0$ or because E_y is large (in this case the sign of a_2 is irrelevant).

For now let us focus on the second case, where there is one *real* solution.

Exact analytical case

The exact case is analytically tractable and Cardano's formula prescribes a solution. The potential minimum location is given by

$$y_0 = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right)^{\frac{1}{3}} + \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right)^{\frac{1}{3}} \quad (\text{S33})$$

$$= \left(\frac{eE_y}{8a_2} + \sqrt{\frac{-D}{108}} \right)^{\frac{1}{3}} + \left(\frac{eE_y}{8a_2} - \sqrt{\frac{-D}{108}} \right)^{\frac{1}{3}} \quad (\text{S34})$$

In evaluating this expression, we discard any complex expressions from taking the cube root, since we are interested in real values of y_0 only.

Estimate of the minimum electron frequency

To estimate the minimum electron frequency predicted by this model, we expand the square root in Eq. (S33) to first order in the small quantity p/q , which is valid because near the minimum of the electron frequency curve $p/q \propto a_1/E_y \ll 1$. This yields:

$$y_0 \approx \left[\frac{p^3}{27q} \right]^{\frac{1}{3}} + \left[-q \left(1 + \frac{p^3}{27q^2} \right) \right]^{\frac{1}{3}} \quad (\text{S35})$$

$$\approx -q^{\frac{1}{3}} + \frac{q^{\frac{2}{3}}}{3} \left(\frac{p}{q} \right) - \frac{p q^{\frac{1}{3}}}{3 \cdot 27} \left(\frac{p}{q} \right)^2, \quad (\text{S36})$$

which is consistent with the zero-th order solution $y_0 = -q^{1/3}$, obtained by setting $p = 0$ in Eq. (S28). Plugging this value for y_0 into Eq. (S26) gives an estimate for the minimum frequency due to in-plane fields

$$\tilde{\omega}_{\min}^2 \approx \frac{12a_2}{m_e} \left(q^{\frac{2}{3}} - \frac{2p}{3} \right) \approx \frac{12a_2^{\frac{1}{3}}}{m_e} \left(\frac{eE_y}{4} \right)^{\frac{2}{3}}, \quad (\text{S37})$$

where in the last simplification we have used the definition of q . Note that if either $E_y = 0$ or $a_2 = 0$, $\omega_{\min} = 0$.

In Fig. S11 we show that the minimum electron frequency increases for larger in-plane fields E_y , and for very large E_y a crossing with the resonator becomes impossible. The

experimentally observed minimum electron frequency of approximately 6 GHz corresponds to a moderate value of E_y . This model qualitatively reproduces the main features of the electron frequency curve observed in the experiment, namely the deviation of a square-root dependence and the observation of a nonzero minimum frequency. It also provides further evidence of an anharmonic trapping potential *and* the presence of an in-plane field $E_y \neq 0$.

S7 Purcell decay

Decay through the resonator

The Purcell decay through the resonator depends on the coupling strength g , resonator linewidth κ and electron-resonator detuning $\Delta_{er} = \omega_e - \omega_r$ [15, 16, 17]:

$$\gamma_1 = \frac{1}{T_1} = \frac{g^2 \kappa}{\Delta_{er}^2 + (\kappa/2)^2} \quad (\text{S38})$$

There are two limiting cases of interest. First, on resonance ($\omega_e = \omega_r$) $\gamma_1 = 4g^2/\kappa$ resulting in a sharp drop in T_1 . Secondly if $\Delta_{er} \gg \kappa$, $\gamma_1 = (g/\Delta)^2 \kappa$.

We numerically evaluate Eq. (S38) in Fig. S12 for $g/2\pi = 110$ MHz, $\kappa/2\pi = 23$ MHz and $\omega_r/2\pi = 7.163$ GHz. The predicted T_1 values drop below 1 μ s for $\Delta_{er} < 1.1$ GHz and the agreement between experiment and theory suggests that T_1 is limited by Purcell decay for such small Δ_{er} . However, the discrepancy for $\Delta_{er} > 1.1$ GHz suggests there are other mechanisms limiting the measured T_1 for large values of detuning.

Decay through other bias electrodes

We calculate the T_1 contribution due to a single dot electrode from the circuit model in Fig. S13 a. This model assumes the electron couples capacitively to a bias electrode with capacitance C_c , and has additional capacitance to ground, or other electrodes specified by C_{other} . As in the experiment, the bias electrode also contains an on-chip filter and is connected to a load impedance $Z_0 = 50 \Omega$. Similar to the calculation of a Purcell effect in superconducting qubits [18], we calculate the decay rate through the bias electrode via the effective bias electrode admittance Y_{eq} (see Fig. S13 a):

$$\gamma_1 = \frac{1}{T_1} = \frac{\text{Re}(Y_{\text{eq}}(\omega_e))}{C_{\text{other}}}, \quad (\text{S39})$$

where we neglect a renormalization of the capacitance C_{other} due to the imaginary part of Y_{eq} . After expressing Y_{eq} in terms of circuit parameters, we find

$$\gamma_1 = \omega_e \frac{C_c}{C_{\text{other}}} \frac{\omega_e Z_0 C_c}{(1 - \omega_e^2 L_f (C_f + C_c))^2 + (\omega_e Z_0 (C_f + C_c))^2}, \quad (\text{S40})$$

where $0 \leq C_c/C_{\text{other}} \leq 1$. It is clear that γ_1 depends sensitively on C_c , so it is important to determine the value of C_c accurately.

We determine a reasonable value for the capacitance C_c following an argument presented in Ref. [2]. Suppose the electron is confined at the center of a one-dimensional harmonic potential with confinement energy $\frac{1}{2}m_e\omega_e^2 y^2$. A small voltage ΔV on the bias electrode produces an electric field E at the location of the electron which changes its position by Δy . Newton's second law states

$$m_e\omega_e^2 \Delta y = eE = e\Delta V \frac{\partial \alpha}{\partial y}, \quad (\text{S41})$$

where α is the coupling constant of the bias electrode at the electron's location. From this we find

$$\Delta y = \frac{e\Delta V}{m_e\omega_e^2} \frac{\partial\alpha}{\partial y}. \quad (\text{S42})$$

The induced charge due to the change in electron position is given by

$$\Delta q = e \frac{\partial\alpha}{\partial y} \Delta y = \frac{e^2}{m_e\omega_e^2} \left(\frac{\partial\alpha}{\partial y} \right)^2 \Delta V. \quad (\text{S43})$$

The capacitance of an electron to a bias electrode is therefore

$$C_c = \frac{\Delta q}{\Delta V} = \frac{e^2}{m_e\omega_e^2} \left(\frac{\partial\alpha}{\partial y} \right)^2. \quad (\text{S44})$$

In principle, Eq. (S44) applies to arbitrary bias electrodes (including the microwave resonator), however Eq. (S44) is only valid for resonator bias electrode if $\omega_e \gg \omega_r$, because the coupling to the resonator modifies Newton's second law (Eq. (S41)) [2].

From finite element modeling we find that the Guard electrodes have the dominant capacitance with $\partial\alpha/\partial y \approx 0.03 \mu\text{m}^{-1}$, which is small compared with $\partial\alpha/\partial y \approx 0.46 \mu\text{m}^{-1}$ for each resonator electrode. The Bottom and Unload electrodes are even less sensitive with $\partial\alpha/\partial y \approx 0$, suggesting the Guards are the only relevant bias electrodes to consider. With a filter inductance of $L_f = 12 \text{ nH}$ and capacitance of $C_f = 0.8 \text{ pF}$, the resulting electron capacitance and T_1 contribution (conservatively assuming $C_c/C_{\text{other}} = 1$) are shown in Fig. S13 b,c. The predicted $T_1 > 1 \text{ ms}$ for $\omega_e/2\pi > 3 \text{ GHz}$, with a slight dip in predicted T_1 due to a filter resonance at $\omega = 1/\sqrt{L_f C_f} \approx 2\pi \times 2 \text{ GHz}$. The coupling parameter $\partial\alpha/\partial y$ and on-chip filters with lower cut-off frequencies $1/\sqrt{C_f L_f}$ should be optimized in future device designs to reduce the microwave leakage through dc bias electrodes.

S8 Classical helium vibrations

Fig. S14 a shows the noise spectral density of vibrations of our cryostat acquired using a geophone when the active stabilization is on and off. Although the low-frequency noise spectral density increases by an order of magnitude when the active stabilization is turned off, we observe no change in the measured two-tone linewidth (Fig. S14 b).

S9 Temperature-dependent observation of vacuum Rabi splitting

We see experimentally that the prominence of the vacuum Rabi splitting we observe in resonator spectroscopy measurements improves as both the temperature decreases (Fig. S15), and as the negative offset applied to the dot electrodes increases (Fig. S16). The temperature-dependent manifestation of this vacuum Rabi splitting is consistent with coupling of the trapped electron to a thermally activated bath, which could originate from thermal fluctuations of the helium, or from a collection of additional charges in the vicinity of the dot.

S10 Prospects for spin readout

It is natural to ask what the results in this manuscript imply for coupling to the electron on helium spin state [19]. One way to couple a single spin to a microwave resonator is

by applying a magnetic field gradient to hybridize the charge motion and spin degrees of freedom in a manner analogous to that which has been achieved previously in semiconducting quantum dots [20, 21]. In addition, this approach has been studied theoretically for both semiconducting quantum dots, as well as for electrons on helium [15, 22] and neon [23] and represents a promising path toward achieving electron spin readout.

An experimental demonstration of spin-photon coupling involves sweeping the spin qubit frequency through the microwave resonator frequency while the charge qubit frequency is detuned off-resonance to isolate the spin state from charge noise. Below, we show this detuning dilutes the spin-photon coupling by a factor ($g_{cs}/\Delta_{cs} < 1$), but the suppression of charge noise scales as $(g_{cs}/\Delta_{cs})^2$ [24, 23], which implies a large Δ_{cs} is generally preferable.

The spin-photon coupling strength g_s can be written as

$$g_s = g_c \frac{\mu_B a_x}{\hbar} \frac{\partial B_z}{\partial x} \frac{\sqrt{2}}{\omega_c(1 - \omega_s^2/\omega_c^2)} \quad (\text{S45})$$

$$= g_c \frac{\mu_B a_x}{\hbar} \frac{\partial B_z}{\partial x} \frac{\sqrt{2}\omega_c}{(\omega_c - \omega_s)(\omega_c + \omega_s)} \quad (\text{S46})$$

$$\approx g_c \frac{\mu_B a_x}{\sqrt{2}\hbar} \frac{\partial B_z}{\partial x} \frac{1}{\omega_c - \omega_s} \quad (\text{S47})$$

$$= g_c \frac{g_{cs}}{\Delta_{cs}}, \quad (\text{S48})$$

where μ_B is the Bohr magneton, $g_c/2\pi$ is the charge-photon coupling strength, $a_x = (\hbar/m_e\omega_c)^{1/2} \approx 50$ nm is the spatial extent of the charge ground state wavefunction, $\omega_c/2\pi$ is the charge qubit frequency, $\omega_s/2\pi$ is the spin qubit frequency set by the Zeeman splitting, and $\partial B_z/\partial x$ is the magnetic field gradient.

Eq. (S48) shows that the spin-photon coupling is proportional to the charge-photon coupling, and is reduced by a numerical factor g_{cs}/Δ_{cs} , where

$$\frac{g_{cs}}{2\pi} = \frac{\mu_B a_x}{\sqrt{2}\hbar} \frac{\partial B_z}{\partial x} \quad (\text{S49})$$

Note that Eq. (S49) can also be derived from the theory developed for semiconducting double quantum dots [25, 23], by setting $2t_c/\hbar = \omega_c$ and taking the approximation $g_c/\Delta_{cs} \ll 1$. Using $\partial B_z/\partial x = 0.1$ mT/nm for the field gradient (see e.g. Refs. [15] and [23]), the spin-charge coupling evaluates to $g_{cs}/2\pi \approx 50$ MHz. From the value of $g_c/2\pi \simeq 120$ MHz measured in this work, we infer that the charge qubit can be detuned by $\Delta_{cs}/2\pi \approx 2$ GHz while still achieving spin coupling rate of $g_s/2\pi \approx 3$ MHz. As such, microsecond spin-qubit readout should be within reach.

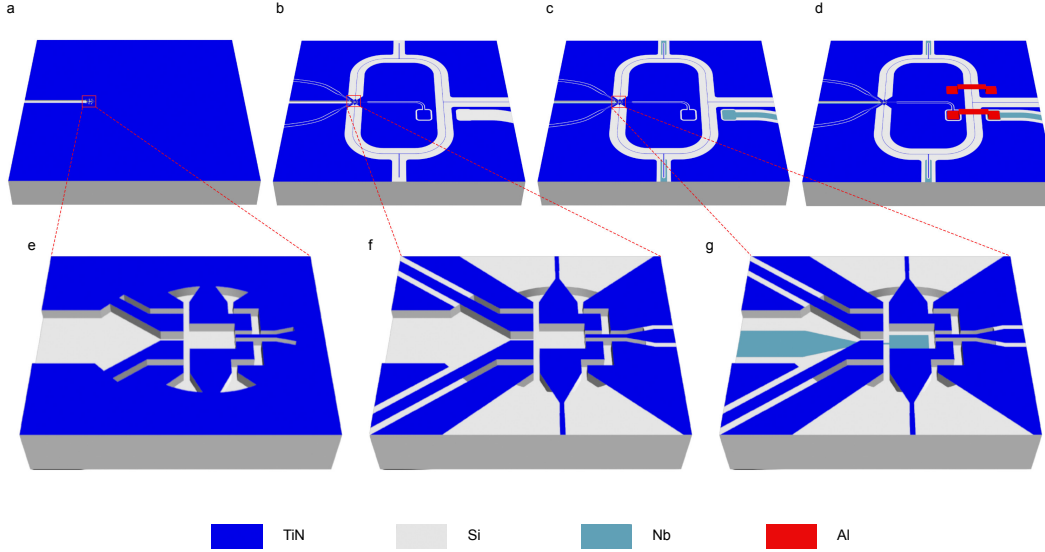


Figure S1: Process flow of device fabrication **a**, Si trenches with TiN ground plane in the electron dot region. **b**, Patterning CPW resonator, Guard electrodes, and Unload electrode on TiN ground plane. **c**, Adding Nb electrodes at the bottom of the Si trench and in TiN etched region. **d**, Adding Al airbridges (red) to make the connection between TiN Unload electrode (blue) and Nb DC bias line (light blue), as well as connection between two separate TiN ground planes. **e**, **f**, **g** are zoom-in images of **a**, **b**, **c**.

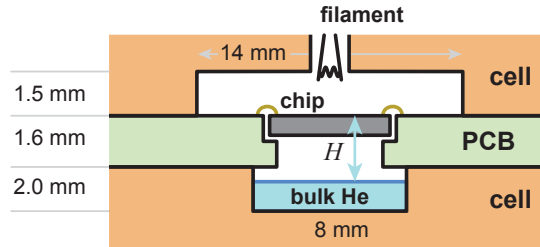


Figure S2: Device enclosure. Cross-sectional schematic view of the chip integrated within the hermetically sealed sample enclosure indicating the bulk helium reservoir region and tungsten filament used for electron emission.

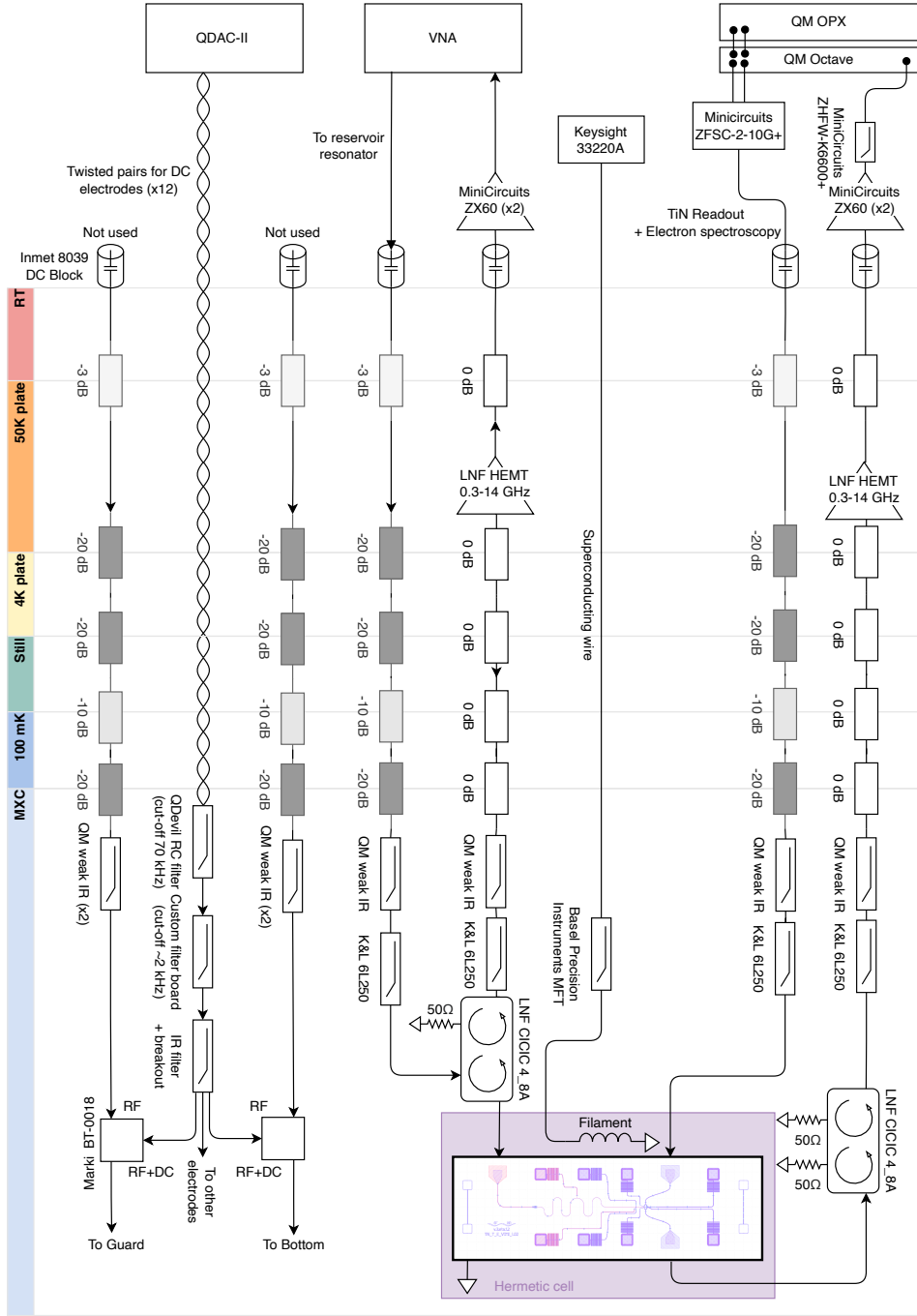


Figure S3: Dilution refrigerator wiring diagram and experimental instrumentation. Refrigerator stages are color coded from room temperature (red, RT) to the mixing chamber plate (blue, MXC). The device is located inside a hermetic sample enclosure. For complete details see Methods.

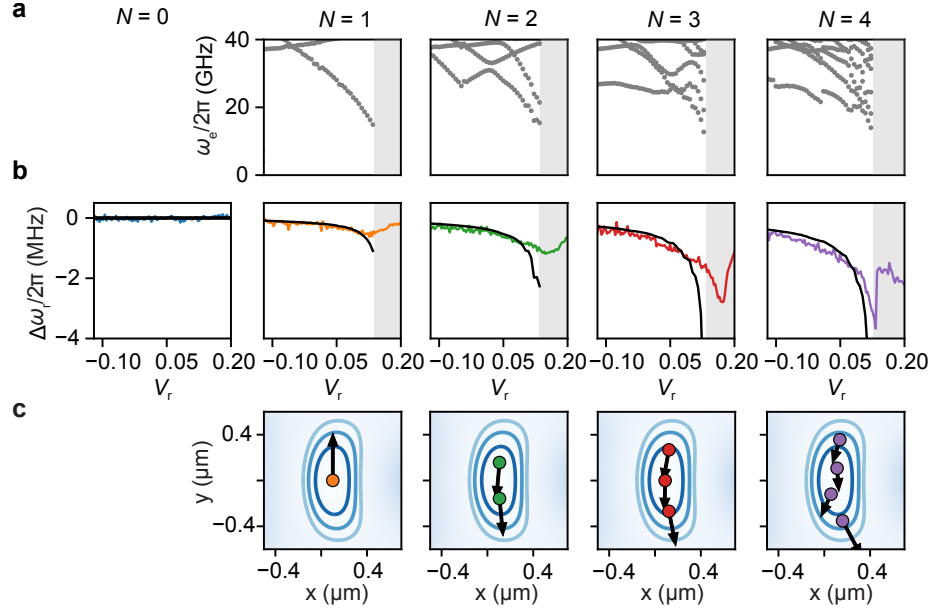


Figure S4: Resonator voltage dependence for electron clusters of various sizes. **a**, Top row: in-plane motion frequencies for electron clusters consisting of $N = 1$ to $N = 4$ electrons. Abrupt changes in the electron frequency ω_e can be observed due to rearrangement of the electrons within a cluster. **b**, Bottom row: measured (colored data) and modeled resonance frequency shift (black lines) show good agreement. The gray shaded voltage regions indicate that the simulated potential can no longer hold the electron cluster, potentially explaining the kinks in the measured resonator frequency shift $\Delta\omega_r$. **c**, Electron configurations at $V_r = 0.08$ V for $N = 1$ through $N = 4$ with strongest coupled eigenvector (black arrows) pointing predominantly in the y -direction.

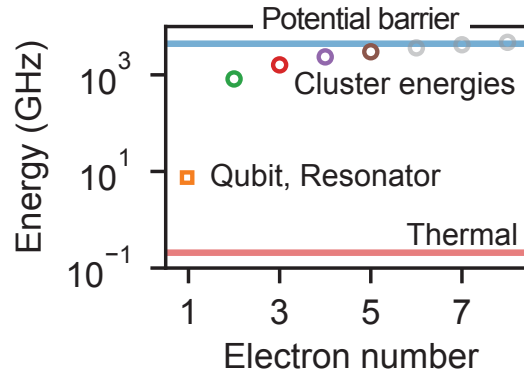


Figure S5: Relative energy scales of the dot-confined electron system. The depth of the electrostatic trap of the quantum dot ($\sim 10^3$ GHz) exceeds both the electron motional state frequency ($\simeq 4-8$ GHz) of $N \leq 4$ electrons, the resonator energy, and the thermal energy ($\simeq 0.2$ GHz) by several orders of magnitude. Therefore, quantum tunneling does not play a role and electron confinement times are practically infinite.

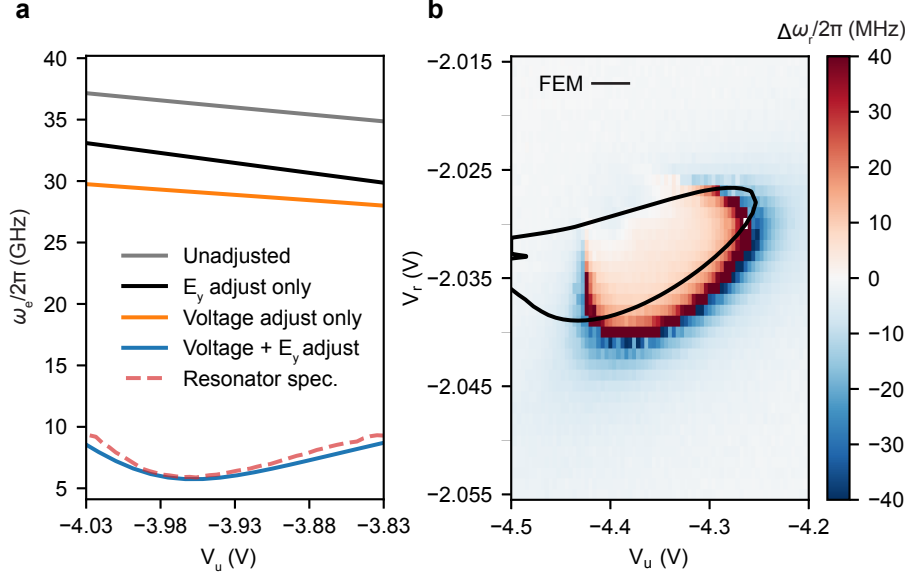


Figure S6: Electron motional transitions extracted from FEM. **a**, Simulated electron frequencies for experimental voltages and adjusted FEM parameters. Adjustment includes both changes to the dot electrode voltages, and inclusion of a compensating field E_y , likely due to nearby stray charges. Best agreement between simulated values and experimental resonator spectroscopy (“Resonator spec.”) found when both voltage adjustments and a compensating field are included (“Voltage + E_y adjust”). Adjusted parameters found in Table S1. Electron frequency values are simulated by solving the 2D Schrödinger equation for the electrostatic potential. **b**, Experimental resonator spectroscopy data from Fig.2 b overlaid with FEM simulation of electron resonance condition. Black line indicates where FEM simulation predicts electron frequency to be equal to resonator frequency at 7.162 GHz. Adjusted parameters used to produce black line are provided in Table S2.

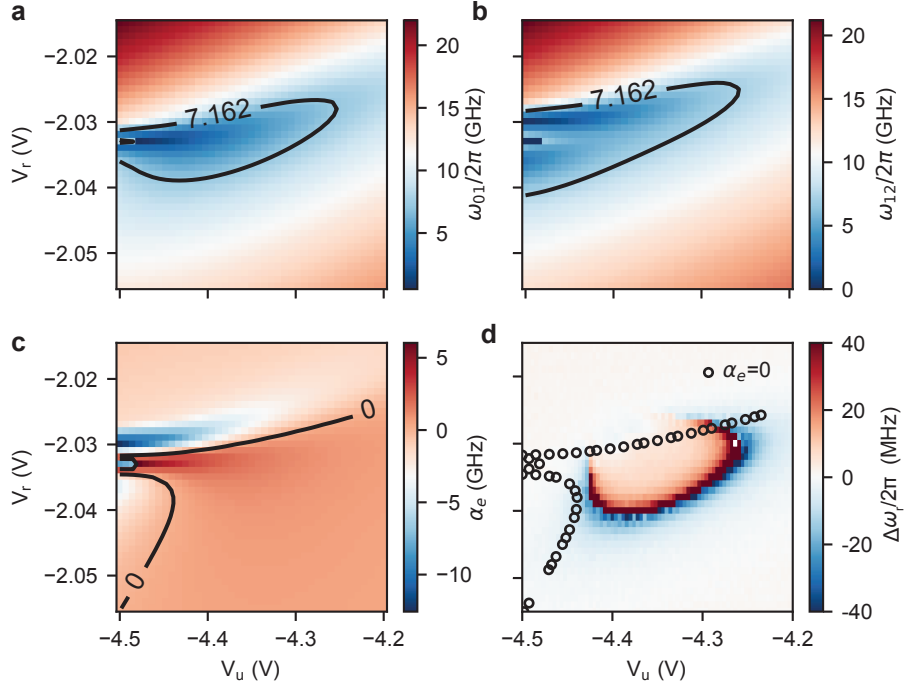


Figure S7: FEM modeling of higher order motional state transitions. **a**, FEM simulation of ground to first excited motional state transition frequency $\omega_{01}/2\pi$. Contour indicates where $\omega_{01}/2\pi = 7.162$ GHz. **b**, FEM simulation of first to second excited motional state transition frequency $\omega_{12}/2\pi$. Contour indicates where $\omega_{12}/2\pi = 7.162$ GHz. **c**, Anharmonicity of the electron excited states $\alpha_e = \omega_{12}/2\pi - \omega_{01}/2\pi$. Contour indicates where $\alpha_e = 0$ and the two transitions are degenerate. **d**, Experimental resonator spectroscopy data shown in Fig.2b overlaid with contours from **a** and **c** where $\omega_{01}/2\pi = 7.162$ GHz and $\alpha_e = 0$, respectively. Presence of additional features in the spectra close to $V_r = -2.03$ V agree well with where the two contours overlap, suggesting these features arise from coupling to higher order excited state transitions. All simulations performed using the parameters in Table S2 to solve the 2D Schödinger equation and calculate the quantized electron energies.

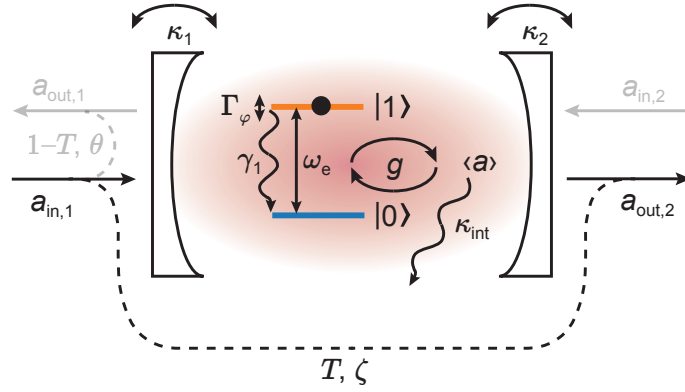


Figure S8: Modeling experimental transmission spectra. Schematic displaying the resonator with input and output coupling rates $\kappa_{1,2}$, internal loss rate κ_{int} and intra-cavity field $\langle a \rangle$. The electron is represented as a two-level system with dephasing Γ_φ and transverse decay γ_1 , which couples to the resonator with rate g . The resonator is probed with strength $a_{\text{in},1}$ and transmission is measured via $a_{\text{out},2}$. In this experiment, we do not apply an input to $a_{\text{in},2}$ and do not detect at $a_{\text{out},1}$, so we may neglect these ports in our analysis. We include additional transmission crosstalk with amplitude T and phase ζ , and reflection crosstalk with amplitude $1 - T$ and phase θ .

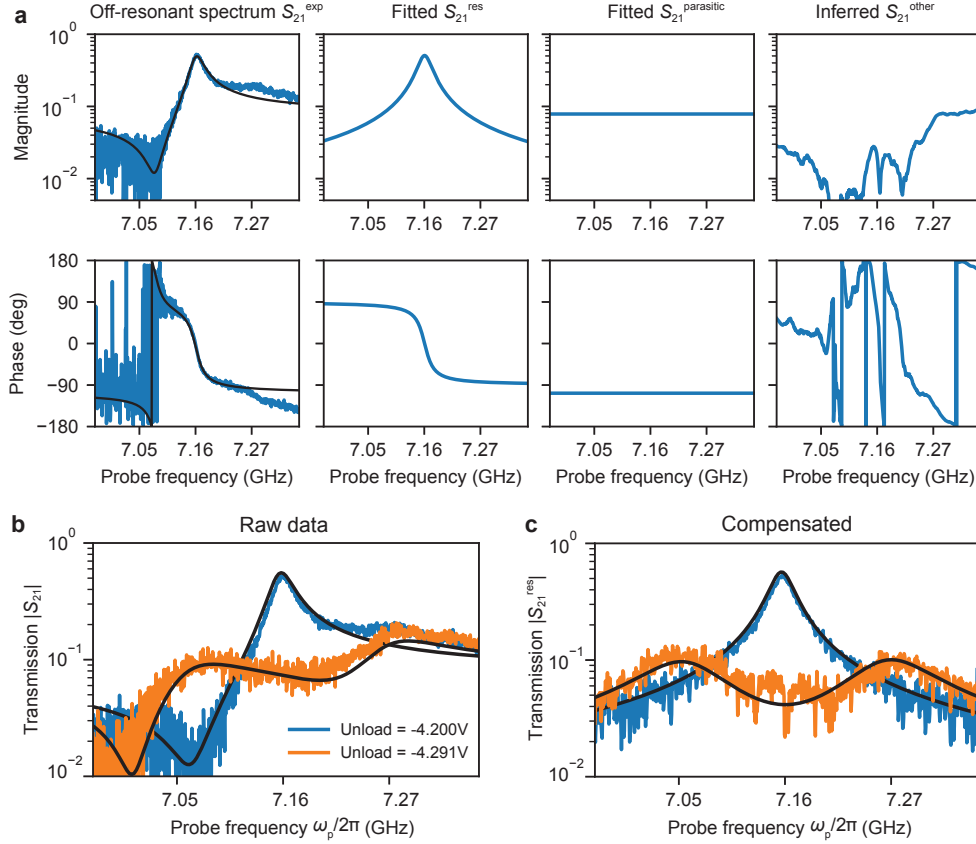


Figure S9: Method for compensating for background transmission. **a**, Far detuned resonator transmission spectrum (blue) with fit to Eq. (S20) (black), yielding $T = 0.008 \pm 0.002$, $\zeta = -0.30 \pm 0.02$ rad. From the fit we reconstruct S_{21}^{res} and $S_{21}^{\text{parasitic}}$ and infer S_{21}^{other} as described in Eq. (S21). **b**, Raw resonator spectra for two voltages where $\omega_e \gg \omega_r$ (blue) and $\omega_e \approx \omega_r$ (orange). The solid black lines are predictions from Eq. (S20) with values of T and ζ from **a**. **c**, Resonator transmission spectra from **b**, compensated for the parasitic transmissions, isolating the resonator-electron subsystem. Solid black lines are predictions from Eq. (S20) with values $T = \zeta = 0$.

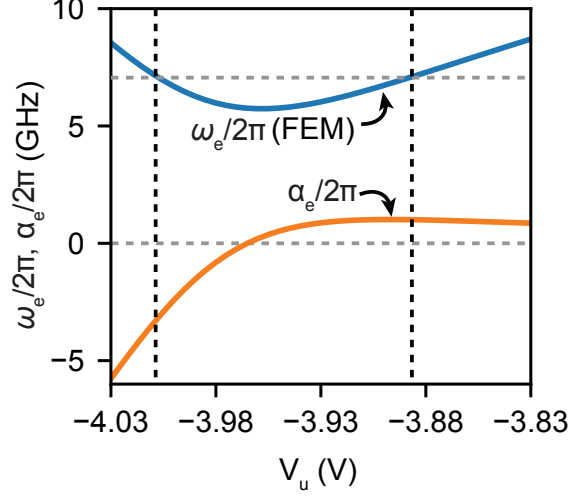


Figure S10: Anharmonicity of the trapping potential of the quantum dot. Anharmonicity $\alpha_e = \omega_{12} - \omega_{01}$ and simulated electron frequency ω_e of the compensated electrostatic trapping potential from Fig.3 b in the main text. Values are calculated from FEM using the values from Table S1. Horizontal dashed line indicate where $\alpha_e = 0$ and $\omega_e = \omega_r$, and vertical dashed lines indicate the voltages where $\omega_e \approx \omega_r$. The anharmonicity is large at most voltages, supporting the use of a two-state model for the electron used to produce the fit from the spectroscopy data shown in Fig.3 b in the main text. The anharmonicity only vanishes near the minima of the electron frequency, which may explain some of the broadening in the two-tone signal seen in Fig.3 b in the main text.

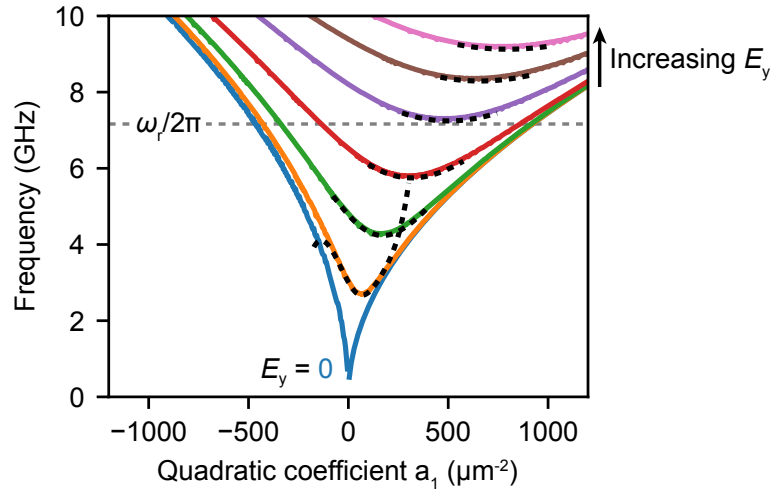


Figure S11: Qualitative model for the measured electron spectrum. Modeled electron frequency as function of harmonic coefficient a_1 , for fixed a_2 . Colored curves represent increasing values of E_y , which show a lifting of the minimum electron frequency from zero. Black dashed lines are first order approximations from Eq. (S36). This simple model reproduces the key experimental observations in Fig.3 b in the main text.

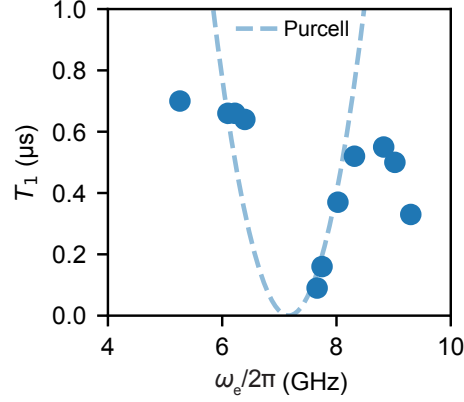


Figure S12: T_1 dependence on electron frequency. Measured T_1 times at various electron frequencies. In these measurements the electron frequency is tuned by varying V_u . The dashed line is the expected Purcell decay rate through the resonator calculated from Eq. (S38). Good agreement for $\Delta_{er} < 1.1$ GHz suggests that T_1 is limited by Purcell decay for these values of the detuning, but that other mechanisms limit T_1 for $\Delta_{er} > 1.1$ GHz.

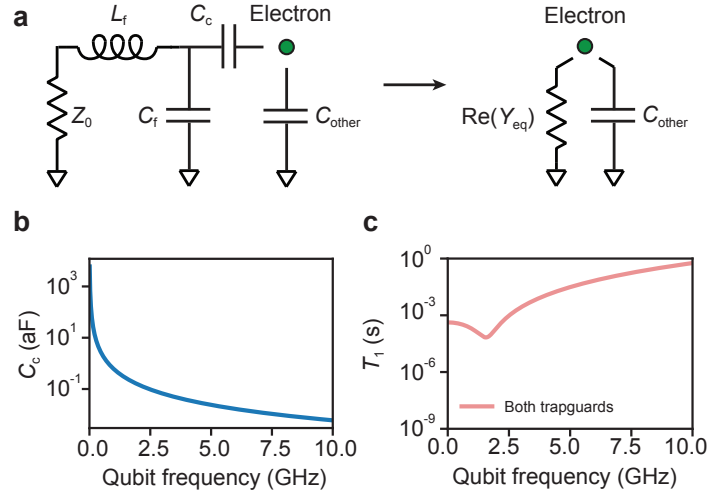


Figure S13: Purcell decay through the bias electrodes. **a**, Circuit model for calculating the T_1 contribution due to bias electrodes. The trapped electron is capacitively coupled to electrodes with an on-chip filter (inductance L_f and capacitance C_f) and the load impedance of the voltage source is Z_0 . The real part of the equivalent admittance acts as a resistor which affects T_1 . **b**, Calculated coupling capacitance for the Guard electrodes **c**, Corresponding T_1 contribution for the Guard electrodes.

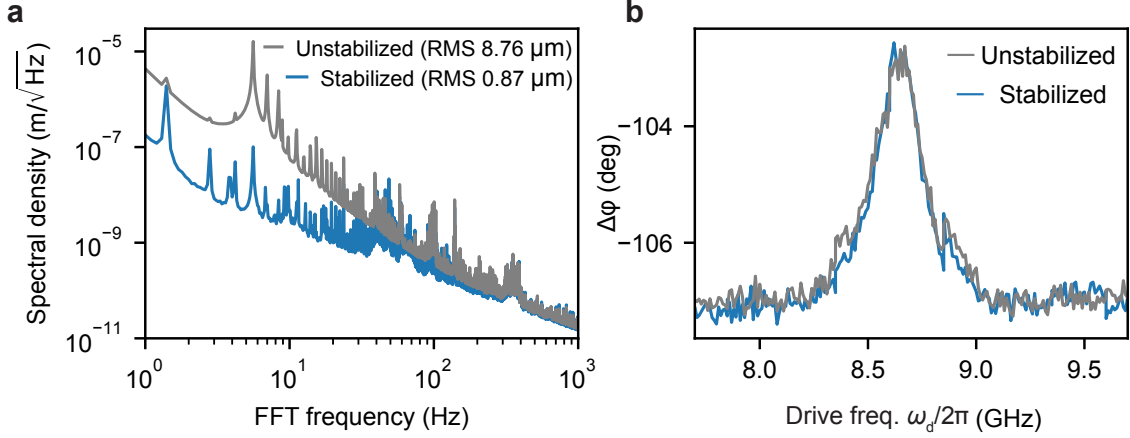


Figure S14: Vibrational noise spectra and electron state two-tone linewidth with and without cryostat vibration stabilization. **a**, Vibrational noise spectra acquired using a geophone sensor when cryostat active vibration cancellation system is on and off. Peaks in both spectra occur at harmonics of the pulse tube at 1.4 Hz. RMS noise amplitude undergoes a ten-fold decrease from 8.76 μm to 0.87 μm when stabilization is turned on. **b**, Despite the ten-fold decrease in the vibration noise spectral density when the active stabilization is turned on, the measured two-tone linewidth of the electron motional state γ does not change.

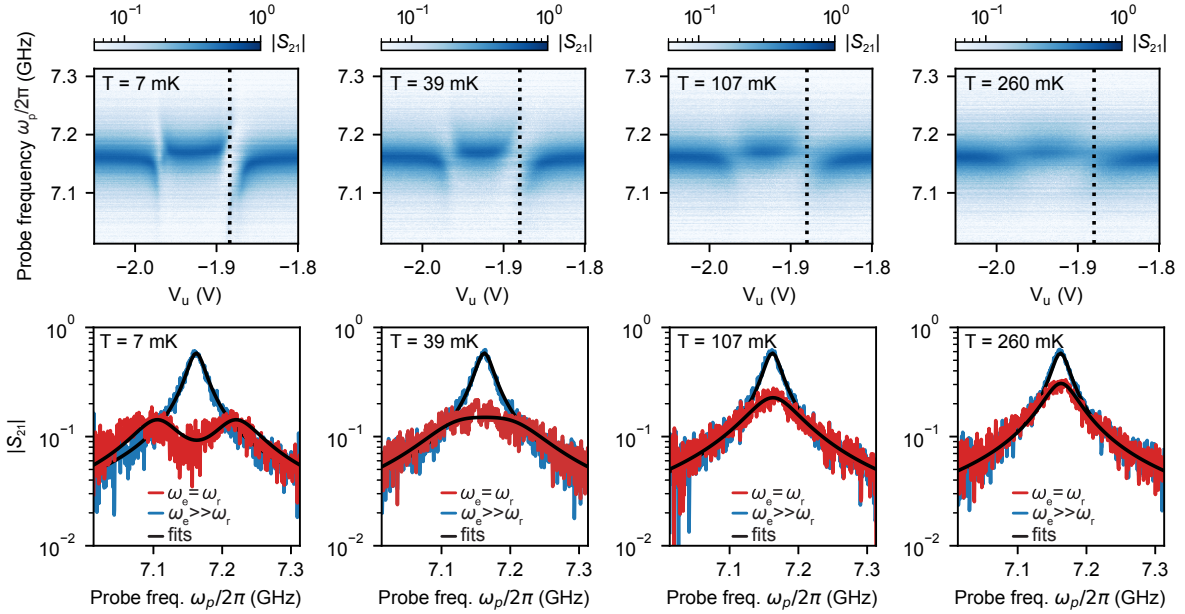


Figure S15: Emergence of the vacuum Rabi splitting with decreasing temperature. (Top) Resonator spectroscopy measurements taken at $T = 7, 39, 107$, and 260 mK. The prominence of the anticrossings decreases as the system gets warmer. Dashed vertical line is where the electron frequency is on resonance with the resonator. (Bottom) Resonator transmission spectrum taken off-resonance (blue) and on-resonance at the dashed black line (red) for the spectroscopy measurement taken at each temperature. The Rabi splitting is only clearly visible at the low temperatures. The spectra are compensated for parasitic transmission and fit to the transmission model as described in the Supplementary Information, SS4.

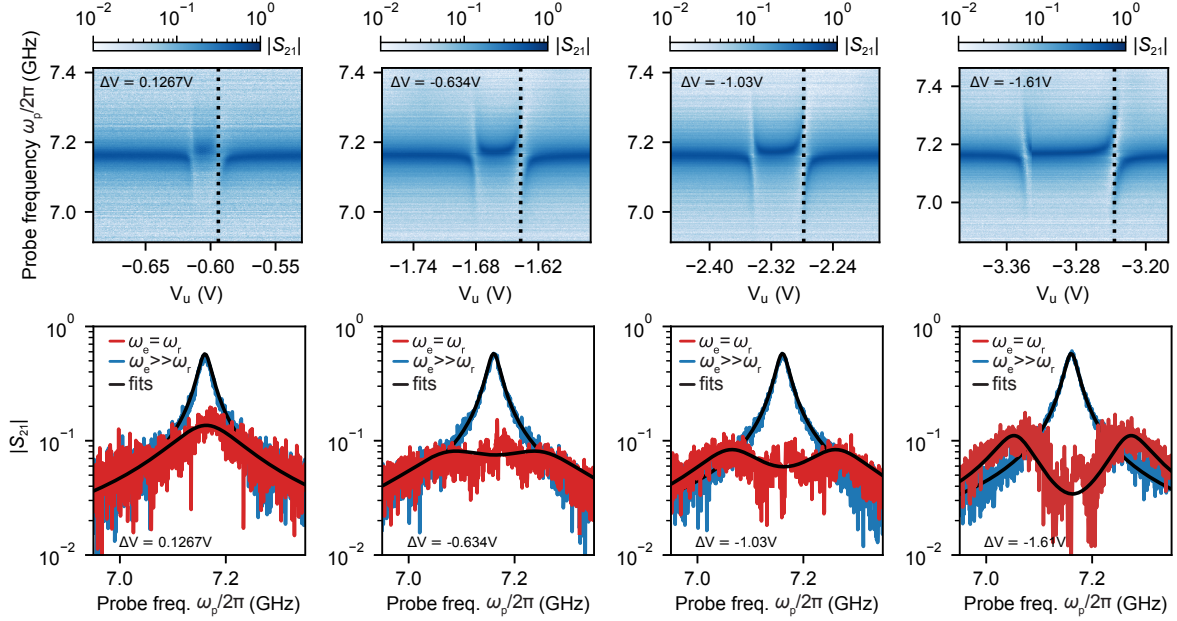


Figure S16: Emergence of the vacuum Rabi splitting with increasing voltage offset. (Top) Resonator spectroscopy measurements taken at $\Delta V = 0.1267, -0.634, -1.03, -1.61$ V. The prominence of the anti-crossings increases as the voltage offset becomes more negative. Dashed vertical line is where the electron frequency is on resonance with the resonator. (Bottom) Resonator transmission spectrum taken off-resonance (blue) and on-resonance at the dashed black line (red) for the spectroscopy measurement taken at each temperature. The Rabi splitting is only clearly visible at more negative offsets. The spectra are compensated for parasitic transmission and fit to the transmission model as described in the Supplementary Information, SS4.

Table S1: Voltage and E-field parameters used in FEM simulation of electron frequency. “Unadjusted” refers to experimentally used values, “Voltage only” refers to adjustments in FEM with offsets in dot electrode voltages only, “ E_y only” refers to adjustments in FEM with compensating in-plane electric field only, “Voltage + E_y ” refers to adjustments in both voltage offset and electric field used to produce “Simulated” best fit in Fig.3 b in the main text.

FEM params.	Unadjusted	Voltage only	E_y only	Voltage + E_y
V_b (V)	-1.68	-1.58	-1.68	-1.58
$V_{g,up}$ (V)	-3.35	-3.35	-3.35	-3.35
$V_{g,down}$ (V)	-2.85	-2.85	-2.85	-2.85
V_r (V)	-2.013	-2.013	-2.013	-2.013
V_u (V)	(-4.03, -3.83)	(-4.0, -3.8)	(-4.03, -3.83)	(-4.0, -3.8)
E_y (V/cm)	0	0	142	142
E_x (V/cm)	0	0	0	0

Table S2: Voltage and E-field adjustments used in FEM simulation of electron frequency. “Experimental” column values are those used in Fig.2 b. “Adjusted” column values are parameters used to produce dashed line resonance condition in Fig. S6 b.

FEM params.	Experimental	Adjusted
V_b (V)	-1.67	-1.555
$V_{g,up}$ (V)	-3.35	-3.35
$V_{g,down}$ (V)	-2.85	-2.85
V_r (V)	(-2.055, -2.015)	(-2.055, -2.015)
V_u (V)	(-4.5, -4.2)	(-4.55, -4.25)
E_y (V/cm)	–	172.9
E_x (V/cm)	–	-105.0

Table S3: Experimental voltage configurations used when applying global offset. Voltages used in resonator spectroscopy sweeps from which Γ_2 values are extracted and presented in Fig.4 c in the main text. The ΔV parameter presented as the x-axis of Fig.4 c in the main text is defined as V_r .

ΔV (V)	V_b (V)	V_r (V)	$(V_{g,up}, V_{g,down})$ (V)	V_u (V)
0.1267	0.4	0.1267	(-0.6, -0.6)	-0.594
-0.253	0	-0.253	(-1.0, -1.035)	-1.104
-0.634	-0.4	-0.634	(-1.4, -1.44)	-1.637
-1.03	-0.8	-1.03	(-1.8, -1.87)	-2.278
-1.415	-1.2	-1.415	(-2.2, -2.275)	-2.887
-1.61	-1.4	-1.61	(-2.4, -2.477)	-3.236

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