
The extraction of work from hidden degrees of freedom

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Supplementary Note 1: Experimental setup and synchronization

Optical trapping. Optical confinement was realized using a 532 nm diode-pumped solid-state laser (Coherent[®] Verdi-V2). The laser intensity was modulated by an acousto-optic modulator (AA Opto-Electronic[®] MT80-A1.5-VIS) and directed into the back focal plane of a 100 $\times$ oil-immersion objective (Zeiss[®] EC Plan-Neofluar M27, NA = 1.3), creating a harmonic trapping potential. For double-well experiments, the laser focus was oscillated at 700 Hz using a galvanometric mirror to generate a time-shared potential (Supplementary Fig. 1a). This modulation frequency significantly exceeds the particle’s corner frequency, ensuring that the tracer experiences a stable, time-averaged conservative potential rather than a dynamically flickering trap.

Imaging and tracking. Particle positions were measured using bright-field imaging with a 470 nm LED illumination source and a 20 $\times$ condenser objective (Nikon[®] EPI Plan SLWD, NA = 0.35). Scattered light was collected by the trapping objective and imaged onto a CCD camera (Basler[®] a2A2600-64umBAS), operated at 100 frames per second with a 150 $\times$ 150 pixel region of interest. Tracer particles appeared as bright Gaussian-like spots on a dark background (Supplementary Fig. 1b). Their positions were extracted using a custom tracking algorithm adapted from Ref. [1]. Briefly, raw images were smoothed with a Gaussian kernel, the brightest pixel was identified, and the local intensity distribution was fitted with a second-order polynomial. This procedure yielded a sub-pixel localization accuracy of approximately 6 nm.

Potential reconstruction. The trap stiffness κ and potential geometry were reconstructed from equilibrium position fluctuations via Boltzmann inversion, $V(x) = -k_B T \ln P(x)$, yielding a harmonic potential $V(x) = \frac{1}{2}\kappa x^2$ (Supplementary Figs. 1c,d). The maximum laser power at the sample corresponded to a trap stiffness of approximately 40 μ N/m.

Stage and temperature control. The sample chamber was mounted on a piezoelectric translation stage (Piezoconcept[®] LT3.300), allowing precise and reproducible relative motion between the optical trap and the viscoelastic medium. Temperature stabilization was achieved using an objective heater (Okolab[®]), which maintained the oil-immersion objective at 25°C. Because the objective is in direct thermal contact with the capillary, the sample region remained thermally stable throughout the experiments.

Hardware synchronization. Real-time feedback protocols required precise temporal synchronization between imaging, laser modulation, and mechanical actuation. This was achieved using two dual-channel function generators (Tektronix[®] AFG31000), which served as a shared master clock for the experiment. One generator controlled the acousto-optic modulator and piezoelectric stage motion, while the second provided hardware trigger signals for camera acquisition, ensuring deterministic timing and eliminating software-induced jitter between measurement and actuation. To validate synchronization and ensure data integrity, a data acquisition card (National Instruments[®] USB-6003) continuously recorded the camera exposure signal and

the piezo-stage position monitor at a sampling rate of 3 kHz, well above the imaging rate. This high-bandwidth hardware logging enabled each video frame to be unambiguously mapped to the instantaneous trap position, providing a robust temporal reference for state classification, feedback execution, and work calculation.

Supplementary Note 2: Viscoelastic micellar solutions and timescales

Solution preparation. Viscoelastic micellar solutions were prepared by dissolving cetylpyridinium chloride monohydrate (CPyCl, Sigma[®], 99%) and sodium salicylate (NaSal, Sigma[®], 99.5%) in equimolar proportions in deionized water. Two surfactant concentrations were used in this study: 8 mM for single-well experiments and 5 mM for double-well experiments. For a total solvent mass of 104.4 g, the 8 mM solution was prepared using 286.4 mg of CPyCl and 128.1 mg of NaSal, whereas the 5 mM solution contained 179.0 mg of CPyCl and 80.1 mg of NaSal.

All mixtures were stirred continuously at 100 rpm for 24 h at 45 °C to ensure complete dissolution and homogenization. After preparation, the solutions were cooled to 25 °C and maintained under gentle stirring until use. Under these conditions, the surfactants self-assemble into worm-like micelles forming an entangled network, giving rise to pronounced viscoelastic behavior.

Sample preparation. Silica microspheres of diameter 2.73 μm (microParticles GmbH[®]) were used as tracer particles and dispersed at low volume fraction to avoid interparticle interactions. For optical trapping experiments, approximately 1 μL of a concentrated microsphere suspension was added to 1 mL of the micellar solution. The suspension was loaded into rectangular borosilicate glass capillaries (VitroTubes[®], thickness 100 μm) by capillary action. The open ends were sealed with bee wax (Hampton Research[®]) and reinforced with a two-component epoxy resin to ensure mechanical stability and chemical inertness. Samples were equilibrated for at least 24 h on a temperature-controlled stage maintained at 25 °C prior to measurements.

Recoil-based characterization. To obtain an estimate of the characteristic viscoelastic relaxation timescales relevant to our experiments, we performed recoil measurements following established protocols reported in our previous works [2, 3]. In these experiments, a trapped particle is translated at a constant velocity $v = 0.3 \mu\text{m/s}$ for a duration $t = 60 \text{ s}$ using an optical trap, which is then abruptly switched off. These parameters were chosen to ensure we remain within the linear response regime of the viscoelastic bath. Upon release, the particle exhibits a recoil toward its initial position due to the relaxation of elastic stresses stored in the micellar network. The recoil trajectories display a clear bi-exponential relaxation, reflecting the coexistence of short- and long-time stress relaxation of the micellar network. This recoil-based characterization is experimentally efficient, as it requires significantly less statistics compared to

equilibrium measurements, which rely on long trajectories to accurately estimate mean-squared displacements and related steady-state observables.

The mean recoil trajectory is well described by a double exponential,

$$\langle x(t) \rangle = a - a_1 e^{-t/\tau_1} - (a - a_1) e^{-t/\tau_2} \quad (1)$$

The extracted fit parameters for the recoil curve shown in Supplementary Fig. 1 d are $a = 4.03 \mu\text{m}$, $a_1 = 1.98 \mu\text{m}$, $\tau_1 = 0.173 \text{ s}$, and $\tau_2 = 6.04 \text{ s}$. The black lines indicate ensemble-averaged displacements and the shaded regions denote the standard deviation across 18 realizations. In the context of the present study, the longer relaxation time provides a relevant estimate of the memory timescale of the bath.

Supplementary Note 3: Statistical uncertainty and control-ensemble construction

Bootstrap resampling. Statistical uncertainties for all experimental observables were quantified using a bootstrap resampling procedure to estimate the error in the mean [4, 5]. For the state-conditioned relaxation curves, which were subsampled from long continuous trajectories, we generated 500 bootstrap realizations by randomly selecting 10^3 segments with replacement. For the work extraction data, where each curve corresponds to a specific measurement delay δt , bootstrapping was performed by generating 2000 resampled datasets. Each dataset was constructed by selecting the maximum number of independent trajectories with replacement from the original ensemble. In both cases, the shaded bands represent the standard deviation of the bootstrap distribution, providing a robust estimate of the standard error of the mean.

Construction of control ensembles. We implement a resampling procedure to isolate the contribution of hidden degrees of freedom ($h\text{DoF}$) by constructing a control distribution $\tilde{\rho}_{|ij\rangle}(x, 0)$ that reproduces the positional distribution of $\rho_{|ij\rangle}(x, 0)$ at $t = 0$, while removing any dependence on the measurement history. In our notation, the state $|j\rangle$ always refers to the outcome of the second measurement performed at time $t = 0$, whereas the index i denotes the outcome of a previous measurement performed at time $t = -\delta t$.

For a given second-measurement outcome $|j\rangle$, the corresponding ensemble $\rho_{|j\rangle}$ is decomposed into history-conditioned subsets $\rho_{|ij\rangle}$ according to the earlier outcome i . For example as shown in Fig. 5, the ensemble $|1\rangle$ at $t = 0$ is partitioned into the subsets $|01\rangle$ (blue circles) and $|11\rangle$ (orange diamonds), which contain trajectories that both satisfy the condition $x(0) \in |1\rangle$, but differ by their measurement outcome at the earlier time $t = -\delta t$.

To construct the history-blind reference ensemble $\tilde{\rho}_{|ij\rangle}$, the single-measurement ensemble $\rho_{|j\rangle}$ is

partitioned into spatial bins. Within each bin b , the total number of trajectories (n_b) belonging to $\rho_{|j\rangle}$ and m_b number of trajectories belonging to the history-conditioned subset $\rho_{|ij\rangle}$. To construct $\tilde{\rho}_{|ij\rangle}$, we then randomly select exactly m_b trajectories from the full set of n_b trajectories in the corresponding bin, irrespective of their previous outcome i . Consequently, the resampled ensemble contains trajectories originating from both history-conditioned subsets (e.g. both $|01\rangle$ and $|11\rangle$ when constructing $\tilde{\rho}_{|11\rangle}$), but with the same spatial distribution as $\rho_{|11\rangle}$ at $t = 0$.

This procedure preserves the marginal positional distribution at the second measurement while eliminating correlations with the earlier measurement outcome. By construction, the control ensemble therefore satisfies $\tilde{\rho}_{|ij\rangle}(x, 0) = \rho_{|ij\rangle}(x, 0)$ and consequently $\tilde{I}_{|ij\rangle}(0) = I_{|ij\rangle}(0)$. Any subsequent deviation between $I_{|ij\rangle}(t)$ and $\tilde{I}_{|ij\rangle}(t)$ must therefore originate from hidden history-dependent degrees of freedom that influence the future dynamics.

Supplementary Note 4: Minimal micromechanical model of hidden degrees of freedom

To corroborate our experimental observations, we perform numerical simulations based on a minimal micromechanical model incorporating h DoFs. This framework has been experimentally validated in a range of viscoelastic fluids, including the micellar system considered here [2, 3, 6, 7]. While such models can be systematically extended to include multiple bath particles, we restrict ourselves to the minimal case of a single bath particle for simplicity. This approximation remains valid when the fluid is at equilibrium [8] or subject to small deformations [9].

In this model, h DoFs are introduced via a hidden bath particle labeled by the index b . This bath particle is coupled to the colloidal particle (which carries no index) through a harmonic spring with stiffness κ_b (see Supplementary Fig. 1, d inset). The colloidal and bath particles are characterized by their respective friction coefficients γ and γ_b . Within an optical trap with stiffness κ and trap center position $\lambda(t)$ their dynamics are governed by the following set of overdamped Langevin equations:

$$\gamma \dot{x}(t) = -\kappa [x(t) - \lambda(t)] - [x(t) - x_b(t)] + \xi(t) \quad (2)$$

$$\gamma_b \dot{x}_b(t) = -\kappa_b [x_b(t) - x(t)] + \xi_b(t) \quad (3)$$

where (ξ, ξ_b) are independent Gaussian white noise terms with zero mean, respectively. The above equations are equivalent to a single generalized Langevin equation with an exponential memory kernel [3]. To obtain the model parameters, we start from the recoil fit, which can be described by a two-bath model with frictions γ_1, γ_2 and spring constants κ_1, κ_2 , coupled to a tracer with friction γ . The fit parameters are then mapped to the following ratios [2, 10] $\gamma_1/\gamma = 6.97$, $\kappa_1/\gamma = 3.95$, $\gamma_2/\gamma = 37.40$, and $\kappa_2/\gamma = 1.39$. For equilibrium behavior, this two-bath description can be simplified to an effective single bath particle with $\gamma_b/\gamma = (\gamma_1 + \gamma_2)/\gamma$ and

$\kappa_b/\gamma = (\kappa_1 + \kappa_2)/\gamma$, while keeping a good agreement with experimental data [8]. These effective ratios are scaled by the experimentally measured tracer friction $\gamma = 0.34 \times 10^{-6} \text{ kg/s}$ [9]. This way, we yield as effective parameters of the above single-bath particle model $\gamma_b = 15 \text{ }\mu\text{N s/m}$ and $\kappa_b = 1.8 \text{ }\mu\text{N/m}$ which are used for all the numerical simulations presented in our work.

Based on this minimal single-bath model, we perform numerical simulations of the energy relaxation of the colloidal particle following a double-measurement protocol (Supplementary Fig. 3a). We also quantify the information decay and compute the non-Markovian order parameter Φ (Supplementary Fig. 3b). In both cases, the simulations reproduce the experimental observations with good accuracy.

Supplementary Note 5: Markovian Baseline and Numerical Simulations

To confirm that the observed information backflow is a signature of non-Markovian dynamics, we compare our experimental results with a memory-less baseline. In a Markovian system, the particle’s evolution is governed solely by the instantaneous potential and thermal fluctuations, with no dependence on the history of the hidden degrees of freedom (hDoF).

Langevin Dynamics Simulation. We modeled the Markovian limit using a standard overdamped Langevin equation:

$$\gamma \dot{x} = -\nabla V(x) + \sqrt{2k_B T \gamma} \xi(t), \quad (4)$$

where γ is the friction coefficient, $V(x)$ is the harmonic potential, and $\xi(t)$ is Gaussian white noise. The parameters were matched to the experimental single-well trap stiffness (κ), temperature (T) and γ [9] given in Supplementary Table. 2. We applied the identical double-measurement protocol used in the experiments to the simulated trajectories. As shown in Supplementary Fig. 6a, the state-averaged information $I(t)$ (solid line) and the history-blind control $\tilde{I}(t)$ (open circles) are indistinguishable in the simulation. This collapse confirms that for a Markovian system, once the initial position distribution is matched ($\tilde{\rho}_{|ij\rangle}(x, 0) = \rho_{|ij\rangle}(x, 0)$), the future information decay is identical regardless of the system’s history.

Validation of Non-Markovianity. To rigorously confirm that the observed information backflow arises from non-Markovian dynamics, we compare the experimental results with a memoryless baseline. In a Markovian system, the future evolution of the particle is determined solely by its instantaneous state, implying that each state-conditioned component $\Phi_{|ij\rangle}$ should vanish. To provide a stringent test, we compute the sum of the absolute values of the components, $\sum_{ij} P_{ij} |\Phi_{|ij\rangle}|$, for the Markovian case. This ensures that each component of the order parameter tends to zero.

As illustrated in Supplementary Fig. 6b, the simulated Markovian sum remains near zero across all delay times δt . In contrast, the experimental data for the 8 mM micellar solution exhibits a pronounced peak in the state-averaged $\Phi(\delta t)$ at $\delta t \approx 1$ s. This maximum aligns with the characteristic relaxation timescale of the micellar network, where information stored in the hidden degrees of freedom (hDoF) most strongly influences the particle’s subsequent trajectory. The clear divergence between the simulated Langevin dynamics and the experimental observations provides unambiguous evidence for non-Markovian information backflow.

Supplementary Note 6: Work extraction

To extract work following the detection of a state $|ij\rangle$, we translate the trap at a constant velocity $\dot{\lambda}$ over a duration $t_f = 3$ s (see Supplementary Fig. 2). The value of $\dot{\lambda}$ is adjusted for each state to optimize the extraction protocol (Supplementary Table 1).

In the experiments, we showed that for the $|11\rangle$ state, the extracted work depends strongly on the delay δt . In contrast, Langevin simulations in a purely Markovian environment (see above) yield a work ratio W_m/W_m^* that is independent of δt (Supplementary Fig. 4a). This demonstrates that the observed δt dependence originates from the hidden hDoF of the environment, confirming that part of the extracted work arises from these degrees of freedom.

Implementing the same protocol in the non-Markovian numerical simulations yields very good agreement with the experimental results (Supplementary Fig. 4b). In particular, the ratio W_m/W_m^* follows the non-Markovian order parameter Φ , and exceeds unity for the $|10\rangle$ state, demonstrating that energy is extracted from the hDoF. The absolute extracted work instead reflects the relaxation dynamics, with the higher-energy states ($|11\rangle$ and $|01\rangle$) yielding the largest work output.

Supplementary Tables

Supplementary Table 1 Speed of trap for different states

State	$\dot{\lambda}$ (nm/s)	t_f (s)
$ 00\rangle$	45	3 s
$ 10\rangle$	25	3 s
$ 01\rangle$	45	3 s
$ 11\rangle$	45	3 s

Supplementary Table 2 Simulation parameters for Markovian system

Parameter	Symbol	Value
Temperature	T	298 K
Trap stiffness	κ	1.9×10^{-6} N/m
Friction coefficient	γ	6.6×10^{-6} kg/s

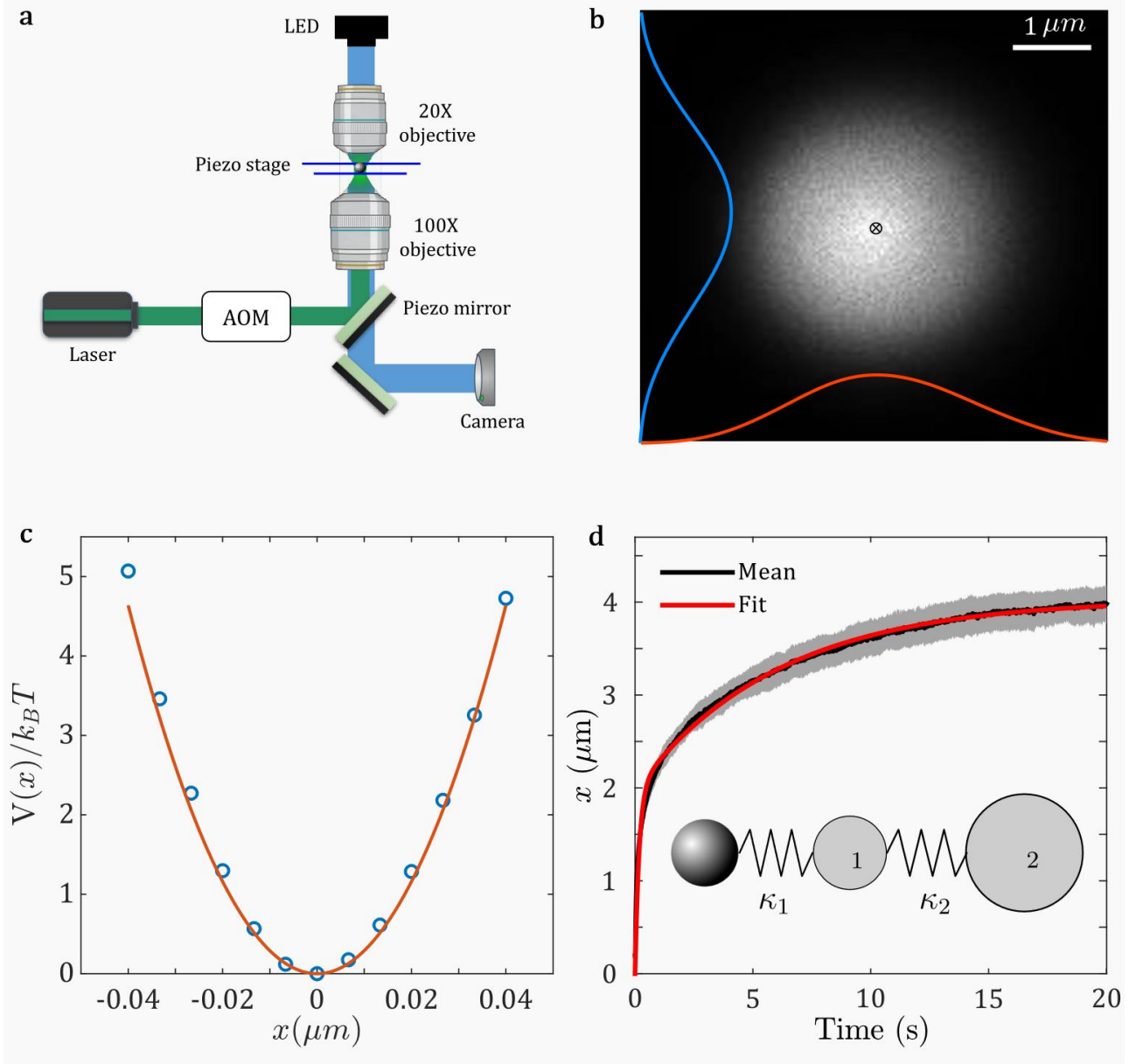
Supplementary Table 3 Simulation parameters for non-Markovian system

Parameter	Symbol	Value
Temperature	T	298 K
Trap stiffness	κ	1.47×10^{-6} N/m
Tracer friction	γ	0.34×10^{-6} kg/s
Bath Particle friction	γ_B	15×10^{-6} kg/s
Bath Particle stiffness	κ_B	1.8×10^{-6} N/m

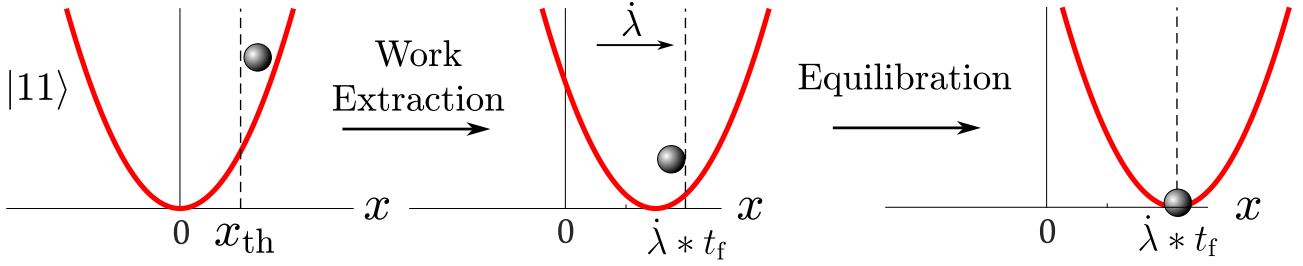
Supplementary Figures

References

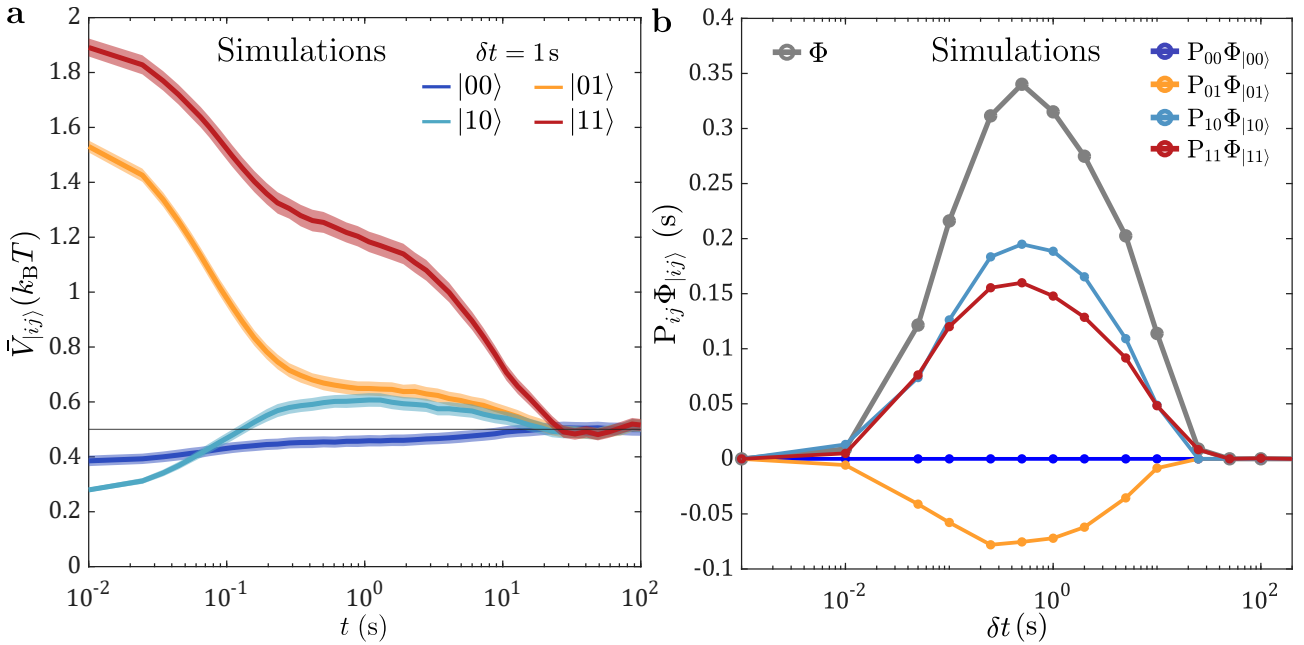
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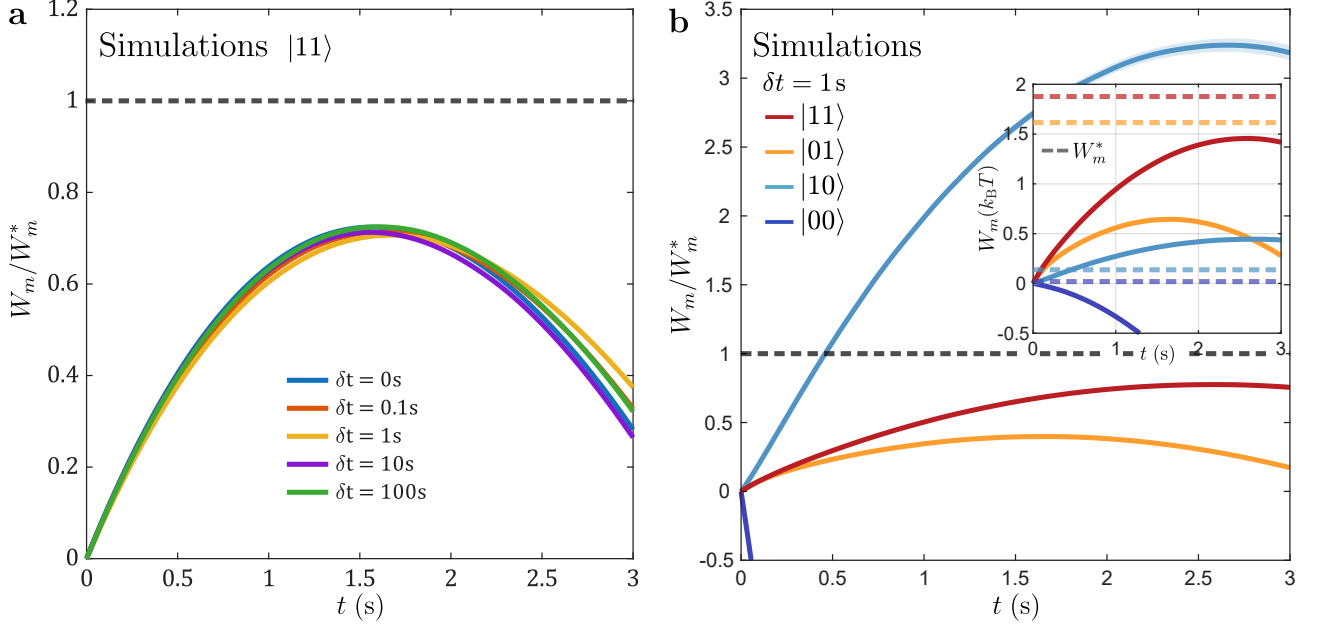
Supplementary Fig. 1 Real-time tracking and potential reconstruction. **a**, Schematic of the experimental setup showing the optical path and imaging system. **b**, CCD image of a single trapped particle with its Gaussian intensity profile, peaking at the particle center. **c**, Reconstructed single-well optical trapping potential **d**, Mean recoil trajectory (black line) of an optically trapped particle in the 8 mM micellar solution. The particle was driven at $v = 0.3 \mu\text{m/s}$ for 60 s. The shaded area indicates the standard deviation obtained from 18 realizations. The recoil exhibits bi-exponential relaxation, with long relaxation times of approximately 6 s. **Inset**, Sketch of the micromechanical model, where the colloidal particle with friction γ is connected to a fictitious "bath" particle with friction γ_B , through a harmonic spring with stiffness κ_B .



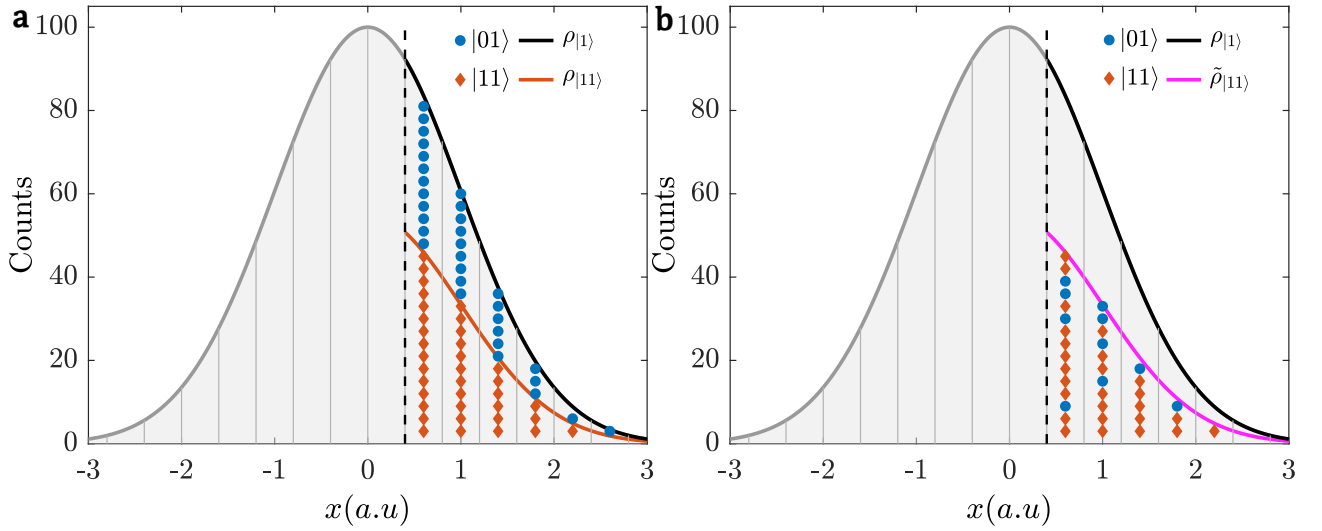
Supplementary Fig. 2 Information to work protocol. **a**, Sketch of the work extraction protocol from a $|11\rangle$ state within a single well potential. After measuring a $|11\rangle$ state, the trap is translated at constant velocity $\dot{\lambda}$ during $t_f = 3$ s.



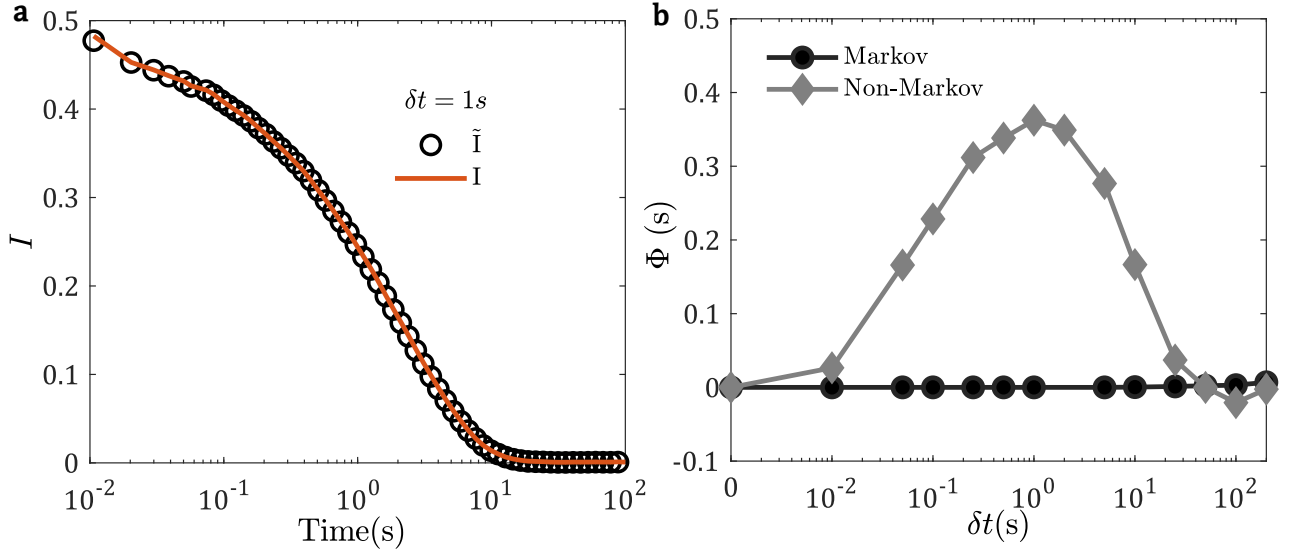
Supplementary Fig. 3 State-conditioned relaxation and order parameter for simulations. **a**, Simulations of state-conditioned relaxation curves $\bar{V}_{ij}(t)$ following a two-measurement protocol at $\delta t = 1$ s, showing qualitative agreement with the experiments. Notably the energy back-flow from $|10\rangle$ is clearly visible. **b**, Simulation of the integrated order parameters weighted by their relative probabilities $P_{ij}\Phi_{ij}(\delta t)$ for individual measurement-conditioned states, together with the state-averaged quantity $\Phi(\delta t)$. As for the experiments, we observe a pronounced maximum at $\delta t \approx 1$ s.



Supplementary Fig. 4 Work extraction from $h\text{DoF}$ through two-measurement protocols in simulations **a**, Averaged normalized work W_m/W_m^* extracted from a $|11\rangle$ state and for increasing δt , within a Markovian environment. Opposite to the experiments in a non-Markovian fluid, here we observe no influence of δt , and all curves collapse together. **b**, Averaged normalized work W_m/W_m^* extracted for a fixed value of $\delta t = 1\text{s}$ and states $|11\rangle$ (red), $|01\rangle$ (light orange), $|10\rangle$ (teal), and $|00\rangle$ (blue) in simulations. For both the ratio and absolute curves (inset) simulations show very similar results to the experiments.



Supplementary Fig. 5 Construction of history-blind control ensembles. **a**, A sketch showing the distribution (histogram) of the single-measurement state $\rho_{|1\rangle}$ (black curve) and the history-conditioned subset $\rho_{|11\rangle}$ (orange curve). The ensemble $\rho_{|1\rangle}$ is decomposed into subsets conditioned on the previous outcome, such that in each spatial bin b , m_b trajectories (orange diamonds) correspond to $\rho_{|11\rangle}$, while the remaining trajectories (blue circles) correspond to $\rho_{|01\rangle}$. Within each spatial bin to the right of the threshold, the total vertical stack represents n_b trajectories. **b**, Construction of the control ensemble $\tilde{\rho}_{|11\rangle}$. The distribution is formed by randomly subsampling m_b trajectories from n_b in each bin. This results in a mixed composition of circles and diamonds, ensuring the ensemble is history-blind while maintaining a spatial density and initial information $I(0)$ identical to the original two-measurement state.



Supplementary Fig. 6 Validation of non-Markovianity against Langevin simulations. **a**, State-averaged information decay $I(t)$ for the Markovian case. The solid line represents the double-measurement information $I(t)$, while open circles represent the history-blind control $\tilde{I}(t)$ derived from simulated Langevin trajectories. The perfect overlap confirms that history does not influence information decay in memory-less systems. **b**, Integrated order parameter $\Phi(\delta t)$ as a function of measurement delay. The black markers (Markov) represent the sum of the absolute values of state-conditioned components $\sum |\Phi_{|ij\rangle}|$, which remains zero across all δt . The gray markers (Non-Markov Exp.) show the state-averaged experimental data $\Phi(\delta t)$ for the 8 mM solution. The pronounced maximum at $\delta t \approx 1$ s identifies the temporal window where the hidden degrees of freedom of the micellar network exert the strongest influence on the particle's relaxation.