
Probing picosecond depairing currents in type-II superconductors

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S1. Sample characterizations

S1.1 Characterization of the NbN thin film

NbN samples were deposited by dc magnetron sputtering on MgO substrates with a substrate temperature of 350 °C, with a thickness of ≈ 20 nm and a superconducting critical temperature of $T_c \approx 15$ K [1]. The thin films were characterized by XRD shown in Fig. S1. A clear separation between the Bragg peaks of NbN and MgO can be observed at $(\bar{6}00)$ and $(\bar{4}00)$. The other Bragg peaks cannot be clearly separated because of the strong intensity of the peaks from MgO, and merge as stripes stretched towards the origin as expected for such a configuration. This confirms the epitaxial growth of monocrystalline NbN thin films.

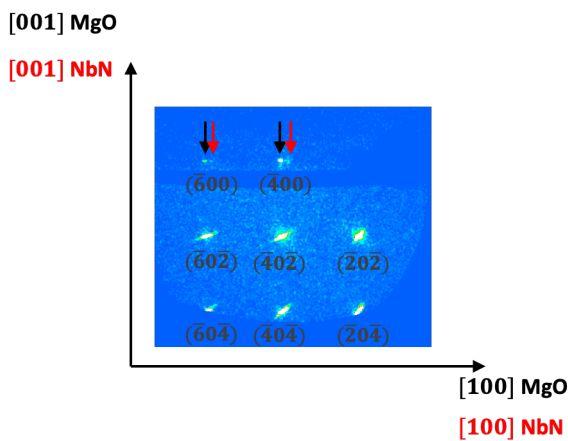


Figure S1| XRD pattern of the NbN film grown on MgO. NbN and MgO both have cubic lattice structure but with different lattice constants ($a_{\text{NbN}} \approx 4.4$ Å and $a_{\text{MgO}} \approx 4.2$ Å). A clear separation between the Bragg peaks of NbN and MgO can be observed at $(\bar{6}00)$ and $(\bar{4}00)$. The other Bragg peaks cannot be clearly separated because of the strong intensity of peaks from the MgO substrate. Instead, these Bragg peaks are stretched towards the origin as expected for such a configuration.

S1.2 Characterization of the YBCO thin film

$\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) samples were deposited by dc magnetron sputtering on sapphire substrates with 20 nm CeO_2 buffer layer, with a thickness of ≈ 50 nm and a critical temperature of ≈ 85 K by Ceraco ceramic coating GmbH. 50 nm Au was deposited on top of YBCO in situ after sputtering to get good electrical contact. The bare YBCO samples were characterized by XRD shown in Fig. S2. This confirms the epitaxial growth of monocrystalline YBCO thin film.

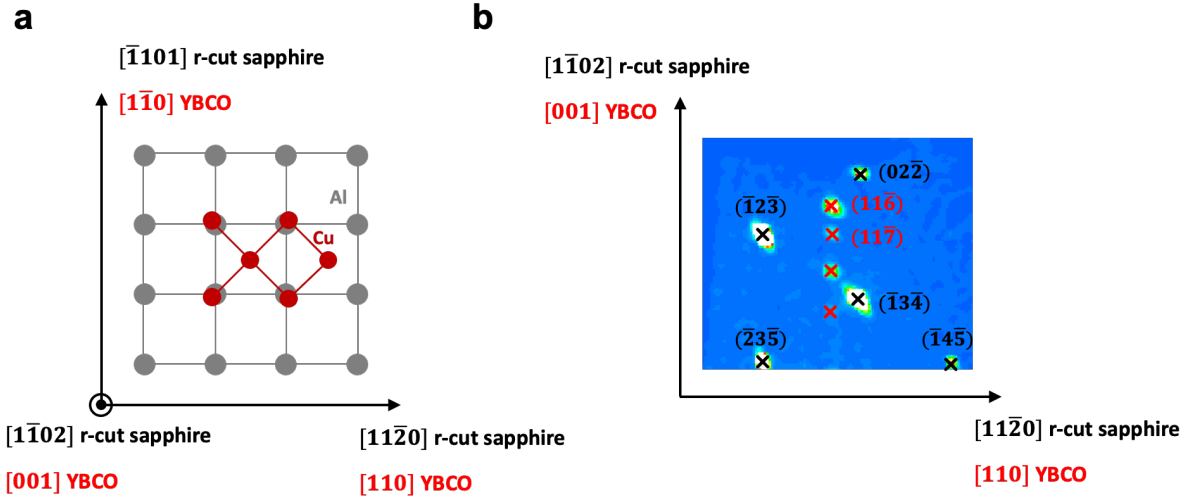


Figure S2| XRD pattern of YBCO film grown on r-cut sapphire with CeO_2 buffer layer. a, Illustration of the arrangement of Cu atoms from YBCO on Al atoms from Al_2O_3 . b, Bragg peaks in $(1\bar{1}0)$ plane of YBCO. The peaks indicated in black correspond to the r-cut sapphire while the red peaks relate to YBCO. Because the thickness of the CeO_2 buffer layer is much thinner than YBCO, its pattern is not observable.

S2. Device fabrication procedures

NbN thin films were patterned with laser lithography and reactive ion etching (RIE) using SF_6 . Photoconductive switches and coplanar waveguides were fabricated on MgO substrates with pre-patterned NbN thin films using laser lithography and e-beam deposition. Specifically, 200 nm thick silicon was deposited to form photoconductive switches and 10 nm Ti / 260 nm Au was deposited to form coplanar waveguides. In order to get good electrical contact between the waveguide and the patterned NbN thin film, the contact area of the NbN thin film was cleaned with Argon plasma and was then capped with 50 nm Au in situ, before depositing Ti/Au to form a waveguide. With such procedures, a contact resistance of $\approx 3 \Omega$ for each contact was obtained.

YBCO together with the top Au layer was patterned with laser lithography and wet etching methods. A solution of KI and I_2 in DI water was used to etch the Au layer, and a mixture of phosphoric acid and DI water was used to etch the YBCO thin film. Photoconductive switches and coplanar waveguides were fabricated on sapphire substrates with pre-patterned YBCO thin films using the same method as discussed above. A contact resistance of $\approx 2.5 \Omega$ for each contact was obtained.

In both cases, the negligible contact resistance is critical for the measurements shown in this work. In measurements of critical current J_c , this is important to avoid heating at contacts, which undermines the value of J_c . In measurements of depairing current J_c^* , this guarantees that the observed dynamics are from the sample itself instead of the change of contact resistance.

S3. Characterization of NbN device

An optical microscope image of the NbN ultrafast device used for the measurements is shown in Figure S3. The sample size is, width $\approx 10\text{ }\mu\text{m}$ $\times$ length $\approx 30\text{ }\mu\text{m}$ $\times$ thickness $\approx 20\text{ nm}$.

In order to directly compare the conventional critical current density J_c and the intrinsic depairing current density J_c^* , both DC and picosecond transport measurements were conducted on the same device.

In DC transport measurements, there were three contributions to the measured resistance: Au signal line of the coplanar waveguide, contact area and sample. To eliminate the contribution from the Au signal line, during Ti/Au deposition for fabricating the coplanar waveguides, a narrow line was deposited on another MgO substrate. This narrow line was separately characterized with a Physical Property Measurement System (PPMS) to extract its temperature-dependent resistivity. This served as a reference to calculate the resistance contribution of the Au signal line on the ultrafast device (yellow curve in Fig. S4a). The resistance versus temperature dependence is shown in Fig. S4b after subtracting this contribution. At 7 K, below the superconducting transition temperature, the measured resistance is $\approx 6\text{ }\Omega$, with $\approx 3\text{ }\Omega$ from the left and right contact. In Fig. S5, the sample resistance R_{sample} is presented after subtracting the contact resistance.

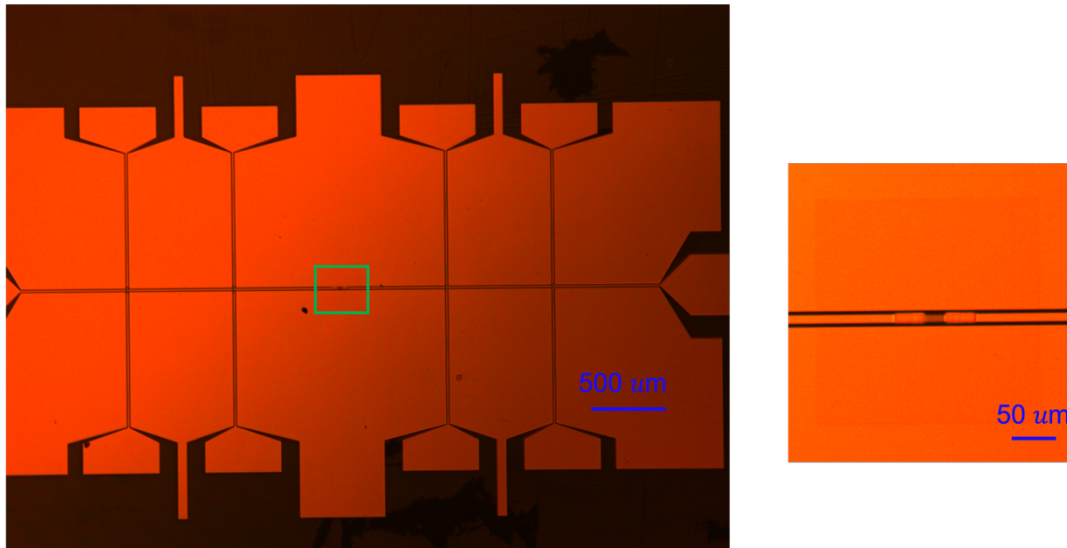


Figure S3| Optical micrograph of NbN ultrafast device.

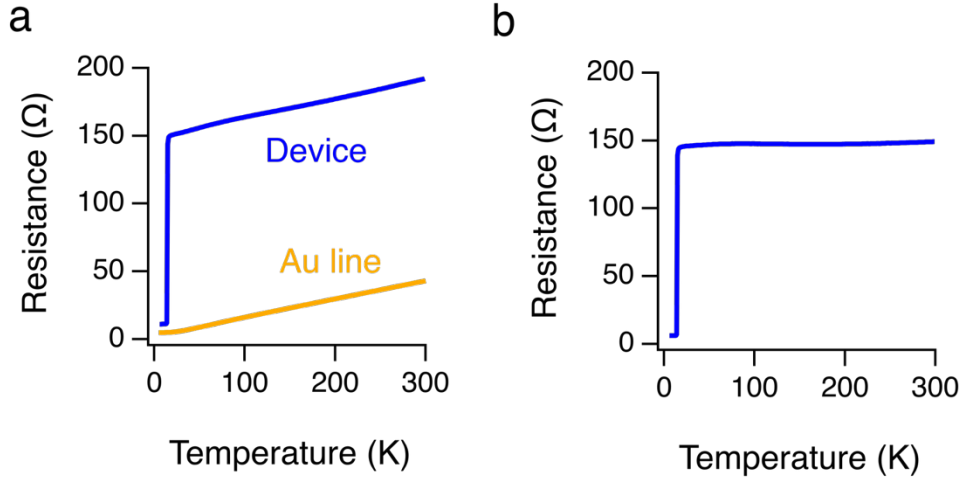


Figure S4| Resistance contribution from the Au signal line of the coplanar waveguide. a, Device resistance (blue) and calculated Au line resistance (yellow) versus temperature. b, Resistance versus temperature after subtracting the contribution of the Au line.

To exclude possible heating effects in the DC transport measurement of the conventional critical current density J_c , we used the measurement configuration shown in Fig. S6. A Keithley 6221 current source and a Keithley 2182 nanovoltmeter were combined to launch μ s-long current pulses. The measurement of the sample resistance was conducted in a fixed 400 μ s time window with a tunable delay after the onset of the applied current. As shown in Fig. S4b, the critical current value — at which sample resistance starts rising — remained constant when increasing the delay time. Rather, the sample resistance increased faster after the current was above the critical value, due to the accumulated heating after the sample became dissipative. In this way, we can exclude a possible heating effect at the contacts while measuring the DC critical current density.

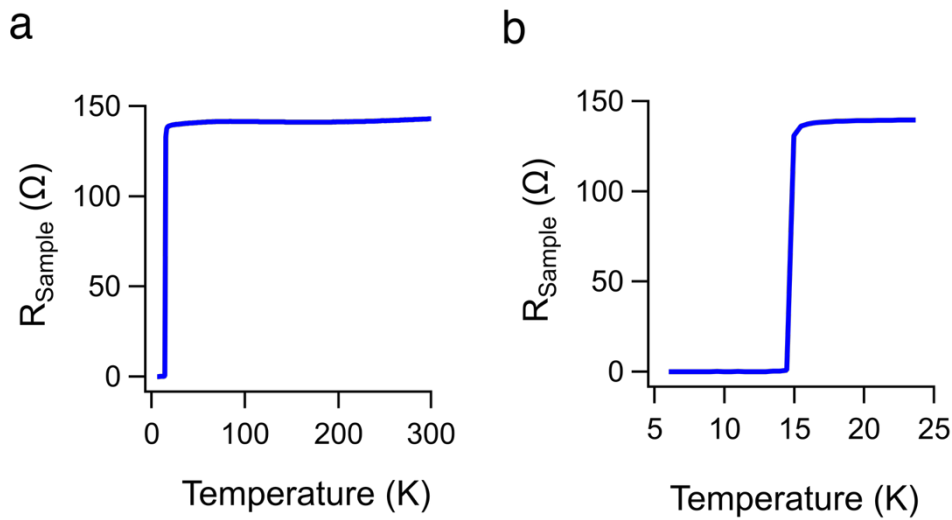


Figure S5| Superconducting transition of NbN sample. a, b, Sample resistance versus temperature.

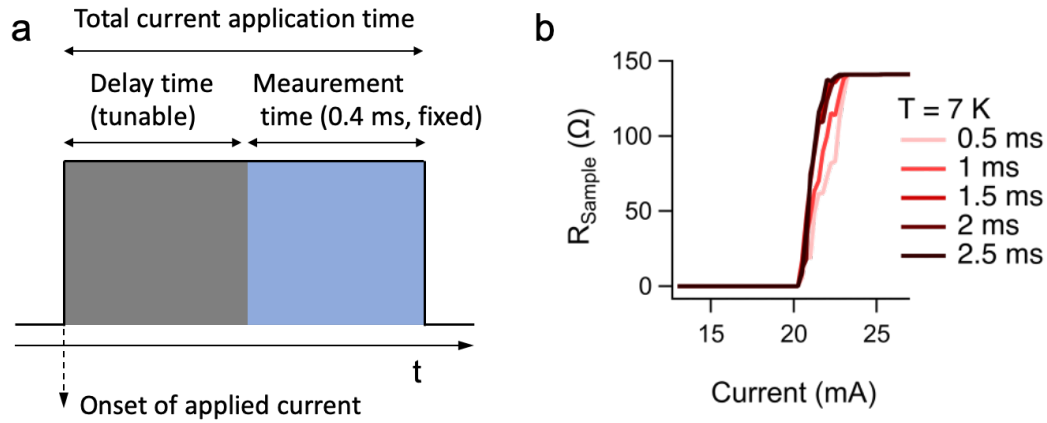


Figure S6| Dependence of the DC critical current measurements on delay time. a, Configuration of measurement setup. b, NbN sample resistance versus current for five different delay times.

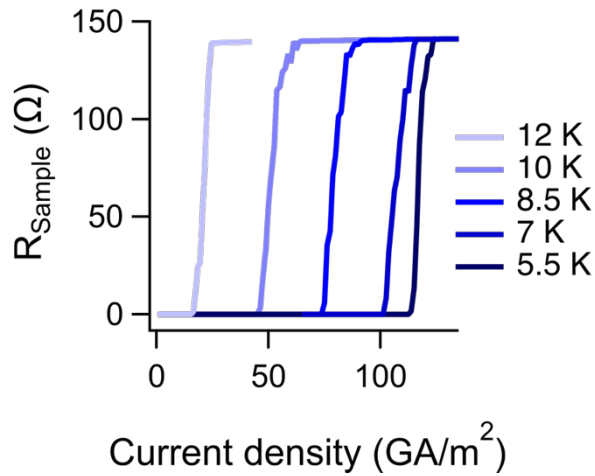


Figure S7| DC critical current density measurements of the NbN sample at different temperatures.

S4. Characterization of YBCO devices

An optical microscope image of the YBCO ultrafast device used for the measurements is shown in Figure S8. The sample size is, width $\approx 10 \text{ } \mu\text{m}$ $\times$ length $\approx 30 \text{ } \mu\text{m}$ $\times$ thickness $\approx 50 \text{ nm}$.

The same procedures as described for the NbN device were performed to identify the resistance contribution from the Au signal line, contact and sample (Figures S9 and S10). The resistance of each contact is $\approx 2.5 \text{ } \Omega$. In Fig. 10b the sample resistance R_{sample} is presented after subtracting the contact resistance. Compared with measurements on NbN, the measurement temperature is at 50 K higher than 7 K and the contact resistance is smaller, so we expect there is also no heating effect in the measurement of DC critical current density J_c .

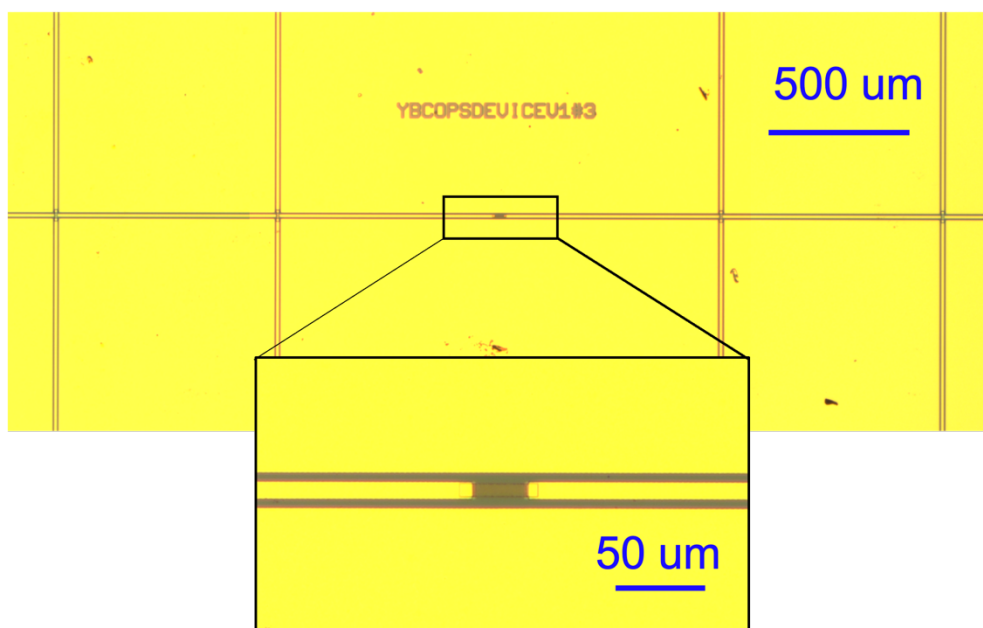


Figure S8| Optical micrograph of YBCO ultrafast device.

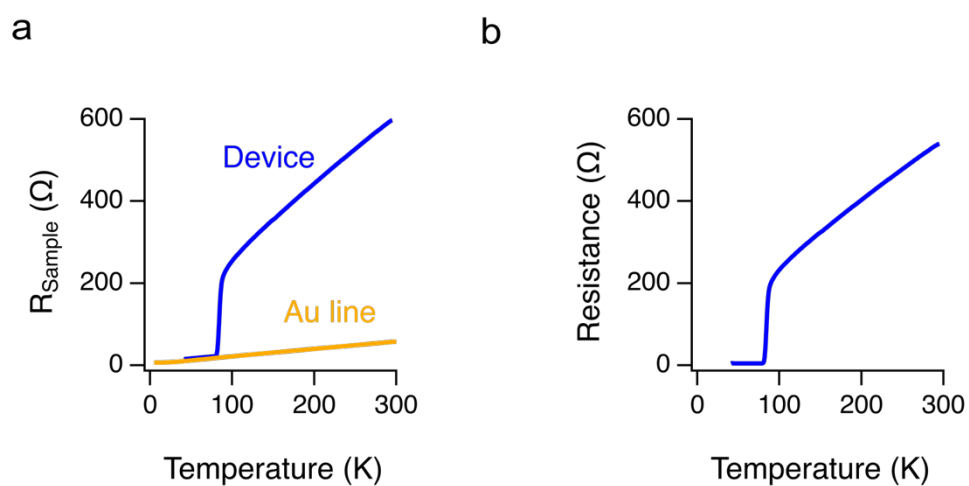


Figure S9| Resistance contribution from the Au signal line of the coplanar waveguide. a, Device resistance (blue) and calculated Au line resistance (yellow) versus temperature. b, Calibrated resistance versus temperature after subtracting the contribution of the Au line.

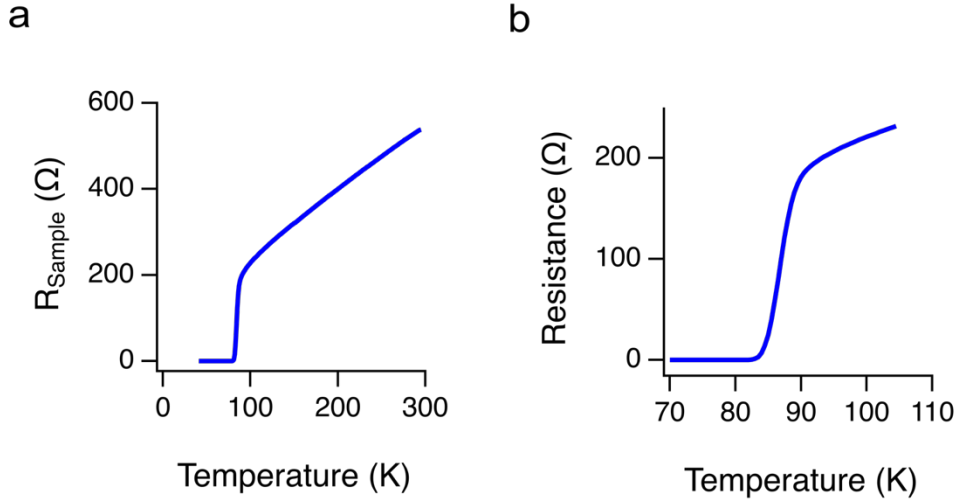


Figure S10| Superconducting transition of the YBCO sample. a, b, Sample resistance versus temperature.

S5. Calibration procedure

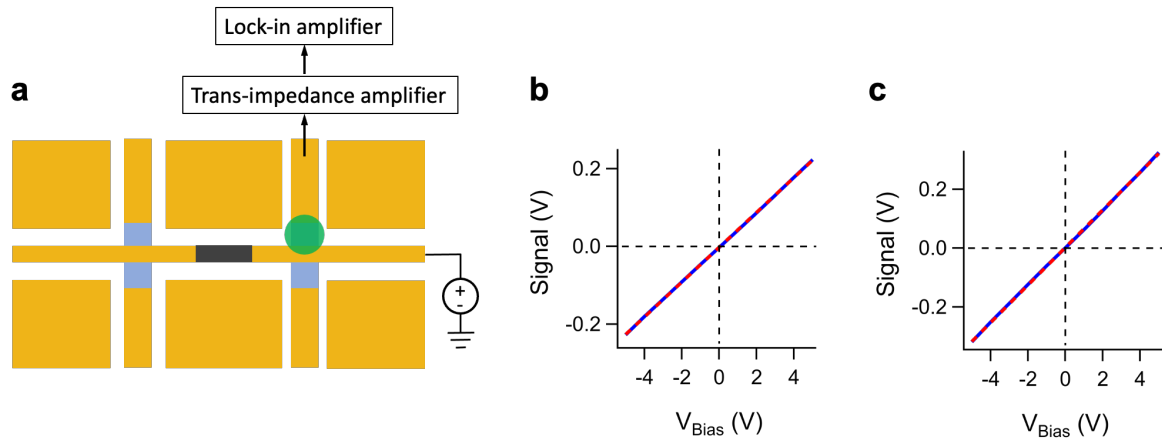


Figure S11| Switch calibration. a, Sketch for calibration process. b, Calibration of the photoconductive switch used for sampling incoming/reflexive pulses. The measured curve is shown in blue and the linear fitting in dashed red. c, Calibration for the photoconductive switch used for sampling the transmitted pulses. The measured curve is shown in blue and the linear fitting in dashed red.

The calibration procedures are discussed in detail in Ref. [2, 3]. Here the sensitivity of each switch used to probe the current pulses was calibrated with the configuration shown in Fig. S11a. The signal line was biased with a voltage source, leaving the other end open. The switch that was calibrated was illuminated with a 1kHz-chopped 515-nm laser beam and the signal was measured with a lock-in amplifier after a custom-built ultralow-noise trans-impedance amplifier. A linear signal versus V_{bias} dependence was obtained. For example, calibrations performed for the switch measuring the incoming/reflected and the transmitted current pulses at 7 K are shown in Fig. S11b

and Fig. S11c, respectively. All measured quantities using photo-excited switches were normalized with the slope α of the calibration curve.

The peak current amplitudes of the incoming and transmitted pulses were calculated as follows,

$$I_{peak} = 1.414 \times \frac{V_{signal,peak}}{\alpha \cdot Z_0 \cdot 0.85} \quad (1)$$

Here, $V_{signal, peak}$ is the measured peak signal from the amplifier, α is the slope of the switch calibration curve as discussed above, Z_0 is the wave impedance of the coplanar guide $\approx 50 \Omega$, the factor 0.85 is taking into account the average value between the pulses reaching the photoconductive switches and the pulses transmitting through the switches ($\approx 30\%$ reflection due to local impedance mismatch, see Ref. [2], which gives $(1+0.7)/2 = 0.85$). As illustrated in Fig. S12, the peak signal under DC bias is 1.414 times larger than that under picosecond pulse bias. Therefore, a prefactor of 1.414 is taking into account the correlation effect between the actual current pulse time profile and the switch response time profile, which are assumed to be very similar due to the high bandwidth of the coplanar waveguide (0.7 THz with -3dB), compared with the full half maximum width of current pulse (FHMW ≈ 2 ps).

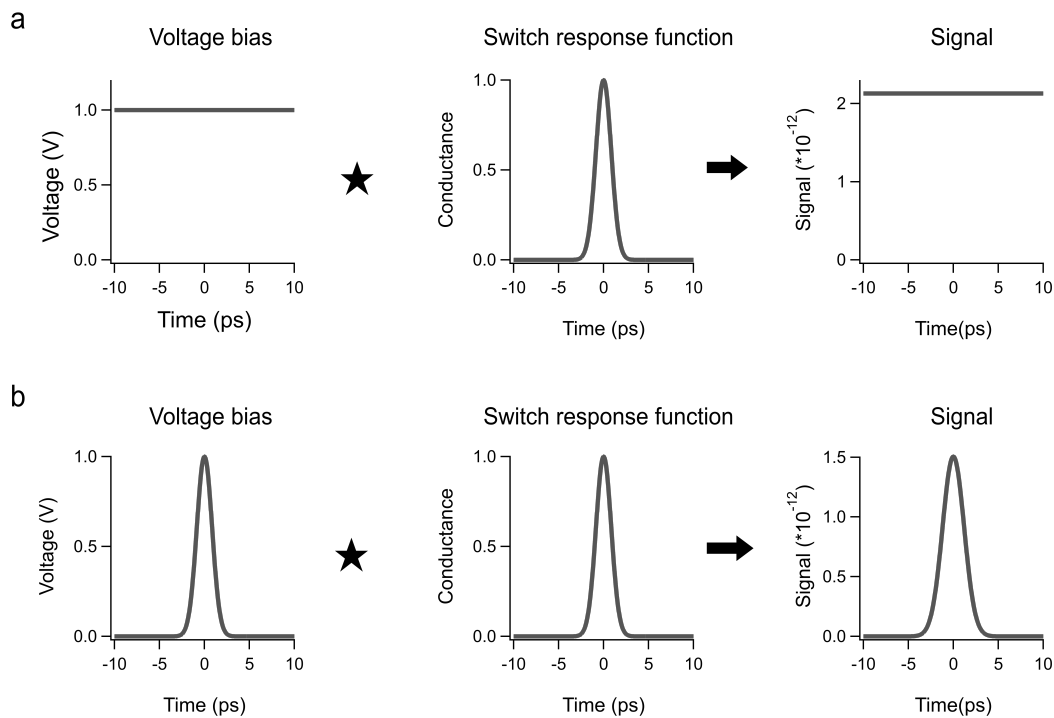


Fig.S12| Illustration of measured signal from photoexcited switch under (a) DC and (b) picosecond voltage bias. “ $\star$ ” denotes the cross-correlation operation.

Here we did not adopt another calibration method by calculating the injected charge from photo-excited switches as discussed in Ref. [2,3], because this requires to acquire full temporal profiles for each peak current density shown in Fig. 4 and Fig.

S15, and is impractically time-consuming. These two calibration approaches have been compared in Ref. [3] and been shown to be consistent.

We have further validated the above calibration method by applying a bias current $I_{\text{bias}} < I_c$ and checked how much the depairing process shifted. In Fig. S13 data with $I_{\text{bias}} = 19.25$ mA is shown. As can be seen the shift of the transmitted peak current is around 19.45 mA, which validates our calibration method for the peak currents of picosecond pulses and also indicates the different origins of J_c and J_c^* .

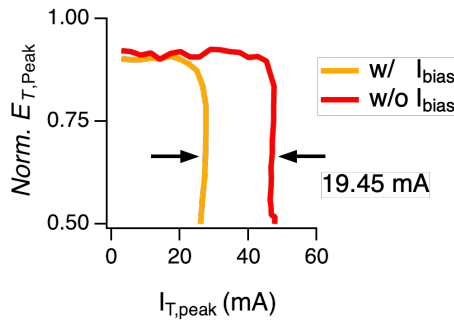


Figure S13| Offset of the depairing process by applying the bias current I_{bias} .

S6. Picosecond ultrafast transport measurements on YBCO

The picosecond ultrafast transport measurements on YBCO are shown in Figure S14. The results are similar to the ones on NbN, demonstrating the inductive response at $T = 40$ K and the resistive response at $T = 110$ K, below and above $T_c = 85$ K, respectively. By increasing the incoming peak current density, the YBCO sample also developed a resistive response. However, this transition is a slow and gradual process, as shown in Fig. 4c and 4d. For example, with $J_{T,\text{peak}}$ reaching $6 \times J_c$, the normalized transmitted pulse (peak $\approx 43\%$) is stronger than the one measured at $T = 110$ K (peak $\approx 23\%$), right above T_c , indicating incomplete suppression of superconductivity.

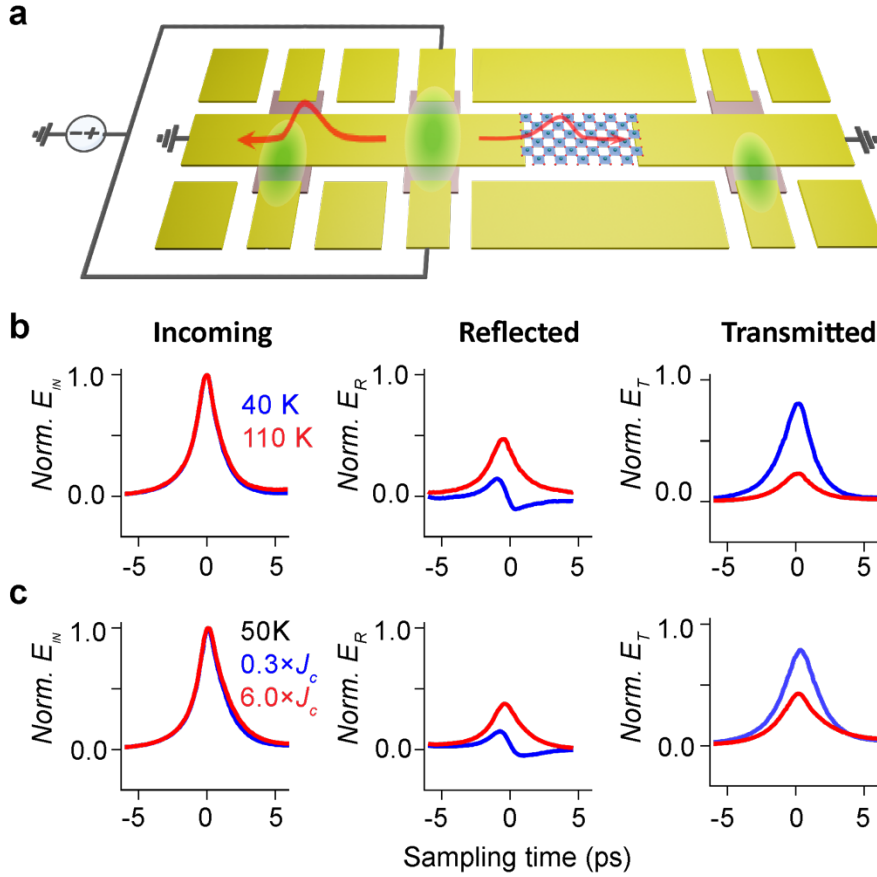


Figure S14| Picosecond ultrafast electrical transport measurements on YBCO. a, Illustration of the device architecture, which is the same as the one shown in Fig. 3 of the main text. b, Measurements at 40 K and 110 K, below and above $T_c = 85$ K, respectively. At 40 K, the reflected pulses show an inductive feature and ~80% of the incoming pulses transmits through. At 110 K, the incoming pulses partially reflect and partially transmit through, due to sample's resistive response. c, Measurements at 50 K with different peak current densities of the incoming pulses. Here the labels indicate the peak current densities of the transmitted pulses for direct comparison. At $6.0 \times J_c$, the sample shows a resistive response, but with a larger transmittance.

S7. Temperature dependence of J_c and J_c^* in NbN thin film

The temperature dependence of J_c is extracted from Fig. S7. More picosecond transport measurements on NbN at different temperatures are shown in Figure S15. The temperature dependence of J_c^* is extracted from Fig. S15d.

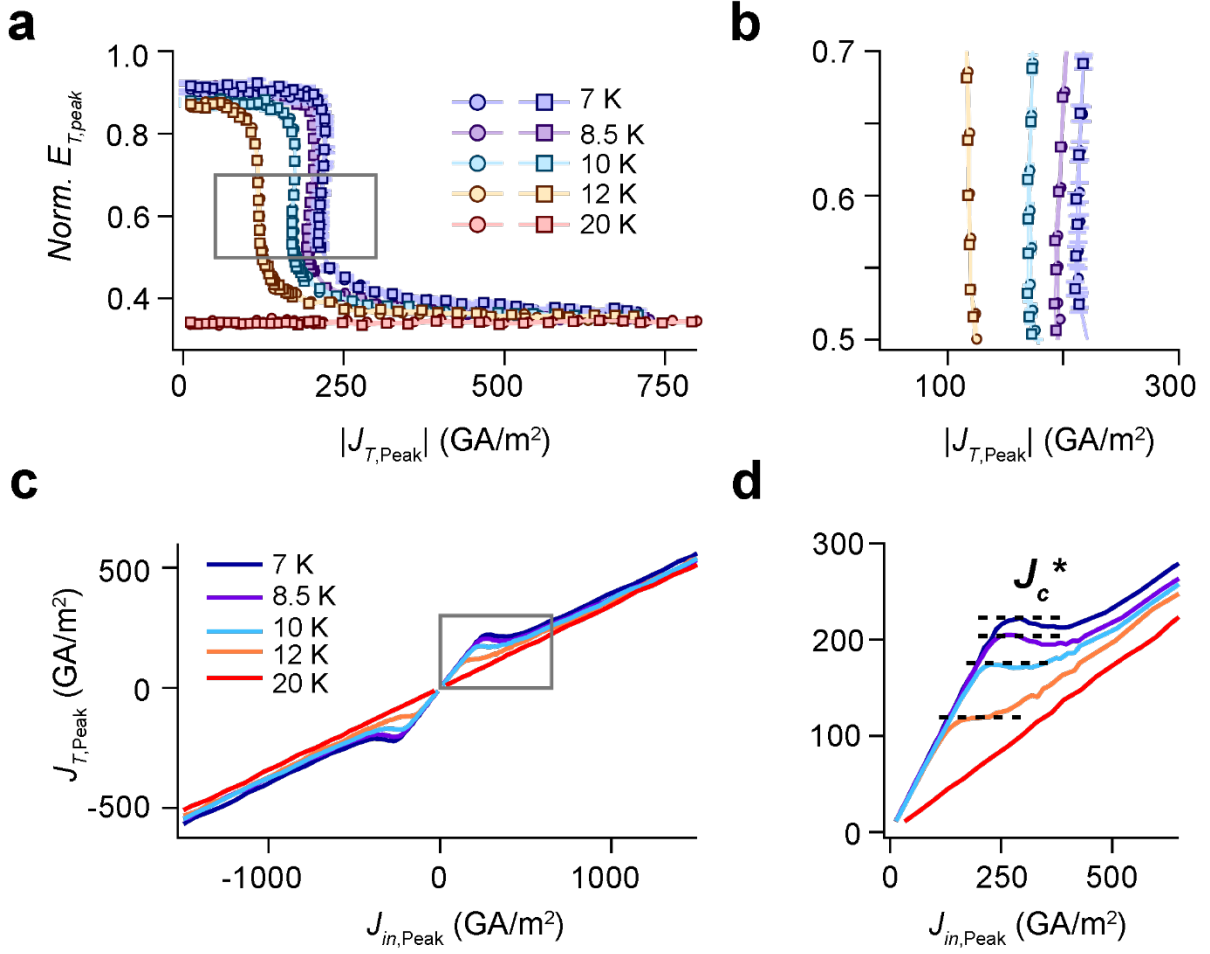


Figure S15| Temperature dependence of depairing current density J_c^* in NbN. a, normalized transmitted electric peak field (normalized by the peak electric field of the incoming pulse) versus the transmitted peak current densities at 7, 8.5, 10, 12, 20 K. • and ■ symbols represent the cases with positive and negative picosecond pulses. b, Zoom-in plot of (a) to show the sharp drop feature. c, transmitted peak current density versus incoming peak current density at 7, 8.5, 10, 12, 20 K. d, Zoom-in plot of (c). Depairing current densities J_c^* at different temperatures are indicated by dashed lines. The error bars shown in (a-b) are defined as standard errors.

S8. Extraction of superconducting coherence length (ξ), magnetic penetration depth (λ) and Pearl length P_L of NbN strip

S8.1 Extraction of superconducting coherence length (ξ)

In order to extract the coherence length $\xi(0)$, we measured the superconducting transition in a magnetic field in order to extract $B_{c2}(0)$, from which $\xi(0)$ can be directly calculated. The results are shown in Fig. S16. We took the magnetic field when the sample resistance reaches half of the normal state resistance as B_{c2} and follow the Werthamer-Helfand-Hohenberg (WHH) theory [4].

$$B_{c2}(0) = -0.69T_c \frac{dB_{c2}}{dT} \big|_{T=T_c} \quad (2)$$

$$\xi(0) = \left[\frac{\Phi_0}{2\pi B_{c2}(0)} \right]^{1/2} \quad (3)$$

We extract $B_{c2}(0) = 12 \text{ T}$ and $\xi(0) = 5.24 \text{ nm}$, close to 5.3 nm from Ref. [5].

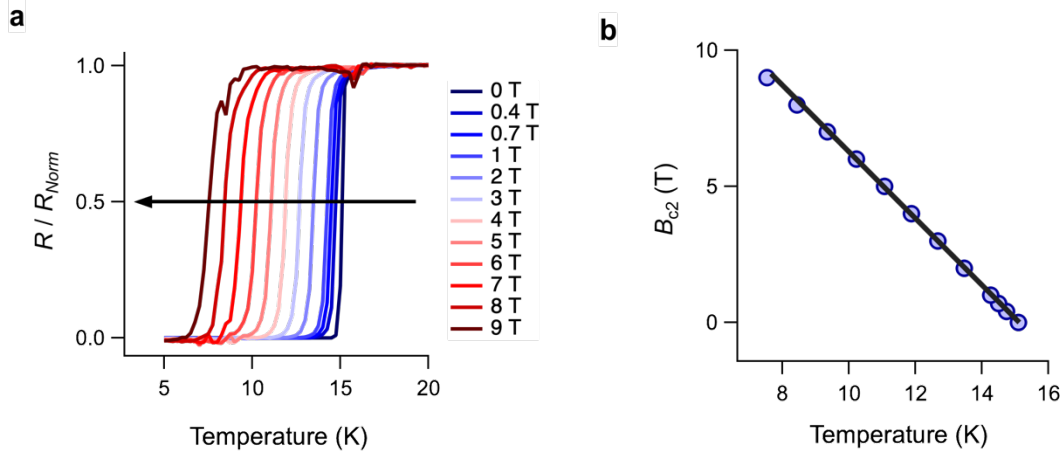


Figure S16| Measurements of B_{c2} at different temperatures. (a) Superconducting transition for various applied magnetic fields. (b) Extracted B_{c2} values versus temperature.

S8.2 Extraction of magnetic penetration depth (λ)

Regarding the magnetic penetration depth $\lambda(0)$, we extracted its value from the sample's kinetic inductance from the equation below [6].

$$L_{kin} = \frac{m}{2n_s e^2} \cdot \frac{l}{s} = \frac{\mu_0 \lambda^2 l}{s} \quad (4)$$

Where m is the effective mass of the electron; n_s is the superfluid density; e is electron charge; l is the length of strip; s is the cross section of the strip; μ_0 is the vacuum magnetic permeability; and λ is the magnetic penetration depth.

The kinetic inductance of the sample is extracted from the measured reflected pulse shown in Fig. 3b of the main text. Fig. S17 shows the data and the simulated reflected signal with $L_{kin}(0) = 23 \text{ pH}$, with matching amplitude. This corresponds to $\lambda(0) = 350 \text{ nm}$, which is close to the value of 360 nm in Ref. [5]. The detailed simulation procedure is described below.

The equivalent circuit of the sample together with the coplanar waveguide is shown in Fig. S18. The sample is treated using a two-fluid model. R_n and L_n are the resistance and kinetic inductance of the normal channel; L_s is the kinetic inductance of the superconducting channel and Z_0 is the impedance of the coplanar waveguide, which is 50Ω . R_n , L_n and L_s are directly related with the superfluid density as $R_n = R_{n0}/(1 - n_s)$, $L_n = L_{n0}/(1 - n_s)$ and $L_s = L_{s0}/n_s$, where R_{n0} and L_{n0} are the

sample's resistance and kinetic inductance when all carriers are normal quasiparticles, and L_{s0} is the sample's kinetic inductance when all carrier are Cooper pairs. We take $R_{n0} = 140 \Omega$, from DC transport measurement. $L_{n0} = L_{s0}$ is treated as a variable. We note that the assumption $L_{n0} = L_{s0}$ is based on the assumption that the effective mass of Cooper pairs is double that of electrons. Although this assumption can be relaxed, the exact value of L_{n0} have a negligible effect on the extracted kinetic inductance of the superconducting state from which the magnetic penetration depth λ is calculated. At 7 K, the superfluid fraction is approximately 95%, and the impedance of the normal channel ($R_n + i\omega L_n$) is two orders of magnitude larger than that of the superconducting channel. Specifically, $R_n = R_{n0}/(1 - n_s) \approx 2800 \Omega$, while $i\omega L_s \approx 24 \Omega$ at 1 THz, even without considering the contribution from L_n . Therefore, the sample response is dominated by the superconducting channel, making the precise value of L_{n0} uncritical.

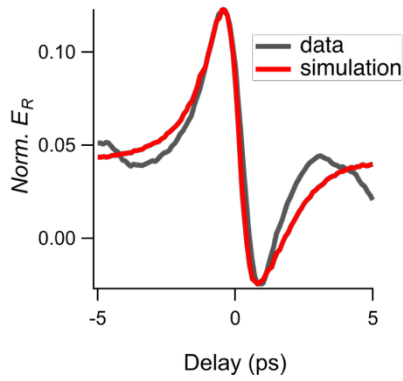


Figure S17| Simulation of the reflected pulse on NbN at 7 K.

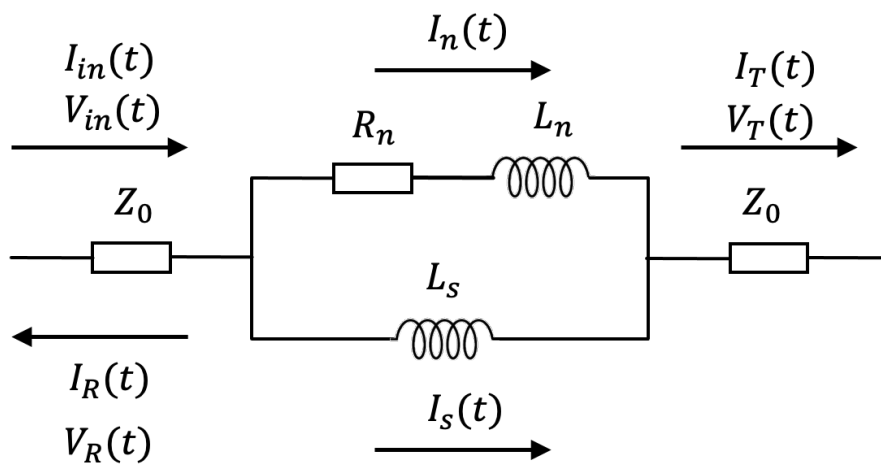


Figure S18| Circuit model of the sample.

The reflection and transmission of current pulses at the sample are described by the equations below.

$$I_{in}(t) - I_R(t) = I_s(t) + I_n(t) \quad (5)$$

$$I_T(t) = I_s(t) + I_n(t) \quad (6)$$

$$V_{in}(t) + V_R(t) - V_T(t) = L_s \cdot \frac{dI_s(t)}{dt} \quad (7)$$

$$V_{in}(t) + V_R(t) - V_T(t) = L_n \cdot \frac{dI_n(t)}{dt} + R_n \cdot I_n(t) \quad (8)$$

Where, $V_{in}(t) = I(t) \cdot Z_0$, $V_R(t) = I_R(t) \cdot Z_0$ and $V_T(t) = I_T(t) \cdot Z_0$.

With the incoming current $I_{in}(t)$ shown in Fig. 3b as input, the reflected current $I_R(t)$ is calculated from the differential equations above. The curve with $L_{s0} = 23 \text{ pH}$ fits the data best and is shown in Fig. S16.

S8.3 Extraction of Pearl length (P_L)

In order to check whether the current flowing through sample's cross section is uniform, we calculate the Pearl length (P_L), which characterize the current penetration into superconductor thin films [7]. P_L is directly related with λ with $P_L = \frac{2\lambda^2}{d}$, where d is the film thickness. From the extracted value of λ above, we get $P_L(0) = 12.2 \text{ } \mu\text{m}$ and $P_L(7 \text{ K}) = 13 \text{ } \mu\text{m}$, which is larger than the sample's width. Therefore, the current distribution across the sample is uniform. From the same procedure (see Fig. S19), we get $P_L(40 \text{ K}) = 19.4 \text{ } \mu\text{m}$ and $P_L(50 \text{ K}) = 21 \text{ } \mu\text{m}$ for YBCO, therefore the current flowing through YBCO at 50 K is also uniform.

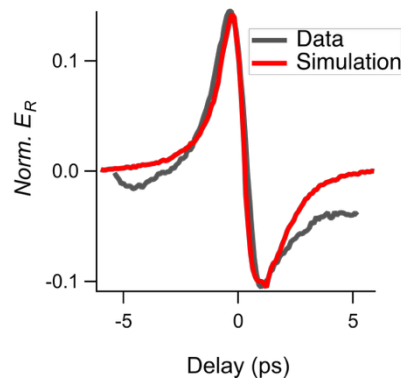


Figure S19| Simulation of the reflected pulse on YBCO at 40 K.

S9. Calculation of J_c^*

As the depairing current J_c^* is intrinsically determined by the microscopic properties of the s-wave superconductor, it can be estimated from the microscopic superconducting parameters. As shown in the main text, $J_c^* = n_s v_s \cdot (2e)$, where $v_s = \frac{\Delta}{\hbar \cdot k_F} = \frac{\hbar}{\pi m \xi}$. Therefore,

$$J_c^*(T) = \frac{2\hbar e}{\pi m} \frac{n_s(T)}{\xi(T)} \quad (9)$$

Here, we treat the temperature-dependent superfluid density $n_s(T)$ and coherence length $\xi(T)$ in dirty limit as follows [8]:

$$n_s(T) = \frac{m}{2e^2 \mu_0 \lambda^2(T)}, \text{ where } \lambda^2(T) = \frac{\lambda(0)^2}{1 - (\frac{T}{T_c})^4} \quad (10)$$

$$\xi(T) = \frac{\xi(0)}{\sqrt{\tanh\left(1.74 \sqrt{\frac{T_c}{T} - 1}\right)}} \quad (11)$$

We take $\lambda(0) = 350$ nm and $\xi(0) = 5.24$ nm extracted above. The results are shown in Figure 5 of the main text and exhibit remarkable agreement with the experimental results.

S10. Further experimental evidence of picosecond depairing dynamics

In order to provide direct experimental evidence on different mechanisms of current thresholds observed on DC and picosecond timescales, we fabricated a NbN strip with notches on one side (width ~ 8 μm , thickness ~ 20 nm, notch depth ~ 2 μm), and measured J_c and J_c^* along opposite directions under an external magnetic field.

When the magnetic field generated by the bias current aligns with applied magnetic field, the critical current is suppressed. This is interpreted as local magnetic field enhancement which facilitates vortex penetration and motion [9-11]. Opposite effect is observed when the two fields are anti-aligned (see Fig. S20b).

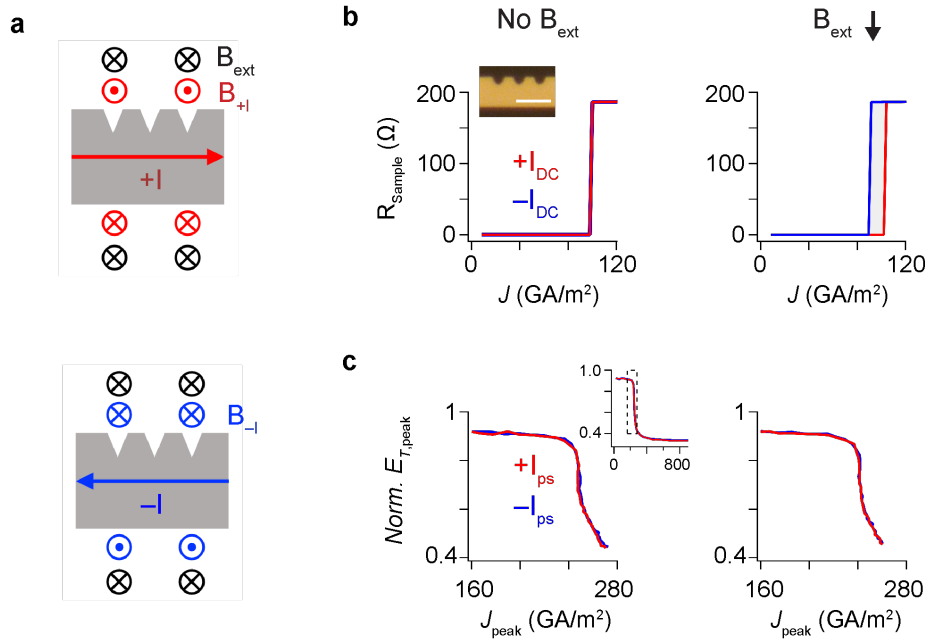


Figure S20 | Comparison of magnetic field effect on critical and depairing currents. (a) Illustration of external magnetic field (B_{ext}) and the magnetic field from current ($B_{\pm I}$). (b) Critical currents along both directions with zero B_{ext} and downward pointing B_{ext} . (c) Depairing currents along both directions with zero B_{ext} and downward pointing B_{ext} .

On the other hand, when the current threshold is measured with picosecond pulses, the response is independent on the relative alignment of externally applied and current generated magnetic fields (see Fig. S20c). This further confirms that J_c^* does not result from vortex penetration but from other effects — microscopic depairing dynamics.

Here, we attribute the DC critical current to be from the vortex entry and subsequent depinning. With sample width of $\sim 10 \mu m$ used in the main text, which is smaller than the Pearl length, other effects, such as phase slip centers or lines, cannot be totally excluded from our experimental observations. We want to note that, regardless of the exact mechanism responsible for the DC critical currents observed in NbN strip, this mechanism is irrelevant for the picosecond dynamics observed in our experiments. This is evidenced by the distinct DC and picosecond transport measurement results on the notched NbN strip shown above.

S11. Theoretical simulations of depairing process

S11.1 Dynamics model within BCS framework

The dynamics of the superconductor is assumed to be homogeneous throughout the sample geometry. This is confirmed by the Pearl length $P_L = \frac{2\lambda_L^2}{t}$ determined above. In NbN, $P_L \approx 13 \text{ } \mu\text{m}$ at $T = 7 \text{ K}$ and in YBCO, $P_L \approx 21 \text{ } \mu\text{m}$ at $T = 50 \text{ K}$, both larger than the width of the sample $\sim 10 \text{ } \mu\text{m}$. Therefore, in both cases, the current distribution is uniform across the sample's cross section.

A vortex, if present, introduces an additional local circulating screening current around its core, which produces only a spatially localized perturbation to this otherwise nearly uniform background current.

Quantitatively, the vortex-induced screening current decreases rapidly with distance from the vortex core, given by $J(r) = \frac{\Phi_0}{4\pi\mu_0\lambda^2 r}$ [7]. For the parameters of our NbN films, its magnitude is approximately 10 GA/m^2 at 100 nm from vortex core and approximately 1 GA/m^2 at $1 \text{ } \mu\text{m}$. These values are substantially smaller than the depairing current density of order 100 GA/m^2 . Although the screening current can therefore produce appreciable local nonuniformity very close to a vortex core, this perturbation is confined to a small region and does not substantially modify the current distribution over the majority of the sample.

In addition, our picosecond measurements show the disappearance of the superconducting vortex-diode response (see Sec. 10), consistent with vortex motion being strongly suppressed on the picosecond timescale. The vortices therefore remain effectively confined near their nucleation sites at the sample edges during the current pulse. Consequently, when the bulk of the NbN strip approaches the depairing threshold, its response is governed predominantly by the approximately uniform transport supercurrent.

In discussions below, all quantities are in units with $\hbar = k_B = 1$.

We also assume that the order parameter relaxes sufficiently fast that we can treat the dynamics as essentially adiabatic — that is they follow the instantaneous equilibrium behavior at each point in time as a function of the current. From the Usadel equation [12], we are able to compute the evolution of the supercurrent J_s and gap Δ as a function of temperature T/T_c and Cooper pair Doppler shift energy $\mathbf{Q}_s \mathbf{v}_F / T_c$, where $\mathbf{Q}_s = \mathbf{A} - \frac{\phi_0}{2\pi} \nabla \theta$ is the gauge-invariant momentum of Cooper pairs.

The equilibrium Usadel equation is given by

$$[(\epsilon + i\eta)\hat{\tau}_3 - i\Delta\hat{\tau}_2, \hat{g}^R] + ie^2 D A^2 [\hat{\tau}_3, \hat{g}^R [\hat{\tau}_3, \hat{g}^R]] = 0 \quad (12)$$

where the angled brackets represent a commutator, $\hat{g}^R$ is the spectral function, satisfying the normalization condition $(\hat{g}^R)^2 = 1$, D is the diffusion constant of the

material and A is the vector potential, ignoring spatial gradients in the sample geometry. Here η represents the Dynes broadening factor, which gives finite quasiparticle life-time due to non-elastic scattering processes (magnetic impurities, phonons, etc.). The gap is determined self-consistently through the gap equation

$$\Delta = -\frac{\pi\lambda}{2} \int_{\epsilon} \mathbf{tr} \hat{t}^- \hat{g}^K \quad (13)$$

where λ is the BCS coupling constant and $\mathbf{tr}$ represents the trace over Pauli matrices $\hat{t}$. Here

$\hat{g}^K = (\hat{g}^R - \hat{g}^A) \tanh \frac{\epsilon}{2T}$ and $\hat{g}^A = -\hat{t}_3 (\hat{g}^R)^\dagger \hat{t}_3$. Accordingly, we expand the current in the Usadel equation up to first order in time derivatives, capturing the non-linear static response of the superconductor as well as the non-linear static response of the normal state, resulting in a non-linear two-fluid model.

$$J(w) = -n_s(A)A + \sigma(A)E \quad (14)$$

where

$$n_s = i \frac{\pi\sigma_n}{4} \int_{\epsilon} \mathbf{tr} [\hat{t}_3 \hat{g}^R(\epsilon) \hat{t}_3 \hat{g}^K(\epsilon) + \hat{t}_3 \hat{g}^A(\epsilon) \hat{t}_3 \hat{g}^K(\epsilon)] \quad (15)$$

is the superfluid density and

$$\sigma = -\frac{\pi\sigma_n}{4} \int_{\epsilon} \mathbf{tr} \left[\hat{t}_3 \hat{g}^R(\epsilon) \hat{t}_3 \frac{\partial \hat{g}^K(\epsilon)}{\partial \epsilon} - \hat{t}_3 \hat{g}^A(\epsilon) \hat{t}_3 \frac{\partial \hat{g}^K(\epsilon)}{\partial \epsilon} \right] \quad (16)$$

is the conductivity. Here $E(t) = -\frac{\partial Q_s}{\partial t}$ is the electric field.

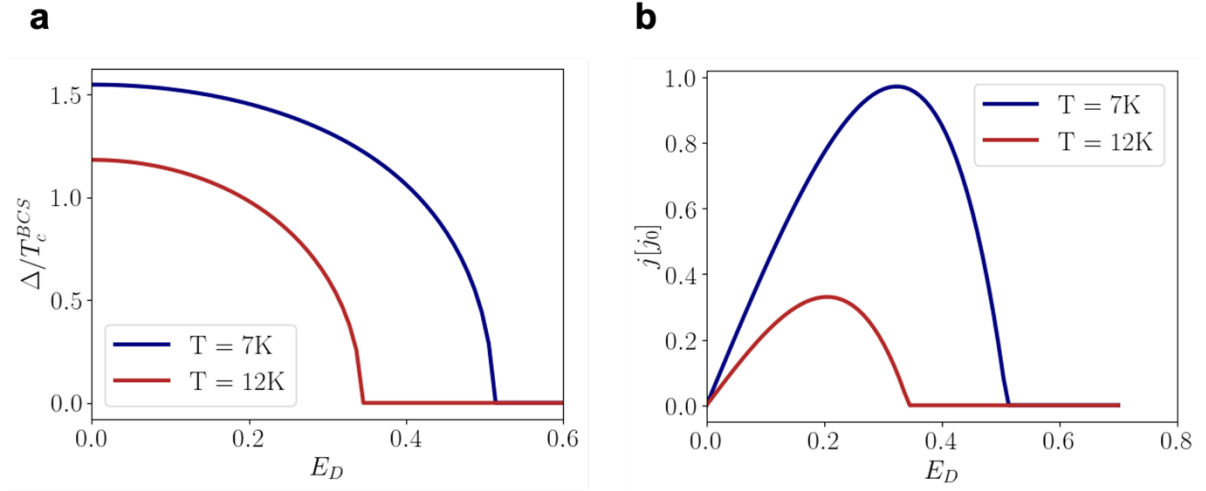


Figure S21| Nonlinear response of s-wave superconductor under strong current drive. a, Gap in units of T_c^{BCS} as a function of the Doppler shift E_D for 7 and 12K. b, Normalized supercurrent in units of j_0 (see text) as a function of Doppler shift ($\eta = 0.08 T_c^{BCS}$).

S11.2 Dynamics model within tDGL theory

To simulate the response of the system within tDGL theory, we assume a two fluid model.

The superfluid response is given by the usual time-dependent GL equations

$$\tau_{GL} \partial_t f = \left[1 - f^2 - \left(\frac{2\pi\xi}{\phi_0} \right)^2 Q_s^2 \right] f + \xi^2 \nabla^2 f \quad (17)$$

where f is the normalized superconducting gap $f = \Delta(\mathbf{r}, t)/\Delta_0$, Φ is the voltage potential and ξ is the coherence length of the system. Here, Q_s represents the gauge invariant vector potential

$$Q_s = \frac{1}{2e} \nabla \theta - A \quad (18)$$

and θ is the superfluid phase. The superfluid current and the normal state current within the thin sheet are given as

$$J_s = \frac{d}{\lambda^2} \delta(z) \theta \left(\frac{w}{2} - |y| \right) f^2 Q_s \quad (19)$$

$$J_n = d \delta(z) \theta \left(\frac{w}{2} - |y| \right) \sigma_n E \quad (20)$$

and the total current is the summed contributions from superfluid and normal fluid currents.

For the calculations in the tDGL framework, the magnetic penetration lengths are extracted from the measurements $\lambda = 350 \text{ nm}$ at 7K and $\lambda = 400 \text{ nm}$ at 12K for NbN as discussed in

S.8.2. The normal state resistivity is set to match the transmittance at $T > T_c$. We take the microscopically predicted depairing current values at two temperatures as input.

S11.3 Simulating sample's response under strong ps current drive

The sample's response when incorporated into coplanar waveguide, is modeled under the assumption that the sample is a lumped element, which is valid with sample length $\sim 30 \mu m \ll$ pulse wavelength $\sim 150 \mu m$. This means that at the sample position, we can write for incoming, reflected and transmitted current/voltage respectively,

$$I_{in}(t) - I_r(t) = I_t(t) = I(t) \quad (21)$$

$$V_{in}(t) + V_r(t) = V_t(t) + V(t) \quad (22)$$

Where $I_{in}(t)$, $I_r(t)$, $I_t(t)$ are the incoming, reflected and transmitted current pulse, $V_{in}(t)$, $V_r(t)$, $V_t(t)$ are the incoming, reflected and transmitted voltage pulse. $I(t)$ and $V(t)$ are the current and voltage across the sample.

The incoming, reflected and transmitted voltage pulses have a linear relation to the current, $V_{in,r,t} = Z_0 I_{in,r,t}$, where Z_0 is the wave impedance of coplanar waveguide. Combining everything together we get a relation between the current $I(t)$ and voltage $V(t)$ across the sample:

$$I_{in}(t) = I(t) + \frac{V(t)}{2Z_0} \quad (23)$$

This can be re-cast in the form of a differential equation which can be solve numerically $V(t) = E(t)L = \partial_t Q_s L$, where L is the sample length.

$$\frac{\partial Q_s}{\partial t} = 2Z_0 (I_{in}(t) - I(Q_s(t))) \quad (24)$$

The normal state resistivity is set to match the transmittance at $T > T_c$. We choose the BCS coupling constant to match the observed T_c of 15 K. Within the dirty superconductor framework and Dynes η of zero, the superfluid density is given as

$$\pi \sigma_n \Delta \tanh\left(\frac{\beta \Delta}{2}\right) = n_s \quad (25)$$

Therefore, the samples kinetic inductance within the superconducting phase for low pulse strength is completely determined by the normal state resistivity and gap size

$$L_s = \frac{R_n}{\Delta \tanh\left(\frac{\beta\Delta}{2}\right)} \quad (26)$$

The successful application of our theory at weak pulses proves the validity of the dirty limit approximation and the lumped circuit element model. The only remaining parameter within our framework is then the depairing current. To extract this, we use the experimentally measured value of coherence length at 0K and use the dirty superconductor relation

$$\xi(T) = \sqrt{\frac{D}{2\Delta(T)}} \rightarrow \xi(T) = \xi(0) \sqrt{\frac{\Delta(0)}{\Delta(T)}} \quad (27)$$

Where we obtain the gap as a function of T from the self-consistency equation from above. To relate the coherence length to the depairing current, we go back to the current - vector potential relation and the Usadel equation. We define a dimensionless Doppler shift as $E_D^2 = e^2 D A^2 / T_c$ and the full current in the DC limit can be re-written as

$$j = \sigma_n T_c \frac{1}{e\sqrt{D/T_c}} \left[\frac{\pi E_D}{4} \int_{\epsilon} \text{tr} \left(\hat{\tau}_3 \hat{g}^R(\epsilon, E_D) \hat{\tau}_3 \hat{g}^K(\epsilon, E_D) + \hat{\tau}_3 \hat{g}^A(\epsilon, E_D) \hat{\tau}_3 \hat{g}^K(\epsilon, E_D) \right) \right] \quad (28)$$

Importantly, we see from the Usadel equation that the spectral functions only depend on E_D and do not contain additional factors or dependence on the diffusion constant. Therefore, the depairing current can be re-written as

$$I_c^* = \frac{L}{R_n} \frac{T_c}{\xi(T)} \sqrt{\frac{T_c}{2\Delta(T)}} \times C_0 \quad (29)$$

where C_0 is the maximum of the expression in the square bracket above as a function of E_D , at fixed temperature. We define I_0 as the $I_c^* = I_0 * C_0$. This allows us to extract the critical current from the zero temperature measurements of B_{c2} , leaving only one undetermined parameter in our theory which is the Dynes broadening η . In our simulations, we choose the value for Dynes broadening equal to 10% of the T_c^{BCS} , to ensure numerical stability and convergence. The finite value of η causes corrections in the gap temperature dependence and critical temperature. Therefore, in our fitting procedure, we choose the BCS coupling to correspond to $T_c(\eta = 0) = T_c^{BCS} = 1$ and represent all the quantities in units of T_c^{BCS} . For the chosen value of Dynes broadening, we obtain $T_c = 0.89 T_c^{BCS}$, therefore $T_c^{BCS} = \frac{1}{0.89} * 15 K$ and in the above equation for depairing current we replace T_c by T_c^{BCS} , since that is the natural unit of gap.

Below we show the simulations based on microscopic BCS theory in dirty limit and tDGL theory (Fig. S22). As we can see the tDGL cannot capture the sharp drop of transmittance at

$T = 7$ K, while the microscopic theory can reproduce the observed current density threshold (J_c^*) and the abrupt collapse of superconductivity above this threshold much better. This is because, the Ginzburg-Landau theory relies on the expansion in the power of order parameter and is valid only near T_c [13, 14]. As expected, at $T = 12$ K, closer to T_c , the results from both theories start to coincide. We note that, the piecewise-linear features shown in the simulation results are attributed to the numerical discretization.

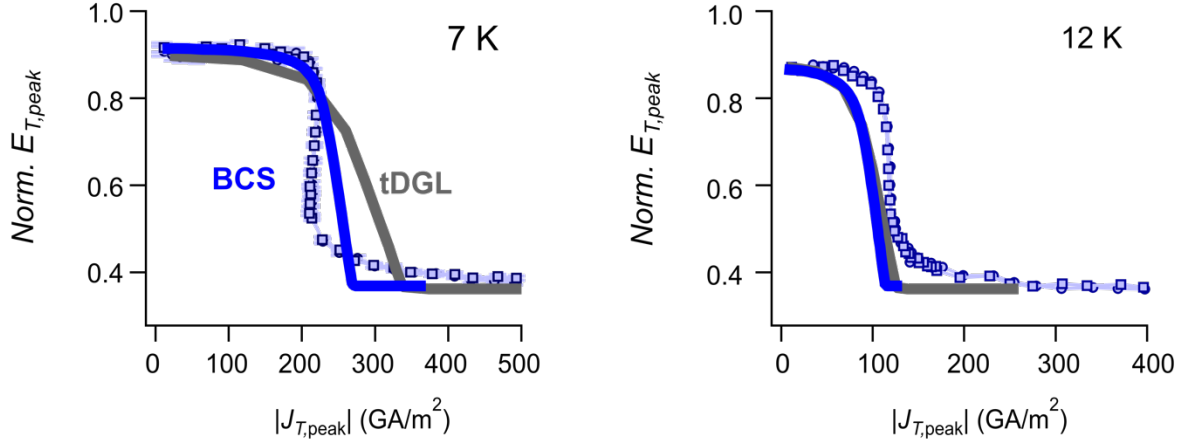


Figure S22| Comparison of simulation results based on BCS theory in dirty limit and tDGL theory. The simulation results based on BCS theory in dirty limit and tDGL theory are plotted in blue and grey colors. The measurement data is plotted in light blue.

Our instantaneous theory provides estimates of J_c^* in equilibrium. In a dynamical setting, the value of the maximum current that can be passed through the superfluid might be different. Experimentally, we see that in the case of NbN the dynamical depairing current is very close to the theoretically calculated depairing current as demonstrated in Fig. S20. Since the two values are consistent, we can justify the use of instantaneous equilibrium theory, in which depairing dynamics is faster than the timescale of picosecond current pulse.

We also note that experimentally the transmittance gradually approaches the normal-state response for stronger pulses above the depairing threshold. However, this slow suppression is not captured within the current adiabatic theoretical framework. We attribute this discrepancy to non-equilibrium effects and additional relaxation mechanisms beyond our adiabatic model.

Several processes can contribute on relevant timescales. First, both cold-atom experiments and BCS theory suggest that, following a quench, the superconducting gap relaxes on a timescale set by the gap magnitude [15-17], which for $T_c \sim 15$ K corresponds to the picosecond regime. Second, phonon-mediated relaxation processes occur on comparable

timescales. Third, in the dirty limit applicable to NbN, elastic scattering introduces additional fast relaxation channels.

These mechanisms can jointly contribute to the observed gradual suppression, including gap retardation effects and quasiparticle relaxation. While our experimental platform provides access to these ultrafast dynamics, a quantitative disentanglement of the individual contributions would require a more complete non-equilibrium theoretical framework, which is beyond the scope of the present work.

S11.4 Discussion of depairing dynamics in YBCO

Unlike the case of NbN, the microscopic theory of cuprate superconductors is not yet well established even in equilibrium, and remains an active area of research. As a result, there is currently no widely accepted theoretical framework that allows a quantitative description of non-equilibrium depairing dynamics in *d*-wave systems comparable to the BCS/Usadel approach used for *s*-wave superconductors. This limitation makes it challenging to construct a fully microscopic model for the YBCO response under strong current injection.

Nevertheless, the gradual suppression of superconductivity observed in YBCO is qualitatively consistent with the presence of gap nodes, where quasiparticles can be excited at arbitrarily low energies. Within a phenomenological framework, this can be qualitatively captured by the GL theory — $n_s \propto 1 - \frac{J^2}{J_c^{*2}}$. Therefore, we have employed tDGL theory described in section 11.2 to simulate the dynamics in YBCO under picosecond current bias (Fig. S23). The value of J_c^* is derived from $J_c^* = \frac{1}{\lambda^2} \frac{2}{3\sqrt{3}} \frac{\Phi_0}{2\pi\mu_0\xi}$, where λ is the magnetic penetration depth, Φ_0 is the magnetic flux quanta, μ_0 is the vacuum magnetic permeability and ξ is the superconducting coherence length. The value of λ is ~ 810 nm as described in Section 8 and the value of ξ is taken to be ~ 1.4 nm [18]. We note that although the exact value of ξ of our YBCO thin film is not directly characterized, the value of ξ is on the order of 1 nm [18-20] and small value variation does not affect the overall features of our tDGL simulations.

We emphasize that while this approach captures the overall trend, it does not fully resolve the microscopic dynamics. A more complete theoretical treatment would require a robust microscopic description of the *d*-wave superconducting state under non-equilibrium conditions.

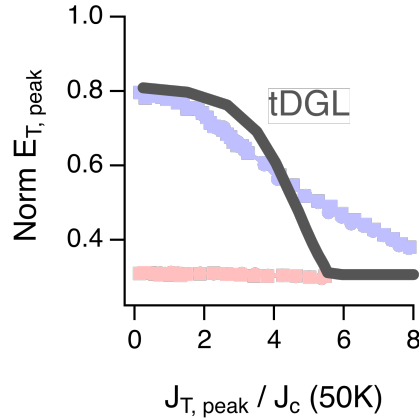


Figure S23| Comparison of tDGL simulation results and measurements on YBCO at 50 K. The simulation results based on tDGL theory are plotted in dark grey. The measurement data is plotted in light blue (50 K) and light red (110 K).

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