nature
SUPPLEMENTARY INFORMATION
https://doi.org/10.1038/s41586-018-0460-0

In the format provided by the authors and unedited.

Sensing with tools extends somatosensory processing beyond the body

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Supplementary Data to **Sensing with tools extends somatosensory processing beyond the body** by L. E. Miller, L. Montroni, E. Koun, R. Salemme, V. Hayward, and A. Farnè

THE goal of our study was to investigate whether humans can sense the location of an impact on a tool. Six behavioural experiments provided strong evidence that humans are highly accurate at this task when using hand-held wooden rods. This striking result raises the question of the sensing mechanism that might be employed by the participants in the absence of a sheet of mechanoreceptors on a tool considered as a sensory extension of the body. We explored this question in the second part of the main text, leveraging ideas from structural mechanics and computational neuroscience. In essence, the participants solved an inverse problem. They mapped a rod's mechanical response to the location of an impact. These supplementary data are intended to clarify the underpinnings of this inverse problem and how the nervous system might solve it.

Inverse problems generally refer to the determination of the causes (e.g., impact location) from a set of observations (e.g., vibrations on the hand). Mathematically, inverse problems are posed in terms of a 'forward' model that predicts observations from parameters, and 'inverse' problems are solved when one finds the parameters from observations [39]. When the object under study is a dynamic system, such as a vibrating rod, inverse problems can take the form of the determination of the initial and/or boundary conditions of one or several differential equations.

We argued in the main text that the determination of the location of an impact on a hand-held rod given observed vibrations is akin to the determination of initial conditions, given an impulse response and a set of prior assumptions. It is important that the impact location be encoded in the vibratory signal in a manner that is largely invariant to such factors as the rod's material, its geometry, and the manner in which it is held. Here, we expand this proposal by going into greater theoretical detail than was possible there. The inverse problem considered here differs from problems typical in engineering where the source of excitation (i.e., the contacting object) is generally assumed to be known [40].

Section 1 of these Supplementary Data describes how contact location is encoded into the vibratory response of a rod. In Section 2 we develop a model of how contact location can be efficiently decoded by a process that could be supported by early stage neural computations. In Section 3, we describe the results of a pilot study that lent empirical support to these models.

1 Impact Location Encoding

1.1 The Euler-Bernouilli Beam

The vibrations of a straight, slender rod can be modelled by the solutions of a differential equation that is classically derived from the Euler-Lagrange minimisation principle of least action. This equation is found by considering the distribution of transverse kinetic energy and bending potential energy along a rod of length, L. If $x \in [0, L]$ designates the location along the rod and if only small deflections from the resting position, $u(x, t)$, are considered, this equation is the basis of the so-called the Euler-Bernouilli beam theory [41],

$$\underbrace{\frac{\partial^2}{\partial x^2}\left(E(x)I(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right)}_{\text{from bending energy}} + \underbrace{\left(\mu(x)\frac{\partial^2 u(x,t)}{\partial t^2}\right)}_{\text{from kinetic energy}} = q(x,t). \tag{1}$$

The quantities $E(x)$ and $\mu(x)$ represent the mechanical properties of the rod. The quantity $E(x)$ is the elastic modulus of the material from which the rod is made. The elastic modulus captures the rod's material resistance to deformation. The quantity $\mu(x)$ is the rod's mass per unit of length, which depends on the rod's material and on its geometry. The quantity $I(x)$, called the second moment of area, depends on the shape of the cross section. In general, these quantities depend on the location, x, along the rod. The quantity $q(x,t)$ presents the action of external objects at particular places on the rod and at particular times. It is hard to find the solutions of this equation in the general case.

However, if the rod is homogeneous and uniform (as was the case in Experiments 1-5) then E, I, and μ are constants. If, in addition, the rod is left free to oscillate, then the equation becomes an unforced ordinary differential equation. For ease of writing, the dependency of the displacement u on location and time can be implied in the foregoing and a damping term is added to model the fact that in most materials energy dissipates. Thus, (1) becomes,

$$EI\frac{\partial^4 u}{\partial x^4} + \mu\frac{\partial^2 u}{\partial t^2} + \lambda\frac{\partial u}{\partial t} = 0. \tag{2}$$

A differential equation that depends on time cannot be solved without the specification of initial conditions, that is of the state, $u(x,0)$, of the modelled object when $t = 0$. When it also depends on space, boundary conditions must specify how the state of the object and a sufficient number of derivatives should be treated at the boundary of the domain inside which the equation applies.

1.2 Canonical boundary conditions

Imagine a uniformly rigid rod that is clamped (i.e., held stationary) at one end and free to oscillate at the other. At the clamped end, the deflection is zero at $x = 0$, that is, $\forall t$, $u(0,t) = 0$, and the rod remains tangent to the clamp, $(\partial u/\partial x)(0,t) = 0$. At the free end, the rod does not bend since there is no moment applied, so the curvature and the change of curvature are zero, $(\partial^2 u/\partial x^2)(L,t) = 0$ and $(\partial^3 u/\partial x^3)(L,t) = 0$, which gives the four needed boundary conditions. A rod is said to be free when the free-end boundary conditions applies at $x = 0$ and at $x = L$, which also gives four boundary conditions.

A rod held in the hand can neither be modelled as perfectly clamped nor completely free. It would therefore be hazardous to attempt to model its effect using the aforementioned boundary conditions. The governing equation would need to be augmented with additional terms that take into the account the complex structural and mechanical properties of the hand's soft tissues, muscles, and bone structure. However, as highlighted in the foregoing, the solutions of (2) have similarities with boundary conditions as different as the clamped rod or the free rod. These cases may be viewed at the limit cases of a tight and a loose grip hand, giving boundary conditions that are "in between".

1.3 Modes

The solutions of (2) can be shown to have the form,

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t) w_n(x). \tag{3}$$

Solutions thus decompose into an infinite sum of countable, discrete components. These components, called "modes", have the physical interpretation of the "standing waves" that give to physical systems in bounded domains, mechanical or otherwise, the propensity to oscillate at specific frequencies. In other words, the solutions of the unforced equation (2) exist only for certain frequencies called natural frequencies, ω_n, $n = 1, \ldots, \infty$. For small displacements, the modes are harmonic and thus their amplitudes have the form,

$$a_n(t) = A_n \sin(\omega_n t + \phi_n)\, e^{-\frac{1}{2}\frac{\lambda}{\mu}t}\,. \tag{4}$$

Modal solutions can in certain cases be truncated to a few terms, which is true for rods and other types of objects, such as strings and plates. Most slender rods tend to be underdamped, meaning that the modes do not decay too fast in time. We can take $\phi_n = 0, \forall n$ for the case of the unforced rod with an impulsive initial condition.

1.3.1 Modal Frequencies

The specific values of the natural frequencies depend on the geometry and on the material properties of the rod. If n is the mode number, the natural frequencies are,

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\mu}}\,, \quad \text{where} \quad \beta_n = \eta_n \frac{\pi}{L}\,, \tag{5}$$

The η_n are constants close to unity or a few multiples of unity. They are specific to each boundary condition and inversely proportional to the rod length, L. The modal frequencies thus depend in the same way on the rod length so their ratios are *length-invariant*. The specific frequencies of each mode are predictable if and only if the geometry and the material of the rod are known, features that the human haptic system is specifically tuned towards detecting. It would be unsurprising if the nervous system was able to immediately predict the modal frequencies for a rod without prior experience wielding it. This prediction was indeed supported by the results of our fifth experiment.

The application of the boundary conditions of the free rod or of the clamped rod amounts to solving, $\cosh(\beta_n L)\cos(\beta_n L) = 1$ or -1, respectively, for the solutions to exist. The roots of these equations gives possible values for the β_n and hence for the η_n. For a free rod, the η_n are 1.5, 2.5, 3.5, 4.5, …; for a clamped rod, they are 0.6, 1.49, 2.5, 3.5, 4.5, …. Clamping a rod rigidly at one end shifts the modal frequencies downward compared to a free rod. The influence of the boundary conditions is great in the low frequencies only (i.e., the lowest mode), but diminishes rapidly with increasing mode order.

We surmise that a human grip provides a boundary condition somewhere between these limit cases. Tightening a grip shifts the modal frequencies downward and promotes the occurrence of a low frequency "whipping mode" that does not exist in the free case. Mode 1 of the free case aliases to mode 2 of the clamped case, mode 2 to mode 3, and so on. Apart from the whipping mode, the natural frequencies, however, are indistinguishable between cases (Extended Fig. 3a). This is a critical point, because the grip strength of the tool user will likely change from moment-to-moment during tool-extended sensing. Thus, given that the frequencies of the higher modes are invariant to grip strength, the user will continually be "in tune" with the rod. As we will see in the next section, the encoding of impact location is also grip strength-invariant.

1.3.2 Mode Shapes

We now attend to the mode shapes, $w_n(t)$, in (3). It is these components that contains the impact location information. They have the form,

$$w_n = \underbrace{C_1 \cosh(\beta_n x) + C_2 \sinh(\beta_n x)}_{\text{non periodic part}} + \underbrace{C_3 \cos(\beta_n x) + C_4 \sin(\beta_n x)}_{\text{periodic part}}\,. \tag{6}$$

For the free rod and for the clamped rod, these mode shapes can be factored into simpler forms,

$$\text{free rod:} \quad w_n = C^{\text{free}} \Big[(\sin\beta_n x + \sinh\beta_n x) - D_n^{\text{free}}(\cos\beta_n x + \cosh\beta_n x)\Big]\,, \tag{7}$$

$$\text{clamped rod:} \quad w_n = C^{\text{clamp}} \Big[(\cosh\beta_n x - \cos\beta_n x) + D_n^{\text{clamp}}(\sin\beta_n x - \sinh\beta_n x)\Big]\,. \tag{8}$$

The factors C^{free} and C^{clamp} are scaling factors that can be taken to be equal to one since the initial amplitude of each mode depend solely on the initial conditions. The factors D_n^{free} and D_n^{clamp}, however, are functions of the boundary conditions. For the canonical cases at hand,

$$D_n^{\text{free}} = \frac{\sin\eta_n\pi - \sinh\eta_n\pi}{\cos\eta_n\pi - \cosh\eta_n\pi}, \quad \text{and} \quad D_n^{\text{clamp}} = \frac{\cos\eta_n\pi + \cosh\eta_n\pi}{\sin\eta_n\pi + \sinh\eta_n\pi}\,. \tag{9}$$

Calculations done, the D_n^{free} and the D_n^{clamp} turn out to all be close to one, except for D_1^{clamp} which is 0.734. We plot in Extended Figure 3b-c the dimensionless mode shapes, $\bar{w}_n(l)$, as a function of the dimensionless location $l = x/L$. As can be seen in these plots, like the modal frequencies, the shape of mode 1 of the free case is nearly identical to the shape of mode 2 of the clamped case, mode 2 to mode 3, and so on. Mode 1 of a clamped rod, the so-called 'whipping mode', does not have a counterpart in the vibrations of a free rod. Its natural frequency is low and is unlikely to be strongly excited in hand-held rod. It is therefore not taken into account in the analysis.

The mode shapes are notoriously sensitive to deviations from the assumptions of uniformity outlined in Section 1.1 , particularly as far as the mass distribution is concerned and less so from smooth variations of the cross section [41]. Mode shapes will be predictable to a tool user as long as the assumption of uniformity is not violated. This observation explains the results of the sixth experiment where, unbeknownst to the participants, we violated the uniformity assumption (see Methods and main text). The first half of the rod, the rigid portion, fulfilled the uniformity assumption and was usable as a sensor. The non-rigid portion, on the other hand, violated this assumption. Given that the predictable and unpredictable portions of the rod had very different modal responses (Extended Fig. 2h), it was impossible for users to sense impact location on the non-rigid portion.

1.4 Response to an Impact

The complex time domain response that we observe when we strike a rod arise from the peculiar distribution of modal frequencies that, unlike with strings, are not in integer multiples of a fundamental frequency. Like with strings, however, the different modes are selectively excited according the location of the excitation. For example, if an impact occurs at a location where the value of a mode shape is zero (see Extended Fig. 3b-c), called a 'node', this mode is not excited. If it is at an 'antinode' the excitation is maximum.

An impact is modelled by forcing an initial condition on velocity at one location, l^*. The condition, $(\partial u/\partial t)(l^*, 0) = v_0$, corresponds to the impulsive acceleration of an ideal inelastic collision at l^*. To model how the impact energy distributes among the different modes, consider that an impacted rod continues to vibrate subsequently to the impact where each mode oscillates with a decaying amplitude, initially of values A_n. From (4), the magnitude of the velocity of a particular mode at time 0 is $A_n\omega_n$. Thus, the total kinetic energy stored by mode n at time 0 is,

$$H_n(0) = \tfrac{1}{2}\mu L A_n^2 \omega_n^2 \int_0^1 \bar{w}_n^2(\zeta)\mathrm{d}\zeta \,. \tag{10}$$

The integral factor represents the contribution of a particular mode shape to the total vibration energy. Because these integrals evaluate to values close to one for all modes and for the two boundary conditions cases, the initial energy of a particular mode depends on its natural frequency and initial amplitude, but not on its shape. Call $\bar{H}_n(t)$ the energy of mode n of a rod of unit length and unit mass density, $\forall n$,

$$\bar{H}_n(0) \simeq \tfrac{1}{2} A_n^2 \omega_n^2, \quad \text{since} \quad H_n(0) \simeq \tfrac{1}{2} A_n^2 L \mu \omega_n^2 \,, \tag{11}$$

which is like the energy of a vibrating mass impulsed at time 0. The modal superposition (3) is truncated at some rank, although this step is not strictly necessary to keep the total energy finite. Touch being insensitive to frequencies higher than 1,000 Hz, we can select a truncation of rank, N, accordingly. An impact could well excite many higher modes, which could be heard, but only a few will drive the somatosensory system. Just after an impact at location, l^*, the total kinetic energy of a dimensionless rod is the sum of kinetic energy of each mode, $\bar{H}_n$, weighted by the square of the value of the mode shapes at that location,

$$\bar{H}(0) \simeq \sum_{n=1}^{N} \bar{w}_n^2(l^*) \bar{H}_n(0) = \tfrac{1}{2} \sum_{n=1}^{N} \bar{w}_n^2(l^*) A_n^2 \omega_n^2 \,. \tag{12}$$

Thus, we find that the modal amplitudes decay proportionally to the modal frequencies. Upon impact, each mode receives an initial velocity that is weighted by the value of the mode shape at the place of the impact and initial modal amplitudes are weighted the same way. The sign must be kept to preserve the total momentum.

1.5 Summary and Discussion

We can summarise the preceding discussion by the following observations.

1. Rods vibrate as a superposition of discrete modes, each of which is the product of a spatial component, called a mode shape, and a temporal component, which is a decaying harmonic oscillation at a natural frequency. The mode shapes and the natural frequencies depend on the boundary conditions.
2. The frequencies of each mode are not the multiples of a fundamental. For a given free rod, the frequencies are proportional to 1.5^2, 2.5^2, 3.5^2, 4.5^2, ... For the same rod clamped, the frequencies are proportional to 0.6^2, 1.49^2, 2.5^2, 3.5^2, 4.5^2, ... Clamping a rod adds a low frequency 'whipping' mode that is absent in the free case. However, the frequency of the first mode in the free case aliases the second mode in the clamped case, the second aliases the third, and so on (Extended Fig. 3a).
3. The influence of a hand grip on the modes can be thought of as somewhere in between the free and clamped cases. Given this, we can observe that a grip will 'tune' the modal frequencies of the first mode yet have little-to-no impact on higher modes.
4. The specific modal frequencies do however depend on how rods are made (Extended Fig. 3a), namely their material (i.e., elastic modulus and density), length, and cross-section shape, but not their ratios.
5. The amplitude of excitation of each mode depends on the relative location of a strike along the length of the rod (i.e., its mode shape). For equal initial excitations, the vibratory amplitude of each mode is inversely proportional to its frequency. Mode shapes have nodes where displacement is zero, which divide the length of the rod into coarse regions vibrating at different frequencies after an impact. The modes of the free rod are symmetrical about the centre, a symmetry that is broken for the clamped rod (Extended Fig. 3b-c).
6. When rods are underdamped, the spatial organisation of the vibrations is completely independent of how rods are made. Furthermore, mode shapes for a given condition (i.e., free or clamped) are invariant to grip strength, with the exception of the first mode. The shape of mode 1 of a free rod resembles mode 2 of a clamped rod, and so on.

Different conditions of mode excitation are shown in Extended Figure 3d-g to demonstrate the diversity of the temporal responses that can result from different boundary conditions and initial conditions. Extended Figure 3d and 3e shows the modal response of a free rod that has been contacted at l^* = 0.33 and 0.50, respectively. The plots show the vibration's displacement (upper panel) and its first two derivatives, velocity (middle panel) and acceleration (lower panel). Identical plots for the clamped case can be seen in Extended Figure 3f and 3g. Notice that in all cases the modal response is almost completely independent of whether the rod is free or clamped. Further, because differentiation emphasises the high frequencies, velocity and acceleration patterns (i.e., strain and strain rate) tend to be less influenced by the boundary condition than displacement patterns all the while preserving sensitivity on the location of impact (see results in the main text; Fig. 2). The tick marks above the plots of velocity and acceleration correspond to the point in time that they cross zero (i.e., zero crossing), a signal that could reduce the dimensionality of the motifs while preserving information about impact location (see Section 2).

The mode frequencies and shapes in the free case are aliased by the next highest mode in the clamped case (Extended Fig. 3a-c). The influence of grip strength on the modal response is somewhere in between these two limit cases. While grip strength will change over time during sensing, this is unlikely to have a large influence on the velocity and acceleration of motifs. This point is underscored by our classification analysis (see main text) that combined the vibrations recorded from every participant, each of whom likely held the rod somewhat differently. Despite this, classification accuracy remained very high (Fig. 2e). Vibratory motifs therefore form a location-specific feature space (Fig. 2b) whose geometry is invariant across the material and structural properties of uniform rods and the boundary conditions set by the user's control policies.

Grip strength will, however, influences the decay rate of each mode. To explore grip behaviour during sensing, we recorded grip strength during the active and passive conditions. During active sensing, participants relaxed their grip when bringing the rod into contact with the object, a strategy that caused the vibrations to last longer. This is not a strategy used during passive sensing, because the participants could not anticipate when the impact occurred and therefore could not modulate their grip. This observation contributes to explain why location sensing was significantly more accurate during active sensing (Extended Fig. 2g).

2 Decoding the Location of Impact

2.1 Brief Recall of The Somatosensation of Vibrations

Having reviewed how impact location is encoded in the vibratory response of a rod, the question now is how it might be decoded by the nervous system. In vibrations, the displacement, the velocity, or the acceleration responses are different representations of the same signal. Whether they or any other derivative of the signal is used for analysis is a matter of convenience and of availability of appropriate sensors. In mammalian touch, certain sensor populations seem to be more sensitive to the strain and others to the strain rate of the tissues in which they are embedded [42, 43]. Selective sensitivity, however, depends on numerous factors, chief among them are the time scale of the evolution of strain and its magnitude, but sensitivity depends weakly on frequency. It would hazardous to assign a single sensor population to the task of providing the raw sensory input that enables observers to decode the impact location on a rod from its vibratory response.

Nevertheless, there exists a receptor population, known as Pacinian corpuscules, that is well suited for this task [44, 45]. Indeed, it is the location-specific responses of this population that we modelled using the TouchSim simulation package. In mammals the signals of different sensors are integrated as early as in second-order neurones and propagate through at least five neuronal layers before they can guide behaviour [46]. It is therefore worthwhile to consider the computational aspects of the decoding task from a broad perspective.

2.2 Computational Considerations

The primary aim of sensory information neural encoding is the reduction of tremendous dimensionality inherent to the raw physics of the world. A time-honoured signal processing approach that is favoured by engineers is frequency, and more recently, time-frequency analysis. The obvious advantage of these analyses is the dimensionality reduction they afford. A signal of infinite dimensionality can be reduced to a small number of components, provided that the signal has statistical properties that change slowly with time (or space), such as structural vibrations. While it is believed that the auditory system deploys a type of time-frequency analysis (i.e., 'gammatones') when processing sounds [47], a similar computational process does not seem to have any counterpart in the somatosensory system. Even if the somatosensory system can be coaxed into performing crude spectral discrimination, it seems ill-equipped to performing the fine decoding of impact location on a rod in the frequency or in the time-frequency domain, even if some measure of it could be called upon to enhance performance. Time domain analysis is a more suitable candidate and is more in line with known physiological, neuroanatomical, and behavioural data. It is for this reason that we focused on the time domain signals in our study.

2.3 Time Domain Decoding

In the time domain, the waveforms adopt different shapes as the order of differentiation increases since the high frequency modes become emphasised, see Extended Figure 3d-g. The simplest but perhaps the least computationally efficient approach taken by the nervous system would be to analyse the raw time domain signals. Such an approach ought not to be discarded completely since the mode shapes (Extended Fig. 3b-c) vary smoothly (in the mathematical sense of 'differentiability') with the location of impact. Thus, two impact locations which are neighbours on the rod also give rise of signals that are also neighbours in a feature space based on raw waveforms. Indeed, our experiment focused exclusively on accelerations signals in the time domain, demonstrating that they robustly encode impact location. While nothing in principle would preclude the realisation of such an approach by the nervous system, a computational process needed to extract impact location directly from the waveform has to contend with the infinite dimensionality of the raw signal space, although it is possible that the nervous system may use some criterion to discretise the signals. It is therefore enticing to consider more computationally efficient approaches to the decoding problem.

Other than by amplitude, information in a signal can be encoded by phase. The phase of an oscillatory signal is simply the argument of the sinusoidal functions that compose the complete signal. As per (4) there is one phase per mode considered, the $\omega_n t$, $n = 1, \cdots, N$. Thus, it is tempting to consider a feature space that is based on the phase differences between modes, since, like amplitudes, the phases differences encodes impact location. A naive approach to the computational process, then, would entail phase-accurate disentanglement of the different modes. The first step in such an analysis would resemble a short-term, phase sensitive spectral analysis, a process whose difficulty makes it unlikely to be implemented by the somatosensory system.

2.4 Extrema in Signals Encode Phase Interferences Between Modes

A more economical approach is to consider how the modes lead to the constructive and destructive interferences that determine the different waveforms. A well-known proxy for the occurrence of interferences in vibrations is the sequence of extrema in the signal. The somatosensory system might instead analyse the direct consequences of the *phase interferences* between modes. Here, the issue of selecting the appropriate derivative of the signal cannot be avoided. Given the known physiology of mechanoreceptors, it is reasonable to assume that the central nervous system receives information closely related to the occurrence of extrema of strain or of strain rate, which is equivalent to considering the zero-crossings of velocity and acceleration, respectively. Since the location-specific distribution of modal frequencies is reflected in the motif's phase differences (Extended Fig. 3d-g), analysing the timing of zero crossings would be a computationally efficient method for decoding impact location. This is reflected by phase-locking to the peaks of the next lowest derivative (i.e., displacement) (Extended Fig. 5a-b).

To assess the plausibility of this coding scheme, we re-analysed our data using the temporal patterns of zero crossings in the vibratory motifs. We found that the responses of the simulated Pacinian mechanoreceptors were almost perfectly phase-locked to the zero crossings of the velocity (i.e., peaks of displacement; Extended Fig. 5a-b). This remarkable result suggests that information-preserving dimensionality reduction occurs at the initial stages of somatosensory processing in the nervous system. We further found that (i) impact location could be accurately decoded from the temporal patterns of zero crossings of acceleration alone (Extended Fig. 5c); and (ii) that this pattern precisely predicted participant's trial-by-trial behaviour (Extended Fig. 5d). Thus, decoding impact location from the extrema in the time domain signal of a motif is not only a computationally efficient method of dimensionality reduction, but it is also physiologically plausible. Where and how this analysis is implemented in the nervous system [46] is a question the warrants future investigation.

3 Initial pilot study

As was discussed in Section 1, the time course of the displacements of all the points on the object is highly dependent upon the initial conditions, on the boundary conditions, and on the location of impact. As a pilot experiment to initially address the question of the informativeness of the vibratory response to impact, we probed the wooden rod's structural dynamics while it was clamped in place with a bench vice. The body of the rod was impacted at four evenly-spaced locations (13 to 43 cm) using a spring-loaded device. Vibrations were recorded (Analog Devices; Model ADXL335) from the handle (2.5 cm from the beginning of the handle) and the index finger (identical location) in separate datasets. Another accelerometer was attached to either the handle of the tool or the middle phalanx of the index finger from one of the authors (L.E.M.) as it pressed against the handle. For each surface, we recorded thirty trials per impact location (order randomized). All signal processing and data analyses were identical to those reported in the main article.

Multiple impacts at the same location producing consistent vibratory motifs both on the handle (median r=0.81, IQR=0.15) and on the index finger (median r=0.96, IQR=0.05). This result provided the initial hints for the existence of vibratory motifs. We then used support vector classification to characterise how location-specific information accumulated within these motifs over time (temporal window sizes: 2 to 100 ms, by steps of 2 ms). Classification accuracy for mechanical vibrations on the handle increased rapidly, reaching 90% accuracy within 28 ms (Fig. 2e). Remarkably, classification accuracy for cutaneous vibrations reached 90% almost five times faster (6 ms post-impact). It is interesting to note that this decoding speed is almost identical to what we observed in the experiment reported in the main article (see results and Fig. 2e), suggesting that the biomechanical properties of the hand actually enhance the location information encoded in motifs.

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