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Vowel recognition with four coupled spin-torque nano-oscillators

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Supplementary information: numerical simulations

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In this supplementary document, we present the numerical simulations that were performed to investigate the important features that oscillators should possess to classify accurately.

1. Model description

For simulating ideal oscillators, we consider the van der Pol model of non-linear dynamics that captures the essential features of spin-torque nano-oscillators coupled dynamics and can be generalized to other non-linear oscillators[1]. We consider four identical oscillators which only differ by a relative mismatch of 2% in their natural frequencies f_0, and which dynamics are modified by two microwave input signals. This leads to the following differential equations in polar coordinates $\binom{s_i}{\theta_i}$, where (2) index i (i=1,2,3,4) represents the i^{th} oscillator:

$$\frac{ds_i}{dt} = -\alpha\omega_{0,i}\left(1-\frac{I}{I_{th}}+Qs_i^2\right)s_i+F_e\cos\theta_i(\cos(\psi_{e,A}-\omega_{e,A}t)+\cos(\psi_{e,B}-\omega_{e,B}t))+\varepsilon\, F_e\cos\theta_i\sum_{j=1}^{N=4}s_j\cos\theta_j$$

$$\frac{d\theta_i}{dt} = \omega_{0,i}(1+N_0s_i^2)+\frac{F_e}{s_i}sin\theta_i(\cos(\psi_{e,A}-\omega_{e,A}t)+\cos(\psi_{e,B}-\omega_{e,B}t))+\ \varepsilon\frac{F_e}{s_i}\sin\theta_i\sum_{j=1}^{N=4}s_j\cos\theta_j$$

In this equation, $\omega_{0,i}$ is the natural angular frequency of the oscillator, $\alpha = 0.013$ is the damping coefficient, $Q = 3.02$ is the nonlinear damping parameter, I is the dc current injected in the oscillator, $I_{th} = 1\ mA$ is the threshold dc current of self-sustained oscillations of the magnetization, N_0 is the nonlinear frequency shift normalized by the natural angular frequency, $\omega_{e,A}$ and $\omega_{e,B}$ are the respective angular frequencies of the two external microwave inputs A and B, $\psi_{e,A}$ and $\psi_{e,B}$ are their relative phase shifts (Here $\psi_{e,A} = \psi_{e,A}$ = 0), $F_e = 1.3 \times 10^{-3}$ is the coupling strength to each external microwave input signal A and B, and ε the mutual coupling strength between oscillators, normalized by the coupling to the inputs.

2. Recognition performances

In this study we evaluate the impact of different oscillator characteristics (tunability and mutual coupling) on the classification performance of the network. With this purpose, some oscillator parameters are modified and a new optimized classification rate is calculated for each new set of material parameters. Each set of oscillator parameters corresponds to different oscillator behaviors and thus give rise to different synchronization maps. In particular the range of operation of the oscillators is modified and, in consequence, the linear combination previously applied to the formants to obtain two characteristic frequencies in the range of operation of the oscillators is no longer optimal. The linear combination of the formants should therefore be adapted for each point of Fig. 3a and c in order to determine the best recognition rates with the newly considered oscillator parameters. In this kind of network the recognition rate is optimized when:

(i) The free-running frequency difference between oscillators is similar:

$$|\omega_1 - \omega_2| \approx |\omega_2 - \omega_3| \approx |\omega_3 - \omega_4|$$

(ii) The width of the injection locking range of all 4 oscillators is similar:

$$\Delta_1 \approx \Delta_2 \approx \Delta_3 \approx \Delta_4$$

Thus, we first estimate which values of ω_i fulfil these requirements and we calculate the linear transformation of the formants whose final frequencies (inputs to the network) better fit the synchronization map expected from these ω_i and Δ_i.

Finally, for each oscillator parameters and associated linear combination of the formants, we simulate numerically the learning process and find the optimum recognition rate.

Following this procedure, we study the influence of oscillator tunability and mutual coupling on the classification ability of our network.

In Fig. 3a-b, the oscillator locking ranges are modified by tuning their normalized non-linear frequency shift coefficient N_0 from 0.00 to 0.26 with a step of 0.02, in the absence of mutual coupling. The optimum recognition rate is calculated for each value of N_0. From the experimental frequency-current dependence the experimental nonlinear frequency shift can be obtained and averaged to $N_0^{exp} \approx 0.18$.

In Fig. 3c-d, the tunability is maintained fixed (N_0 = 0.08, corresponding to a value of locking range/frequency difference of 0.58 in Fig. 3a), and the normalized mutual coupling ε is varied. The experimental value of the mutual coupling between oscillators coupling ε is 1.6.

3. Synchronization maps

In the simulations of the synchronization maps, the dc currents applied to the 4 oscillators are kept constant and two external signals with a fixed external force are applied (we keep the same external force $F_e = 1.3 \times 10^{-3}$). We swept the frequency of two external sources A and B. Thus, each simulated synchronization map (see Fig.3) is constituted by 300x300=90 000 simulated points. These simulated points are independent from each other. This allows us to run simulations in parallel on GPUs.

Each simulated point in the map is calculated by numerically solving the system of coupled differential equations (1) using a fourth order Runge-Kutta scheme at $T = 0K$ (no thermal noise). Using the simulated cartesian trajectory and velocity of each vortex core in the dot plane (x, y), the instantaneous frequency of each oscillator is extracted through the instantaneous angular evolution $\omega(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$. The steady state frequency of each oscillator is obtained by computing the temporal average of the instantaneous frequency over only the last 20% of the simulated time trace. The synchronization between oscillators and microwave signals is detected by analyzing the frequency difference between oscillators and external sources:

If $|\omega_i - f_A| \leq f_{th}$, oscillator i and external source A are considered to be synchronized.

If $|\omega_i - f_A| > f_{th}$, oscillator i and external source A are considered to be not synchronized.

Where f_{th} is a threshold set to $f_{th} = 0.5$ MHz.

4 Evaluation of the injection locking range normalized by the frequency difference between oscillators

The ratio between the injection locking range and the frequency difference used in Fig.3a is obtained by averaging the injection locking range of the 4 oscillators $\bar{\Delta} = \frac{1}{4}(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)$ and the frequency difference between oscillators $\bar{\delta} = \frac{1}{3}(\delta_{12} + \delta_{23} + \delta_{34})$ where $\delta_{12} = |\omega_1 - \omega_2|$, $\delta_{23} = |\omega_2 - \omega_3|$, $\delta_{34} = |\omega_3 - \omega_4|$. The ratio defined in Fig. 3 A (denoted ϱ here) is thus : $\varrho = \frac{\bar{\Delta}}{\bar{\delta}}$.

Supplementary information references

1. Slavin, A. & Tiberkevich, V. Nonlinear Auto-Oscillator Theory of Microwave Generation by Spin-Polarized Current. *IEEE Trans. Magn.* **45,** 1875–1918 (2009).