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Supplementary information

Controlled flight of a microrobot powered by soft artificial muscles

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Yufeng Chen, Huichan Zhao, Jie Mao, Pakpong Chirarattananon, E. Farrell Helbling, Nakseung Patrick Hyun, David R. Clarke & Robert J. Wood

Supplementary Information

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Supplementary Information contains:
Supplementary Notes 1-5
Supplementary References
Description of Supplementary Videos

*equations and reference numbering follows from the main text

S1: Self-sensing of in-flight collisions and robot damage

In the main text, Figure 2e-h show examples of collision robustness and sensing. In these experiments, the robot is fixed in place while an obstacle is brought in close proximity to collide with the robot wing or the DEA. Through monitoring the DEA current, the robot can sense in-flight collisions and robot damage. Extended Data Figure 3 reports five additional in-flight collisions and characterization experiments on collision sensing. Composite images and raw current measurements are shown to illustrate collision sensing. Extended Data Figure 3a and b show a short takeoff flight free of collisions. Extended Data Figure 3c and d show a takeoff flight that experiences a collision 330 ms after liftoff. The corresponding spike in DEA current is circled in red. Similarly, Extended Data Figure 3e and f show the composite image and change in current of another takeoff flight with a collision 450 ms after liftoff. In Extended Data Figure 3g and h, we show a flight in which the robot is placed into a small transparent box. The robot pulls its tether at takeoff, makes a collision with the ground at 210 ms, and then makes sustained collisions from 380 ms to 450 ms. These collisions are observed in the DEA's current response. Furthermore, we show a flight characterization test (Extended Data Figure 3i and j) in which the robot loses a wing during flapping. This causes an increase in the DEA's current due to changes in the load. The current output is measured with an offboard amplifier (Trek Model 677B) at 10 kHz, and the raw data are plotted in Extended Data Figure 3.

Although collisions can be reliably detected by measuring the instantaneous current, we cannot differentiate collisions with the wing and with the DEA. In addition, collision detection is easier to identify in open-loop flights because the driving voltage amplitude remains constant. Future studies on sensing in-flight collisions in closed-loop flights should develop a model that accounts for the instantaneous change of DEA current due to a change of driving voltage.

S2: Cause and influence of DEA nonlinear actuation

In the main text, Figure 3b illustrates the DEA's actuation nonlinearity. Here we explain the cause of this observation in detail. The static relationship between the DEA free displacement and its input voltage is nonlinear because the strain is proportional to the input voltage squared:

$$\frac{\Delta L}{L} = \frac{1}{2Y}\epsilon_p\left(\frac{V}{d}\right)^2. \quad (11)$$

Here ΔL is the DEA extension, L is the DEA length, Y is the elastomer Young's modulus, ϵ_p is the elastomer's permittivity, and d is the thickness of a single elastomer layer. Given a sinusoidal voltage input:

$$V = A_0 + A_1 \sin wt, \quad (12)$$

the DEA response is given by the equation:

$$\Delta L = \frac{L}{2Yd^2}\epsilon_p\left(A_0^2 + \frac{A_1^2}{2} + 2A_0A_1 \sin wt - \frac{A_1^2}{2}\cos 2wt\right). \quad (13)$$

In equation (13), we use trigonometric identities to expand the term $\sin^2(wt)$. For our experiments, the values of A_0 and A_1 are the same, which implies the magnitude of the second harmonic is 25% that of the driving frequency. The second harmonic also has a 90° phase shift. Dynamically, the magnitude of each harmonic is modified differently by the system transfer function. Figure 2b shows that the system resonance is around 500 Hz. At a driving frequency of 250 Hz, we observe a secondary peak because the second harmonic is near the system resonance (500 Hz) and is thus amplified by the Q of the system.

As shown in Figure 3b, this actuation nonlinearity also affects the flapping kinematics. Extended Data Figure 4a shows the FFT of Figure 3b. The x-axis is normalized by the driving frequency. We observe that the magnitude of the second harmonic is substantial for the cases of 1 Hz and 100

Hz driving. In contrast, the second harmonic amplitude is reduced significantly when the system operates around the resonant frequency (the 280 Hz case). This shows that actuation nonlinearity can be attenuated by driving near system resonance.

Further, dynamic buckling of the DEA substantially affects flapping kinematics. Figure 3f shows the left wing stroke amplitude as a function of driving frequency and voltage. It further illustrates that circumferentially constraining the DEA reduces the dynamic buckling magnitude and consequently increases the wing stroke amplitude. Extended Data Figure 4b shows the tracked data for the robot's right wing. Figure 3f and Extended Data Figure 4b show the same effect of dynamic buckling. Further, the buckling motion reduces the separation distance between electrodes. The DEA's elastomer layers have defects due to air bubbles or particulates trapped in the fabrication process. The reduction of separation distance causes the electric field to exceed the elastomer's dielectric strength, especially at locations near these defects. Consequently, dynamic buckling leads to local dielectric breakdown, which causes excessive electrode self-clearing. This results in large and irregular current spikes, which reduces DEA lifetime. Hence, we switch off the robot once dynamic buckling is observed, and we do not conduct experiments for operating conditions higher than 1400 V for the unconstrained cases.

S3: Passively stable ascending flight of a four-wing robot

Unstable takeoff flights without precession

In the main text, Figure 4a shows an unstable takeoff flight of a robot with one DEA. Due to small fabrication errors, the robot's left and right wings produce slightly different lift and drag forces, and they lead to unstable takeoffs. While placing the robot's center of mass below its center of pressure slows down the pitch and roll dynamics, the robot remains unstable during ascending flights. Extended Data Figure 5a-c show three additional takeoff flights of the same robot. These flights are unstable, and the robot flips over within 200 ms after liftoff. The maximum takeoff height is less than 1.5 cm in all flights.

Figure 4b shows a stable ascending flight of a robot driven by two DEAs. This robot is designed to achieve passive stability through precession around its body z-axis during ascending flight. The robot is unstable if it does not generate a sufficient yaw torque around its body z-axis. Extended Data Figure 5d-i show six unstable takeoff flights in a two-DEA robot. Small fabrication errors may cause the one module to generate smaller lift forces than the other module. Consequently, the robot pitches and eventually flips over (Extended Data Figure 5d-f). In addition, there may be asymmetries between the left and right wing lift and drag forces. This causes the robot to roll sideways and eventually flip over (Extended Data Figure 5g-i). The maximum robot takeoff height is less than 3 cm in these experiments.

Robot yaw torque generation

To achieve precession along the body z-axis, the robot needs to generate yaw torque. Several existing flapping wing robots[3,41,42] have demonstrated yaw control. In two piezoelectric-driven robots[3,41], yaw torque is generated by commanding asymmetric up stroke and down stroke motions. The wings move faster during half of the flapping period to generate asymmetric drag forces in up stroke and down stroke. In another design[42], the authors offset flapping-wing modules such that the thrust from different modules are not parallel with each other.

In our design, each one-DEA robot module can generate a yaw torque by offsetting the wing pitch angle during assembly. Extended Data Figure 6a illustrates the wing pitch offset and the effect on yaw torque production. This offset allows a wing to generate different drag forces during its up stroke and down stroke (Extended Data Figure 6b). Since the net drag force is non-zero, the robot experiences a yaw torque. The magnitude of this yaw torque can be changed by varying the wing stroke amplitude, which corresponds to the input voltage amplitude. Generating yaw torque is crucial for achieving passive upright stability in the two-DEA robot. This is achieved by attaching two modules with yaw torque offsets in the same direction (Extended Data Figure 6c). Likewise, our four-DEA, eight-wing robot is also capable of generating yaw torque. We attach two two-DEA robots together with modules that have yaw torque offsets in opposing directions (Extended Data Figure 6d). While the four DEA robot can generate net yaw torque in either positive or negative direction, we do not implement yaw control because it is not necessary for demonstrating controlled hovering flights. Future studies can develop a quadrotor-like controller for this robot to demonstrate yaw control.

Notation

We use the following notation for the derivations:

- We use bold letters to denote vector quantities and a hat to denote unit vectors. $\mathbf{0}$ denotes a vector of all 0s. For the special unit vector that is parallel to a particular coordinate axis, we write the vector as $\mathbf{e_i}$ without a hat. This means that the vector is 1 at the i^{th} entry and 0 everywhere else.

- We use upper case letters to represent matrix quantities. Specifically, we use R to denote a 3×3 rotation matrix. Further, we let R_i denote the i[th] column of R.
- For vectors $\mathbf{v} \in \mathbb{R}^3$, $\mathbf{a} \in \mathbb{R}^3$, we let $S_{\mathbf{v}}$ denote the skew symmetric matrix generated by $\mathbf{v}$ such that $S_{\mathbf{v}}\boldsymbol{a} = \mathbf{v} \times \mathbf{a}$.

Dynamical model and simulation

We derive a dynamical model, conduct numerical simulations, and run open-loop flight experiments to demonstrate passively stable ascending flight in the four-wing robot. The coordinate system definition for this vehicle is shown in Extended Data Figure 7a. The rotational and translational dynamics of the system are given by the following equations:

$$\dot{R} = RS_{\boldsymbol{w}}, \tag{14}$$

$$I\dot{\boldsymbol{w}} + \boldsymbol{w} \times I\boldsymbol{w} + \boldsymbol{\tau_D} = \boldsymbol{\tau_A}, \tag{15}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{v}, \tag{16}$$

$$\dot{\boldsymbol{v}} = \frac{1}{m}(-mg\boldsymbol{e_3} + R\boldsymbol{F_A} + \boldsymbol{F_D}). \tag{17}$$

Here I is the robot's moment of inertia tensor, $\boldsymbol{w}$ is the body angular velocity, $S_{\boldsymbol{w}}$ is the skew symmetric matrix generated by $\boldsymbol{w}$, $\boldsymbol{\tau_D}$ is the damping torque, $\boldsymbol{\tau_A}$ is the applied torque from the robot's wings, R is the body rotation matrix (relative to an inertial coordinate system), $\boldsymbol{x}$ is the center of mass position, $\boldsymbol{v}$ is the robot's translational velocity in the inertial frame, m is the robot's mass, g is the acceleration due to gravity, $\boldsymbol{F_A}$ is the applied force from the robot's wings, and $\boldsymbol{F_D}$ is the damping force on the robot body. The applied and damping forces and torques are given by:

$$\boldsymbol{F_A} = \sum_{i=1}^{4}\left[0, F_{D,i}, F_{L,i}\right]^T, \tag{18}$$

$$\boldsymbol{F_D} = -b_f||\boldsymbol{v}||\boldsymbol{v}, \tag{19}$$

$$\boldsymbol{\tau_A} = \sum_{i=1}^{4} \boldsymbol{r_i} \times \boldsymbol{F_{A,i}}, \tag{20}$$

$$\boldsymbol{\tau_D} = -b_t||w||w. \tag{21}$$

Here F_D and F_L are the time averaged drag and lift forces of each wing, b_f and b_t are the body damping force and torque coefficients, and $\boldsymbol{r}$ is the displacement from the robot's center of mass to each wing's center of pressure. This displacement vector (l_x, l_y) is illustrated in Extended Data Figure 7b. We intentionally induce a net drag force by biasing the resting wing pitch angle during robot assembly. This net drag force induces a body yaw torque, which causes the robot to precess around its body z-axis. The values of these parameters are reported in Extended Data Table 2.

Equations (14) – (17) can be solved numerically to simulate the ascending flight of the four-wing robot. After the variables R, $\boldsymbol{w}$, $\boldsymbol{x}$, and $\boldsymbol{v}$ are solved, we compute the pitch-roll-yaw (ϕ, θ, ψ) Euler angles by solving the inverse kinematics:

$$\phi = \arctan\left(\frac{R_{21}}{\cos\theta}, \frac{R_{11}}{\cos\theta}\right), \tag{22}$$

$$\theta = \arctan(-R_{31}, \cos\theta), \tag{23}$$

$$\psi = \arctan\left(\frac{R_{32}}{\cos\theta}, \frac{R_{33}}{\cos\theta}\right). \tag{24}$$

Here, the two possible representations of $\cos\theta$ are: $cos\theta = \pm\sqrt{1 - R_{13}^2}$. In our calculation, we choose the positive value. Our simulation (Supplementary Video 5, part 2) shows that the robot can achieve open-loop stable ascending flight even if the lift forces generated by the wings differ by up to 10%. The robot can maintain upright stability by undergoing precession along its body z-axis. Extended Data Figure 7c-f show a sample simulation and compares this with the experiment shown in Figure 4b. By inducing a rotational speed of 17.2 rev/s around the body z-axis (Extended

Data Figure 7d), the robot can compensate for the lift force differences of its four wings. Extended Data Figure 7d shows good agreement between the simulated and the measured robot yaw motion. In simulation, the robot's roll and pitch oscillation amplitudes are less than 3° (Extended Data Figure 7e). Extended Data Figure 7f further shows the simulated robot displacement (x, y, and z) and the measured displacement in the z-direction. Both simulation and experiment show that the robot reaches 23.5 cm after 0.8 s of takeoff, and its terminal ascending speed is 50 cm/s. All the parameters used in this simulation are reported in Extended Data Table 2.

Further, we illustrate the stabilizing effect of precession in simulation. We conduct eleven takeoff simulations with the same robot parameters and vary the induced yaw torque. Extended Data Figure 7g shows the robot altitude as a function of time. Extended Data Figure 7h shows the same simulation result for time less than 0.3 second, corresponding to the inset shown in Extended Data Figure 7g. The red dashed line shows the maximum robot altitude for unstable takeoffs. For a body yaw torque smaller than 1 mN·mm, the robot's maximum altitude cannot exceed 10 mm before it starts to fall. The red arrows on top of the trajectories in Extended Data Figure 7g and h show that increasing yaw torque leads to an increase in the robot's altitude. Extended Data Figure 7i shows the robot's altitude one second after takeoff increases with increasing body yaw torque.

Extended Data Figure 7j-o show simulation results for body pitch and roll orientations. Extended Data Figure 7k and n show insets of j and m, respectively. The red arrows in j, k, m, and n illustrate that an increase of body yaw torque leads to a reduction of pitch and roll oscillation amplitudes. Extended Data Figure 7l and o show the robot's maximum pitch and roll deviations in the one-second takeoff flight, which decrease as body yaw torque increases.

Proof of vehicle stability

Here we show the robot is passively upright stable during steady state flight. We define passive upright stability as the projection of the robot body-z axis on the inertial z-axis remains positive for all time. The robot attitude dynamics are given by the following equations:

$$\dot{R} = RS_{\boldsymbol{w}}, \tag{25}$$

$$I\dot{\boldsymbol{w}} = -\boldsymbol{w} \times I\boldsymbol{w} + \boldsymbol{\tau}_{\mathbf{A}} - b\boldsymbol{w}. \tag{26}$$

The damping force is linearized with respect to the robot angular velocity during steady state. To show robot passive stability, we impose two conditions on the robot moment of inertia and applied torque: $I_{zz} > I_{xx} \approx I_{yy}$, and $\tau_z \gg \tau_x, \tau_y$. At steady state, the analytical solution is given by:

$$w_x^e = \frac{1}{b^2+\alpha^2}(b\tau_x - \alpha\tau_y), \tag{27}$$

$$w_y^e = \frac{1}{b^2+\alpha^2}\left(\alpha\tau_x + b\tau_y\right), \tag{28}$$

$$w_z^e = \frac{\tau_z}{b}, \tag{29}$$

where $\alpha = \frac{(I_z - I_x)}{b}\tau_z$. The subscript e denotes that the solution is solved for the equilibrium condition. To show asymptotic stability, we define the Lyapunov function:

$$V(\boldsymbol{w}) = \frac{1}{2}\overline{\boldsymbol{w}}^T I \overline{\boldsymbol{w}}, \tag{30}$$

where $\overline{\boldsymbol{w}} = \boldsymbol{w} - \boldsymbol{w}^e$ is the error term. Next, we calculate the derivative of V:

$$\dot{V} = \overline{\boldsymbol{w}}^T(-\boldsymbol{w}^e \times I\overline{\boldsymbol{w}} - b\overline{\boldsymbol{w}}) \leq -b\left|\left|\overline{\boldsymbol{w}}\right|\right|^2 + I_{zz}\left|\left|\overline{\boldsymbol{w}}\right|\right|^2\left|\left|\boldsymbol{w}^e\right|\right|. \tag{31}$$

From equation (31), we obtain the stability condition of the system. The angular velocity $\boldsymbol{w}$ converges asymptotically to $\boldsymbol{w}^{\boldsymbol{e}}$ if $\dot{V} < 0$, which requires the condition:

$$\frac{b}{I_{zz}} > \sqrt{\frac{1}{b^2+\alpha^2}\left(\tau_x^2 + \tau_y^2\right) + \frac{\tau_z^2}{b^2}}. \tag{32}$$

Equation (32) gives requirements on the values of the robot's moment of inertia and applied torque such that the robot's angular velocity can reach a constant steady state solution.

To show the robot is passively upright stable, we need to analyze the robot attitude dynamics at the steady state angular velocity. Suppose that at $t = 0$ the inertial z-axis points in an arbitrary direction $\boldsymbol{q_0} = \left[q_x,\ q_y,\ q_z\right]^T$ in the body frame. In other words, if $\boldsymbol{q_0} = \boldsymbol{e_3}$, then the robot body is in upright position. The robot is said to be passively upright if $e_3^T\boldsymbol{q}(t) > 0$ for all time. Assuming the angular velocity is constant, we obtain the analytical solution of the body z-axis by using the Rodrigues formula:

$$\boldsymbol{q}(t) = \left(I - \frac{S_{\boldsymbol{w^e}}}{\lambda}\sin\lambda t + \frac{S_{\boldsymbol{w^e}}^2}{\lambda^2}(1-\cos\lambda t)\right)\boldsymbol{q_0}, \tag{33}$$

where we define $\lambda = ||\boldsymbol{w^e}||$. We let $J = e_3^T\boldsymbol{q}(t)$ and solve for the minimum of this function:

$$J_{\min} = \frac{1}{\lambda^2}\left(w_z\boldsymbol{q_0^T}\boldsymbol{w^e} - \sqrt{\alpha_1^2+\beta_1^2}\right). \tag{34}$$

Here α_1 and β_1 are defined as:

$$\alpha_1 = -\lambda\boldsymbol{q_0^T}S(\boldsymbol{e_3})\boldsymbol{w^e}, \tag{35}$$

$$\beta_1 = \boldsymbol{q_0^T}\boldsymbol{w^e}w_z^e - \lambda^2\boldsymbol{q_0^T}\boldsymbol{e_3}. \tag{36}$$

The system is passively upright stable if $J_{\min} > 0$. We can simplify this expression into a quadratic form:

$$w_z^2\left(\boldsymbol{q_0^T}\boldsymbol{w^e}\right)^2 - (\alpha_1^2+\beta_1^2) = \lambda^2(w_z^e)^2\left(\boldsymbol{q_0^T}P\boldsymbol{q_0}\right) > 0, \tag{37}$$

where the symmetric matrix P is given by:

$$P = \begin{pmatrix} -a_1^2 & a_1 a_2 & a_1 \\ a_1 a_2 & -a_2^2 & a_2 \\ a_1 & a_2 & 2 - a_3^2 \end{pmatrix}. \quad (38)$$

The elements $a_1, a_2,$ and a_3 are defined as:

$$\begin{cases} a_1 = \frac{w_x^e}{w_z^e} \\ a_2 = \frac{w_y^e}{w_z^e} \\ a_3 = \frac{\lambda}{w_z^e} \end{cases}. \quad (39)$$

If $Tr(P) > 0$, then P has at least one positive eigenvector. This implies there exists a set containing feasible initial conditions that guarantees stability:

$$U = \{\boldsymbol{q_0} \in S^2 | \boldsymbol{q_0^T} P \boldsymbol{q_0} > 0\}. \quad (40)$$

Here S^2 is a unit spherical shell embedded in $\mathbb{R}^3$. To ensure $Tr(P) > 0$, we need to satisfy the expression:

$$Tr(P) = 3 - 2a_3^2 > 0. \quad (41)$$

Equation (41) simplifies to:

$$\frac{|w_z^e|}{||\boldsymbol{w^e}||} > \sqrt{\frac{2}{3}}. \quad (42)$$

This analysis shows that if the z-component of the robot angular velocity is dominant, and that the initial condition $\boldsymbol{q_0}$ is close to the upright condition ($\boldsymbol{e_3}$), then the robot's body z-axis will point upward ($e_3^T \boldsymbol{q}(t) > 0$) for all time.

Experimental validation

In addition to the takeoff flight shown in Figure 4b, we show three additional passively stable ascending flights. These flights (Extended Data Figure 8a-c) are one-second long due to the constraint of our flight arena (about 35 cm high). These flights highlight the difference in performance between stable flights with precession and unstable takeoffs. With the precession effect, the robot can reach a maximum height of approximately 25 cm within one second of takeoff. Without precession, the robot takeoff height cannot exceed 3 cm (Extended Data Figure 5d-i). These flight tests are done in a motion capture system to measure attitude and position data. We place reflective trackers on the robot for motion tracking and control the robot open loop. The corresponding tracking of the robot position and orientation is shown in Extended Data Figure 8d-o. Extended Data Figure 8d-g, 12h-k, and 12l-o correspond to the flights shown in a, b, and c, respectively. The tracking becomes inaccurate when the robot flies out of the views of some or all cameras (i.e., the flat regions in h, i, l, and m correspond to the time that Vicon cameras cannot see some markers). In all three cases, we observe oscillation along the pitch and the roll axes (f, j, n), but they do not grow unboundedly because of the yaw rotation.

One advantage of having passive upright stability is that the robot can reject a small disturbance or imbalance. We further present two experiments in which the robot's left and right sides are purposely adjusted such that the lift forces are substantially different (about 10-15%). Extended Data Figure 9a shows a 0.5 second flight in a smaller flight chamber. The video is recorded at 3000 Hz. We highlight the robot's orientation at 85 ms after its takeoff using a red colored label. The robot has a roll orientation of approximately 30°. In addition to the large lift imbalance, the robot has a small yaw angular velocity at liftoff. This causes the robot to experience a large initial roll orientation offset. As the robot continues to fly upward, its yaw rotational speed increases and the robot reduces the pitch and the roll oscillation amplitude through precession. The robot reaches a maximum height of 10 cm in 0.5 seconds of takeoff.

Further, we conduct a second flight using the same robot in a motion tracking arena (Extended Data Figure 9b). The tracked position and robot orientation are shown in Extended Data Figure 9c-f. Similar to the previous flight, the robot experiences large roll and pitch deviations during liftoff (approximately 100 ms). The tracked pitch and roll angles in Extended Data Figure 9e show the first peaks are approximately 35°. These peaks are highlighted by a red circle in Extended Data Figure 9e. The oscillation of the robot's pitch and roll angles decreases as the robot's yaw velocity increases. This experiment shows that precession allows the robot to reject small disturbances and initial offsets. However, the robot can be destabilized under a large disturbance. The robot is pulled by its tether when it reaches a height of approximately 18 cm (red regions in Extended Data Figure 9c-f), and it is quickly destabilized despite continuing to precess about its body z-axis.

In the main text, we show an example flight with collision recovery (Fig. 4d and video S6). This is enabled through a combination of the vehicle's passively stability and collision robustness. However, passive collision recovery does not happen each time, and in many cases the robot is destabilized after experiencing multiple collisions. Extended Data Figure 9g shows a collision recovery experiment (with the same robot as in Fig. 4d) that fails. The robot collides against the wall three times after liftoff, and it is destabilized by the third collision. The robot falls to the ground, and then it exits through an opening on the cylinder's wall (the opening was initially left for the tether). The robot then demonstrates a short takeoff before it is switched off. In-flight passive collision recovery depends on many factors such as the robot's flight speed, yaw angular velocity, incident angle, and the geometry of the obstacle. In addition, the robot may be destabilized after making repeated collisions with the same obstacle. Further studies can investigate the statistics of passive collision recovery or develop a feedback controller for collision recovery.

S4: Feedback controlled flight of an eight-wing robot

We designed a feedback controller to enable hovering flight in the robot with four DEAs and eight wings. Extended Data Figure 10a defines the robot's coordinate system and the rotational axes. Here x, y, z are the translational degrees of freedom and ϕ, θ, ψ are Euler angles following the pitch, roll, and yaw convention. Extended Data Figure 10b illustrates geometric parameters such as the distances (l_x, l_y) between the DEAs and the robot's center of mass. The values of l_x and l_y are 20.3 mm and 7.0 mm, respectively.

We design a controller by first deriving the robot's equation of motion:

$$\ddot{\boldsymbol{x}} = -g\boldsymbol{e}_3 + \frac{F}{m} R\boldsymbol{e}_3, \tag{43}$$

where $\ddot{\boldsymbol{x}}$ is the robot's acceleration, g is gravity, F is the net lift force, m is the robot's mass, and R is robot rotation matrix relative to the inertial frame. The robot mass m is 660 mg. We differentiate equation (43) and make two simplifying assumptions: $\frac{F}{m} \approx g \approx constant$ and $w^2 \ll ||\dot{\boldsymbol{w}}||$. These simplifications lead to the following equations:

$$\boldsymbol{x}^{(3)} = \frac{F}{m}(w_2 R_1 - w_1 R_2), \tag{44}$$

$$\boldsymbol{x}^{(4)} = \frac{F}{m}(\dot{w}_2 R_1 - \dot{w}_1 R_2), \tag{45}$$

where $\boldsymbol{x}^{(i)}$ is the ith order derivative of $\boldsymbol{x}$, and R_j is the jth column of the robot's rotational matrix. Next, we define the error vector:

$$\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x_d}, \tag{46}$$

where $\boldsymbol{x}$ and $\boldsymbol{x_d}$ are the displacement and the desired displacement of the robot's center of mass, respectively. Our controller aims to drive the error vector $\boldsymbol{e}$ and its derivatives to zero:

$$\boldsymbol{e}^{(4)} + \lambda_3 \boldsymbol{e}^{(3)} + \lambda_2 \boldsymbol{e}^{(2)} + \lambda_1 \dot{\boldsymbol{e}} + \lambda_0 \boldsymbol{e} = 0, \tag{47}$$

where λ_i are the coefficients of the controller. We obtain the controller for attitude and lateral position by taking the inner product of equation (47) with the first two columns of the rotation matrix R, and this operation implies equation (47) only applies to the x and y directions. In this derivation, we make the approximation that $|\boldsymbol{w} \times I\boldsymbol{w}| < |\boldsymbol{\tau}|$. Here I is the robot's moment of inertia tensor of the eight-wing vehicle and $\boldsymbol{\tau}$ is the applied body torque. The principal moment of inertia I_{xx}, I_{yy}, and I_{zz} are 4.8×10^4 mg·mm^2, 2.7×10^5 mg·mm^2, and 3.2×10^5 mg·mm^2, respectively. The equations of the attitude and lateral position controller are given by:

$$\frac{\tau_x F}{I_{xx} m} = -R_2^T \boldsymbol{x_d}^{(4)} - \lambda_3 \left(\frac{F}{m} w_1 + R_2^T \boldsymbol{x_d}^{(3)} \right) - \lambda_2 R_2^T (g\boldsymbol{e_3} + \ddot{\boldsymbol{x}}_{\boldsymbol{d}}) + \lambda_1 R_2^T \dot{\boldsymbol{e}} + \lambda_0 R_2^T \boldsymbol{e}, \tag{48}$$

$$\frac{\tau_y F}{I_{yy} m} = -R_1^T \boldsymbol{x_d}^{(4)} - \lambda_3 \left(\frac{F}{m} w_2 + R_1^T \boldsymbol{x_d}^{(3)} \right) - \lambda_2 R_1^T (-g\boldsymbol{e_3} - \ddot{\boldsymbol{x}}_{\boldsymbol{d}}) - \lambda_1 R_1^T \dot{\boldsymbol{e}} - \lambda_0 R_1^T \boldsymbol{e}. \tag{49}$$

The values of λ_0, λ_1, λ_2, and λ_3 need to satisfy the Routh-Hurwitz stability conditions. In equations (48) and (49), the terms $\ddot{\boldsymbol{x}}, \boldsymbol{x}^{(3)}$, and $\boldsymbol{x}^{(4)}$ have been substituted with the expressions from equations (43)-(45) with the previous approximation $\frac{F}{m} \approx g$.

To achieve controlled flight, we designed a proportional-derivative (PD) controller for the robot's altitude. The altitude controller is given by the equation:

$$\frac{F}{m} R_{33} = g + \ddot{z_d} - \Lambda_1(\dot{z} - \dot{z_d}) - \Lambda_0(z - z_d), \tag{50}$$

where Λ_1 and Λ_0 are the controller gains. Torque control (equations (48)-(49)) and force control (equation (50)) are executed in parallel, similar to the previous geometric control approach[43]. The values of the robot parameters and the controller gains are reported in Extended Data Table 3. The controller gains in (47) and (50) were tuned experimentally, based on the results from the previous research[43]. The values were verified to satisfy the Routh-Hurwitz stability condition before implementation.

This controller takes the robot position and attitude from an external motion tracking system as inputs. The position error ($\boldsymbol{e}$) was obtained directly from the position feedback and the velocity error ($\dot{\boldsymbol{e}}$) was computed using a filtered first-order derivative, while the rotational matrix (R) and the angular rate (w) were derived from the attitude feedback in a similar fashion. For hovering flights, $\dot{\boldsymbol{x}}_{\boldsymbol{d}}$, $\ddot{\boldsymbol{x}}_{\boldsymbol{d}}$, $\boldsymbol{x}_{\boldsymbol{d}}^{(\mathbf{3})}, \boldsymbol{x}_{\boldsymbol{d}}^{(\mathbf{4})} = 0$. The accuracy, tracking frequency, and latency of this existing motion tracking system are described in detail in a previous study[3]. The controller calculates the net thrust force (F) and pitch and roll torques (τ_x, τ_y) as outputs. We do not control the robot yaw motion because it is not necessary for hovering flight demonstration.

The decision not to control the yaw motion is similar to a previous work[43]. In principle, the eight-winged robot is capable of yaw control. As demonstrated by the four-winged robot, yaw torque can be generated by biasing the resting wing pitch angle. Subsequently, an eight-winged robot can be fabricated with two pairs of wings biased to generate positive yaw torque, while the other two pairs are biased to generate negative yaw torque. The configuration of the robot then becomes similar to that of a conventional quadrotor (with two propellers generating positive yaw torque, two propellers generating negative yaw torque). A robot in such a configuration is capable of producing four outputs (total thrust and torque about three body axes) independently.

Given the desired force and torque outputs (F, τ_x, τ_y), we need to calculate the input voltage amplitude for each DEA. First, we determine the thrust force (T_i) for each DEA through a least-squares regression:

$$T_i = \frac{F}{4} + L^{+}[\tau_x; \tau_y]. \tag{51}$$

In equation (51), L^{+} is the pseudoinverse of the displacement matrix L. L has the dimension $\mathbb{R}^{4\times 2}$ and its transpose is given by:

$$L^{T} = \begin{bmatrix} l_x & l_x & -l_x & -l_x \\ -l_y & l_y & l_y & -l_y \end{bmatrix}. \tag{52}$$

Having obtained the thrust force for each DEA, we calculate the input voltage amplitude. The relationship between lift force production and input voltage is given by a linear approximation:

$$V_i = \alpha T_i + \beta + \gamma_i\ . \tag{53}$$

Here α and β are mapping parameters that are identical for all DEAs, and γ_i is an offset parameter unique to each DEA. The parameter γ_i accounts for the fabrication variation between each DEA. The values of these parameters are reported in Extended Data Table 3.

To achieve the 16-second flight shown in the main text and in Figure 4e-i, we need to conduct several trimming flights to set the controller parameter values. These flights demonstrate robot flight endurance and repeatability. The robot has achieved a cumulative flight time of over 150 seconds (45000 flapping cycles) without needing any repair. The tracked robot position and altitude for five 10-second hovering flights are shown in Extended Data Figure 10c-q, and the corresponding controller parameter values are reported in Extended Data Table 3. We find the robot operation time is mainly limited by either the robot wing or the wing hinge. In takeoff experiments with robots that are powered by either one or two DEAs, the robot wings need to be repaired or replaced after 30-50 flights (each last between 0.5 to 1 second) because of uncontrolled collisions with the flight chamber wall or crash landings (Supplementary video 6). For static characterization tests that do not involve any collisions, we find the wing hinges soften and tear after approximately 10 minutes of operation at takeoff conditions. Our DEA endurance tests show that the DEA can operate for approximately 30 minutes at flight conditions. We never observe failure in the robot transmission or airframe. In this robot design, the DEA, the wing hinge, and the wing can be independently replaced during robot repair.

S5: Comparison with existing flapping-wing MAVs

In this paper, we present the first flapping-wing MAV powered by soft actuators that can demonstrate takeoff and hovering flight. There are several existing MAVs that have similar mass (<1 g) and size (<10 cm), and they are powered by piezoelectric[3], electromagnetic[5], and electro-hydrodynamic[44] actuators. Among these vehicles, only the ones powered by piezoelectric actuators have demonstrated hovering flight capabilities. Here we compare our DEA powered vehicle with the state-of-the-art piezoelectric powered MAV and describe its advantages and shortcomings.

The DEA-powered robot has two advantages over the robot powered by piezoelectric actuators. First, the DEA fabrication process is more scalable than that of the piezoelectric actuators. As long as all DEAs can be fit within a template (whose size is limited by the spin coater's diameter), the time spent in the multi-layering process is independent of the number of DEAs being fabricated. In contrast, fabrication of the piezoelectric actuators requires laser machining of every actuator, so the fabrication time scales proportionally with the number of piezoelectric actuators being fabricated. Currently, the fabrication speed of the DEAs is two times faster than that of the piezoelectric actuators. In addition, the robot assembly and repair processes are simpler. Since the DEA actuates along a linear axis, the robot transmission simplifies to a planar four-bar. In contrast, the piezoelectric actuators driven robots require a spherical four-bar transmission or an extra slider-crank mechanism added to the planar four-bar. Having a simpler transmission relaxes the requirement on assembly precision from 20 μm to approximately 100 μm, and also reduces the assembly time. Further, the DEA-powered robot is repairable because it only attaches to the robot transmissions. In the case of wear or damage, any component from the DEA-powered robot can be replaced. In contrast, the base of the piezoelectric actuators needs to be securely attached to the robot airframe with permanent epoxy to minimize serial compliance (and thus to maximize power output), consequently it is impossible to replace damaged actuators or robot transmissions.

One main advantage of the DEA-powered robot is its robustness to collisions. Collision resilience is contributed by many robotic components including the actuator, the transmission linkages, and the wings. Piezoelectric actuators have low fracture toughness and failure strain, and may not be as resilient to in-flight collisions. While introducing additional compliance in the robot may improve collision resilience[45], this may adversely influence flight quality because of the effects on resonance matching. In addition, the DEA-powered robot can sense collisions when the wing or the DEA hits an obstacle. The change of DEA current is due to the impact forces, which may cause damage if the DEA is replaced by piezoelectric actuators. Examples of collision robustness shown in Figure 2e-h and Extended Data Figure 3 are challenging for the MAV powered by piezoelectric actuators.

On the other hand, the DEA-powered robot has several shortcomings compared to the piezoelectric-powered robot. The main disadvantage is that the DEA requires high voltage actuation and is substantially less efficient compared to piezoelectric actuators. In the main text we mention that this DEA-driven robot is 15 times less efficient and requires a 6.5 times higher voltage compared to a recent power autonomous piezoelectric driven MAV[32]. Achieving power autonomous flight in soft-powered robots requires improvements of soft actuators and robot design. Section 3 of Methods discusses future directions on soft actuator improvements, and here we explain directions on robot design and scaling trends. In the state-of-the-art power autonomous MAV[32], the net power dissipation consists of three parts: boost and drive circuit loss, actuator transduction loss, and aerodynamic dissipation from drag forces. The relative power dissipation in these processes are 77%, 3%, and 20%, respectively. Most power dissipation comes from the circuit losses associated with the reduced efficiency of small scale, low mass magnetic components and high voltage topologies. These losses can be reduced by reducing actuation voltage (as discussed in Methods S3) and improving circuit efficiency. Since a MAV's net payload increases as its size increases (or as more small modules are assembled together to make a larger vehicle), having a larger vehicle will accommodate a heavier and more sophisticated circuit topology with higher efficiency. For instance, the efficiency of the boost circuit in the piezoelectric driven MAV[32] will increase with a larger, commercially-available transformer that has lower winding resistance and core losses. Off-the-shelf components such as a blocking-diode can be added to the circuit to improve energy recovery. In addition, we can adapt a multi-wing design similar to the recent MAV[32] to further improve aerodynamic efficiency. While scaling up the vehicle has benefits, new challenges also arise. A larger DEA may experience more complex buckling modes under large aerodynamic loads.

In addition, the DEA's nonlinear actuation creates challenges for controller design and reduces power efficiency. In this paper, we linearize the DEA's motion around its resonance by attenuating higher harmonics through post-resonance inertial roll off. This simple design creates two problems. First, the DEA cannot use higher harmonics to fine-tune flapping kinematics. This limits a flight module's ability to generate roll, pitch, and yaw torques for flight control. Second, damping out higher harmonics leads to high power dissipation and reduces DEA efficiency. Instead of using this simple linearization approach, future studies should develop methods that exploit the higher harmonics in a DEA's nonlinear motion. Alternatively, future work can also develop nonlinear transduction models and design nonlinear driving inputs to command a DEA's actuation.

Supplementary References

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Description of Supplementary Videos

Supplementary Video 1: Robot flapping kinematics
This high speed video shows a robot flapping its wings at 280 Hz. The left-wing stroke and pitch amplitudes are 42° and 59°, respectively. The right-wing stroke and pitch amplitudes are 41° and 62°, respectively. The robot is driven with a 1300 V, 280 Hz sinusoidal signal.

Supplementary Video 2: Collisions with obstacles of a stationary robot
This video shows collisions between an obstacle and a robot that is driven at 1300 V and 300Hz. In part 1, an obstacle makes collisions with the robot's left wing. While repeated collisions reduce the wing stroke amplitudes, the DEA continues to operate without experiencing failure. In part 2, the obstacle directly presses against the DEA during its operation. This contact reduces the amplitude of the wing motion, but does not break the DEA.

Supplementary Video 3: DEA nonlinear actuation
This video illustrates the influence of nonlinear DEA actuation on flapping wing kinematics. Part 1 compares the flapping kinematics at 1 Hz, 100 Hz, and 280 Hz. At operating frequencies that are substantially lower than system resonance, the wing stroke motion is asymmetric due to nonlinear actuation. The motion becomes purely sinusoidal near system resonance (280 Hz) because higher order terms are attenuated by viscoelasticity. Part 2 illustrates how DEA dynamic buckling substantially reduces wing stroke amplitudes. The buckling frequency is half that of the driving frequency.

Supplementary Video 4: Liftoff of a robot driven by one DEA
The robot powered by one DEA demonstrates open-loop liftoff. This robot can produce sufficient lift force to take off from the ground. However, the robot is intrinsically unstable, and it has no control authority. A carbon fiber rod with a point mass is attached to the robot's airframe to adjust its center of mass position. The robot is unstable in both the body roll and pitch axes. These two flights show that the robot quickly falls over after liftoff.

Supplementary Video 5: Flight of a robot driven by two DEAs
This video shows flight demonstrations of a four-wing robot powered by two DEAs. Part 1 shows unstable liftoff flights in which the robot does not rotate with respect to its body yaw axis. The robot is unstable if it does not generate sufficient yaw torque to precess. Part 2 compares experimental and simulation results of the robot's open-loop, passively upright stable ascending flights. Quantitative flight trajectory comparisons are shown in Extended Data Figure 7. Part 3 shows three different open-loop takeoff flights. Although the driving frequency and phase of each DEA are changed, the robot is still passively upright stable. This result shows that wing-wing interactions do not substantially influence robot stability.

Supplementary Video 6: Robustness against in-flight collisions
This video shows the robot can survive in-flight collisions. Part 1 shows a robot flying out of a transparent cylindrical shell. The robot collides with the shell wall as it exits the cylinder. It passively recovers from this collision and continues to fly upward. Part 2 shows simultaneous flights of two robots. These robots are intentionally flown into a wall, and they do not experience

damage after experiencing head-on collisions and crash landings. Part 3 shows two robots colliding with each other in mid-air. One robot recovers and flies upward after this collision.

Supplementary Video 7: Hovering flight demonstration through feedback control
This video shows controlled hovering flights of an eight-wing robot powered by four DEAs. Part 1 shows a 10-second flight and part 2 shows a 16-second flight. In both demonstrations, the robot's yaw motion is not controlled. Detailed flight data is reported in Figure 4e-i and Extended Data Figure 10i-k.