Peer Review File

Manuscript Title: Floating under a levitating liquid

Editorial Notes:

Reviewer Comments & Author Rebuttals

Reviewer Reports on the Initial Version:

Referee #1:

In this manuscript the authors conduct a series of rather spectacular experiments to demonstrate the effect of vertical vibration onto (i) the sinking or rising of air bubbles in liquid columns (ii) the dynamic stabilization of the Rayleigh-Taylor instability in liquid layers, and (iii) the upside down floating of immersed solid bodies.

The first series of experiments shows an air bubble sinking to the bottom of an oscillating container if the bubble is created below a certain depth. The second experiment demonstrates a levitating layer of a heavy liquid supported by a gas layer. The Rayleigh-Taylor instability at the lower interface of the liquid layer is fully suppressed if the horizontal dimension of the container is smaller than a certain critical size. The estimate for the critical container size is obtained and experimentally verified as a function of the vibration amplitude and frequency. The last set of experiments is a striking demonstration of the reversed floating of small objects: if the acceleration is sufficiently high, an immersed solid body will float upside down at the lower interface of a levitating fluid layer.

In my opinion, the jewel of the manuscript is the last set of experiments that demonstrate reversed floatation of small objects at the lower interface of a levitating layer. These spectacular results are novel to the best of my knowledge. The observed phenomenon will undoubtedly be of extreme importance to the researchers in the fluid dynamic community and well beyond. The offered analytical model seems plausible. I feel that the value of the manuscript will be greatly improved if the authors could experimentally verify the stability condition - eq.(23), derived in the supplement.

I have a few reservations about the other two experimental situations, discussed in the manuscript. Thus, the dynamics of air bubble in oscillating liquid columns has been studied theoretically and experimentally in the past. The experimental results in Fig.1 are qualitatively similar to hose, reported in Ref.[20]. It is somewhat difficult to see how the demonstration of the past results contributes to the value of the manuscript.

Also, the possibility to dynamically stabilize the Rayleigh-Taylor instability by means of a vertical vibration is well known. Apart from the classical works such as Refs.[12,13,14], there are other more recent studies, where the interplay between the Faraday and the Rayleigh-Taylor instability has been discussed. For example:

1) In one-layer films: "Mechanism of the Faraday instability in viscos liquids ", Kumar, Phys. Rev. E, 62 1416 (2000)
2) In one-layer films linear and nonlinear: "Rayleigh-Taylor instability in thin liquid films subjected to harmonic vibration", Stermna-Cohe, Bestehorn, Oron, Phys. Fluids, 29(5):052105 (2017)
3) In two-layer films: "Faraday instability of a two-layer liquid film with free upper surface", Pototsky, Bestehorn, Phys. Rev. Fluids, 1, 023901 (2016), "Faraday instability and nonlinear pattern formation of a two-layer system: A reduced model", Bestehorn, Pototsky, Phys. Rev. Fluids 1, 063905 (2016).
4) For isolated liquid drops: "Vibration-induced floatation of a heavy liquid drop on a lighter liquid film", Pototsky, Bestehorn, Oron, Phys, Fluids, 31, 087101 (2019).
5) FYI, an experiment with the dynamic stabilization of the Rayleigh-Taylor instability by vibration is now available on youtube: https://www.youtube.com/watch?v=XFAEhui7ygM

A better review of the literature would be desirable, especially around Eqs.(8,9) in the Supplement.

Additionally, it would be interesting to know if the boundary effects at the container walls play any role in the observed stabilization phenomenon. After all, the estimate for the largest container size, where the interface can be stabilized, was obtained by neglecting all boundary effects.

In summary, I was really impressed with the striking experiments on the upside down floating as well as with the quality of the experimental work. I am inclined to recommend the publication after a major reshuffling of the results and revision. I suggest to hide (move) the results in Fig.1 either towards the end of the manuscript, or into the Supplement. The discussion around Fig.2 should be revised, according to the comments outlined above. The results in Fig.3 should be made central. Finally, a paragraph with conclusive remarks must be added at the end of the manuscript to improve its readability.

<u>Referee #2:</u>

The submitted manuscript is an experimental study exploring the floating liquid in air by oscillations. The results demonstrated that the vertical shaking can have a dynamical stabilizing effect and this can float immersed object upside down even on the lower interface—This is the novelty of this work. Although the work seems very interesting, it needs to be improved according to the following comments.

1.Why did the authors use both glycerol (actually glycerol solution?) and silicon oil? Doesn't it have any difference on the phenomenon? One would think that the surface tension can be different for glycerol and silicon oil so that the wettability may affect the stability effect. Is the friction with the side wall negligible for the stable levitation of the liquid layer?

2.Fig. 1 b: How did the authors define and measure “the critical sinking depth d*”? (from the air/liquid interface to the center of gravity of the bubble?) Also, experimental data in Fig. 1 b needs error bar with the description of uncertainty analysis in the paper or supplementary information. Fig 1 b represents the d* as a function of shaking amplitude A. How about the effect of the bubble volume and the frequency ω/2π on the d*? Experimental data was fitted by the model “c/A with c= 5.5 mm3”. The authors should mention the physical meaning of c = 5.5 mm3. What about the effect of the gas property? (Did the authors use different gas (not air)?)

3.Fig. 2f: Can the authors levitate the liquid layer beyond L= 18 cm? If so, how about the maximum volume (mass) by this system? How long can the liquid layer be levitated at maximum? The authors also need to state the dye for colorizing the liquid layer.

4.Fig. 3b: Although the authors provided the model by the force balance in the paper and supplementary information, there is no information and analysis on the potential energy (represented in x axis). This should be further clarified, and relevant analysis may be added if necessary.

5.Fig. 3d: How did the authors measured the immersed volume? For upper and lower case (dynamic), spheres can move the liquid surface (as inferred from Supplementary Movie 6). If so, the immersed volume is a function of time.

6.Figs. 3d and e: Did the authors visualize and discuss the flow field in the liquid layer? The interaction between floating object and flow fields needs to be discussed(induced flow fields are clearly seen in Supplementary Movie 2).

7.Is it possible to provide the potential application with this phenomenon? It would be helpful for the community of scientists and engineers working in fluid dynamics and relevant area.

Minor improvements:

Conditions: What was the temperature and humidity condition in the experiments? The temperature and humidity information can be helpful for the future readers.

Fig. 1a: How did the authors control the injection rate of the bubble into the container?

p.6, line 17: “Fig. 2c and 2c” => “Fig. 2c and 2d”

p.7, line 8: “Figure 2e” => ““Figure 2f””

Fig. 3 legend: “f, Boats…” => “e, Boats…”

References 17 title: “Parametric instabilit of the interface betwwen two fluids” => “Parametric instability of the interface between two fluids”

Supplementary information:
p.4 line4: Reference “(23)” => “23(superscript)”.
p.5 Figure S2: “Figure 2” => “Figure S2”

Referee #3:

The paper concerns experimental and theoretical analysis of several counter-intuitive phenomena arising in fluid-filled volumes under the action of vertical vibration. In particular, the phenomenon of gas (air) bubbles sinking is observed and explained, as well as the phenomenon of levitating fluid layers caused by the vertical vibration. The last part of the paper concerns the phenomenon of the "upside-down floating" and discusses the dynamics of objects at the inverted interface of the levitating fluid layer.
I believe that the presented phenomena are very interesting and deserve to be known by a wider scientific community. Although some of the discussed phenomena, e.g. gas bubble sinking, have been described in other publications in the field, in my opinion, the paper has enough novelty to warrant publication in Nature. I would also recommend the authors to include references to some relevant papers:
1. V.N. Chelomeĭ, Sov. Phys. Dokl. 28, 387 (1983) [Dokl. Akad. Nauk SSSR 270 (1), 62 (1983)].
2. Blekhman I.I., Blekhman L.I., Sorokin V.S., Vasilkov V.B., Yakimova K.S. Surface and volumetric effects in a fluid subjected to high-frequency vibration // Proc. of the Institution of Mechanical Engineers, Part C, Journal of Mechanical Engineering Science – London. 2012. Vol. 226. Issue 8. August. P. 2028–2043
3. Blekhman, I.I., Blekhman, L.I., Vaisberg, L.A. et al. “Anomalous” phenomena in fluid under the action of vibration. Dokl. Phys. 53, 520 (2008).
4. Zen'kovskaja S.M., Novosjadlyj V.A. Influence of vertical oscillations on bilaminar system with a non-rigid interface [Влияние вертикальных колебаний на двухслойную систему с деформируемой поверхностью раздела]. Zhurnal vychislitel'noj matematiki i matematicheskoj fiziki, 2008, vol. 48, no. 9, pp 1710-1720.

I believe that the paper is well-written and presents the results in a clear manner. However, the supplementary material needs a bit of revision, for example, the material in Section 3a appears to be too trivial to be included. Also when discussing equation (3) "the third term is damping due to the relative motion between the container and the liquid." should be replaced by "the fourth term is damping due to the relative motion between the container and the liquid."
To conclude, in my opinion, the paper is well-written and nicely presents and explains some very interesting dynamic phenomena that deserve to be known by a wider scientific community. However, appropriate credit to the previous work should be given as is suggested above.

Author Rebuttals to Initial Comments:

Reply Referee #1 (Remarks to the Author):

In this manuscript, the authors conduct a series of rather spectacular experiments to demonstrate the effect of vertical vibration onto (i) the sinking or rising of air bubbles in liquid columns (ii) the dynamic stabilization of the Rayleigh-Taylor instability in liquid layers, and (iii) the upside down floating of immersed solid bodies.

The first series of experiments shows an air bubble sinking to the bottom of an oscillating container if the bubble is created below a certain depth. The second experiment demonstrates a levitating layer of a heavy liquid supported by a gas layer. The Rayleigh-Taylor instability at the lower interface of the liquid layer is fully suppressed if the horizontal dimension of the container is smaller than a certain critical size. The estimate for the critical container size is obtained and experimentally verified as a function of the vibration amplitude and frequency. The last set of experiments is a striking demonstration of the reversed floating of small objects: if the acceleration is sufficiently high, an immersed solid body will float upside down at the lower interface of a levitating fluid layer.

In my opinion, the jewel of the manuscript is the last set of experiments that demonstrate reversed floatation of small objects at the lower interface of a levitating layer. These spectacular results are novel to the best of my knowledge. The observed phenomenon will undoubtedly be of extreme importance to the researchers in the fluid dynamic community and well beyond. The offered analytical model seems plausible. I feel that the value of the manuscript will be greatly improved if the authors could experimentally verify the stability condition – eq. (23) derived in the supplement.

We thank the reviewer for his enthusiasm and appreciation of the findings presented in the article. We followed the referee's suggestion and performed additional experiments to verify the stability condition derived in the supplement. We now include these new results in the main body text together with a new figure (Fig. 3 in the new version). For clarity, we reproduce below the corresponding text:

In the discussion of figure 2:

> Taking into account the dynamical effect, i.e. the time averaged effect of the oscillations, provides an additional stabilizing dynamical potential around the two equilibrium positions (see inset Fig. 2b). Averaged small displacements Z_b of the floater around the two equilibrium positions satisfies the same dynamical equation $Z_b + \omega_b^2(1+\alpha)Z_b = 0$, ω_b being the angular frequency associated with the buoyancy force and $\alpha = \frac{\omega_b^2}{2}\left(\frac{A_l\omega}{g}\right)^2$ the correction induced by the averaged dynamical effects (see Supplementary Information for details). While, the dynamical effects increase the stability of the equilibrium at the upper interface ($\omega_b^2 > 0$, $\alpha > 0$), the unstable static equilibrium at the lower interface ($\omega_b^2 < 0$) is stabilized by the dynamical effects ($\alpha < 0$). The floater stability is reached for liquid velocity v_l above a critical velocity satisfying $v_b^* = \sqrt{2}g/|\omega_b| < v_l$

and at the end of the article:

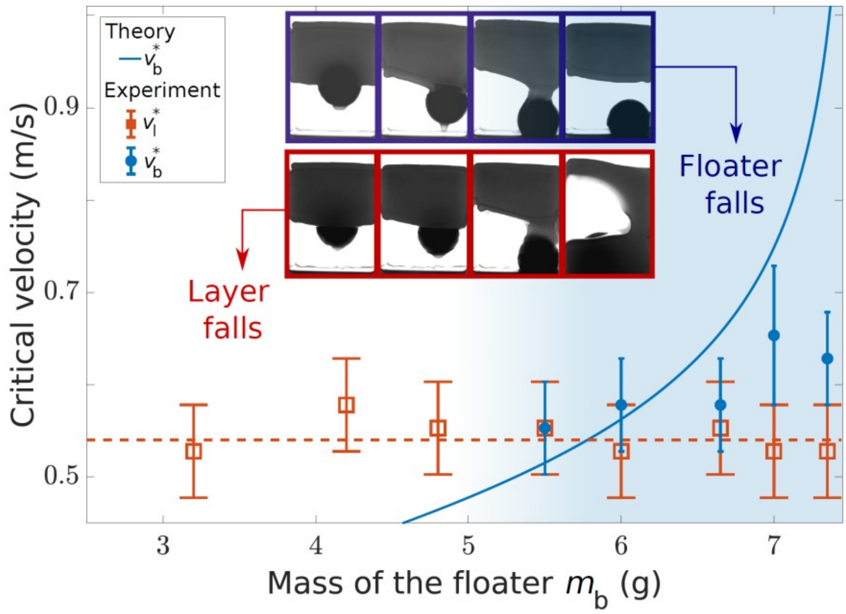

Figure 3 Floater and liquid layer stability. Critical velocities for the stability of the liquid layer v_l^* and for the floater v_b^* as a function of the floater mass m_b. The experiments are performed in a 4x5 cm^2 container with silicon oil and a spherical floater of 2.5 cm with various masses. The model (blue line) based on dynamic stabilization uses the liquid density measured experimentally $\rho = 0.92$ kg/L and no adjustable parameter. Above a certain floater mass $v_b^* > v_l^*$ (blue area) the floater can fall while the liquid layer remains stable. The error bars correspond to extremal values over 3 measurements. The dashed red line is the mean value of the v_l^* measurements. Inset image sequences of the experiment showing the layer falling at v_l^* for $m_{fl} = 4.8$ g and showing the floater falling at v_b^* for $m_b = 6.6$ g (see Supplementary Movie 7).

The previous stability conditions suggest that the critical fluid velocity to stabilize a floater v_b^* and the liquid layer v_l^* are different and that sufficiently massive floaters should fall before the layer collapses ($v_b^* > v_l^*$). We performed the experiments in silicon oil with spherical floaters of increasing mass, the heaviest one being almost neutrally buoyant (Figure 3). Contrary to light floaters which fall with the liquid layer as the excitation amplitude is decreased, heavier floaters fall before the levitating layer gets destabilized (see inset Figure 3 and Supplementary Movie 7). The theoretical critical velocity v_b can be exactly computed for spheres without any adjustable parameters (solid blue line). The expected range of masses for which $v_b^* > v_l^*$ is consistent with the experimental findings (blue area in Fig. 3) and the values v_b^* are in reasonable agreement. Discrepancy occurs for almost neutrally buoyant floaters (as also for buoyant equilibrium position). In this limit, new phenomena are observed such as a small relative motion of floaters with respect to the surrounding fluid layer which seems to play a significant role in the floater equilibrium.

This counter-intuitive upside-down buoyancy phenomenon suggests that the stabilization of Rayleigh-Taylor instability through vibrations can be considered not only in itself but also as a playground for new experiments in unexplored conditions. Under this perspective, many phenomena that occur at the interface between air and liquids could be investigated and reformulated in this new exotic configuration such as transport and segregation.

I have a few reservations about the other two experimental situations, discussed in the manuscript. Thus, the dynamics of air bubble in oscillating liquid columns has been studied theoretically and experimentally in the past. The experimental results in Fig.1 are qualitatively similar to those reported in Ref. [20]. It is somewhat difficult to see how the demonstration of the past results contributes to the value of the manuscript.

We agree with the referee that the experimental results are similar to those previously reported. Following the referee advice, we moved Figure 1 and the associated text describing the motion of the air bubble in the supplementary material. We have only maintained Fig. 1a of the experimental setup and inserted it in Fig. 2 (which now becomes Fig. 1).

Also, the possibility to dynamically stabilize the Rayleigh-Taylor instability by means of a vertical vibration is well known. Apart from the classical works such as Refs. [12,13,14], there are other more recent studies, where the interplay between the Faraday and the Rayleigh-Taylor instability has been discussed. For example:

1) In one-layer films: "Mechanism of the Faraday instability in viscous liquids ", Kumar, Phys. Rev. E, 62 1416 (2000)
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4) For isolated liquid drops: "Vibration-induced floatation of a heavy liquid drop on a lighter liquid film", Pototsky, Bestehorn, Oron, Phys. Fluids, 31, 087101 (2019).

5) FYI, an experiment with the dynamic stabilization of the Rayleigh-Taylor instability by vibration is now available on youtube: https://www.youtube.com/watch?v=XFAEhui7ygM

A better review of the literature would be desirable, especially around Eqs.(8,9) in the Supplement.

We thank the referee for pointing out these references, that are now included in the manuscript. We also added the corresponding references in the Supplement around Eqs. (8,9).

Additionally, it would be interesting to know if the boundary effects at the container walls play any role in the observed stabilization phenomenon. After all, the estimate for the largest container size, where the interface can be stabilized, was obtained by neglecting all boundary effects.

Concerning the boundary effects and the very discernable flow fields generated there, we believe they are not necessary for stabilization. The boundaries are relevant when the container and the liquid are out of phase and there is relative motion between them. For low frequencies, due to the nature of the forced oscillator, the liquid and container move together and the layer remains stable. Video footage of this can be provided, where the dynamics at the boundary are negligible, and the layer is still held up, even though the boundary effects are reduced. A more thorough investigation of the flows in themselves is definitely required.

We added a discussion about the secondary flows induced by the boundaries and their influence on the equilibrium position of the floaters in the Supplementary Information.

In summary, I was really impressed with the striking experiments on the upside down floating as well as with the quality of the experimental work. I am inclined to recommend the publication after a major reshuffling of the results and revision. I suggest to hide (move) the results in Fig.1 either towards the end of the manuscript, or into the Supplement. The discussion around Fig.2 should be revised, according to the comments outlined above. The results in Fig.3 should be made central. Finally, a paragraph with conclusive remarks must be added at the end of the manuscript to improve its readability.

We agree that the article should focus on the results presented in figure 3 and we reshuffled in depth the manuscript accordingly.

To summarize the main changes in the manuscript:

- we have moved Figure 1 and the associated text describing the motion of the air bubble in the supplementary material. We have only maintained Fig. 1a of the experimental setup and inserted it in Fig. 2 (which now becomes Fig. 1).

- We made central the results presented in Fig. 3 (which now becomes Fig. 2).

- We added a new figure (Fig. 3 of the new manuscript) where we tested experimentally the stability condition. This is now included in the main manuscript.

We would really like to thank the reviewer for his incentive to focus on the main findings of the paper and to perform these additional experiments which clearly improve the paper extending the validity of the model and pushing the experimental findings a step further.

Reply Referee #2 (Remarks to the Author):

The submitted manuscript is an experimental study exploring the floating liquid in air by oscillations. The results demonstrated that the vertical shaking can have a dynamical stabilizing effect and this can float immersed object upside down even on the lower interface —This is the novelty of this work. Although the work seems very interesting, it needs to be improved according to the following comments.

We thank the reviewer for his interest in the findings of the paper and his detailed remarks which have helped us improved the manuscript as well as given us an opportunity to clarify it. We agree to all the comments and answered all the questions (see below the detailed point by point answers). It is important to mention that we have reshuffled in depth the manuscript following suggestions made by another referee to focus on the novelty of the paper. We hope that the reviewer will agree with the changes that do not contradict his own comments and that, we think, helped improved the manuscript.

To summarize the main changes:

- We have moved Figure 1 and the associated text describing the motion of the air bubble in the supplementary material. We have only maintained Fig. 1a of the experimental setup and inserted it in Fig. 2 (which now becomes Fig. 1).

- We made central the results on the upside down buoyancy presented in Fig. 3 (which now becomes Fig. 2).

- We added a new figure (Fig. 3 of the new manuscript) where we tested experimentally the stability condition for the inverted floating position initially in the supplementary. This is now included in the main manuscript.

1. Why did the authors use both glycerol (actually glycerol solution?) and silicon oil? Doesn't it have any difference on the phenomenon?

We used both glycerol and silicon oil in order to check the generality of the phenomenon that we report in this manuscript. Both liquids are viscous but present significant differences in terms of density, hydration or wettability. The robustness of our experimental observations shows, in first approximation, that these differences have little influence on the obverved phenomenon.

When looking in more details, glycerol leads to a larger dispersion of the data when measuring the critical velocity as a function of the length of the container (fig. 1f). We believe this is linked to the hydration of glycerol that occurs during our experiments leading to a slow drift in the viscosity of the solution. We nevertheless used glycerol to test buoyancy effects (fig. 2) as its specific gravity allows the *use of wider range of floaters. Results are comparable when using silicon oil.*

We added the following sentence in the text describing the new fig. 1:

> "Though they have different physical properties, both liquids exhibit similar behavior provided that their viscosity is large enough."

One would think that the surface tension can be different for glycerol and silicon oil so that the wettability may affect the stability effect.

The surface tension of the two liquids is indeed different, as silicon oil totally wets the container while glycerol is in a partial wetting situation. It is worth noticing that there is a huge dynamical effect on the contact line behavior due to the vertical vibration. In our experimental configuration, silicon oil becomes dynamically non-wetting (Although, this is not fully surprising from previous studies — see for example Laudau & Levich, Acta Physicochim. USSR 17:42, 1942; Huh & Scriven, J. Colloid Interface Sci.35:85–101, 1971; Voinov Fluid Dyn.11:714–21, 1976; Cox, J. Fluid Mech. 168:169–94, 1986 – it seems that this is a new configuration worth investigating).

Since both liquids appear to be (dynamically) non-wetting, we do not consider this to have an effect on the stability, at most it is a secondary one, and would come into play for containers that approach the capillary length.

We added the following sentence in the text describing the new fig. 1:

> "In particular, the wetting conditions appear to have limited influence thanks to dynamical effects on the contact line."

Is the friction with the side wall negligible for the stable levitation of the liquid layer?

The effect of the friction on the layer is an interesting one, since we are dealing with very viscous liquids. We can estimate the friction forces on the wall with simple scaling arguments. We are interested in the σ_{xz} *component of the stress tensor, giving the force on the wall as* $F_z = S\sigma_{xz}$*, where* S *is the surface of the liquid layer touching the wall. The component of the tensor is given by* $\sigma_{xz} = \frac{\eta}{2}(\partial_x v_z + \partial_z v_x)$*,* *where we can neglect the derivative over z since we consider the velocity to be constant in that direction. We can approximate the derivative over x as $\frac{v_z}{\delta}$ where $\delta = \sqrt{\frac{2\eta}{\rho\omega}}$ is the penetration depth (a few millimeters here). The force is then given $F_z = A\, v_z \frac{\eta}{2\delta}$. Taking $\eta = 0.5$ Pa.s and a typical speed of 1cm/s, we get a force of $1.4\ 10^{-3}$N. This means that friction with the side wall is not sufficient to vertically sustain the liquid layer, even if it has important consequences in terms of secondary flows as can be observed in the movies.*

We added this discussion in the Supplementary Informations (end of section 3) and the following sentence in the text:

> "Other effects might have an influence on the stability, such as friction or the flows formed at the boundary, but there influence is limited (see Supplementary Information)"

2. Fig. 1 b: How did the authors define and measure "the critical sinking depth d*"? (from the air/liquid interface to the center of gravity of the bubble?)

We define the critical depth as the distance between the barycenter of the bubble (computed using image analysis tools) and the air/liquid interface. A single bubble of 0.4 mL was injected at the bottom of the bath where it stayed due to a large enough amplitude of shaking. The amplitude was lowered so that we can place the bubble at the critical depth (which itself is an equilibrium point for the bubble). By changing the amplitude, we can change the position of the bubble in the liquid. A video of the liquid was taken to extract the position of the bubble and amplitude of shaking at each critical depth. A comment describing the method has been added to the supplementary.

Also, experimental data in Fig. 1 b needs error bar with the description of uncertainty analysis in the paper or supplementary information.

Error bars have been added as ½ of the height of the bubble and explicated in the figure caption (now in the supplementary materials Fig. S1)

Fig 1 b represents the d as a function of shaking amplitude A. How about the effect of the bubble volume and the frequency ω/2π on the d*?*

The dependence of the critical depth on frequency, as is shown by our model, is ω^{-4}, which is much stronger than the dependence on the amplitude. In essence, the depth depends on the acceleration of the liquid, which can be modified by changing the amplitude or frequency. We chose to look at the amplitude since the depth of the liquid is limited, we have a greater range of amplitudes that we can explore than frequencies, allowing for greater precision.

We have not seen that the size of the bubble in our case had a significant effect, but for larger ones it surely does, as has been investigated previously by (Sorokin 2012). This provides an interesting experimental avenue. The reference has been added to the supplementary.

Experimental data was fitted by the model "c/A with c= 5.5 mm3". The authors should mention the physical meaning of c = 5.5 mm3.

We thank the referee for pointing this out. Using the expressions developed in section 1 of the revised supplementary, we have $c = \frac{p_0}{\rho\omega^2}\frac{2g}{\omega^2}$ This volume should be split in two parts. The first term corresponds to the typical size of a bubble with resonant frequency ω (Baird 1963, Jameson 1966), and the second corresponds to a characteristic length coming from gravity (eg. the pendulum). When the bubble is put in the liquid, we have two important factors, one is the modulation of the bubble's *radius, represented by the first term, and the second is the action of the liquid above the bubble, represented here as the second term. These two scales are then combined to form the characteristic volume c.*

We have added this discussion at the end of section 1 of the Supplementary Information.

What about the effect of the gas property? (Did the authors use different gas (not air)?)

We did not use any other gas than air, but the literature suggests that the gas property can impact the critical depth in particular concerning its compressibility (Blekhman 2008).

We have added a short comment at the end of section 1 of the Supplementary Information.

3. Fig. 2f: Can the authors levitate the liquid layer beyond L= 18 cm? If so, how about the maximum volume (mass) by this system?

We managed to float a liquid layer of 20 cm x 4 cm or a 12 cm x 12 cm layer containing 0.5 L of glycerol. Beyond these dimensions the air layer is very delicate to create as the bubble tends to reach the surface before filling the whole bottom of the tank. This difficulty can be overcome by adding more liquid in the tank at cost of an increased mass to shake. At that point we are limited by the maximal acceleration that can be provided by our shaker. We believe that using a larger shaker we could levitate larger layers provided the air layer can be created and the Faraday instability on the lower interface is not started.

How long can the liquid layer be levitated at maximum?

In our experiments we maintained the levitation of liquid layers for tens of minutes and we did not see any limitation concerning a maximum duration for this phenomenon.

We added the following sentence in the text:

Provided the spring-mass oscillation is properly tuned, layers can be sustained for tens of minutes and there are no restriction in the number of levitating layers that can be piled up.

The authors also need to state the dye for colorizing the liquid layer.

The colorization was made numerically using graphics software, the liquid was initially transparent or translucent depending on the amount of air bubbles in it.

To account for this comment, we modified the caption of Fig. 1e:

> Digitally colorized three-quarter views of the oscillating containers with one and two levitating liquid layers of silicon oil (see Supplementary Movies 4).

4.Fig. 3b: Although the authors provided the model by the force balance in the paper and supplementary information, there is no information and analysis on the potential energy (represented in x axis). This should be further clarified, and relevant analysis may be added if necessary.

We thank the referee for pointing this out. A detailed discussion has been added in the Supplementary Informations (section 4b) and modified the main text:

> "Taking into account the dynamical effect, i.e. the time averaged effect of the oscillations, provides an additional stabilizing dynamical potential around the two equilibrium positions (see inset Fig. 2b). Averaged small displacements Z_b of the floater around the two equilibrium positions satisfies the same dynamical equation $Z_b + \omega_b^2(1+\alpha)Z_b = 0$, ω_b being the angular frequency associated to the buoyancy force and $\alpha = \frac{\omega_b^2}{2}\left(\frac{A_l\omega}{g}\right)^2$ the correction induced by the averaged dynamical effects (see Supplementary

> Information for details). While, the dynamical effects increase the stability of the equilibrium at the upper interface ($\omega_b^2 > 0, \alpha > 0$), the unstable static equilibrium at the lower interface ($\omega_b^2 < 0$) is stabilized by the dynamical effects ($\alpha < 0$). The floater stability is reached for liquid velocity v_l above a critical velocity satisfying $v_b^* = (\sqrt{2}\, g)/|\omega_b| < v_l$."

5. Fig. 3d: How did the authors measured the immersed volume? For upper and lower case (dynamic), spheres can move the liquid surface (as inferred from Supplementary Movie 6). If so, the immersed volume is a function of time.

The immersed volume plotted in figure 3 (which has become fig. 2 in the revised manuscript) is indeed a mean immersed volume along time. It was measured by taking the height between the top of the sphere and the interface. For small oscillations of the sphere at the interface, the mean immersed volume can be computed by inserting the mean height in the cap sphere formula. Moreover, each point of the curve corresponds to an average over several realizations of the experiment for a given floater. The immersed volume depends on the amount of liquid sticking to the floater and this changes at each realization. The error bars take into account the motion of the sphere in the liquid and the different results over the realizations.

To account for this comment, we modified the caption of Fig. 2d:

> "Time-averaged equilibrium positions for 2 cm diameter spheres with various masses as a function of the immersed volume at the upper (squares) and lower interface (diamonds). Circles are equilibrium positions obtained without shaking. The dashed line is given by the Archimedes' principle with $\rho_l = 1.1$ kg/L for glycerol. The error bars correspond to extremal values over 5 measurements."

6. Figs. 3d and e: Did the authors visualize and discuss the flow field in the liquid layer?

The flow field can indeed be visualized thanks to the small air bubbles that are sucked in the liquid during shaking (such as in Movie 2). We observe liquid recirculations at the edges. These recirculations always take the same shape of two counter-propagating vortices parallel to the upper and lower contact lines. These recirculations seem to be related to the relative motion between the liquid layer and the container, triggered by the strong shearing. These recirculations seem to increase at the resonance of the liquid layer motion (see Fig. 1c).

We now discuss this point in the Supplementary Information (section 3) and added the following sentence in the text:

> "Other effects might have an influence on the stability, such as friction or the flows formed at the boundary, but there influence is limited (see Supplementary Information)"

The interaction between floating object and flow fields needs to be discussed (induced flow fields are clearly seen in Supplementary Movie 2).

We can evaluate the consequences of the secondary flow (see Fig. 1b and Movie 2) on the floating object by estimating the drag force on the floater. From the movie, we have $V \approx 1$ cm/s and a *floater of size $R \approx 1$ cm, and thus a Reynolds number $Re \approx 0.3$. This means that viscous effects (slightly) dominate in this flow. From this we deduce the viscous drag force $F_v \approx \eta RV \approx 5.10^{-5}$ N, much smaller than the weight of the floater 10^{-2} N. The secondary flow should thus not significantly change the vertical equilibrium of the floaters.*

We now discuss this point in the Supplementary Information (section 4b) and added the following sentence in the text:

> "Note that the drag force induced by secondary flows due to recirculation in the liquid layer should not significantly change the vertical equilibrium position"

7. Is it possible to provide the potential application with this phenomenon? It would be helpful for the community of scientists and engineers working in fluid dynamics and relevant area.

This phenomenon suggests that the stabilization of Rayleigh-Taylor instability through vibrations can be considered not only for itself but also as a playground for new experiments. Under this perspective, many 'classical' phenomena that occur at the interface between air and liquids could be investigated and reformulated in this new situation. In particular many well-known equilibriums at such interfaces may be changed in this exotic configuration, as it is the case for buoyancy (a floating object cannot always float at the downward interface). Transport or segregation at this interface could also lead to unexpected behaviors. It is possible that there are also some industrial applications to this experiment although we do not see any yet. As the theory is simple and the experiment impressive, this could also be used to promote physics to a non-specialist audience.

We have added as a concluding paragraph in the manuscript:

> "This counter-intuitive upside-down buoyancy phenomenon suggests that the stabilization of Rayleigh- Taylor instability through vibrations can be considered not only for itself but also as a playground for new experiments in unexplored conditions. Under this perspective, many phenomena that occur at the interface between air and liquids could be investigated and reformulated in this new exotic configuration such as transport and segregation."

Minor improvements:

Conditions: What was the temperature and humidity condition in the experiments? The temperature and humidity information can be helpful for the future readers.

The temperature during the experiments was room temperature (20 °C). We did not observe any effect of humidity (which is roughly constant in our laboratory, around 50% +/- 10% depending on the season). We made sure to change the glycerol after using it since it is very susceptible to hydration.

We added this information in the caption of fig. 1.

Fig. 1a: How did the authors control the injection rate of the bubble into the container?

The injection was made by hand using a graduated syringe to ensure all bubbles had the same volume. In each measurement of the critical depth, the same bubble was always used as it can be kept in the liquid continuously. It seems that the injection rate is not critical provided it is not too high to disrupt the liquid layer.

p.6, line 17: “Fig. 2c and 2c” => “Fig. 2c and

2d” p.7, line 8: “Figure 2e” => ““Figure 2f””

Fig. 3 legend: “f, Boats…” => “e, Boats…”

We have corrected these 3 points according to the renumbering of the figures.

References 17 title: “Parametric instabilit of the interface betwwen two fluids” =>
“Parametric instability of the interface between two fluids”

We have corrected this.

Supplementary information:
p.4 line4: Reference “(23)” => “23(superscript)”.
p.5 Figure S2: “Figure 2” => “Figure S2”

We have corrected this.

Reply referee #3 (Remarks to the Author):

The paper concerns experimental and theoretical analysis of several counter-intuitive phenomena arising in fluid-filled volumes under the action of vertical vibration. In particular, the phenomenon of gas (air) bubbles sinking is observed and explained, as well as the phenomenon of levitating fluid layers caused by the vertical vibration. The last part of the paper concerns the phenomenon of the "upside- down floating" and discusses the dynamics of objects at the inverted interface of the levitating fluid layer.

I believe that the presented phenomena are very interesting and deserve to be known by a wider scientific community. Although some of the discussed phenomena, e.g. gas bubble sinking, have been described in other publications in the field, in my opinion, the paper has enough novelty to warrant publication in Nature.

We thank the reviewer for his interest in the findings of the paper and his detailed remarks which have helped us improved the manuscript as well as given us an opportunity to clarify it. We agree to all the comments and answered all the questions (see below the detailed point by point answers). It is important to mention that we have reshuffled in depth the manuscript following suggestions made by another referee to focus on the novelty of the paper. We hope that the reviewer will agree with the changes that do not contradict his own comments and that, we think, helped improved the manuscript.

To summarize the main changes:

- We have moved Figure 1 and the associated text describing the motion of the air bubble in the supplementary material. We have only maintained Fig. 1a of the experimental setup and inserted it in Fig. 2 (which now becomes Fig. 1).

- We made central the results on the upside down buoyancy presented in Fig. 3 (which now becomes Fig. 2).

- We added a new figure (Fig. 3 of the new manuscript) where we tested experimentally the stability condition for the inverted floating position initially in the supplementary. This is now included in the main manuscript.

I would also recommend the authors to include references to some relevant papers:
1. V.N. Chelomeĭ, Sov. Phys. Dokl. 28, 387 (1983) [Dokl. Akad. Nauk SSSR 270 (1), 62 (1983)].

2. Blekhman I.I., Blekhman L.I., Sorokin V.S., Vasilkov V.B., Yakimova K.S. Surface and volumetric effects in a fluid subjected to high-frequency vibration // Proc. of the Institution of Mechanical Engineers, Part C, Journal of Mechanical Engineering Science – London. 2012. Vol. 226. Issue 8. August. P. 2028–2043
3. Blekhman, I.I., Blekhman, L.I., Vaisberg, L.A. et al. “Anomalous” phenomena in fluid under the action of vibration. Dokl. Phys. 53, 520 (2008).
4. Zen'kovskaja S.M., Novosjadlyj V.A. Influence of vertical oscillations on bilaminar system with a non-rigid interface [Влияние вертикальных колебаний на двухслойную систему с деформируемой поверхностью раздела]. Zhurnal vychislitel'noj matematiki i matematicheskoj fiziki, 2008, vol. 48, no. 9, pp 1710-1720.

We have added these references which are indeed much relevant in particular Blekhman et al. 2008 is a really profound review which is much inspiring (and that we unfortunately missed in our bibliography search!).

I believe that the paper is well-written and presents the results in a clear manner. However, the supplementary material needs a bit of revision, for example, the material in Section 3a appears to be too trivial to be included. Also when discussing equation (3) "the third term is damping due to the relative motion between the container and the liquid." should be replaced by "the fourth term is damping due to the relative motion between the container and the liquid."

We have removed the supplementary section 3a and corrected the reference to the fourth term in equation (3) of the supplementary.

To conclude, in my opinion, the paper is well-written and nicely presents and explains some very interesting dynamic phenomena that deserve to be known by a wider scientific community. However, appropriate credit to the previous work should be given as is suggested above.

We hope that the changes will clearly give the appropriate credits to previous works and focus on the novelty of the present work.

Reviewer Reports on the First Revision:

Referee #1:

All criticism, raised by this referee has been satisfactory addressed in the new version of the manuscript. I recommend the publication in the present form.

Referee #2:

Thank you for your detailed response. I was happy to be the reviewer for this outstanding work. I would like to recommend acceptance of the manuscript after addressing minor corrections below.

page 1 Figure 1b caption: "Supplementary Movie2 " =>"Supplementary Movie1 " (also need to rename other Supplementary Movie#)
page 5, line 6: "Fig. 2e" => "Fig. 1e"

Supplementary Information:
page 1 Figure S1: "a.(underline)" => "a.(no underline)"
page 1: "Fig. 1a" => "Fig. S1a"
page 1: "Fig. 1b" => "Fig. S1b"
page 4: "Figure 2c and 2d" => "Figs. 1c and 1d"
page 5: Reference (23) =>31(superscript & revised ref. #)
page 6: "Fig. 2f" => "Fig. 1f"

Referee #3:

The authors have addressed all my comments in the revised version of the paper. I would like to note, however, that the paper [Chelomei, V. N. Mechanical paradoxes caused by vibrations. Soviet Physics Doklady 28, 387 (1983)] concerns generating levitating liquid layers by applying external vibrations, not the sinking of air bubbles. I recommend the authors to discuss this reference explicitly in their manuscript.

The new section of the paper concerned with the floater and liquid layer stability appears to be very relevant and interesting. I recommend the authors to do an additional grammar check. Otherwise the paper can be accepted, in my opinion.

Author Rebuttals to First Revision:

Reply to Referee #1 (Remarks to the Author)

All criticism, raised by this referee has been satisfactory addressed in the new version of the manuscript. I recommend the publication in the present form.

We thank the referee for his/her recommendation and we would once again like to express our deep gratitude for his/her previous suggestions that have led to significant improvements to the paper.

Reply to Referee #2 (Remarks to the Author)

Thank you for your detailed response. I was happy to be the reviewer for this outstanding work. I would like to recommend acceptance of the manuscript after addressing minor corrections below.

We thank the referee for his/her enthusiasm and we would once again like to express our deep gratitude for his/her previous suggestions that have led to significant improvements to the paper.

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page 4: "Figure 2c and 2d" => "Figs. 1c and 1d"
page 5: Reference (23) =>31(superscript & revised ref. #)
page 6: "Fig. 2f" => "Fig. 1f"

We have corrected all these mistakes.

Reply to Referee #3 (Remarks to the Author)

The authors have addressed all my comments in the revised version of the paper. I would like to note, however, that the paper [Chelomei, V. N. Mechanical paradoxes caused by vibrations. Soviet Physics Doklady 28, 387 (1983)] concerns generating levitating liquid layers by applying external vibrations, not the sinking of air bubbles. I recommend the authors to discuss this reference explicitly in their manuscript.

The new section of the paper concerned with the floater and liquid layer stability appears to be very relevant and interesting. I recommend the authors to do an additional grammar check. Otherwise the paper can be accepted, in my opinion.

We thank the referee for his/her recommendation and we would once again like to express our deep gratitude for his/her previous suggestions that have led to significant improvements to the paper.

We have added explicit reference to Chelomei's paper in the abstract:

> *"Vibrations of liquids also induce other paradoxical phenomena such as the sinking of air bubbles[14–19] or stabilization of the liquid layer above sinking objects[20]".*

and in the discussion (about the Chelomei's pendulum):

> *"It is interesting to note that similar dynamical stabilizations were observed with a washer mounted on a vibrated inverted pendulum[20]"*

We also had the paper proofread by someone whose mother tongue is English and corrected the grammatical errors.