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Supplementary information

Emergent electromagnetic induction in a helical-spin magnet

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Supplementary Information

Current-induced dynamics of helix

The current-driven motion of a spin-helix can be described by two collective coordinates (Refs. 3 and 18 in the main text): the position along the Q-direction of the helix (X) and the angle (φ) representing a tilt of spins from the helical plane. The equation of motion can be described as

$$\lambda\dot{\phi} + \alpha\dot{X} = \zeta\beta j - v_{\mathrm{pin}} \sin\left(\frac{X}{\lambda}\right) \tag{1}$$

$$\dot{X} - \alpha\lambda\dot{\phi} = \frac{1}{2} v_{\mathrm{int}} \sin 2\phi + \zeta j, \tag{2}$$

where λ is the helical period, j is the current density, α is the Gilbert damping parameter, β is a dimensionless quantity representing the strength of the non-adiabatic effect, and v_{pin} and v_{int} represent electron velocity at the threshold current density for extrinsic pinning (j_{pin}) and for intrinsic pinning (j_{int}), respectively. Here, we note the extrinsic pinning arises from defects and impurities, and the intrinsic pinning results from the magnetic anisotropy; since X and φ are canonical conjugates, the translational motion X of a helix accompanies a tilt of the spin from the helical plane (i.e. a change in φ). Below j_{int}, spins cannot be tilted due to the cost of magnetic anisotropy energy, and consequently, the helix is pinned. (Ref. 18 in the main text). The coefficient ζ is given by $\zeta = \frac{a^3 P}{2eS}$, where a, P, and S are the lattice constant, the spin polarization of the conduction electron, and the local spin moment, respectively. By using j_{pin} and j_{int}, v_{pin} and v_{int} are described as $v_{\mathrm{pin}} = \zeta\beta j_{\mathrm{pin}}$ and $v_{\mathrm{int}} = 2\zeta j_{\mathrm{int}}$, respectively. Because there are nonlinear terms with respect to ϕ and X in Eqs. (1) and (2), the dynamics of the helix are generally nonlinear. In particular, the first term in the right-hand side of Eq. (2) is derived from the magnetic anisotropy, which is assumed to be $K_{\perp}S^2$ here. However, more generally, higher order contributions of the magnetic anisotropy and j exist in Eqs. (1) and (2), altering the form of this nonlinear term. Since the solution of Eqs. (1) and (2) depends on the form of these higher order terms, for generality we expand ϕ in a power series of *j* as follows in this work:

$$\phi = Aj + Bj^3 + Cj^5 + \cdots. \tag{3}$$

Therefore, discussion below do is not affected by the detailed form of Eqs. (1) and (2). Because the motion of helix is symmetric with respect to the current direction in the centrosymmetric systems, ϕ is an odd function of current.

Derivation of emergent inductance

Here, we derive the emergent inductance (Ref. 3 in the main text) considering the current-nonlinear term of ϕ. The *x*-component of the emergent electric field can be described as follows (Refs. 3, 6, and 7 in the main text):

$$e_x = \frac{h}{2\pi e}\mathbf{n}\cdot(\partial_x \mathbf{n} \times \partial_t \mathbf{n}), \tag{4}$$

where $\mathbf{n}$ is a unit vector parallel to the direction of spins. In the case of a proper-screw helix with magnetic modulation vector $(Q = 2\pi/\lambda)$ parallel to the *x*-axis, $\mathbf{n}$ is given by

$$\mathbf{n} = \mathbf{e}_2 \cos(Qx + \psi) + \mathbf{e}_3 \sin(Qx + \psi) + m\boldsymbol{e}_1\,, \tag{5}$$

where $\boldsymbol{e}_{1,2,3}$ are unit vectors parallel to the *x*, *y*, and *z* axes, respectively, and ψ and *m* are the phase of the helix and the magnetic moment parallel to *Q*, respectively. Here, we consider small *m* and approximate the normalization constant of Eq. (5) as $\sqrt{1+m^2} \approx 1$. Then, by inserting Eq. (5) into Eq. (4), we obtain

$$e_x = \frac{PhQ}{2\pi e}\partial_t m = \frac{Ph}{e\lambda}\partial_t m. \tag{6}$$

Here, P is spin polarization, which arises because the sign of emergent electric field is opposite for majority and minority electrons (Ref. 13 in the main text). For small m, m can be replaced with ϕ, which represents a tilt of the spin from the helical plane (see also Fig. 1c in the main text for the definition of ϕ). Hence, the emergent electric field is described as

$$e_x = \frac{Ph}{e\lambda}\partial_t\phi. \tag{7}$$

As discussed in the above section, in the case of helices driven by current, ϕ can be written in Eq. (3). Then, inserting Eq. (3) into Eq. (7), we obtain

$$e_x = \frac{Ph}{e\lambda}(A + 3Bj_\mathrm{x}^2 + 5Cj^4 + \cdots)\partial_t j_x. \tag{8}$$

Since $j_x = I/S_\mathrm{c}$, where I and S_c are the current and cross-section of the device, respectively, we can rewrite Eq. (8) as

$$V = \frac{Phd}{e\lambda S_\mathrm{c}}\left(A + 3B\frac{I^2}{S_\mathrm{c}^2} + 5C\frac{I^4}{S_\mathrm{c}^4} + \cdots\right)\frac{dI}{dt}, \tag{9}$$

where d is the distance between the electrodes used for detection, and $V = e_x d$ is the recorded voltage. Here, the coefficient of dI/dt corresponds to the inductance. Equation (9) indicates that the inductance increases with decreasing the cross-section of the device (S_c) in contrast to conventional inductors. Although we assume a proper-screw helix in the above, this discussion is also valid for other non-collinear spins structures such as transverse conical, cycloidal helices, and Bloch and Neel domain walls.

In lock-in measurements, we input a sine-wave current ($I = I_0 \sin \omega t$) and measured the out-of-phase components of the linear voltage (V_Y^{1f}) and the third harmonic voltage (V_Y^{3f}). By inserting $I = I_0 \sin \omega t$ into Eq. (9) and considering ϕ up to the third order (i.e., $\phi = Aj + Bj^3$), we obtain

$$V = \frac{Phd\omega}{e\lambda S_c}\left(A + 3B\frac{I_0^2\sin^2\omega t}{S_c^2}\right)I_0\cos\omega t = \frac{Phd\omega}{e\lambda S_c}\left[\left(A + \frac{3BI_0^2}{4S_c^2}\right)\cos\omega t - \frac{3BI_0^2}{4S_c^2}\cos 3\omega t\right]I_0. \quad (10)$$

Here, the coefficient of $\cos\omega t$ and $\cos 3\omega t$ corresponds to V_Y^{1f} and V_Y^{3f}, respectively. Hence, the imaginary part of the impedance ($\mathrm{Im}\Delta\mathrm{Z}^{1f}$) and the imaginary part of the third harmonic impedance ($\mathrm{Im}Z^{3f}$) can be described as

$$\mathrm{Im}\Delta\mathrm{Z}^{1f} = \frac{V_Y^{1f}}{I_0} = \frac{Phd\omega}{e\lambda S_c}\left(A + \frac{3BI_0^2}{4S_c^2}\right) \quad (11)$$

$$\mathrm{Im}Z^{3f} = \frac{V_Y^{3f}}{I_0} = -\frac{Phd\omega}{e\lambda S_c}\frac{3BI_0^2}{4S_c^2}. \quad (12)$$

Here, we discuss the magnitude of $\mathrm{Im}\Delta\mathrm{Z}^{1f}$ and $\mathrm{Im}Z^{3f}$. According to Eqs. (11) and (12) , the ratio between $\mathrm{Im}\Delta\mathrm{Z}^{1f}$ and $\mathrm{Im}Z^{3f}$ can be described as $r = \mathrm{Im}\Delta\mathrm{Z}^{1f}/\mathrm{Im}Z^{3f} = -(1 + 4A/3Bj_0^2)$, where $j_0 = I_0/S_c$. By determining *A* and *B* from the fit to the *j* dependence of $\mathrm{Im}\Delta\mathrm{Z}^{1f}$ (Fig. 3a in the main text), we obtain $r^{\mathrm{fit}} = -0.7$ at *T* = 5.6 K and $j_0 = 3.3 \times 10^8\ \mathrm{Am}^{-2}$. On the other hand, by using the experimentally measured values of $\mathrm{Im}\Delta\mathrm{Z}^{1f}$ and $\mathrm{Im}Z^{3f}$ (Figs. 3b,c in the main text), we obtain $r^{\mathrm{measured}} = \mathrm{Im}\Delta\mathrm{Z}^{1f}/\mathrm{Im}Z^{3f} \sim -3$, which is comparable with r^{fit}. We also estimate the amplitude of ϕ from *A* and *B* determined from the fit to the *j*-dependence of $\mathrm{Im}\Delta\mathrm{Z}^{1f}$; at $j_0 = 1.0 \times 10^8\ \mathrm{Am}^{-2}$, *f* = 10 kHz, and *T*= 10 K, for example, the amplitude of ϕ is approximately 15 degree.

The sign of emergent inductance

The sign of emergent inductance is determined from the product of (1) the sign of the emergent electric field and (2) the direction of motion of the spin structures. (1) The sign of emergent electric field is determined from the sign of the coupling constant between the local moment and conduction electrons. This is described by the spin polarization *P*. (2) In the present case, because the helix is driven by the spin-transfer torque (STT), the sign of direction of the motion of the spin structures is linked to the sign of STT. Spin transfer torque is also related to the coupling between local moment and conduction electrons, and the sign of STT is determined by the sign of the same coupling constant to the case of emergent electric field, i.e. the spin polarization factor (P) in the Eqs. (1) and (2) in the Supplementary Text. This indicates, even if we change the sign of P by tuning the position of Fermi level, the sign of the emergent inductance does not change when the spin structure is driven by STT. The positive emergent inductance can be realized by independently controlling the direction of the motion of the spin structure. For example, this can be achieved by using the spin-orbit torque (Ref. 30 in the main text). In a bilayer structure of heavy metal and magnetic material, the torque arising from pure spin current injected from the heavy metal layer due to the spin Hall effect, which called spin-orbit torque. In this case, the direction of the motion of the spin structures can be controlled by choice of the heavy metal; because the sign of the spin Hall effect (i.e. the sign of torque) depends on material, the direction of motion of the spin structures can be arbitrary controlled by appropriate choice of the heavy material (Ref. 30 in the main text). Hence, we would control the direction of motion of the spin structure and thus the sign of emergent inductance. Here, we note the direction of motion of spin structure is related the sign of the coefficient of Eq. (3) in Supplementary Text.

Frequency dependence of the current-induced dynamics of a helix

Here, we discuss the frequency dependence of the current-driven dynamics of a spin-helix. Equations (1) and (2) can be linearized as follows (Ref. 3 in the main text):

$$\lambda\dot{\phi} + \alpha\dot{X} = \zeta\beta j - \frac{v_{\mathrm{pin}}X}{\lambda} \tag{10}$$

$$\dot{X} - \alpha\lambda\dot{\phi} = v_{\mathrm{int}}\phi + \zeta j. \tag{11}$$

By solving these equations, we obtain

$$\phi = \frac{\zeta j}{\lambda} \frac{i\omega(\beta - \alpha) + f_{\mathrm{pin}}}{(1+\alpha^2)\omega^2 + i\alpha\omega(f_{\mathrm{int}} + f_{\mathrm{pin}}) - f_{\mathrm{int}} f_{\mathrm{pin}}}, \tag{12}$$

where $f_{\mathrm{int}} = \frac{v_{\mathrm{int}}}{\lambda}$ and $f_{\mathrm{pin}} = \frac{v_{\mathrm{pin}}}{\lambda}$ represent the characteristic frequency for the intrinsic pining and the extrinsic pinning, respectively. For small ω and $f_{\mathrm{int}} \gg f_{\mathrm{pin}}$, ϕ can be approximated as

$$\phi \approx \frac{\zeta j}{\lambda f_{\mathrm{int}}} \frac{f_{\mathrm{pin}}}{i\alpha\omega - f_{pin}}. \tag{13}$$

Equation (13) is Debye-type relaxation as observed in the experiment (Fig. 4b in the main text). The typical angular frequency (ω_{Debye}) can be described as

$$\omega_{\mathrm{Debye}} = \frac{f_{pin}}{\alpha} = \frac{\zeta \beta j_{\mathrm{pin}}}{\alpha\lambda} = \frac{a^3 P}{2eS} \frac{\beta j_{\mathrm{pin}}}{\alpha\lambda}. \tag{14}$$

Taking the following parameters: $\lambda = 3 \times 10^{-9}$ m, $e = 1.6 \times 10^{-19}$ C, $a = 1 \times 10^{-9}$ m, S = 1, $P = 0.1$, $\alpha \sim \beta$, and $j_{\mathrm{pin}} = 1 \times 10^6$ Am^{-2}, we estimate the order of magnitude of the typical frequency as $f_{\mathrm{Dybye}} = \omega_{\mathrm{Debye}}/2\pi \sim 10$ kHz, which is consistent with the experimentally observed value.