Peer Review File

Manuscript Title: The time programmable frequency comb and its use in quantum-limited ranging

Reviewer Comments & Author Rebuttals

Reviewer Reports on the Initial Version:

Referees' comments:

Referee #1 (Remarks to the Author):

The authors demonstrated an agile programmable frequency comb where the pulse time and phase can be flexible controlled with a fast speed and a high precision. The concept is very interesting and novel to the best of my knowledge. It breaks the paradigm that the frequency comb is only used as a fixed frequency ruler or reference oscillator. As an example application, nearly quantum-limited ranging with an unprecedentedly high precision is demonstrated when such an agile comb is used to coherently track the received signal comb and demodulate the timing and phase information. Many other applications may also benefit from this novel technique, such as time/frequency metrology, pump-probe experiments, etc. I think the paper fits within the scope of Nature in terms of novelty and potentially wide impact. The following are some places which may need further clarification.

1. It is mentioned that the novel ranging method reduces the detection threshold by ~5,000 fold compared to the conventional dual-comb ranging. I understand that this threshold is defined as the received power level with a SNR ~10. Then why is this level chosen as a “threshold”?
2. When the received signal power is above 10 nW, the ranging precision reaches a floor of 0.7 nm. What is this floor limited by?
3. It is shown that the relative range can also be retrieved from the unwrapped carrier phase. A higher precision can then be achieved compared to that with the pulse timing (as in ref. 7). Does it mean that the ultimate ranging precision is actually better than Eqn. (1) as well as the results shown in Figs. 3c and 5?
4. What is the offset frequency of the signal comb compared to the TPFC?
5. On page 7, in the paragraph starting with “Here, as shown in Figure 3…”, there is an extra “and” in the sentence “In both modes, the relative time and phase between the TPFC pulses and the strongly attenuated signal-comb pulses %and% are detected by a power-insensitive…”.
6. In Fig. 3b, measurement error bar is indicated in the legend, but no error bars are shown in the plot.
7. In Fig. 2a, it’s better to clearly mention that the surface plot represents the pulse envelope repeatedly monitored with linear optical sampling. And what are the small weak pulses that can also be clearly observed?
8. The labels in Fig. 1c are kind of confusing. I can figure out that Deta_X(t) and Delta_theta(t) mean the increments relative to the last state. Perhaps it is clearer to use them to represent the relative changes compared to the initial state.
9. On page 7, in the paragraph starting with “In tracking mode…”, it’s better to keep writing the mean photons per pulse as 1/77 just as in the other places.

10. The labels in Fig. 4b are kind of confusing. Both "R(t)-FMCW" and "R(t)-R_carrier(t)" point to the same grey trace. And what is the meaning of "0.4s smooth"?
11. In Fig. 5, it's better to replace the vertical lines aside "SNR limit" with dashed lines because they are actually not part of these traces.
12. In the caption of extended data figure 1, there is a typo about the order of the curves. "from 980 nW (top curve) down to 0.33 pW (bottom curve)" should be "from 980 nW (bottom curve) down to 0.33 pW (top curve)".

Referee #2 (Remarks to the Author):

The work given by Caldwell et al. demonstrates a time programmable frequency comb with which one can set the pulses' timing and phase in an arbitrary manner. The authors use the comb laser and demonstrate significant improvement in dual-comb ranging measurement. By adjusting the timing of the comb pulse to that of the measurement target pulse, one can efficiently use the photons for the measurement by removing the dead time, which gives a significant advantage in SNR compared to the conventional dual-comb linear sampling. I find this work provides quite a big step in the field of frequency comb ranging. As the authors describe in the manuscript, it can also be useful for other applications such as long-distance free-space time transfer or compressive spectroscopic sensing. The manuscript is very well written with details, and the experimental demonstrations are made at a very high level, which is convincing enough to be published with several minor modifications. The followings are my concerns, which I request the authors to address.

1. The authors quite often use the expression "agile" programmable frequency comb. However, in the main body of the manuscript, it is unclear how fast one can set and achieve the programmed time and phase of the pulses. How fast can it be settled and what makes the current limitation, and how can we improve it? The bandwidth of the electronic servo system is described in the Methods section, but it does not easy to pick up the above-mentioned specifications. I would request the authors explain it in the main body and discuss its current limitation and possibility of future improvement. It is important for readers to consider future applications.
2. In the abstract, there is a statement saying, "reducing the detection threshold by ~5000-fold, …." It is unclear compared to what it reduces by 5000-fold if one reads the abstract only. I would suggest the authors make it clear in the abstract already.
3. In Fig.3c, the sigma_R hits the limit around the received signal comb power of 10 nW and becomes worse above that power (10nW-1 uW). There is no explanation of it in the description of Fig.3 in the current manuscript. I would request the authors to provide some discussions on it.
4. The Methods sections can be improved by separating them into more subsections. For example, there are long sections of "Optical Hardware" and "Digital Electronics System". It would be easier to read if these long sections had sub-sections with proper headings.
5. The reason for selecting the 26-kHz bandpass filter following the timing discriminator is unclear in the main body.
6. In the Methods section, the acronym "LOS" is not spelled out when it appears first. It is spelled out afterward.

Referee #3 (Remarks to the Author):

Dear Editor,
the authors demonstrate how the relative coherence and pulse arrival time (timing) of two combs can be actively controlled and stabilised, to produce a comb (a "tracking comb" or Time-Programable Frequency Comb) that is a very accurate replica of the original one. This brings immediate advantages in ranging applications, where it allows beating diffraction or turbolence-induced photon losses and perform high-resolution, high-rate ranging even with very little received power. The paper describes how the comb tracking features can be controlled using advanced FPGA and contains the description of an interferometric time-discriminator, suitable to detect the lead/lag between two interfering comb pulses and unambiguiusly correct for it within the stabilization process.
The achievement is novel and original and is of undoubtful interest in the field of comb-based time/frequency transfer, ranging and (I guess) imaging-related applications of ranging. The quality of presentation is good and clear, although in some cases it required to consult references to fully understand some of the specialised content. The technical quality of the paper is very high; the description of the problem, its threatment and methodology are solid and supported by results. It was indeed a very interesting paper. Concerning the experimental results, although the authors explicitly address ranging applications and mention advancements compared to their previous work in the field (which is indeed the case as far as photon losses are concerned), there is no real-field test, as the paper was mainly intended to demonstrate the novel time-programmable comb technique. As such, the characterization is performed on a laboratory testbed, and application in ranging is demonstrated with a moving rail.
Concerning the interest of this work in other fields, connections are suggested by the authors, tough not explicitly addressed and detailed. This makes it difficult for me to fully appreciate the broad-reaching applications of the time programmable comb in fields not related (directly or indirectly) to ranging/timing (see more detailed comment below).
From my point of view, which is barely a technical one, the paper should be published provided some minor aspects are more widely discussed or clarified, for which I attach a detailed list here below.
Regarding the editorial suitability to the Nature journal, potential concerns are: 1) the not clear connection to applications outside ranging, even tough this is by itself a quite variegated one (time-transfer, satellite tracking, radar, imaging...); 2) the main focus of the paper is the technique description, with results showed only on a lab-based test setup. Even if the authors provide scaling rules to meet the more stringent requirements of real world (target motion and turbolence), it was not always clear how the obtianed results and test conditions compare to the one of real world, if one has no numbers already in mind (as for a non-technical readership).
A final judgement on its suitability for this journal could be more wisely addressed by the Editor.

The following list better details previous comments and reports minor technical concerns for which I would appreciate clarification in the final manuscript:

- Abstract and first paragraph, discussion of applications:

the authors mention " sensing application", but it is not immediately clear what "sensing" implies in this context beyond ranging. In the end of abstract, the authors mention that other applications "should be possible" but these are not addressed further in the text and therefore the statement remains a bit vague. I suggest the authors either go in more detail, or remove from the abstract. Regarding dual-comb spectroscopy applications: to my (non-expert) knowledge, a whole class of comb-based spectroscopy is based on the linear optical sampling of a comb after it passes through a sample, and the information is contained in the amplitude of the comb tooths after the sample, that show the signature of absorbtion. But to my understanding, in the TPFC the information about amplitude and pulse-shape of the original comb are lost (only the phase and timing info is preserved). Is my understanding correct? And if yes, can the authors more clearly describe the connection and applicability of their technique to spectroscopy application?

- Fig. 1c: I am confused by the time axis of the plot. If this is the product of LOS, I would expect a time-axis which is magnified by M= 1E6 (200MHz/200Hz). Was the axis in the picture rescaled to the "original" timing? Am I right in assuming that in LOS output an applied Delta X of 5 ps is mapped into 5 microseconds (which is easily resolved, actually?). The author could add a note on this in the caption. Also, can they specify the bandwidth of the photodiode used to detect the linear optical sampling (see also following point), and describe how the red or blue trace in Fig. 1c are obtained?

- Characterization: the authors use LOS to characterize how well the original and TPFC are synchronized, or how accurately a given step can be applied to the TPFC. But to resolve a single-shot timing error of 1E-17 using LOS one still needs about 10 ps of resolution and challenging 100 GHz bandwidth at detection. Can the authors describe better how this is achieved? A similar question holds about results shown in Fig. 2c and give more explanation on how
-Perhaps related to the above question: in line 117-120 it is said that a 5 fs shift is "easily resolved". How this is seen on the pulses timing?

-In line 118 should be: Delta theta_0 =2 pi N (delete f_rep)

- Fig. 2a: I don't get the meaning of the "Amplitude" axis (or color code) in Fig. 2a, and did not find explanation in the text, can the authors comment?

- line 144-148: the authors talk about "resolution" and "precision". I do not get the exact difference between the two concepts. I would tend to say the ultimate resolution depends on pulse-width and SNR according to Eq. 1, where Delta R in turns depend on the pulse width as shown in L. 145. Can the authors clarify ?

- Related to "accuracy" in ranging, is the absolute positioning precision of the target also a concern? I assume this is subject to absolute calibration, that should depend also on any instrumental delay and optical path. Can the authors comment, if this is a quantity of interest, and to what extent in real-world applications?

-line 186-187 and caption of Fig. 3 the authors mention the timing discriminator to be "power-insensitive". I agree on this, but I think the main reason for building the Mach-Zehnder interferometric scheme is indeed to distinguish lead vs lag time, which would otherwise be ambiguous, is it? I would mention this important property in the text, rather than leaving it implicit.

-Line 198: I am a bit confused on the two parameters that are used to lock and synchronizing the TPCF to the signal comb: the pulse-timing and the carrier frequency lock. Here, it is mentioned that both are derived by the timing discriminator output. As far as I understand, the timing alignment is based on the outcome of the timing discriminator, and is achieved by adjusting the phase of the 0th tooth. But I don't see how the carrier-frequency info can be derived from the timing discriminator (there should be a single output from the discriminator, which represents the pulse timing misalignment).
Moreover, in the Methods, it is said (L480) that no action is taken to adjust the phase of the Nth tooth. So, to my understanding, a single degree of freedom is controlled, i.e. the timing misalignment, which seems consisent with the fact that the timing discriminator has a single output. Can the authors clarify this misunderstanding?
A similar question, regarding line 234 and following, is: how is the carrier phase between the two comb pulses extracted from the timing discriminator to yeld carrier-based ranging measurements reported in Fig. 4b?

- Line 187: "26-kHz bandpass filter". Shouldn't the filter following the discriminator be a low-pass filter instead?

-Line 209-211: " momentary loss of signal is not an issue. If brief enough, the object
does not move by more than 100 micron". For an object moving at 1 m/s this implies outages of the order of 100 micro-seconds. How does this number compare to the typical outages in outdoor space? The authors can add a comment on typical numbers or real-world applications.

-Figure 3 and 4: results shown in Fig. 3 are reported for 0.3 pW received power, while those in Fig. 4 mention 3 and 30 pW. Was the power increased to enable measurement with the commerical ranging system? The authors could add a comment on this

-Line 513: the combs have the same cw laser reference. As the pulse timing and optical phase are related, how is the TPFC ranging capability degraded in the case the cw lasers referencing the (incoming) signal comb and TPCF different, as the ranging information is derived from the control signal?

-extended figure 1: I guess the blue curves (upper) refer to lower power and the red ones (lower) refer to higher power, not viceversa as stated in caption

-Extended Figure 2. I guess there is some confusion with colors as to my understanding should be:
called "dark blue" -> blue
called "black" -> "dark blue" (actually, it looks like dark blue on my pdf...)
called "purple" -> ok, purple
please reword or distinguish color in a clearer way

- Some of the most recent work on similar subjects by other groups could be acknowledged where relevant, e.g. :

Gozzard et al Phys. Rev. Lett. 128, 020801 2022
Shen et al., arXiv:2203.11272, 2022
Dix-Matthews et al., Nature Communications 12, 515 (2021)

Author Rebuttals to Initial Comments:

Reply to Referees

First, we wish to thank all the referees for their thoughtful and constructive comments on our manuscript. Following their recommendations, we have modified the document and believe it is stronger as a result.

Please find below specific responses to each referee comment where we have indicated the original comments in black font and our responses in blue font. Where relevant, we indicate changes to the wording of the manuscript with *blue italic* font.

Referee #1 (Remarks to the Author):

The authors demonstrated an agile programmable frequency comb where the pulse time and phase can be flexible controlled with a fast speed and a high precision. The concept is very interesting and novel to the best of my knowledge. It breaks the paradigm that the frequency comb is only used as a fixed frequency ruler or reference oscillator. As an example application, nearly quantum-limited ranging with an unprecedentedly high precision is demonstrated when such an agile comb is used to coherently track the received signal comb and demodulate the timing and phase information. Many other applications may also benefit from this novel technique, such as time/frequency metrology, pump-probe experiments, etc. I think the paper fits within the scope of Nature in terms of novelty and potentially wide impact. The following are some places which may need further clarification.

1. It is mentioned that the novel ranging method reduces the detection threshold by ~5,000 fold compared to the conventional dual-comb ranging. I understand that this threshold is defined as the received power level with a SNR ~10. Then why is this level chosen as a "threshold"?

We realized the use of "detection threshold" in the abstract is slightly misleading since the improvement applies overall. In other words, the tracking dual-comb ranging requires ~5,000 fold less power to achieve the same precision as conventional dual-comb ranging. We modified the abstract to be more precise and it now reads

> "*To highlight its capabilities, we use this programmable comb in a ranging system,* reducing the *required power to reach a given precision by ~5,000-fold compared to a conventional dual-comb system.*"

The SNR of 10 was chosen as a reasonable minimum SNR for a low probability of a false positive detection and matches our experiment. In general, the choice will depend on the usual trade between sensitivity and false positives. As noted above, the 37 dB improvement of the tracking dual-comb versus conventional approach is independent of this choice. To make this clearer in the context of Figure 5, we have also changed the caption in Figure 5 to read

> "*For all three ranging configurations, we require a minimum detection SNR of ~10 for reasonable detection statistics, as indicated by the SNR limit. The 37 dB improvement, however, is independent of this chosen limit.*"

We also adjusted the location of the 37 dB arrow in the figure.

2. When the received signal power is above 10 nW, the ranging precision reaches a floor of 0.7 nm. What is this floor limited by?

This is due to temperature variations acting on differential fiber paths. We added a sentence in the caption Figure 3 which now reads

> *"The measured precision follows the quantum limit (Eq. (1)) from 0.33 ± 0.03 pW to 10 ± 1.0 nW with a penalty of C = 2.16, reaching a systematic noise floor below 1 nm (7 as), which is*

2-10x below previous dual-comb ranging experiments[6–8,42–44] and attributed to residual fiber path-length fluctuations."

In addition, at higher powers there is a slight uptick attributed to optical feedback as discussed in response to Reviewer 2's third question below.

3. It is shown that the relative range can also be retrieved from the unwrapped carrier phase. A higher precision can then be achieved compared to that with the pulse timing (as in ref. 7). Does it mean that the ultimate ranging precision is actually better than Eqn. (1) as well as the results shown in Figs. 3c and 5?

The reviewer is correct that the carrier phase measurement has better precision. However, since we did not implement a handover between timing-based range to carrier-based range we did not feel comfortable claiming this improvement. Rather, we decided to use the carrier-based range as a form of truth data in Figure 4. An accurate handover between the timing and carrier range certainly would be interesting but challenging.

4. What is the offset frequency of the signal comb compared to the TPFC?

The carrier frequencies of the two combs are typically offset by ~10 MHz. This allows for the complex demodulation to be done away from DC. We have added this information to the main text in the context of a much longer paragraph re-write for Referee 3. The relevant sentences read

"*The two heterodyne signals output by the timing discriminator have a nominal carrier frequency of 10 MHz, set by the relative phase locks of the two combs. The signals are then bandpass filtered and demodulated."*

5. On page 7, in the paragraph starting with "Here, as shown in Figure 3…", there is an extra "and" in the sentence "In both modes, the relative time and phase between the TPFC pulses and the strongly attenuated signal-comb pulses %and% are detected by a power-insensitive…".

This typo was removed.

6. In Fig. 3b, measurement error bar is indicated in the legend, but no error bars are shown in the plot.

The statistical error bars are smaller than the dots used in the plot so they did not show up in the figure. We have removed the error bars from the legend.

7. In Fig. 2a, it's better to clearly mention that the surface plot represents the pulse envelope repeatedly monitored with linear optical sampling. And what are the small weak pulses that can also be clearly observed?

We updated the caption for Fig. 2a to more clearly describe the source of the surface plot as well as the small back-reflections the reviewer pointed out. The caption now reads

"Figure 2: Illustration and characterization of the time programmability of the TPFC through LOS, using the setup in Fig 1a. (a) The TPFC pulse train, presented as a surface plot, where each slice in lab time represents a complete LOS measurement as in Figure 1c. The TPFC pulse is located at the LOS signal peak and follows the commanded arbitrary step pattern (red line). Multiple reflections within the setup appear as small satellite pulses."

8. The labels in Fig. 1c are kind of confusing. I can figure out that Deta_X(t) and Delta_theta(t) mean the increments relative to the last state. Perhaps it is clearer to use them to represent the relative changes compared to the initial state"

We have adopted the reviewer's suggestion so that Delta_X(t) and Delta_theta(t) now represent changes relative to the initial state.

9. On page 7, in the paragraph starting with "In tracking mode…", it's better to keep writing the mean photons per pulse as 1/77 just as in the other places.

We have updated the text to read "*only **1/77** mean photons per pulse"* rather than "*only 0.013 mean photons per pulse."*

10. The labels in Fig. 4b are kind of confusing. Both "R(t)-FMCW" and "R(t)-R_carrier(t)" point to the same grey trace. And what is the meaning of "0.4s smooth"?

The "0.4s smooth" was meant to indicate the narrow black trace, which was a 0.4 second smoothing of the light grey trace. However, since this black line was nearly indistinguishable from the zero line and was not discussed further in the text, we removed it from the figure. Since we had not previously in the text defined R_carrier, we deleted the label for "R(t)-R_carrier(t)" as well. (The caption still defines the meaning of this trace.)

11. In Fig. 5, it's better to replace the vertical lines aside "SNR limit" with dashed lines because they are actually not part of these traces.

We agree with the reviewer and implemented this change. We also moved the 37 dB line and arrow down in the figure to clarify that it applies across the full range and not simply at the minimum limit (as noted in the first response.)

12. In the caption of extended data figure 1, there is a typo about the order of the curves. "from 980 nW (top curve) down to 0.33 pW (bottom curve)" should be "from 980 nW (bottom curve) down to 0.33 pW (top curve)".

We thank the reviewer for catching this error. It has been corrected.

Referee #2 (Remarks to the Author):

The work given by Caldwell et al. demonstrates a time programmable frequency comb with which one can set the pulses' timing and phase in an arbitrary manner. The authors use the comb laser and demonstrate significant improvement in dual-comb ranging measurement. By adjusting the timing of the comb pulse to that of the measurement target pulse, one can efficiently use the photons for the measurement by removing the dead time, which gives a significant advantage in SNR compared to the conventional dual-comb linear sampling. I find this work provides quite a big step in the field of frequency comb ranging. As the authors describe in the manuscript, it can also be useful for other applications such as long-distance free-space time transfer or compressive spectroscopic sensing. The manuscript is very well written with details, and the experimental demonstrations are made at a very high level, which is convincing enough to be published with several minor modifications. The followings are my concerns, which I request the authors to address.

1. The authors quite often use the expression "agile" programmable frequency comb. However, in the main body of the manuscript, it is unclear how fast one can set and achieve the programmed time and phase of the pulses. How fast can it be settled and what makes the current limitation, and how can we improve it? The bandwidth of the electronic servo system is described in the Methods section, but it does not easy to pick up the above-mentioned specifications. I would request the authors explain it in the main body and discuss its current limitation and possibility of future improvement. It is important for readers to consider future applications.

We have modified the main text on page 4 to add the following discussion:

> *"Here the maximum slew rate between time steps was conservatively set to 40 ns/s to eliminate the possibility of cycle slips in the PLL during motion. The use of an input tracking filter for the*

PLL signals should enable slew rates as high as 1 µs/s, limited only by the actuators. (See Methods).

Section III.B.3 (newly labeled) has also been modified to read:

III.B.3 Slew Rate

For large time steps, the TPFC response will be limited by the maximum slew rate. A linear slew of the comb pulse time corresponds to a frequency shift of the phase-locked f_0 or f_N frequencies (where the latter is measured by its heterodyne beat signal against the cw reference laser). For a frequency shift of Δf in either of these frequencies, the slew rate is $dX/dt \approx \Delta f / f_N$. *Typically comb actuators can move either of these frequencies by a few modes (for example through the pump current modulation or cavity fiber stretcher). In that case, $\Delta f \sim \pm f_{rep}$, giving a slew rate of* $dX/dt \approx f_{rep} / f_N$. *For the frequency combs used here, this yields a maximum potential slew rate of dX/dt ≈ 200 MHz/200 THz = 1 µs/s.*

To maintain the full accuracy of the comb, there is a second potential constraint to the maximum slew rate. Namely, despite a shift in either of the beat frequencies, f0 or (fN – fCW), the system must reliably track the phase of both beat signals; any cycle slip would result in a loss of accuracy for both the comb pulse time and phase. The resulting limitation on the slew rate depends on a combination of the SNR of the relevant beat signal and sophistication of the demodulation scheme. Indeed, with very high SNR, the beats can be tracked over the full Nyquist window of 0 to ±frep/2 and there is no additional limitation. For a typical SNR, one could implement a tracking filter with feedforward to track dynamic frequency changes and avoid cycle slips even with large deviations in the beat frequency. This approach should minimize any slew rate limitations. However, we did not implement such a filter here. As a result, in the current system, we limited the maximum slew rate to 40 ns/s. This rate was not optimized but was chosen conservatively to avoid cycle slips, which was verified experimentally. The 40 ns/s slew rate was used for the data of Figure 1 and 2. For the ranging data of Figure 3-4, a slower slew rate of 4 ns/s was chosen to match the temporal duration of the overlap between the TPFC and signal comb with the inverse of the 26-kHz measurement bandwidth.

2. In the abstract, there is a statement saying, "reducing the detection threshold by ~5000-fold, …." It is unclear compared to what it reduces by 5000-fold if one reads the abstract only. I would suggest the authors make it clear in the abstract already.

We have adopted the Reviewer's suggestion and with an additional change in response to Reviewer 1 the sentence from the abstract now reads

"*To highlight its capabilities, we use this programmable comb in a ranging system, reducing the required power to reach a given precision by ~5,000-fold compared to a conventional dual-comb system.*"

3. In Fig.3c, the sigma_R hits the limit around the received signal comb power of 10 nW and becomes worse above that power (10nW-1 uW). There is no explanation of it in the description of Fig.3 in the current manuscript. I would request the authors to provide some discussions on it.

We investigated this issue by examining the time traces. The slight uptick of 2x in precision above 10 nW received power is caused by occasional, sudden jumps of 30 to 50 nm in range. We attribute these to nonlinear optical feedback because they only appear at high power, consistent across multiple datasets beyond those used in Fig. 3c. Such jumps are rare; they appear 6 times over the ~6 mins of data covered by the three highest power points, and none are found in the lower power data ever. While investigating and eliminating this odd feedback would be of interest for future work, we considered it out of scope for this paper.

We have updated the figure in the manuscript to show both the previous deviation values with increased noise from these features and additional deviations evaluated from the (majority) portion of the time trances without these jumps, proving that the excess deviation does indeed come from these jumps.

The Figure 3c caption has been modified to read:

> *"At the three high power points, the deviation is given both for the full data trace and for a subset of the data without ~6 sudden ranging/timing jumps of <50-nm amplitude which are attributed to nonlinear optical feedback (and only appear at these higher powers)."*

4. The Methods sections can be improved by separating them into more subsections. For example, there are long sections of "Optical Hardware" and "Digital Electronics System". It would be easier to read if these long sections had sub-sections with proper headings.

We have added subsections to "Optical Hardware" for "Frequency Combs," "Timing Discriminator," and "Ranging Setup." Similarly, we broke up "Digital Electronics System" with subsections for "Digital Platform," "Timing and Phase Commands," "Slew Rate," and "Control Loops in Ranging System."

5. The reason for selecting the 26-kHz bandpass filter following the timing discriminator is unclear in the main body.

To better explain both the general purpose and frequency selection of the 26-kHz filter we have updated the relevant sentences in the main body to read

> *"The two heterodyne signals output by the timing discriminator have a nominal carrier frequency of 10 MHz, set by the relative phase locks of the two combs. The signals are then bandpass filtered and demodulated. The choice of bandwidth is a tradeoff between update rate and sensitivity. Here, we use 26-kHz to be well above typical ~kHz mechanical vibrations while allowing for a fairly long 13 microsecond coherent integration time."*

6. In the Methods section, the acronym "LOS" is not spelled out when it appears first. It is spelled out afterward.

The acronym LOS is now spelled out where it first appears in the Methods section

<u>Referee #3 (Remarks to the Author):</u>

The authors demonstrate how the relative coherence and pulse arrival time (timing) of two combs can be actively controlled and stabilised, to produce a comb (a "tracking comb" or Time-Programable Frequency Comb) that is a very accurate replica of the original one. This brings immediate advantages in ranging applications, where it allows beating diffraction or turbolence-induced photon losses and perform high-resolution, high-rate ranging even with very little received power. The paper describes how the comb tracking features can be controlled using advanced FPGA and contains the description of an interferometric time-discriminator, suitable to detect the lead/lag between two interfering comb pulses and unambiguiusly correct for it within the stabilization process.

The achievement is novel and original and is of undoubtful interest in the field of comb-based time/frequency transfer, ranging and (I guess) imaging-related applications of ranging. The quality of presentation is good and clear, although in some cases it required to consult references to fully understand some of the specialised content. The technical quality of the paper is very high; the description of the problem, its threatment and methodology are solid and supported by results. It was indeed a very interesting paper. Concerning the experimental results, although the authors explicitly address ranging applications and mention advancements compared to their previous work in the field (which is indeed the case as far as photon losses are concerned), there is no real-field test, as the paper was mainly intended to demonstrate the novel time-programmable comb technique. As such, the characterization is performed on a laboratory testbed, and application in ranging is demonstrated with a moving rail.

Concerning the interest of this work in other fields, connections are suggested by the authors, tough not explicitly addressed and detailed. This makes it difficult for me to fully appreciate the broad-reaching applications of the time programmable comb in fields not related (directly or indirectly) to ranging/timing (see more detailed comment below).

From my point of view, which is barely a technical one, the paper should be published provided some minor aspects are more widely discussed or clarified, for which I attach a detailed list here below.

Regarding the editorial suitability to the Nature journal, potential concerns are: 1) the not clear connection to applications outside ranging, even tough this is by itself a quite variegated one (time-transfer, satellite tracking, radar, imaging...); 2) the main focus of the paper is the technique description, with results showed only on a lab-based test setup. Even if the authors provide scaling rules to meet the more stringent requirements of real world (target motion and turbolence), it was not always clear how the obtianed results and test conditions compare to the one of real world, if one has no numbers already in mind (as for a non-technical readership).

A final judgement on its suitability for this journal could be more wisely addressed by the Editor.

The following list better details previous comments and reports minor technical concerns for which I would appreciate clarification in the final manuscript:

- Abstract and first paragraph, discussion of applications:

The authors mention " sensing application", but it is not immediately clear what "sensing" implies in this context beyond ranging. In the end of abstract, the authors mention that other applications "should be possible" but these are not addressed further in the text and therefore the statement remains a bit vague. I suggest the authors either go in more detail, or remove from the abstract.

While we agree that “sensing applications” is a broad phrase, we accompany this sentence with Refs. 3-5 which are review articles each covering in detail the various sensing applications of frequency combs. We also, as the reviewer points out, list these applications later in the abstract with relevant references in the sentence *“Beyond ranging and imaging[6–12], applications in time/frequency metrology[1,2,5,13–23], comb-based spectroscopy[24-32], pump-probe experiments[33], and compressive sensing[34,35] should benefit from coherent control of the comb-pulse time and phase.”* We feel these applications are addressed further in the main text as much of the Discussion, specifically between lines 278 to 302 in the initial submission, is devoted to discussing the applications of the TPFC. We hope further publications will specifically address some of these applications in much more detail.

Regarding dual-comb spectroscopy applications: to my (non-expert) knowledge, a whole class of comb-based spectroscopy is based on the linear optical sampling of a comb after it passes through a sample, and the information is contained in the amplitude of the comb tooths after the sample, that show the signature of absorbtion. But to my understanding, in the TPFC the information about amplitude and pulse-shape of the original comb are lost (only the phase and timing info is preserved). Is my understanding correct? And if yes, can the authors more clearly describe the connection and applicability of their technique to spectroscopy application?

While the frequency-domain output of the TPFC no longer forms a fixed frequency comb, its time-domain pulse remains the same and is merely displaced at will in time. Dual-comb spectroscopy (DCS) can be viewed in either the frequency domain or time domain. In the time domain, DCS makes multiple measurements at evenly spaced time offsets to measure the free-induction decay signal. The Fourier transform of this signal yields the absorption spectrum. In conventional dual-comb spectroscopy, the data points are acquired at an evenly spaced, monotonically increasing time spacing up to the comb pulse separation. The Fourier transform of this signal yields the amplitude of the spectrum at each comb tooth.

However, as in direct analogy with phase-stepped Fourier Transform Spectroscopy, one does not have to acquire the time-domain data in a continuous, linearly increasing manner. With the TPFC, one can take measurements at user-selected time steps (that might be uneven or might not span the full comb ambiguity range). The Fourier transform of this time-domain signal still yields the spectrum although not necessarily with the full resolution. In this way, the TPFC allows the user to tradeoff between acquisition speed and resolution. The references in this section do elaborate on this concept, specifically references 34 and 35.

Fig. 1c: I am confused by the time axis of the plot. If this is the product of LOS, I would expect a time-axis which is magnified by M= 1E6 (200MHz/200Hz). Was the axis in the picture rescaled to the "original" timing? Am I right in assuming that in LOS output an applied Delta X of 5 ps is mapped into 5 microseconds (which is easily resolved, actually?). The author could add a note on this in the caption. Also, can they specify the bandwidth of the photodiode used to detect the linear optical sampling (see also following point), and describe how the red or blue trace in Fig. 1c are obtained?

The reviewer is correct that the time axis is the product of linear optical sampling and has been scaled. To make this clear and better describe how the red and blue traces of Fig. 1c were obtained we have updated the caption for Figure 1c to read

> *"LOS (blue trace) and their envelopes (red trace) for the fixed comb (at X=0) and the TPFC at the given (X, θ) values. The LOS magnification of the time axis is 10^6"*

We use commercial 100 MHz balanced detectors for all measurements of the paper and have added this information to the Methods Section III.A.1 (in the re-labled Methods sections).

Characterization: the authors use LOS to characterize how well the original and TPFC are synchronized, or how accurately a given step can be applied to the TPFC. But to resolve a single-shot timing error of 1E-17 using LOS one still needs about 10 ps of resolution and challenging 100 GHz bandwidth at detection. Can the authors describe better how this is achieved? A similar question holds about results shown in Fig. 2c and give more explanation on ho

Figure 2c was not clear about how we obtained the data points. Each data point represents many LOS measurements, averaged together. We have significantly modified the caption of Figure 2 to read:

> *"(c) The error between the actual and commanded pulse times for the data in 2b (red circles). Each point is a 3-minute average over ~1M individual LOS measurements. This measurement was repeated for multiple different commanded time steps (black circles). The uncertainty bars are based on the LOS measurement noise and residual comb timing jitter. The average difference is 0.66 attoseconds ±1.73 attoseconds (standard error). There is no observed reduction in accuracy or precision despite moving the TPFC over the full 5 ns ambiguity range."*

In revising this figure, we realized that we had omitted three of the data points. Their inclusion modified the average difference, giving the values quoted above that are slightly better than in the original submission, with a standard error of 1.73 attoseconds. We have nevertheless left the abstract to read <2 attoseconds.

Perhaps related to the above question: in line 117-120 it is said that a 5 fs shift is "easily resolved". How this is seen on the pulses timing?

As the reviewer notes, the observation of this shift is related to the above question. Also, the reviewer's comment led us to realize this section of text was slightly out of place and could be clearer. We have moved this text to the Discussion section, and it now reads as follows:

> *"For absolute optical frequency measurements, the TPFC also enables determination of the mode number N by applying a shift* $\Delta\theta_0 = 2\pi N$ *, which will lead to a time shift of exactly* $\Delta X = f_{rep}^{-1}$ *for the correct N. Any integer error in N appears as a 5-fs offset in time, which can be resolved with a second comb and a timing discriminator."*

In line 118 should be: Delta theta_0 =2 pi N (delete f_rep)

Thank you for catching this. This error has been corrected in the text.

Fig. 2a: I don't get the meaning of the "Amplitude" axis (or color code) in Fig. 2a, and did not find explanation in the text, can the authors comment?

We agree that this figure caption was not entirely clear. (This overlapped with comment #7 from reviewer 1.) As mentioned in the response to that comment, we have re-written the caption of fig 2a for clarity. It now reads

> *"Figure 2: Illustration and characterization of the time programmability of the TPFC through LOS, using the setup in Fig 1a. (a) The TPFC pulse train, presented as a surface plot, where each slice in lab time represents a complete LOS measurement as in Figure 1c. The TPFC pulse is located at the LOS signal peak and follows the commanded arbitrary step pattern (red line). Multiple reflections within the setup appear as small satellite pulses.*

We have also updated the z axis of the surface plot to be "*'LOS Signal Amplitude"* instead of the previous *"Amplitude"*

line 144-148: the authors talk about "resolution" and "precision". I do not get the exact difference between the two concepts. I would tend to say the ultimate resolution depends on pulse-width and SNR according to Eq. 1, where Delta R in turns depend on the pulse width as shown in L. 145. Can the authors clarify ?

We have addressed this question in the text by precisely defining our use of the terms "resolution" and "precision". The new text reads:

> "*To demonstrate the advantages of the TPFC in dual-comb sensing, we consider ranging*[6–8]*. In dual-comb ranging, pulses with bandwidth* τ_p^{-1} *from a comb are reflected off an object, and their time-of-flight is detected by coherently heterodyning them against a second comb. This measurement has a resolution of* $\Delta R = c\tau_p / 2$*, which characterizes the ability to distinguish two adjacent reflections. It has a non-ambiguity range* $R_{NA} = c/\left(2 f_{rep}\right)$*, associated with "which pulse" is detected. (This ambiguity can be removed by changing* f_{rep} *and repeating the measurement*[7]*). The accuracy is set by the comb's reference oscillator or knowledge of the index of refraction. In certain applications, absolute calibration and instability of the reference plane will also factor into the accuracy. The precision is the uncertainty in the peak location of the reflection. At best, the precision is equal to the resolution divided by the signal-to-noise ratio (SNR):"*

Related to "accuracy" in ranging, is the absolute positioning precision of the target also a concern? I assume this is subject to absolute calibration, that should depend also on any instrumental delay and optical path. Can the authors comment, if this is a quantity of interest, and to what extent in real-world applications?

Yes, we agree. We addressed this by adding the additional sentence in the rewritten paragraph quoted above in response to the previous question:

> *" In certain applications, absolute calibration and instability of the reference plane will also factor into the accuracy."*

line 186-187 and caption of Fig. 3 the authors mention the timing discriminator to be "power-insensitive". I agree on this, but I think the main reason for building the Mach-Zehnder interferometric scheme is indeed to distinguish lead vs lag time, which would otherwise be ambiguous, is it? I would mention this important property in the text, rather than leaving it implicit.

If the signal power were constant, one could "sit on the side" of the signal and still distinguish between lead and lag time offset and do away with the Mach-Zehnder interferometer. However, in that case, timing noise would be indistinguishable from amplitude noise. We agree the discussion of the timing discriminator needed improvement. We have re-written the paragraph about it in the main body to read

> *"Here, as shown in Figure 3, we replace the sampling frequency comb by the TPFC to overcome these tradeoffs and stiff power penalty. A coherent timing discriminator, shown in Fig. 3b, measures the relative time and phase between the TPFC and signal comb pulses. Creating two time-shifted copies effectively shapes the TPFC pulse to optimize detection of the time of the incoming signal-comb pulses[46,47]. The two heterodyne signals output by the timing discriminator have a nominal carrier frequency of 10 MHz, set by the relative phase locks of the two combs. The signals are then bandpass filtered and demodulated. The choice of bandwidth is a tradeoff between update rate and sensitivity. Here, we use 26-kHz to be well above typical ~kHz mechanical vibrations while allowing for a fairly long 13 microsecond coherent integration time. The amplitudes of the two demodulated signals are combined to compute the time offset between the TPFC and signal comb (see Fig. 3b) while their phase yields the differential carrier phase of the TPFC and signal comb pulses whose derivative is the Doppler shift. Critically, the combination of the two channels from the timing discriminator gives a time offset measurement that is independent of the incoming signal power."*

Line 198: I am a bit confused on the two parameters that are used to lock and synchronizing the TPCF to the signal comb: the pulse-timing and the carrier frequency lock. Here, it is mentioned that both are derived by the timing discriminator output. As far as I understand, the timing alignment is based on the outcome of the timing discriminator, and is achieved by adjusting the phase of the 0th tooth. But I don't see how the carrier-frequency info can be derived from the timing discriminator (there should be a single output from the discriminator, which represents the pulse timing misalignment).

Moreover, in the Methods, it is said (L480) that no action is taken to adjust the phase of the Nth tooth. So, to my understanding, a single degree of freedom is controlled, i.e. the timing misalignment, which seems consisent with the fact that the timing discriminator has a single output. Can the authors clarify this misunderstanding?

A similar question, regarding line 234 and following, is: how is the carrier phase between the two comb pulses extracted from the timing discriminator to yeld carrier-based ranging measurements reported in Fig. 4b?

Line 187: "26-kHz bandpass filter". Shouldn't the filter following the discriminator be a low-pass filter instead?

These questions are all related and we have rewritten our description of the TPFC operation for more clarity. We have re-written the paragraphs previously on Lines 184-203 to read

"Here, as shown in Figure 3, we replace the sampling frequency comb by the TPFC to overcome these tradeoffs and stiff power penalty. A coherent timing discriminator, shown in Fig. 3b, measures the relative time and phase between the TPFC and signal comb pulses. Creating two time-shifted copies effectively shapes the TPFC pulse to optimize detection of the time of the incoming signal-comb pulses[46,47]. The two heterodyne signals output by the timing discriminator have a nominal carrier frequency of 10 MHz, set by the relative phase locks of the two combs. The signals are then bandpass filtered and demodulated. The choice of bandwidth is a tradeoff between update rate and sensitivity. Here, we use 26-kHz to be well above typical ~kHz mechanical vibrations while allowing for a fairly long 13 microsecond coherent integration time. The amplitudes of the two demodulated signals are combined to compute the time offset between the TPFC and signal comb (see Fig. 3b) while their phase yields the differential carrier phase of the TPFC and signal comb pulses whose derivative is the Doppler shift. Critically, the combination of the two channels from the timing discriminator gives a time offset measurement that is independent of the incoming signal power."

This system runs in two modes: acquisition mode and tracking mode, both of which differ from conventional dual comb ranging. In acquisition mode, X(t) is scanned until the tracking comb's timing matches the incoming signal pulse train. While it is possible to scan the entire non-ambiguity range, we can also make use of a priori information to scan the TPFC's over much less than the non-ambiguity range. The information could be provided from external sources or from a Kalman filter if previous range/Doppler measurements are available. Once the system acquires the appropriate reflection, it switches to tracking mode (Fig. 3d). Tracking mode implements a pulse-timing lock and a carrier-frequency lock based on the timing discriminator outputs (see Methods). The combination of the control and error signals from the time and frequency locks in turn yield the range and Doppler velocity of the target object."

The newly labeled Methods Section III.B.4 has also been rewritten for clarity as follows:

III.B.4 Control Loops in Ranging System

For the dual-comb ranging demonstration of Figures 3 and 4, the system operates as nested phase-locked loops. The timing discriminator demodulation is implemented in an FPGA+DSP system with ±13 kHz of bandwidth centered around a demodulation frequency. The 26 kHz bandwidth in the demodulation limits the overall tracking bandwidth, but was chosen to demonstrate the high sensitivity by allowing for coherent integration over ~(26 kHz)$^{-1}$ = 38 us to detect weak return signals. A broader bandwidth would sacrifice some sensitivity but allow for even faster tracking bandwidths. The output of the timing discriminator provides both a time and phase error signal. The time error signal is sent to a PII implemented on the DSP. This PII generates control trajectories for the tracking comb, which are sent via serial port to that comb's controller FPGA that runs the two PLLs for $\theta_0(t)$ and $\theta_N(t)$ at 50-kHz closed-loop bandwidth. The phase error could be similarly fed back directly to these PLLs controlling the TPFC. However, it is simpler, and equivalent, to instead direct the phase error to a PII implemented on the DSP, whose output is fed back to the direct digital synthesizer used to demodulate the ~10-MHz timing discriminator signals. The resulting frequency adjustment is timestamped against the FPGA clock, which allows the accurate unwrapping of the carrier phase to determine the relative *range change, as show in Figure 4. With this choice of phase control, the tracking of the timing is implemented by changing only* $\theta_0(t)$ *and leaving* $\theta_N(t)$ *fixed, which avoids cross talk between the two loops.*

In addition, to continue to address the reviewer's comment on the connection between the timing discriminator output and carrier range measurement, we have modified the text after Figure 4 to read:

"Finally, the coherent timing discriminator also outputs the relative carrier phase between the signal comb and TPFC, whose derivative yielded the velocity above. This phase can also be unwrapped to provide relative range during periods of continuous signal (Fig. 4, yellow trace) as in Ref. 7 and similar to CW interferometry (except avoiding systematic errors from spurious reflections)."

12. Line 209-211: " momentary loss of signal is not an issue. If brief enough, the object does not move by more than 100 micron". For an object moving at 1 m/s this implies outages of the order of 100 micro-seconds. How does this number compare to the typical outages in outdoor space? The authors can add a comment on typical numbers or real-world applications.

While typical numbers will depend on the specific application (outdoor versus indoor, 100m vs 10 km, etc), we agree it might help the reader to give some example numbers. We have modified the body of the text to read

"Finally, momentary loss of signal is not an issue. If brief enough that the object position does not differ from prediction, e.g. from a Kalman filter, by more than $\sim \pm 2\Delta R$ *=* $\sim\pm 100$ *μm, the tracking simply resumes. For even a 10g acceleration, a ~1.5 ms duration loss of signal is tolerable. Sometimes the signal loss may be too long, for example in strong air turbulence[48], in which case the system can transition to acquisition mode using previous data to limit the scan, as discussed above."*

13. Figure 3 and 4: results shown in Fig. 3 are reported for 0.3 pW received power, while those in Fig. 4 mention 3 and 30 pW. Was the power increased to enable measurement with the commerical ranging system? The authors could add a comment on this

The data in Figure 3 (and Extended Data Figure 1) were taken at a different time than Figure 4. The power was increased slightly to maintain signal over the full range of motion of the rail which had slight optical misalignments that caused variations in return power.

14. Line 513: the combs have the same cw laser reference. As the pulse timing and optical phase are related, how is the TPFC ranging capability degraded in the case the cw lasers referencing the (incoming) signal comb and TPCF different, as the ranging information is derived from the control signal?

In the case of locking to two different reference lasers, the incoming signal pulses would have significant additive timing noise due to differential phase noise between the two cavities. We do not think it would be feasible unless there was some other direct connection between the two cw lasers that stabilized their relative phase.

15. extended figure 1: I guess the blue curves (upper) refer to lower power and the red ones (lower) refer to higher power, not viceversa as stated in caption

We thank the reviewer for catching this mistake, and it has been corrected.

16. Extended Figure 2. I guess there is some confusion with colors as to my understanding should be:

called "dark blue" -> blue

called "black" -> "dark blue" (actually, it looks like dark blue on my pdf...)

called "purple" -> ok, purple

please reword or distinguish color in a clearer way

We have made this change.

17- Some of the most recent work on similar subjects by other groups could be acknowledged where relevant, e.g. :

Gozzard et al Phys. Rev. Lett. 128, 020801 2022

Shen et al., arXiv:2203.11272, 2022

Dix-Matthews et al., Nature Communications 12, 515 (2021)

These references have been added to the paper.

Other revisions to the paper:

We also realized that we had omitted a few data points from figure 2c and have corrected that. That in turn changed the mean and standard deviation by a very slight amount in the figure caption (0.77 +/- 2.05 -> 0.66 +/- 1.73 attoseconds). While we corrected this in the text, we left the abstract at the more conservative +/- 2 attoseconds.

Finally, we note that we have also fixed a few typos and shortened the second paragraph in the Discussion to remain below the overall word limit despite the above modifications to the manuscript.

Reviewer Reports on the First Revision:

Referees' comments:

Referee #1 (Remarks to the Author):

The manuscript has been properly revised. I think it's suitable for publication now.

Referee #2 (Remarks to the Author):

The authors properly addressed all my concerns, and I think the manuscript is ready to be published.

Referee #3 (Remarks to the Author):

I thank the authors for addressing my remarks where appropriate, and for clarifying questioned points.
I think the paper is now suitable for publication