

Peer Review File

Manuscript Title: Topological lattices realized in superconducting circuit optomechanics.

Reviewer Comments & Author Rebuttals

Reviewer Reports on the Initial Version:

Referees' comments:

Referee #1 (Remarks to the Author):

The authors have demonstrated 2D and 1D optomechanical arrays based on superconducting circuits. They use the optomechanical toolbox made available by a coupling to localized mechanical modes to deduce the mode shape of optical MW modes (I have some questions regarding their method, see point (2) below).

The paper represents clearly a nice advance for optomechanical arrays demonstrating clean devices with a comparatively large number of cells. The authors demonstrate a good handle on disorder and at the same time their devices are sideband resolved and have optomechanical couplings that are strong enough to allow to reach the regime in which the backaction of the light on the mechanics become significant (although, here, the mechanics is only used as a probe). This appealing features, open the way to other interesting optomechanical experiments. On the other hand, I am not convinced that this experiments represents a clear advance compared to other photonic platforms that have also allowed to realize and measure similar topological edge states. Also, I have to remark that the 2D model they implement is not topological (I provide an explanation below). In summary, I do think that this paper deserves the attention of a broad audience, but I am not sure whether it reaches the high standard of Nature. Beyond the question of the impact, I have noticed some flaw in the interpretation of the experimental results for the 2D array, see point (1) below. These flaws should be corrected before the paper is published in any form.

(1) It is nice that the authors have not restricted themselves to a 1D array but also considered a much more technically challenging 2D example. However, there are some evident mistakes in the interpretation of their results. In particular, the edge states they observe can be smoothly modified to the well known edge states of graphene (for a zig-zag edge). Thus, unlike for the SSH model the asymmetry in the coupling does not play any important role and does not result in any topological phase transition (a more detailed explanation is provided below.) For this reason, the authors should not refer to the edge state they observe as topological and refer to their 2D model as a 2D SSH model. To help the authors, I provide a more detailed explanation below.

Detailed explanation: The model the author refer to as honeycomb ssh model is normally known as the model for strained graphene. This model is well known because for position-dependent strain it leads to pseudomagnetic fields (the authors should add appropriate reference here). In this model, there is only one phase transition that occur for $J'/J=1/2$. For $J'/J<1/2$ the band structure is gapped (for Fermions this would be an insulator phase) while for $J'/J>1/2$ the two bands touch at a pair of Dirac cones (a semimetal phase). This graphene model has some similarity with the SSH model (in 1D) as both models support the edge states that are protected by the chiral symmetry, and can be understood in terms of unpaired sites in the limit $J'<J$. However, there is an important fundamental difference: in the graphene model the edge states are smoothly modified to the well known graphene edge states across the phase transition. Since (unlike for the SSH model or any other topological phase transition) there is no discontinuous

change in the number of edge state, the phase transition is not topological for graphene. [This should actually already be clear by the behaviour of the bulk band structure as a topological phase transitions is a transition between two gapped phases and not between a gapped and a semimetal phase.]

To further help the authors I wish to point out to them that they seem to have lead into a wrong interpretation of their results by mixing up the ratios J'/J and J/J' . The behaviour shown in Fig. 6 (c) with an 'adiabatic phase' that does not wind across the equator corresponds to $J'/J < 1/2$ (for $J' < J$ it reduces to a single point). On the other hand, the behavior in (d) is observed for any $J'/J \geq 1/2$ as any path encircling the Dirac crossing corresponds to a full rotation about the equator.

(2) I have also some doubts regarding the procedure to measure the optical modes shape. The authors deduce the mode shapes from the optically induced mechanical damping. However, this requires an independent estimation of the optomechanical couplings g_{0i} which is technically difficult. As a workaround they try to deduce the mode shape starting from unnormalized reweighted (by g_i) modes and imposing the right normalization via an iterative procedure. However, as far as I can understand this procedure seems to have two separate problems: (i) it does not preserve the orthogonality of the modes, (ii) it is not clear that it would converge to the right modes.

I am skeptical that their procedure would work if the standard deviation σ_g of g_{0i} is not much smaller than the average $\bar{g}$. Is it true that the authors are implicitly making this assumption? If this is the case, they should explicitly warn the readers in the main text and also give as a consistency check $\sigma_g/\bar{g}$ as estimated with their procedure, see cf 9.2.

They should also provide some more convincing numerical evidence about how well the procedure works for $\sigma_g/\bar{g} \ll 1$. Right now some evidence is given in section 3.3. Basically the authors use the orthogonality of the vectors after the iterative procedure as a figure of merit. However, this still leaves open the question how close the estimated normal modes are from the true normal modes (in principle one could obtain normal modes that are orthogonal but far away from the true modes). An appropriate test of their method would consists in considering random values of their system parameters and calculate the deviation of the estimated normal modes from the true normal modes for varying standard deviation of the system parameters. This would allow to estimate the accuracy of their method because the authors can estimate the standard deviation in the system parameters.

Referee #2 (Remarks to the Author):

The paper demonstrates the first electromechanical array able to show topological properties, both in 1D and 2D, with a realization of the SSH model. The experimental results shown here are novel and relevant mainly for two reasons: i) a novel successful nanofabrication technique has been demonstrated capable of extremely low disorder, which is the main challenge in this kind of platforms. ii) a novel measurement scheme of the on-site microwave modes exploiting the optomechanical interaction and the presence of the mechanical degree of freedom. The latter measurement scheme enables to reconstruct the normal modes and more important, the effective Hamiltonian of the system, even beyond the nearest neighbor limit.

The quality of the presentation is high, the methodology is correct, novel and in my opinion extremely useful to the wide community working on the realization of artificial arrays and metamaterials able to show topological, controllable properties.

The data are clearly explained, and all the analysis is carefully and clearly demonstrated in the supplementary material, which are wide and exhaustive in all the many aspects touched by the paper: i) theory, ii) measurement scheme; iii) nanofabrication; iv) design of the electromagnetic and of the mechanical components.

The authors provide many references, and all the main relevant works are cited in my opinion. On the other hand the results presented here will have impact in a broad range of applications (long range interactions, topological transport, highly entangled states) and probably the list of references could be even longer.

One can optionally consider to add some reference related to reservoir engineering (such as for example Y. Yanay and A. A. Clerk, Phys. Rev. Research 2, 023177 (2020) and S. Zippilli and D. Vitali, Phys. Rev. Lett. 126 020402 (2021)) because the platform seems to be particularly suited for that.

A final minor point is related to the adopted measurement scheme. The various couplings have been obtained from the measurement of the induced optomechanical damping on each mechanical resonator and an iterative procedure. However this measurement scheme seems to crucially rely on the separation of frequency of the various modes, so that each local optomechanical system can be treated independently. Can the author comment on this point and eventually better pinpoint the conditions under which this direct measurement method can be applied ?

In conclusion, in my opinion, the paper represents a milestone in the experimental realization of optomechanical arrays and will be very influential for a broad range of scientific topics related to the behavior of quantum many body systems.

I suggest publication in Nature, with the minor amendments suggested above.

Referee #3 (Remarks to the Author):

This is a technically excellent paper that shows how we can use optomechanical coupling to image the topological modes within arrays of superconducting microwave resonators. It definitely addresses an important technical problem! The presentation of the work is very clear and the results are also convincing. I do believe that this is the first time I've seen an experimental blend of well-known optomechanics concepts with newer explorations of topology. The Supplement is also very informative and I very much appreciate the level of detail presented.

While the work is a very nice technical achievement, and very creative, I feel that it does not quite represent a scientific advance or conceptual advance that justifies inclusion in Nature. SSH chains are very well studied in a variety of metamaterials including in microwave structures. What the authors have shown here is the implementation of SSH topology in superconducting microwave arrays, along with an innovative optomechanical probing scheme. Technically brilliant, but it does not bring along any major scientific surprise yet. Perhaps the next iteration of this work will explore deeper concepts and I am very much looking forward to it!

Minor comment: Supplement Fig 21 has a typo "Desined" instead of "Designed"

Reply to Referee Comments

Manuscript “Superconducting circuit optomechanics in topological lattices”

Below we repeat the Referees’ reports verbatim, with our reply in *blue* and actions taken in the manuscript by *bullets*.

Referee #1:

The authors have demonstrated 2D and 1D optomechanical arrays based on superconducting circuits. They use the optomechanical toolbox made available by a coupling to localized mechanical modes to deduce the mode shape of optical MW modes (I have some questions regarding their method, see point (2) below). The paper represents clearly a nice advance for optomechanical arrays demonstrating clean devices with a comparatively large number of cells. The authors demonstrate a good handle on disorder and at the same time their devices are sideband resolved and have optomechanical couplings that are strong enough to allow to reach the regime in which the backaction of the light on the mechanics become significant (although, here, the mechanics is only used as a probe). This appealing features, open the way to other interesting optomechanical experiments.

Reply: We thank the referee for the positive assessment of our work and for the detailed theoretical comments pointed out. We would like to emphasize that multimode optomechanical lattices have been theoretically considered for more than a decade. However, realizing such optomechanical lattices is compounded by the effects of disorder, even in superconducting circuit optomechanics, one of the most mature platforms, where only few-mode systems (two microwave modes; two mechanical modes) have been realized. Extending to a higher number of optomechanical cells has been a major challenge. Our work has overcome this, by introducing a novel method to produce superconducting optomechanical systems, which now enables to realize for the first time both 1D and 2D optomechanical lattices. We believe that this achievement can trigger a wide range of studies on multimode optomechanical physics.

On the other hand, I am not convinced that this experiments represents a clear advance compared to other photonic platforms that have also allowed to realize and measure similar topological edge states.

Reply: We would like to point out that the major advance of our manuscript is not only to demonstrate the measurement of edge states. Indeed, as pointed out, topological edge states have been already realized in photonic, microwave, and electronic structures, however, we believe the main novelty of our work is the realization of large-scale optomechanical lattices, which was a long-lasting challenge in the field for more than a decade. The embedded nonlinear radiation-pressure interactions in our lattices significantly enrich the physics behind – and the modeshape measurement of microwave edge states via the optomechanical interactions is one example of what the rich physics has enabled. This may provide a route to verify several fundamental and applied proposals in the field of quantum optomechanics, including quantum collective dynamics of mechanical oscillators, collective entanglement, reservoir engineering, soliton generation, wide-band nonreciprocity, etc., as cited in the main text.

Another important novelty of our work is to propose and demonstrate the modeshape measurement technique of microwave fields using the optomechanical interactions. A topological model such as a 1D SSH chain was the best test example to show the accuracy of our measurement technique by mapping non-trivial collective microwave states. i.e., topologically protected edge states. Unlike the optical systems (multicore photonic waveguide grids, quantum dots, photonic crystals, etc.), mapping collective modeshapes is challenging in the microwave domain, as mentioned in the main text, and has only been realized by non-direct or field scanning probes schemes. Furthermore, our approach allows to reconstruct the full underlying microwave Hamiltonian without introducing any external perturbation to the lattice, enabling to characterize the frequency disorder of the bare microwave modes, as well as the long-range interactions beyond the nearest neighbor bonds.

Moreover, we would like to emphasize the extremely low fluctuations and disorder in our circuit optomechanical platform. With its flexible controllability and high reproducibility over system parameters, our platform will be straightforwardly applied to a broad range of studies, as mentioned above. To demonstrate the fine tunability, accurate control, and extremely low disorder, we decided to realize a well-known topological model as a proof-of-concept example. Of course, simpler models could also be chosen, however, we decided to demonstrate hybridized topological edge states in a 1D SSH chain to show the minimal frequency disorder in our system ($<0.5\%$) – the hybridization of the edge states on both the edges is sensitive to the frequency disorder, as shown in the SI, i.e., the effect of the disorder can easily break down the hybridization, which is not the case for our 1D optomechanical chain.

At last, we would like to mention some of the special properties of the superconducting circuit optomechanical platform compared to the optical analogs. The superconducting LC resonators demonstrated in our work, which are compact and chip integrated, allow large-scale and site-by-site engineerable optomechanical arrays with negligible frequency fluctuations and disorder. In addition, these superconducting circuit optomechanical lattices can be straightforwardly integrated with superconducting qubits based on Josephson junctions, allowing large-scale qubit-mechanics interactions. Therefore, one of the most appealing aspects of our platform is that it endows the previously demonstrated multimode quantum microwave circuits with rich physics of cavity optomechanics.

Also, I have to remark that the 2D model they implement is not topological (I provide an explanation below).

In summary, I do think that this paper deserves the attention of a broad audience, but I am not sure whether it reaches the high standard of Nature.

Reply: Thanks to the detailed comments especially on our interpretation of the 2D lattice, we believe that our revised manuscript not only demonstrates a novel platform of multimode quantum optomechanics, but in particular also extends into the community of condensed matter physics and topological physics, by realizing a seminal non-trivial physical model for strained graphene. We believe the combination of quantum optomechanics, nano-mechanics, topological physics, and graphene will be of substantial cross-disciplinary interest.

Beyond the question of the impact, I have noticed some flaw in the interpretation of the experimental results for the 2D array, see point (1) below. These flaws should be corrected before the paper is published in any form.

Reply: We sincerely thank the reviewer for pointing out our mistake and one flaw in our interpretation. We have extensively studied over the last months the literature on the strained graphene model, and consulted colleagues and experts in the field, including Prof. Oleg Yazyev and Dr. Pierre Delplace. Based on the seminal work in [Delplace, *et al.*, "Zak phase and the existence of edge states in graphene." *PRB* 2011], we have now modified our interpretation of the edge states and introduced the wavenumber-resolved Zak phase to interpret our results. We have extended in particular the SI for a comprehensive section and believe that the comments of the referee have greatly improved our manuscript's findings and explanations. We address changes in the following.

(1) It is nice that the authors have not restricted themselves to a 1D array but also considered a much more technically challenging 2D example. However, there are some evident mistakes in the interpretation of their results. In particular, the edge states they observe can be smoothly modified to the well known edge states of graphene (for a zig-zag edge). Thus, unlike for the SSH model the asymmetry in the coupling does not play any important role and does not result in any topological phase transition (a more detailed explanation is provided below.) For this reason, the authors should not refer to the edge state they observe as topological and refer to their 2D model as a 2D SSH model. To help the authors, I provide a more detailed explanation below.

Reply: We faithfully thank the referee to point out the issue on our interpretation of the 2D structure. We apologize for using non-standard terminology of "2D SSH model" to describe our model and acknowledge the referee's feedback which led to substantially improve our manuscript. Triggered by the referee's comment, we now use "*strained graphene model*" terminology in the new version of the manuscript (mainly in the SI, where we discuss the 2D model) and cite relevant references as the referee suggested.

Although we found recent highly cited references in the field where they claim the phase transition in the strained graphene model is topological (for example [Rechtsman, *et al.* "Topological creation and destruction of edge states in photonic graphene." *PRL* 2013, Delplace, *et al.*, "Zak phase and the existence of edge states in graphene." *PRB* 2011, and Bellec, *et al.*, "Topological transition of Dirac points in a microwave experiment." *PRL* 2013]), we now fully agree (and understand) with the referee that the phase transition may not be called topological because of the transition between semi-metal and gapped phases which is not comparable with the one between two gapped phases in the 1D SSH model. Therefore, we avoided using "topological phase transition" in the new version.

Changes made:

- We changed our terminology to "*strained graphene model*" and described the "phase transition" at $J'/J=1/2$ in the "Two-dimensional superconducting circuit optomechanical lattice" section of the main text.
- We cited proper references for the strained graphene model, discussing the band structures, the phase transition, and the existence of edge states.

Detailed explanation: The model the author refer to as honeycomb ssh model is normally known as the model for strained graphene. This model is well known because for position-dependent strain it leads to pseudomagnetic fields (the authors should add appropriate reference here). In this model, there is only one phase transition that occur for $J'/J=1/2$. For $J'/J<1/2$ the band structure is gapped (for Fermions this would be an insulator phase) while for $J'/J>1/2$ the two bands touch at a pair of Dirac cones (a semimetal phase). This graphene model has some similarity with the SSH model (in 1D) as both models support the edge states that are protected by the chiral symmetry, and can be understood in terms of unpaired sites in the limit $J'<J$. However, there is an important fundamental difference: in the graphene model the edge states are smoothly modified to the well known graphene edge states across the phase transition. Since (unlike for the SSH model or any other topological phase transition) there is no discontinuous change in the number of edge state, the phase transition is not topological for graphene. [This should actually already be clear by the behaviour of the bulk band structure as a topological phase transitions is a transition between two gapped phases and not between a gapped and a semimetal phase.

Reply: We again thank the referee for the detailed comment here. In the new version, we interpret our edge states in the 2D lattice as “zig-zag edge states” or “tilted armchair edge states” in the strained graphene model (see the SI for more detail) and cite proper references.

As it comprehensively studied in [Delplace, *et al.* "Zak phase and the existence of edge states in graphene." *PRB* 2011], we argue the existence of the edge states in our structure, considered as a truncated graphene ribbon, can be explained by “wavenumber-resolved Zak phase”. In this reference, they provide a method to predict the existence of edge states in graphene ribbons with different boundaries (zig-zag, armchair, etc.) by calculating the Zak phase across an appropriately chosen one dimensional Brillion zone.

To further help the reader to understand the origin of the edge states, we added a section in the SI discussing the 1D SSH model, the strained graphene model, and the existence of the edge states in graphene. We linked the problem of graphene ribbons to a “generalized” 1D SSH model for a given wavenumber parallel to the ribbon direction. Then we show that the existence of the edge states can be predicted by the Zak phase (winding number) of the equivalent 1D SSH problem, as shown in [Delplace, *et al.*, *PRB* 2011].

Changes made:

- We added a comprehensive paragraph in the “Multimode optomechanics in lattices” section of the main text, describing the topological properties of the 1D SSH model.
- We added schematic diagrams in Figure 1 of the main text, describing the winding number for the 1D SSH model.
- We added a paragraph in the “Two-dimensional superconducting circuit optomechanical lattice” section of the main text, explaining the strained graphene model, including the phase transition and the existence of edge states.
- We added a paragraph in the “Two-dimensional superconducting circuit optomechanical lattice” section of the main text, explaining the interpretation of the observed edge states

in our flake using the wavenumber-resolved winding number based on the comprehensive discussion we added in the SI.

- We added numerically calculated edge modes of graphene ribbons in Figure 4 of the main text, showing the overlap of the observed edge modes in the 24-site flake with the theoretically predicted edge modes in the ribbons.
- We added schematic diagrams in Figure 4 of the main text, describing the wavenumber-resolved winding number for our 24-site flake considered as a truncated strained graphene ribbon.
- We added a subsection in Sec. 2A of the SI, discussing the 1D SSH model, including the band structure, the bulk-edge correspondence, the finite-size effect, and the effect of parasitic couplings.
- We added a comprehensive subsection in Sec. 2B of the SI, discussing the strained graphene model, including the band structure, the phase transition, the bulk-edge correspondence for the prediction of the existence of edge states for strain-free and strained graphene ribbons, and the finite-size effect. This subsection follows the formalism introduced by [Delplace, *et al.* PRB 2011].
- We added a subsection in Sec. 2C of the SI, discussing our 24-site lattice considered as a truncated graphene ribbon, including the band structure, the modeshapes, and the effect of parasitic couplings.

To further help the authors I wish to point out to them that they seem to have lead into a wrong interpretation of their results by mixing up the ratios J'/J and J/J' . The behaviour shown in Fig. 6 (c) with an 'adiabatic phase' that does not wind across the equator corresponds to $J'/J < 1/2$ (for $J' < J$ it reduces to a single point). On the other hand, the behavior in (d) is observed for any J'/J ged $1/2$ as any path encircling the Dirac crossing corresponds to a full rotation about the equator.

We thank the reviewer for the helpful comments, and we regret that we made a mistake in the ratio of J'/J .

Changes made:

- We added the bulk band structure of strained graphene for $J'/J = 0.25, 0.5$ (phase transition threshold), and 1 in Fig. 5 of the SI.
- We corrected the phase transition threshold ($J'/J=0.5$) shown as a dashed line in Fig. 11a of the SI, showing the numerically calculated band structures of the 24-site honeycomb lattice.

(2) I have also some doubts regarding the procedure to measure the optical modes shape. The authors deduce the mode shapes from the optically induced mechanical damping. However, this requires an independent estimation of the optomechanical couplings g_{0i} which is technically difficult. As a workaround they try to deduce the mode shape starting from unnormalized reweighted (by g_i) modes and imposing the right normalization via an iterative procedure.

Reply: First, we would like to agree with the referee that the measurement of the local optomechanical coupling rate, g_{0i} , is not a straightforward task. A prior work of our group has established a method to precisely determine g_0 by measuring thermomechanical noise at elevated temperatures (cf. L. D. Toth, *et al.*, *Nat. Physics* 2017). However, this method cannot be straightforwardly applied to determine g_{0i} in our optomechanical lattices, where the local microwave modes are coupled to each other and hybridized.

This is why we developed the novel scheme for deducing the microwave modeshapes without any prerequisite knowledge on g_{0i} but using the iterative normalization method based on unique properties of the unitary transformation from bare to collective microwave modes. We believe the development of such a microwave modeshape measurement is one of the important novelties of our work. We also believe our microwave modeshape measurement is general enough to be applied to many different experimental platforms, which makes our work more attractive to general readers.

Furthermore, it is important to mention that the novel normalization method enables us to realize the independent estimation of the optomechanical couplings g_{0i} , which had been considered a challenging task. Thanks to the iterative normalization method, which allows us to deduce the collective microwave modeshapes, we can now determine the local optomechanical coupling rate, g_{0i} , by using the unnormalized modeshapes and the iteratively normalized ones. This is because the ratio between the unnormalized and normalized modeshapes can be explained by g_{0i} . We showed all the local optomechanical coupling rates for the 1D chain and the 2D lattice in Fig. 25 of the SI.

However, as far as I can understand this procedure seems to have two separate problems: (i) it does not preserve the orthogonality of the modes,

Reply: We do not need to impose orthogonality on our modeshape estimation since our iterative normalization method is capable of deducing the right modeshapes without degrading the orthogonality, which is confirmed by the numerical simulation the referee suggested (see the following reply). Of course, we could impose the orthogonality on the iteratively normalized modeshapes in order to further correct them. However, it requires the phase information of the modes that we can estimate by referring to numerically simulated modeshapes. To make our estimation method as general as possible, we show the modeshapes obtained from our iterative normalization method in the main text.

Instead, we use the orthogonality of the unitary transformation as a benchmark for our method. As shown in Fig. 24 of the SI, our iterative normalization method does not degrade the orthogonality.

Furthermore, we quantitatively calculate the difference in modeshapes between the different estimation steps. As a result, we found that the difference between the raw modeshapes and the iteratively normalized ones is small, which quantitatively shows the minimal disorder in g_{0i} in our optomechanical systems. More importantly, the significantly small difference between the normalized modeshapes and the orthogonalized ones confirms that our iterative normalization method preserves the orthogonality, as numerically verified. Finally, the small difference between the normalized modeshapes and the theoretical modeshapes shows our newly developed fabrication process of circuit optomechanical devices is reliable and reproducible.

Changes made:

- We added a figure (Fig. 1) in Sec. 1B of the SI, showing the numerical confirmation for our iterative normalization method.
- We added two tables (Table I and II) in Sec. 5C of the SI, showing infidelities calculated for different estimation methods and comparison between theory and measured modeshapes.

(ii) it is not clear that it would converge to the right modes.

I am skeptical that their procedure would work if the standard deviation σ_g of g_{0i} is not much smaller than the average $\bar{g}$. Is it true that the authors are implicitly making this assumption? If this is the case, they should explicitly warn the readers in the main text and also give as a consistency check $\sigma_g/\bar{g}$ as estimated with their procedure, see cf 9.2.

Reply: We really thank the referee for pointing out the important concern about our modeshape estimation method. In the current version of the SI, we numerically confirmed that the unnormalized modes always converge to the right ones without any assumption on the local optomechanical coupling rates (see the following detailed explanation).

As the referee suggested, we first prepare a random set of modeshapes and generate the corresponding unnormalized modeshapes by multiplying random prefactors that depend on either site i or mode k . Note that the prefactors should not simultaneously depend on both i and k such that they can be distinguishable from the right modeshapes, which is the case for our experiments. We then apply our iterative normalization method to the unnormalized modeshapes and compare them to the right ones. The numerical results show that the unnormalized modeshapes always converged to the right ones with a sufficiently large number of iterations ($> \sim 50$ iterations), regardless of how large the randomness in the prefactors is. We hope that the numerical simulation will convince the referee of the accuracy of our estimation method.

Although we do not use any assumptions on g_{0i} , we estimate the $\sigma_{g_{0i}}/\bar{g}_{0i}$ to be on the order of a few percent from the gap-size or the microwave frequency disorder, which is one on the features of our robust nanofabrication process, i.e., $(\sigma_{\text{Gap size}}/\bar{\text{Gap size}}) \sim 1\%$.

Changes made:

- We added a paragraph in Sec. 1B of the SI, describing the procedure of the verification of our iterative normalization method.
- We added a figure (Fig. 1) in Sec. 1B of the SI, verifying the convergence of our iterative normalization method, regardless of the $\sigma_{g_0}/\bar{g}_{0,i}$

They should also provide some more convincing numerical evidence about how well the procedure works for $\sigma_g/\bar{g} < 1$. Right now some evidence is given in section 3.3. Basically the authors use the orthogonality of the vectors after the iterative procedure as a figure of merit. However, this still leaves open the question how close the estimated normal modes are

from the true normal modes (in principle one could obtain normal modes that are orthogonal but far away from the true modes). An appropriate test of their method would consist in considering random values of their system parameters and calculate the deviation of the estimated normal modes from the true normal modes for varying standard deviation of the system parameters. This would allow to estimate the accuracy of their method because the authors can estimate the standard deviation in the system parameters.

Reply: We greatly appreciate the referee's suggestion. Using the numerical simulations of the iterative normalization method, we confirmed that the unnormalized modeshapes always converge to the right ones, as explained above. Therefore, in theory, there is no possibility that the modeshapes obtained from the iterative normalization deviate from the true modeshapes within more than the measurement errors.

Referee #2:

The paper demonstrates the first electromechanical array able to show topological properties, both in 1D and 2D, with a realization of the SSH model. The experimental results shown here are novel and relevant mainly for two reasons: i) a novel successful nanofabrication technique has been demonstrated capable of extremely low disorder, which is the main challenge in this kind of platforms. ii) a novel measurement scheme of the on-site microwave modes exploiting the optomechanical interaction and the presence of the mechanical degree of freedom. The latter measurement scheme enables to reconstruct the normal modes and more important, the effective Hamiltonian of the system, even beyond the nearest neighbor limit.

The quality of the presentation is high, the methodology is correct, novel and in my opinion extremely useful to the wide community working on the realization of artificial arrays and metamaterials able to show topological, controllable properties.

The data are clearly explained, and all the analysis is carefully and clearly demonstrated in the supplementary material, which are wide and exhaustive in all the many aspects touched by the paper: i) theory, ii) measurement scheme; iii) nanofabrication; iv) design of the electromagnetic and of the mechanical components.

Reply: We thank the referee for the very positive assessment of our work. We share the opinion that given the ability to now make large 2D optomechanical lattices, a wide range of previously made theoretical proposals can be implemented, ranging from quench dynamics, and non-trivial optomechanical instabilities in lattices, to synchronization or chaos. Likewise, the large multimode nature should enable new schemes for generating mechanically entangled states. Indeed, the key aspects we had to overcome to enable this regime, were to reconsider and solve the main challenges of fabricating large arrays of identical circuit optomechanical cells made from superconducting resonators coupled to vacuum gap capacitors; despite having been used in a wide range of quantum optomechanical experiments, the technology had outstanding issues in terms of yield, fluctuations in mechanical and microwave resonance frequencies that prohibited large arrays of optomechanical cells for well over a decade.

The authors provide many references, and all the main relevant works are cited in my opinion. On the other hand the results presented here will have impact in a broad range of applications (long range interactions, topological transport, highly entangled states) and probably the list of references could be even longer.

Reply: We thank referee for pointing out relevant works.

Changes made:

- We cited relevant proposals in the introduction.

One can optionally consider to add some reference related to reservoir engineering (such as for example Y. Yanay and A. A. Clerk, Phys. Rev. Research 2, 023177 (2020) and S. Zippilli and D. Vitali, Phys. Rev. Lett. 126 020402 (2021)) because the platform seems to be particularly suited for that.

Reply: We thank the referee for pointing out the relevant works.

Changes made:

- We added the two references on reservoir engineering to the introduction and outlook.

A final minor point is related to the adopted measurement scheme. The various couplings have been obtained from the measurement of the induced optomechanical damping on each mechanical resonator and an iterative procedure. However this measurement scheme seems to crucially rely on the separation of frequency of the various modes, so that each local optomechanical system can be treated independently. Can the author comment on this point and eventually better pinpoint the conditions under which this direct measurement method can be applied?

Reply: Yes, this is crucial in the modeshape measurement to be able to identify each physical site in a lattice by a unique mechanical frequency. This requires disorder in the mechanical frequencies to be smaller than the frequency separation in the design, which is the case in our experiment as can be seen in Fig. 2j and Fig. 4d of the main text. Here, we would like to again stress that the reproducibility in the fabrication process is sufficiently high, which is crucial to realize such multimode optomechanical lattices and experiments.

We also would like to emphasize that our scheme does not impose the resolved sideband regime for each collective mode coupled to a mechanical mode. Indeed, the approximation is only that the damped optomechanical mode is spectrally well separated from other mechanical modes in order to retrieve faithfully the optomechanical damping rate, via the full (i.e., also covering the non-resolved sideband) regime. For this reason, the first condition is that the effective mechanical damping rate is smaller than the mechanical frequency separation. The second condition is that the individual collective microwave modes are separated by a frequency larger than their linewidths, such that the optomechanical damping rate of mechanical mode i , dominantly induced by microwave collective mode k . The third condition is that the frequency spacing of the collective microwave modes is much larger than the mechanical frequencies.

Changes made:

- In the “Optomechanical modeshape measurement” section of the main text, we added: “(the disorder in $\Omega_{\mathrm{m},i}$ from the designed value should be smaller than the mechanical frequency increment to ensure correct identification of sites in the lattice)”
- In Sec. 3B of the SI, we added:
“It is important to mention that the mechanical frequency disorder in the fabrication process should be smaller than the mechanical frequency shift introduced by incrementing trench radius in order to keep them in correct designed order. This can be verified by comparing the measured mechanical frequencies from OMIT response and the theoretical $\Omega_{\mathrm{m},i} \propto \frac{1}{R_i}$ relation from the design, as shown in Fig. 2j and Fig. 4e.”
- We added the three abovementioned conditions for a valid modeshape measurement in section 1B of the SI.

In conclusion, in my opinion, the paper represents a milestone in the experimental realization of optomechanical arrays and will be very influential for a broad range of scientific topics related to the behavior of quantum many body systems.

I suggest publication in Nature, with the minor amendments suggested above.

Reply: We really thank Referee #2 for the strong positive assessment.

Referee #3:

This is a technically excellent paper that shows how we can use optomechanical coupling to image the topological modes within arrays of superconducting microwave resonators. It definitely addresses an important technical problem! The presentation of the work is very clear and the results are also convincing. I do believe that this is the first time I've seen an experimental blend of well-known optomechanics concepts with newer explorations of topology. The Supplement is also very informative and I very much appreciate the level of detail presented.

Reply: We thank the referee for his/her feedback about the quality of the work. Indeed, the result has been a culminating of a several years long effort, with numerous challenges and setbacks.

While the work is a very nice technical achievement, and very creative, I feel that it does not quite represent a scientific advance or conceptual advance that justifies inclusion in Nature. SSH chains are very well studied in a variety of metamaterials including in microwave structures. What the authors have shown here is the implementation of SSH topology in superconducting microwave arrays, along with an innovative optomechanical probing scheme. Technically brilliant, but it does not bring along any major scientific surprise yet. Perhaps the next iteration of this work will explore deeper concepts and I am very much looking forward to it!

Reply: We regrettably have to disagree with the referee on this point. The main claim and focus of this work are not realizing a SSH topological chain, but *introducing a novel platform to realize multimode quantum optomechanical systems*. As other referees also pointed out, and we discussed in the manuscript, implementing **optomechanical arrays was a long-lasting challenge in the field of quantum opto- and electromechanics alike, for well over a decade**. Although numerous theoretical studies explored multimode optomechanics searching for interesting physics from collective quantum dynamics to long-range entanglement and topological physics, they could have never been realized in neither optical nor microwave domains. Despite the attractiveness of superconducting circuit optomechanical systems and their success in many quantum optomechanical experiments, these systems were compounded by the frequency disorder and the poor reproducibility in the fabrication process, and could not be used to create 1D or 2D arrays and lattices. We have overcome this challenge.

Here, we implemented large-scale and site-by-site engineerable microwave lattices with nonlinear radiation-pressure interactions to macroscopic mechanical oscillators with minimal disorder. We used a well-known SSH chain as a *proof-of-concept toy model* to demonstrate the accurate control over system parameters and present our modeshape measurement technique to map non-trivial collective microwave modes, i.e., topologically protected edge states. It is worth mentioning that we could demonstrate deep quantum ground-state cooling (<0.05 quanta) and a few milli-second quantum decoherence time of a mechanical oscillator fabricated with this platform [Youssefi, *et al.*, "An ultra-high Q circuit optomechanical quantum memory with 2 milli-second lifetimes." *Bulletin of the American Physical Society* (2022)]. Considering this, we expect a broad range of experiments based on collective quantum control of mechanical oscillators can be implemented in this platform.

Concerning the comments on 'deeper concepts' we would like to make the reviewer aware that the 2D model we have described, actually corresponds to that of strained graphene. In our revised version, we show the beautiful and elegant connection between our 2D lattice and the physics of

strained graphene ribbons, where the existence of edge states can be predicted from the boundary structures of a strained graphene flake. We believe that this deep connection to topological edge states and the physics of strained graphene makes our manuscript theoretically and significantly deeper and nontrivial. Indeed, the bulk-edge correspondence itself is highly nontrivial for ribbon systems of finite size. We, therefore, make a connection to a deep problem in condensed matter physics of graphene in our revised version. We believe we have well overcome the point of deep physics in our revised version.

However, we do agree that the novel platform can be used next to address interesting and in particular emergent nonlinear dynamics that has not yet been theoretically described, and as such also bring ‘surprises’ in the future.

Changes made:

- We modified the “Two-dimensional superconducting circuit optomechanical lattice” section of the main text, showing the possibility of predicting edge states in our device using the wavenumber-resolved Zak phase.
- We added a comprehensive subsection (Sec. 2B) in the SI, describing the physics of the strained graphene model to gain insight into the property of the edge states and their topological origin in our 24-site graphene flake

Minor comment: Supplement Fig 21 has a typo "Desined" instead of "Designed"

Reply: We thank the reviewer for pointing out the error.

Changes made:

- We corrected the error.

Reviewer Reports on the First Revision:

Referees' comments:

Referee #1 (Remarks to the Author):

I am now comfortable with the theoretical presentation of the 2D part of the experiment and I also find convincing the numerical evidences provided by the authors to substantiate the validity of their iterative method to deduce the participation ratios.

I also agree with the authors claims in the reply to my previous report and in the paper.

As I had already acknowledged previously this is clearly a high-quality broad-impact work.

Vittorio Peano

Referee #2 (Remarks to the Author):

I have read the new version of the manuscript, the revised SI, and also the replies and the arguments related to the criticisms of the other referees. The revised version of the paper has further improved the first version which was already of high quality in my opinion. The relevance of the results in the context of topological physics is now better underlined and the paper has been further improved.

Moreover the authors have satisfactorily addressed the points I have raised in my previous report.

I therefore confirm my idea about the paper and I think that the new version of the paper is suitable for publication in Nature. In fact, it is the first nontrivial application of an array of electromechanical systems, and I think it will represent a landmark result

Referee #3 (Remarks to the Author):

I appreciate the response from the authors. However, I still find myself aligned with my original assessment. The technical result is fantastic, but it still is primarily limited to showcasing a new platform. Perhaps the next scientifically interesting study performed with this platform will be the Nature-level paper.

Reply to Referee Comments

Manuscript “Topological lattices realized in superconducting circuit optomechanics”

Referee #1:

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Vittorio Peano

[We thank Dr. Peano for the precious comments, especially on the interpretation of our 2D model which advanced our manuscript.](#)

Referee #2:

I have read the new version of the manuscript, the revised SI, and also the replies and the arguments related to the criticisms of the other referees. The revised version of the paper has further improved the first version which was already of high quality in my opinion. The relevance of the results in the context of topological physics is now better underlined and the paper has been further improved.

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[We thank Referee #2 for supporting our work and his/her precious comments.](#)

Referee #3:

I appreciate the response from the authors. However, I still find myself aligned with my original assessment. The technical result is fantastic, but it still is primarily limited to showcasing a new platform. Perhaps the next scientifically interesting study performed with this platform will be the Nature-level paper.

[We thank Referee#3 for reviewing our manuscript and reflecting positive feedback on our technical achievements. We believe this work, not only solves a long-lasting challenge in the field of quantum optomechanics to scale up such systems but also opens an avenue for a broad range of studies in complex systems and many-body physics which can be realized in our platform.](#)