

Peer Review File

Manuscript Title: The black hole low mass X-ray binary V404 Cygni is part of a wide hierarchical triple, and formed without a kick

Reviewer Comments & Author Rebuttals

Reviewer Reports on the Initial Version:

Referees' comments:

Referee #1 (Remarks to the Author):

Using existing and new observations, the authors demonstrate that the low-mass x-ray binary V404 Cygni which harbors a black hole, has a wide orbit tertiary. The authors manage to characterise the tertiary and show that it has evolved off the main sequence allowing an age estimate of billions of years. The authors argue that the wide orbit of the tertiary sets strong limits on the kick velocity of the binary when the black hole was formed as well as a possible hint for the role of three-body dynamics in the formation, providing new constraints and hints on the formation of stellar mass black holes.

I think that the dynamical association of the binary and the (previously-known visual) companion, which includes proper motion and radial velocity measurements is convincing, that the characterisation of the tertiary is robust and that this discovery is exciting and worth publication in Nature.

I have two main reservations which I think should be addressed before publication.

1. The system is currently several billion years old while the black hole presumably formed ~ 10 million years after the formation of the system. The authors assume that there was no evolution in the orbit of the tertiary in the intermediate time and this is critical to their conclusions regarding the constraints on the kick. While this may likely be true, I think this is not certain and that the authors should be less definitive with their statements. In particular, a slow dynamical unfolding of three body systems may occur with the tertiary gaining energy during multiple close approaches with the inner binary (which could have been much more separated in the past as the authors argue) similar to the process described in Reipurth & Mikkola 2012 (Nature 492, 221). So it could be that following the formation of the black hole, the three body system was much more compact.

Note that the authors are presenting one system and it may have had a unique history with relatively rare events ("canonical" as it is).

2. From the text it is not clear what is known about the existence and properties of tertiaries of other black holes in binaries. Are the authors claiming that (to the best of their knowledge) this is the first example of a distant tertiary? This is hinted at: "several LMXBs are thought to have unresolved companions, which have thus far been interpreted as chance alignments" but not clearly stated. I think the authors should provide more information about what is currently known about

tertiaries to other binaries with black holes and low-mass x-ray binaries in particular.

Minor comments/suggestions:

* I am confused about panel b in figure 4. - what is the initial semimajor axis or period of the tertiary in these simulations? As panel a shows, to allow large kicks small periods are needed.

* how was extended data figure 2 constructed? I didn't see a reference to it in the text.

* In Extended Data Figure 1, the authors show that the peculiar motion of the system is consistent with other stars in the neighbourhood, showing that a low kick is consistent with the motion of the system. I suggest the authors separately provide the value of V_z (in text is enough) which to some extent is more robust as it does not depend on the radial velocity or the distance of the system.

Boaz Katz

Referee #2 (Remarks to the Author):

This is a report on the manuscript by Burdge et al., "The black hole low mass X-ray binary V404 Cygni is part of a wide hierarchical triple, and formed without a kick".

The manuscript describes the discovery that a well-known black hole LMXB is part of a tertiary system. This discovery is certainly interesting and rather unexpected. Overall, the manuscript is readable and clear.

The association of the third star with the binary through a combination of proper motion and radial velocity constraints is convincing, though of course one has to be careful with interpreting an a posteriori statistical analysis that is selectively applied to systems that are already spotted as being interesting or promising. One could also imagine a possibility that unbound but nearby objects may, on average, have more similar proper motions and radial velocities than objects further away. Conceivably, one could imagine a test for whether p values obtained with the methodology used by the authors would indeed be uniformly distributed on [0,1] if applied to a random set of nearby objects -- but, of course, some of those randomly chosen pairs may also prove to be bound, yielding a preference for lower p-values...

The claim that "This discovery reveals that at least some black holes form with nearly no natal kick" (line 18, and similar text elsewhere) is perhaps slightly over-stated and would benefit from providing appropriate context. While the discovery indeed *lends support* to some black holes being born without a natal kick, this fact is rather well established in the literature (see, e.g., Mirabel & Rodrigues 2003, Wong+ 2012, Neijssel+ 2021 on Cygnus X-1; Shenar+ 2022, Vigna-Gomez+ 2024 on VFTS 243).

The hierarchical triple formation scenario is certainly promising. Is it possible to constrain the present-day relative inclination between the inner and outer binaries? In any case, this channel does require some degree of fine-tuning to bring the inner binary together -- but not before the black hole has formed. For example, it would seem that for some of the parameters the authors considered, the Kozai-Lidov timescale may be shorter than the lifetime of the black hole progenitor; I would recommend testing for this as an additional constraint on the formation history (systems should be able to make it to the present-day separation after several Gyr, but the inner binary should remain wider than ~ 10 AU for the first few tens of Myr).

I list some relatively minor comments below in order of appearance:

Line 23: "it is likely that at least half of the mass of the black hole progenitor collapsed into the black hole" -- it is not just likely, but required in order to avoid the binary being disrupted (except, perhaps, for a very fine-tuned natal kick -- but such a scenario is inconsistent with the claim that the natal kick must be small) [see also line 120]

Line 87: "the only way the inner binary could have experienced a large kick and retained the tertiary, is if the tertiary started at a short orbital period, and was kicked into a highly eccentric orbit..." -- I know what the authors meant, but it might confuse the reader into thinking that the tertiary was the

kicked object; consider revising.

Line 103: "This results from matter being ejected in the BH progenitor's rest frame, causing it to be jettisoned out of the binary at the orbital velocity of the BH in the barycentric frame." -- matter in the barycentric frame is moving with a velocity distributed around the SN ejecta velocity $\pm$ the orbital velocity. Perhaps something like the following would be more clear: "This results from matter being ejected **symmetrically** in the BH progenitor's rest frame, causing it to **leave the binary with a net asymmetric velocity equal to** the orbital velocity of the BH in the barycentric frame."

Line 100: "Alternatively, we consider a scenario where the secondary starts with an initial orbital period between 1 and 6 days (its current orbit is 6.4 days)." -- presumably, this refers to the orbit of the supernova; it's not necessarily where the secondary "started" (in the sense of starting its life when the system was first formed); this should be clarified [see also line 182, figure 4b legend & caption]

Line 118: "This would challenge current models for such a scenario, as this black hole has a mass of just $\sim 9M_{\odot}$, which is a smaller BH mass than predicted for such an event." -- the provided reference appears to be an observational paper? In any case, there are certainly a variety of modern models that do propose that such BHs could form through complete fallback (Mandel and Mueller 2020 or Schneider et al. 2021, just to pick two examples -- but see original SN model libraries)

Line 122: "We consider it unlikely that the system formed dynamically in a dense environment such as a globular cluster..." -- what about dynamical a lower-mass, lower-velocity stellar cluster, which may have since evaporated?

Line 172: "If the black hole conservatively accreted this much mass, it could account for the large spin..." -- accreting 0.5 solar masses onto a 9 solar mass non-spinning black hole would only give a spin of about $\chi \sim 0.17$; the mass of the BH would need to be doubled by accretion to bring its spin to 0.9, which seems unrealistic for this system.

Fig 4a: could be useful to indicate the range of plausible current orbital periods of the tertiary, or at least the orbital period of the tertiary if it is assumed to be on a circular orbit.

Might be useful to add a reference for Aladin / Aladin Lite

Discussion of Toomre diagram (extended figure 1): does the suggestion of a low overall velocity relative to the local rest frame further support a small total systemic velocity and, hence, low kick (whether natal or Blaauw)?

Section "Constraining kicks and mass loss":

Again, clarify that "initial" refers to "at the time of the BH formation".

State what kick direction and relative inner/outer binary inclination angle distributions are assumed; I presume isotropic in both cases, but it's perhaps worth a caveat that the latter in particular may not be isotropic, since it is determined by the formation mechanism.

What is M_2' in equation (1) [should it just be M_2]?

On line 267, $M_1 + M_2 \rightarrow M_1 + M_2$.

"We use 100 AU as a lower bound to ensure that the von Zeipel-Lidov-Kozai timescales were short enough that the secondary would have migrated into its current orbit over the few Gyr age of V404 Cygni" -- 100 AU seems to be an unnecessarily strict threshold, as the KL timescale is still many orders of magnitude less than the system lifetime for that inner binary separation.

"where the secondary remains bound in an orbit with a semi-major axis less than the final semi-major axis of the tertiary" -- in principle, why not allow a dynamical interaction that would exchange the secondary and the tertiary? [it's very fine-tuned, but I don't immediately see why it should be forbidden]

What are the units of probability in extended data figure 2? I was guessing per logarithmic interval on the abscissa, but then it does not seem to be normalised...

Typos:

Line 159: "an*d* establish"

Figure 1 caption: "and the separation of 1.43 arcseconds indicated in blue" -- looks like magenta?

Lines 232, 233 -- spurious closing parentheses

Referee #3 (Remarks to the Author):

- A. The authors provide an argument that BH LMXB V404 Cygnus is a wide-triple, provide 2 formation channels, and favor a low-kick formation.
- B. The paper seems sufficiently novel and significant.
- C. The data and methodology is adequately detailed.
- D. Statistics are appropriate with certain points need clarification, below.
- E. The conclusions are robust and excitingly speculative, within means.
- F. A key improvement, mentioned below, would be to identify ways to distinguish the purported formation channels.
- G. Credit is mostly apropos, again with suggestions below.
- H. The text is generally clear.

In summary, the paper is well-presented but would benefit from added insight, clarity, and detail in specific locations mentioned below.

When mentioning formation pathways without kicks or supernovae, the authors should include a wider set of references.

1) <https://ui.adsabs.harvard.edu/abs/2023ApJ...957...68B> provides a good summary, including a counterargument for silent BHs.

<https://arxiv.org/pdf/1710.00838> is also a good counterexample.

Ref 1) above presents two formation channels of BH formation. One of their channels predicts a low kick for imploding stars with remnant masses consistent with that of V404 Cygni. Given a BH/He core mass of $\sim 9 M_{\text{sun}}$, I would expect the model to lie between 12-15 M_{sun} masses, where various efforts see BH formation.

In the discussion lines 88, remove the commas surrounding "is if the territory started at a short orbital period"

The discussion of the two formation channels suggested would benefit from a discussion suggesting ways to disentangle them, assuming for instance a more 'typical' BH mass loss of $\sim 5 M_{\text{sun}}$ in the initial wide orbit + Kozai cycles.

Additionally, CCSNe simulations do not necessarily struggle to produce BHs of $9 M_{\text{sun}}$. The H-envelope is tenuously bound for massive stars that can be unbound in a variety of ways. The He core provides a good mass estimate for complete implosion to a BH, and Ref 1 (and others I encourage the other to investigate) summarizes this argument, and the remnant-progenitor mass relation for successful or failed SNe is an open problem.

The counterarguments paragraph starting line 122 would benefit from an approximate numerical estimate.

In the Methods section, I would like to see more information on how they are able to reduce 1256 sources from proper motion matching to 3. If possible, a short discussion of the El-Badry et al. 2018 methods should suffice. Later (line 229), what if fainter sources were also included in the source (down to Gaia G 20.5)? In line 231, 3 sources should triple the probability for chance alignment, but the number quoted is twice (unless I miss something).

In Eq 1, M_2' should be defined.

One key place the paper could benefit from is mentioning neutrino-driven kicks in CCSNe as one possible channel for low-kicks. A reference to the literature on SN kicks (e.g.

<https://arxiv.org/pdf/2311.12109>

<https://arxiv.org/abs/2310.01509>

<https://arxiv.org/abs/2401.13817>)

would provide helpful context to the paper. I note that, generally, simulations struggle to predict low kicks.

Author Rebuttals to Initial Comments:

Referees' comments:

Referee #1 (Remarks to the Author):

Using existing and new observations, the authors demonstrate that the low-mass x-ray binary V404 Cygni which harbors a black hole, has a wide orbit tertiary. The authors manage to characterise the tertiary and show that it has evolved off the main sequence allowing an age estimate of billions of years. The authors argue that the wide orbit of the tertiary sets strong limits on the kick velocity of the binary when the black hole was formed as well as a possible hint for the role of three-body dynamics in the formation, providing new constraints and hints on the formation of stellar mass black holes.

I think that the dynamical association of the binary and the (previously-known visual) companion, which includes proper motion and radial velocity measurements is convincing, that the characterisation of the tertiary is robust and that this discovery is exciting and worth publication in Nature.

I have two main reservations which I think should be addressed before publication.

1. The system is currently several billion years old while the black hole presumably formed ~ 10 million years after the formation of the system. The authors assume that there was no evolution in the orbit of the tertiary in the intermediate time and this is critical to their conclusions regarding the constraints on the kick. While this may likely be true, I think this is not certain and that the authors should be less definitive with their statements. In particular, a slow dynamical unfolding of three body systems may occur with the tertiary gaining energy during multiple close approaches with the inner binary (which could have been much more separated in the past as the authors argue) similar to the process described in Reipurth & Mikkola 2012 (Nature 492, 221). So it could be that following the formation of the black hole, the three body system was much more compact.

>Yes this is a good point! We have included a bit of text discussing this possibility in lines 129-130, and have softened language to allow for this possibility.

Note that the authors are presenting one system and it may have had a unique history with relatively rare events ("canonical" as it is).

2. From the text it is not clear what is known about the existence and properties of tertiaries of other black holes in binaries. Are the authors claiming that (to the best of their knowledge) this is the first example of a distant tertiary? This is hinted at: "several LMXBs are thought to have unresolved companions, which have thus far been interpreted as chance alignments" but not clearly stated. I think the authors should provide more information about what is currently known about tertiaries to other binaries with black holes and low-mass x-ray binaries in particular.

>Yes, we have included new text and references on this. This is the first definitive confirmed tertiary in an LMXB, but there are references which discuss blended stars much like the older literature on V404 Cygni, and follow-up observations with adaptive optics or JWST/HST could be used to demonstrate that these objects are actually bound to the LMXBs.

Minor comments/suggestions:

* I am confused about panel b in figure 4. - what is the initial semimajor axis or period of the tertiary in these simulations? As panel a shows, to allow large kicks small periods are needed.

>Good catch—we didn't clarify this in methods. We have added the following text "In both cases, the initial orbital period of the tertiary is drawn from a uniform distribution between the orbital period of the secondary and 70000 years." Realistically, the configurations in which the tertiary started out at an orbital period just barely longer than the tertiary are not dynamically stable, but we include them to be conservative (as they generally allow for larger kicks). In any case, either scenario doesn't allow for much of a kick.

* how was extended data figure 2 constructed? I didn't see a reference to it in the text.

>It was constructed using the same simulations used to make Figure 4 (they were originally additional panels in Fig 4 that we chose to remove for the sake of simplifying it). We have added a reference to it in that methods section of the text, indicating that it is meant to illustrate the low survival rate of systems in which the tertiary is kicked from a tight orbit into a wide one.

* In Extended Data Figure 1, the authors show that the peculiar motion of the system is consistent with other stars in the neighbourhood, showing that a low

kick is consistent with the motion of the system. I suggest the authors separately provide the value of V_z (in text is enough) which to some extent is more robust as it does not depend on the radial velocity or the distance of the system.

>Good point! We have included the V_z value in the text, along with the distribution of V_z values among the nearby stars (V404 Cyg is 5 km/s, nearby stars are -4 ± 17 km/s)

Boaz Katz

Referee #2 (Remarks to the Author):

This is a report on the manuscript by Burdge et al., "The black hole low mass X-ray binary V404 Cygni is part of a wide hierarchical triple, and formed without a kick".

The manuscript describes the discovery that a well-known black hole LMXB is part of a tertiary system. This discovery is certainly interesting and rather unexpected. Overall, the manuscript is readable and clear.

The association of the third star with the binary through a combination of proper motion and radial velocity constraints is convincing, though of course one has to be careful with interpreting an a posteriori statistical analysis that is selectively applied to systems that are already spotted as being interesting or promising. One could also imagine a possibility that unbound but nearby objects may, on average, have more similar proper motions and radial velocities than objects further away. Conceivably, one could imagine a test for whether p values obtained with the methodology used by the authors would indeed be uniformly distributed on $[0,1]$ if applied to a random set of nearby objects – but, of course, some of those randomly chosen pairs may also prove to be bound, yielding a preference for lower p-values...

>Yes this is a fair point! We did use all stars within 30' of V404 Cygni to compute the chance alignment probabilities (as indeed, proper motions and RVs for stars in a given part of the sky are not completely random, and tend to be similar due to Galactic motion). This still gives a very small chance alignment probability.

The claim that "This discovery reveals that at least some black holes form with nearly no natal kick" (line 18, and similar text elsewhere) is perhaps slightly

over-stated and would benefit from providing appropriate context. While the discovery indeed *lends support* to some black holes being born without a natal kick, this fact is rather well established in the literature (see, e.g., Mirabel & Rodrigues 2003, Wong+ 2012, Neijssel+ 2021 on Cygnus X-1; Shenar+ 2022, Vigna-Gomez+ 2024 on VFTS 243).

>Yes this is fair—we have softened this language in the abstract to “lends support that” and included the Shenar VFTS 243 reference in the abstract.

The hierarchical triple formation scenario is certainly promising. Is it possible to constrain the present-day relative inclination between the inner and outer binaries? In any case, this channel does require some degree of fine-tuning to bring the inner binary together – but not before the black hole has formed. For example, it would seem that for some of the parameters the authors considered, the Kozai-Lidov timescale may be shorter than the lifetime of the black hole progenitor; I would recommend testing for this as an additional constraint on the formation history (systems should be able to make it to the present-day separation after several Gyr, but the inner binary should remain wider than ~ 10 AU for the first few tens of Myr).

>It currently is not possible to constrain the relative inclinations, but ongoing monitoring with adaptive optics systems on ELTs and optical interferometry with the GRAVITY instrument on the VLT may allow for some orbital constraints, though this observing campaign will be beyond the scope of this work. Good point on the potentially fast Kozai timescales in certain initial configurations. We checked for situations in our simulations where the initial Kozai timescale is faster than the progenitor’s lifetime (Kozai timescale less than 10 Myr), and found that excluding these cases had no appreciable impact on the outcome of the simulations and the resulting kick/mass loss constraints.

I list some relatively minor comments below in order of appearance:

Line 23: "it is likely that at least half of the mass of the black hole progenitor collapsed into the black hole" – it is not just likely, but required in order to avoid the binary being disrupted (except, perhaps, for a very fine-tuned natal kick – but such a scenario is inconsistent with the claim that the natal kick must be small) [see also line 120]

>Good point! We have updated the phrasing to replace the word likely with required.

Line 87: "the only way the inner binary could have experienced a large kick and retained the tertiary, is if the tertiary started at a short orbital period, and was kicked into a highly eccentric orbit..." – I know what the authors meant, but it might confuse the reader into thinking that the tertiary was the kicked object; consider revising.

>Good point! We have clarified this phrasing to make it unambiguous that the tertiary is sent into an eccentric orbit as a result of a kick to the inner binary.

Line 103: "This results from matter being ejected in the BH progenitor's rest frame, causing it to be jettisoned out of the binary at the orbital velocity of the BH in the barycentric frame." – matter in the barycentric frame is moving with a velocity distributed around the SN ejecta velocity \pm the orbital velocity. Perhaps something like the following would be more clear: "This results from matter being ejected **symmetrically** in the BH progenitor's rest frame, causing it to **leave the binary with a net asymmetric velocity equal to** the orbital velocity of the BH in the barycentric frame."

>Good suggestion—we have updated this text.

Line 100: "Alternatively, we consider a scenario where the secondary starts with an initial orbital period between 1 and 6 days (its current orbit is 6.4 days)." – presumably, this refers to the orbit of the supernova; it's not necessarily where the secondary "started" (in the sense of starting its life when the system was first formed); this should be clarified [see also line 182, figure 4b legend & caption]

>Good point—we have clarified the language to indicate this is the orbital period at which the BH formed.

Line 118: "This would challenge current models for such a scenario, as this black hole has a mass of just $\sim 9M_{\odot}$, which is a smaller BH mass than predicted for such an event." – the provided reference appears to be an observational paper? In any case, there are certainly a variety of modern models that do propose that such BHs could form through complete fallback (Mandel and Mueller 2020 or Schneider et al. 2021, just to pick two examples – but see original SN model libraries)

>We have removed this statement, and added several refs that actually support BH formation from complete fallback at these masses (ref #1 provided many examples).

Line 122: "We consider it unlikely that the system formed dynamically in a dense environment such as a globular cluster..." -- what about dynamical a lower-mass, lower-velocity stellar cluster, which may have since evaporated?

>Very good point—we have included a discussion on this (we believe we cannot definitively rule this out, and that this presents a very intriguing possibility).

Line 172: "If the black hole conservatively accreted this much mass, it could account for the large spin..." -- accreting 0.5 solar masses onto a 9 solar mass non-spinning black hole would only give a spin of about $\chi \sim 0.17$; the mass of the BH would need to be doubled by accretion to bring its spin to 0.9, which seems unrealistic for this system.

>Fair enough—we have removed this statement—good catch!

Fig 4a: could be useful to indicate the range of plausible current orbital periods of the tertiary, or at least the orbital period of the tertiary if it is assumed to be on a circular orbit.

>We looked into this, and would prefer not to modify the figure itself, but we have highlighted in the text in several locations that the orbital period of the tertiary must be at least 70000 years if circular (and that hopefully future astrometric work will help further constrain this).

Might be useful to add a reference for Aladin / Aladin Lite

Excellent point! We have added a reference for this.

Discussion of Toomre diagram (extended figure 1): does the suggestion of a low overall velocity relative to the local rest frame further support a small total systemic velocity and, hence, low kick (whether natal or Blaauw)?

>Earlier work suggested that the velocity was anomalous and that this was evidence for a large kick, and our inclusion of this diagram was to illustrate that the velocity actually isn't anomalous. However, we think it's a bit beyond the scope of this paper to model the detailed dynamics and whether that

actually gives us a useful constraint on the kick (it is likely the presence of the bound tertiary gives a much stronger constraint).

Section "Constraining kicks and mass loss":

Again, clarify that "initial" refers to "at the time of the BH formation".

>We have clarified this in all places that it appears in the text.

State what kick direction and relative inner/outer binary inclination angle distributions are assumed; I presume isotropic in both cases, but it's perhaps worth a caveat that the latter in particular may not be isotropic, since it is determined by the formation mechanism.

>Good point! We did assume isotropic in all cases, and we have now explicitly stated this in the text.

What is M_2' in equation (1) [should it just be M_2]?

>Good catch! There should indeed be no prime on M_2

On line 267, $M_1 + M_2 \rightarrow M_1 + M_2$.

>Good catch! We have fixed this

"We use 100 AU as a lower bound to ensure that the von Zeipel-Lidov-Kozai timescales were short enough that the secondary would have migrated into its current orbit over the few Gyr age of V404 Cygni" -- 100 AU seems to be an unnecessarily strict threshold, as the KL timescale is still many orders of magnitude less than the system lifetime for that inner binary separation.

"where the secondary remains bound in an orbit with a semi-major axis less than the final semi-major axis of the tertiary" -- in principle, why not allow a dynamical interaction that would exchange the secondary and the tertiary? [it's very fine-tuned, but I don't immediately see why it should be forbidden]

>This is a fair point. We found that relaxing the 100 AU threshold does not dramatically change our conclusions, and in general our desire is to keep the simulations relatively straightforward to understand, hence our omission of more fine tuned cases like an exchange of the secondary and tertiary.

However, this possibility is an interesting one, and thus we have included some text mentioning it!

What are the units of probability in extended data figure 2? I was guessing per logarithmic interval on the abscissa, but then it does not seem to be normalised...

>These probabilities are the survival fraction of solutions at specific initial configurations that reach a bound orbit with >3500 AU final separation (e.g. one can see solutions that started out that wide and received zero kick

basically have a probability that approaches unity because they all survival and meet this condition).

Typos:

Line 159: "an*d* establish"

>Good catch! Corrected

Figure 1 caption: "and the separation of 1.43 arcseconds indicated in blue" -- looks like magenta?

>Yes very good catch! Thanks for reading this so closely—we have corrected it to magenta.

Lines 232, 233 – spurious closing parentheses

>Good catch! We have corrected this.

Referee #3 (Remarks to the Author):

- A. The authors provide an argument that BH LMXB V404 Cygnus is a wide-triple, provide 2 formation channels, and favor a low-kick formation.
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In summary, the paper is well-presented but would benefit from added insight, clarity, and detail in specific locations mentioned below.

When mentioning formation pathways without kicks or supernovae, the authors should include a wider set of references.

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<https://arxiv.org/pdf/1710.00838> is also a good counterexample.

>Great suggestion! We have incorporated these two references in the discussion during the first paragraph.

Ref 1) above presents two formation channels of BH formation. One of their channels predicts a low kick for imploding stars with remnant masses

consistent with that of V404 Cygni. Given a BH/He core mass of $\sim 9 M_{\text{sun}}$, I would expect the model to lie between 12-15 M_{sun} masses, where various efforts see BH formation.

>Yes indeed! Thank you for suggesting the inclusion of this ref, as it does indeed nicely match what we see in V404 Cygni.

In the discussion lines 88, remove the commas surrounding "is if the territory started at a short orbital period"

>Good catch! We have corrected this.

The discussion of the two formation channels suggested would benefit from a discussion suggesting ways to disentangle them, assuming for instance a more 'typical' BH mass loss of $\sim 5 M_{\text{sun}}$ in the initial wide orbit + Kozai cycles.

>We believe it is actually rather difficult to distinguish between the two channels. The relative inclinations of the outer and inner orbits need not be aligned in either case, nor would one expect the outer orbit to necessarily be circular in either situation. One could argue that Kozai naturally could induced a misaligned spin of the BH with respect to the inner orbit, and we discuss this in the manuscript.

Additionally, CCSNe simulations do not necessarily struggle to produce BHs of $9 M_{\text{sun}}$. The H-envelope is tenuously bound for massive stars that can be unbound in a variety of ways. The He core provides a good mass estimate for complete implosion to a BH, and Ref 1 (and others I encourage the other to investigate) summarizes this argument, and the remnant-progenitor mass relation for successful or failed SNe is an open problem.

>Yes good point. Given all these references we have altered several portions of the text highlighting that some models do indeed predict BH formation from complete implosion at these masses.

The counterarguments paragraph starting line 122 would benefit from an approximate numerical estimate.

>We have actually revised this argument and softened it, and pointed to relevant literature. Namely, we now believe that we cannot easily rule out dynamical capture because this could have occurred in an open cluster, and we cite relevant recent work on this.

In the Methods section, I would like to see more information on how they are able to reduce 1256 sources from proper motion matching to 3. If possible, a short discussion of the El-Badry et al. 2018 methods should suffice. Later (line 229), what if fainter sources were also included in the source (down to Gaia G 20.5)? In line 231, 3 sources should triple the probability for chance alignment, but the number quoted is twice (unless I miss something).

>Good point! We have included the relevant expressions in the discussion section from El-Badry et al 2018.

In Eq 1, M_2' should be defined.

>This was actually a typo—there was not supposed to be a prime on M_2 —good catch!

One key place the paper could benefit from is mentioning neutrino-driven kicks in CCSNe as one possible channel for low-kicks. A reference to the literature on SN kicks (e.g.

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would provide helpful context to the paper. I note that, generally, simulations struggle to predict low kicks.

>Good suggestion, and similar to one made by another ref. We have included the Burrows reference in the discussion of kick mechanisms.