
Supplementary information

Metalasers with arbitrarily shaped wavefront

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Supplementary Information for

Metalasers with arbitrarily-shaped wavefront

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Supplementary Note-1: Geometric phase

For the structure with $r = 0$ nm, the band structure has been calculated using the eigen-mode solver in COMSOL Multiphysics. As shown in Fig. S1A, several resonant modes can be seen in the visible spectral range. The red line corresponds to the TE_1 mode we used. Meanwhile, the Q factor has also been numerically calculated in Fig. S1B with $Q = \text{Re}(\omega)/|2\text{Im}(\omega)|$, where ω is the complex eigen-frequency. The Q factor of the TE_1 mode is divergent at the Γ point, suggesting a symmetry-protected BIC at Γ point.

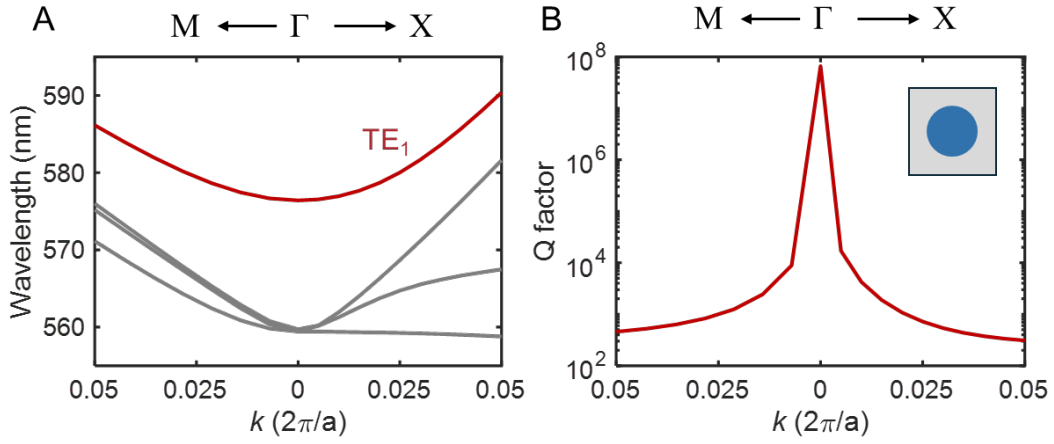


Fig. S1. The band structure and Q factor for the structure with $r = 0$ nm. **A.** The band structure. The red line indicates the TE_1 mode we used. **B.** The Q factor of the TE_1 mode.

When the in-plane inversion (C_2) symmetry is broken, the at- Γ BIC will turn into quasi-BIC with linearly polarized far-field radiation¹. For the cases of $r = 0$ nm and $r = 20$ nm, we calculated the momentum-space distributions of far-field polarization state for the TE_1 mode. As shown in Fig. S2, by introducing the hole perturbation ($r = 20$ nm), two circular polarization states spawn from the polarization vortex of BIC, and the at- Γ quasi-BIC with a maximum Q factor lies on the L line, whose far-field radiation is linearly polarized.

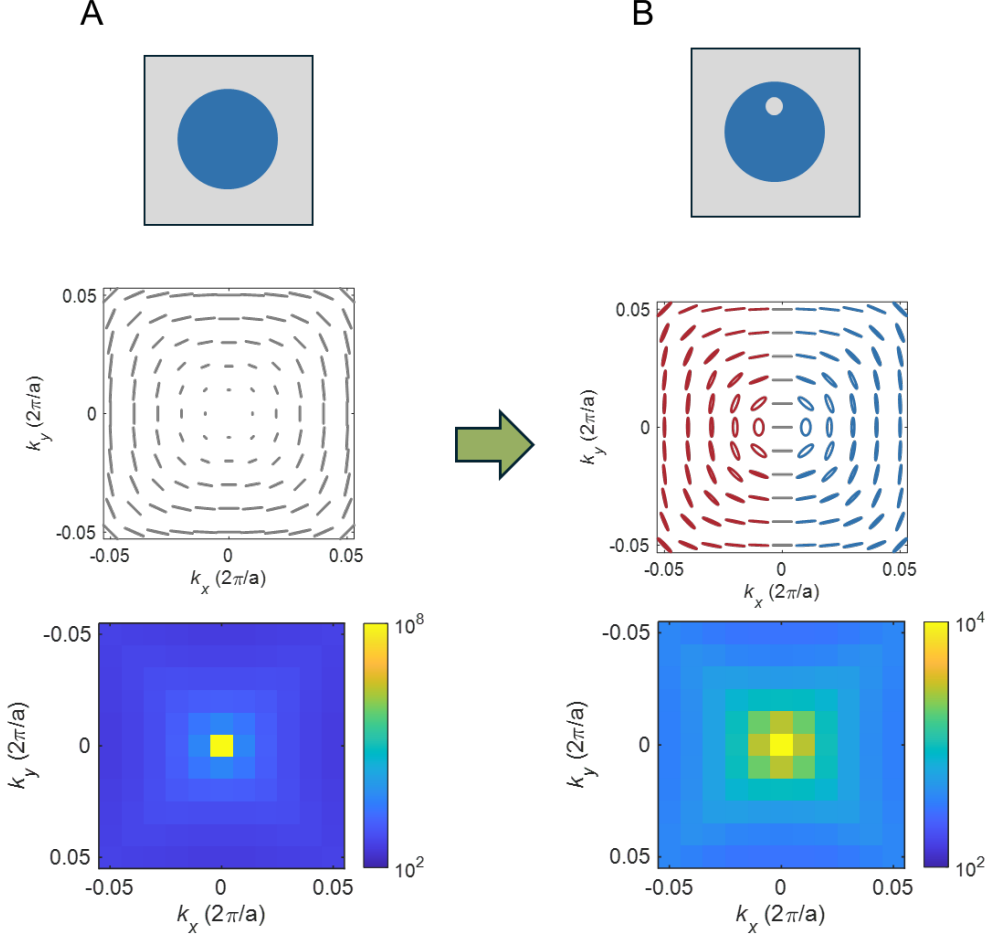


Fig. S2. The far-field polarization states and the Q factors in momentum space for the TE_1 mode. A. The structure with $r = 0$ nm. B. The structure with $r = 20$ nm.

When the azimuthal angle θ of the hole in the nanodisk is changed, the orientation angle θ_F of far-field polarization vector for the quasi-BIC is also varied ($\theta_F \sim \theta - \pi/2$), as shown in Fig. 1b of the main text. As a result, the changes in eigen-polarization angle impart a geometric phase for the circular polarized components of the emission from the quasi-BIC.

Quasi-BICs can be well-captured by the temporal coupled-mode theory (TMCT)^{2,3}. Here, we present a simple model based on the TMCT for studying the emission from the quasi-BIC in the absence of incident waves. The quasi-BIC employed radiates into the lower and upper subspace (labeled by subscript “ l ” and “ u ”) via x - and y -polarization channels. According to TCMT, the dynamic equations are written as

$$\frac{dA}{dt} = (-i\omega - \gamma)A, \quad (1)$$

$$b = DA. \quad (2)$$

In these equations, A is the resonance amplitude. ω is the resonant frequency, while γ is the decay rate due to radiation. $b = (b_x^u, b_y^u, b_x^l, b_y^l)^T$ denotes the outgoing wave. $D = (d_x^u, d_y^u, d_x^l, d_y^l)^T = (d_x, d_y, \sigma_z d_x, \sigma_z d_y)^T$ indicates the coupling coefficient between the resonance and outgoing waves, where σ_z is a parity number linking the upward radiation and the downward radiation because of the z -axis mirror symmetry. According to the conservation of energy, i.e., $d|A|^2/dt = -b^\dagger b$, we can obtain $D^\dagger D = 2\gamma$, which is written as

$$|d_x|^2 + |d_y|^2 = \gamma. \quad (3)$$

Because the far-field radiation of the quasi-BIC at Γ point is linearly polarized, d_x/d_y needs to be real, and thus we can set the coupling coefficient D to be real. By combining this approximation and Eq. (3), the radiation D_{x-y} from the resonance can be written as

$$D_{x-y} = \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} \sqrt{\gamma} \cos(\theta_F) \\ \sqrt{\gamma} \sin(\theta_F) \end{pmatrix}. \quad (4)$$

This form in x - y basis can be written in helical basis:

$$D_{heli} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \sqrt{\frac{\gamma}{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} \cos(\theta_F) \\ \sin(\theta_F) \end{pmatrix} = \sqrt{\frac{\gamma}{2}} \begin{pmatrix} e^{i\theta_F} \\ e^{-i\theta_F} \end{pmatrix}. \quad (5)$$

So, as the eigen-polarization angle varies, the RCP and LCP components of the emission for the quasi-BIC shall acquire opposite geometry phases, as shown in Fig. S3.

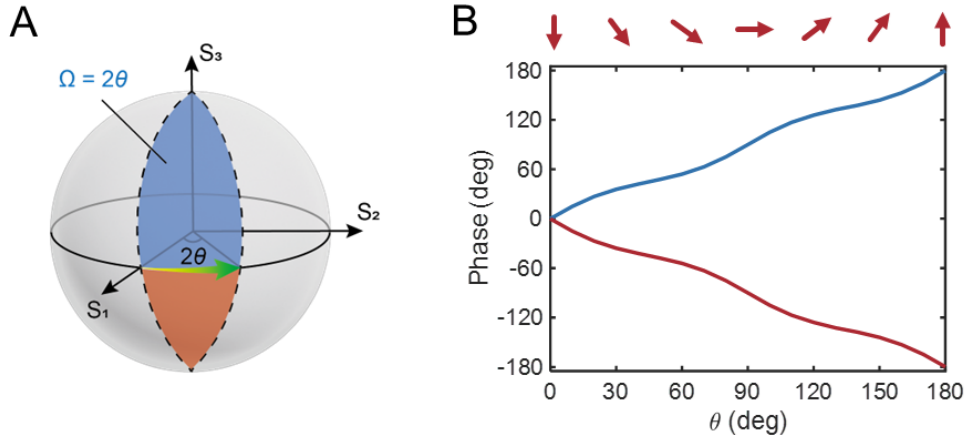


Fig. S3. The geometric phase. **A.** The evolution of polarization states on the Poincare sphere. **B.** The acquired geometric phase of RCP (blue curve) and LCP (red curve) as a function of rotation angle of the hole.

In principle, the geometric phase can also be understood from the point view of dipole approximation. The resonant field of quasi-BIC can be approximated as an in-plane dipole and

produces the geometric phase. In this sense, the field pattern within each unit cell also plays an essential role in determining the geometric phase. In current research, a dye-doped PMMA is employed to endow gain to Si_3N_4 nanostructure. Their refractive indices are $n_{\text{Si}_3\text{N}_4} = 2.02$ and $n_{\text{PMMA}} = 1.49$, respectively. Such a small index difference ($\Delta n \sim 0.53$) prevents the resonant electric field from being completely bound in the nanodisk. As plotted in Fig. S4A, a large portion of electromagnetic field distributes out of the nanodisk. Then the interaction between neighboring nanodisks becomes dramatic and the square-latticed periodic nanostructure affects the field distribution, leading to a square-like distribution of electric fields in the unit cell. Such a field pattern does not match the circular rotation of the hole and results in a nonlinear relationship between the rotation angle of the hole and the polarization angle. Based on the above result, it is easy to know that the nonlinear relationship can be overcome by replacing Si_3N_4 with GaN or GaP. Since GaN and GaP are direct bandgap semiconductors, the dye-doped PMMA is not necessary anymore. With the increase of refractive index difference, we can see that the electric field distribution is more tightly in the nanodisk and closer to a donut. As a consequence, much better linear relationship between the orientation angle of resonant electric dipole and the hole rotation angle can be obtained in Fig. S4B and Fig. S4C.

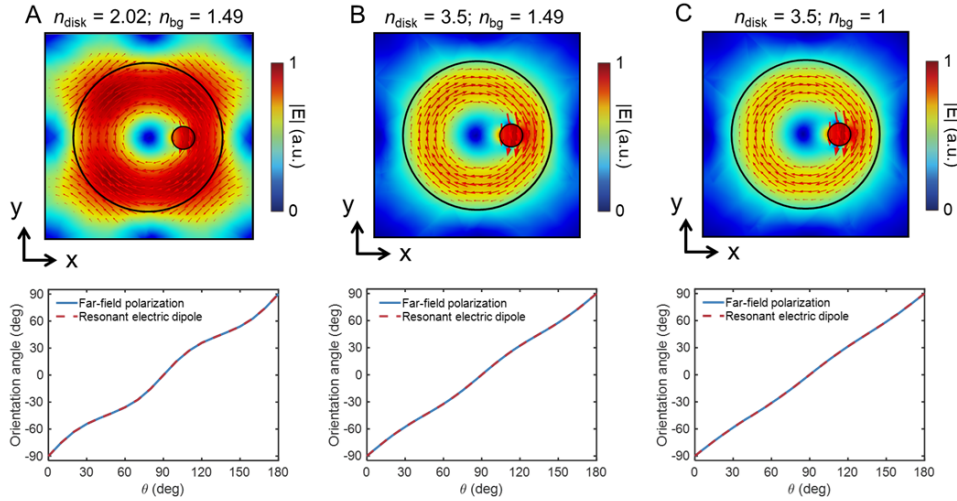


Fig. S4. The electric field distribution of the quasi-BICs, polarization azimuthal angle of emission and the orientation angle of resonant electric dipole as a function of rotation angle of the hole. A. The nanodisk with $n = 2.02$ embedded in PMMA. B. The nanodisk with $n = 3.5$ embedded in PMMA. C. The nanodisk with $n = 3.5$ embedded in air.

In addition, we calculated the resonant wavelengths and Q factors for different rotation angles θ . As plotted in Fig. S5, the rotation of the hole in nanodisk for imparting geometric phase does not affect the resonant wavelength and Q factor of quasi-BIC significantly. Notably, the small variations in the Q factor ensure that the quasi-BIC with encoded geometric phases in the metasurface still has a high Q factor compared to the case of the periodic structure, and can generate laser emission first, avoiding the effects of other guided resonances that have no geometric phase and lie within the dye gain range.

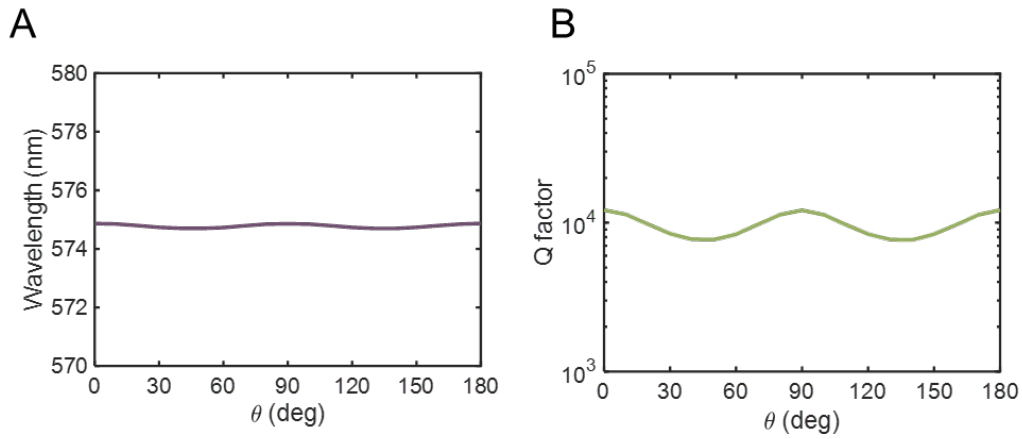


Fig. S5. The resonant wavelength and Q factor of quasi-BIC as a function of rotation angle of the hole.

Supplementary Note-2: Design of holographic images

In the main text, we show a holographic image of “ ψ ” and “ π ” designed and experimentally produced with metasurfaces. Here, we present the details of the phase design for the holographic image. The details are summarized in Fig. S6. The phase distribution of a target image has been calculated with a standard Gerchberg-Saxton (GS) algorithm⁴. In general, serious artifacts (phase dislocations) are often generated when the holographic image is achieved from the designed phase profile following Rayleigh-Sommerfield diffraction theory (inset in Fig. S6). Then a stochastic gradient descent algorithm, Adam, has been employed to eliminate such artifacts during the design⁵. Basically, the gradient descent algorithm back-propagates the errors and updates the phase distribution to minimize the loss function, as depicted in Fig. S6. The iterations of the optimization loops reduce the mean square error (MSE) and the holographic image converge to the target image with minimized phase dislocations.

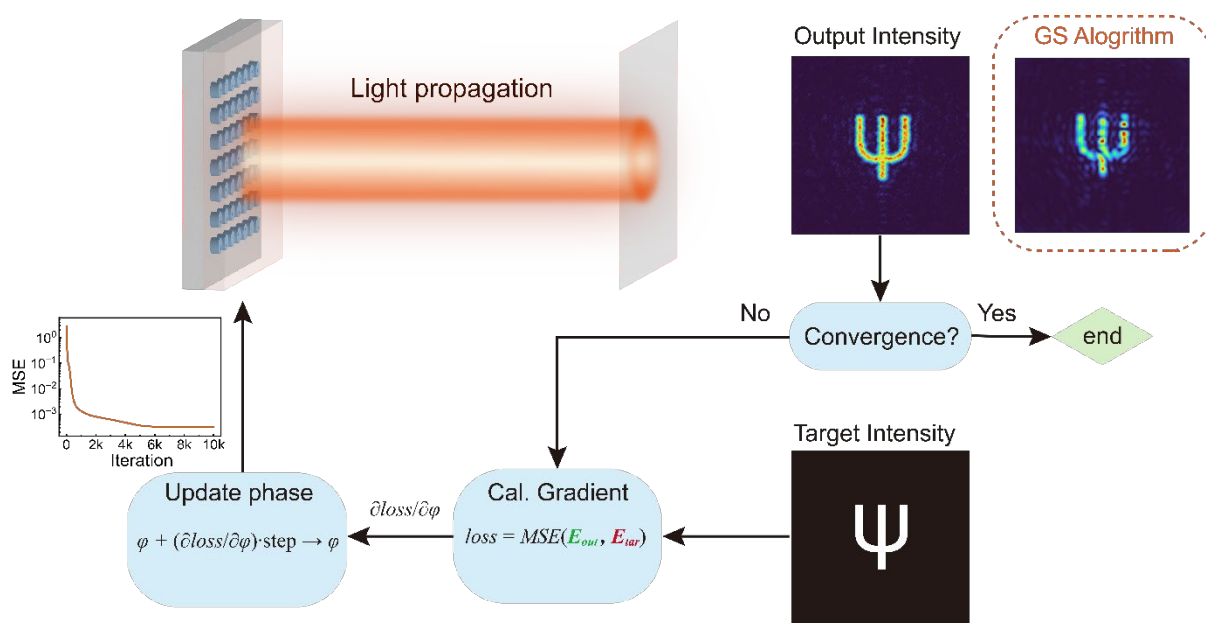


Fig. S6. The optimization process of the holographic image. Insets show the initial and optimized holographic images of “ ψ ”.

Based on the above process, we have computed and optimized the phase profiles of “ ψ ” (see Fig. S7A). Fig. S7B shows the calculated phase distribution with 100×100 pixels for the holographic image. The corresponding intensity distribution of the hologram is plotted as inset in Fig. S6 for a direct comparison. Compared with conventional Gerchberg-Saxton (GS)

algorithm, the optimized phase profile can effectively eliminate the phase dislocations in numerical designs. Similar phenomena hold true for the other holographic image. Another example of π is plotted in Figs. S7C and S7D.

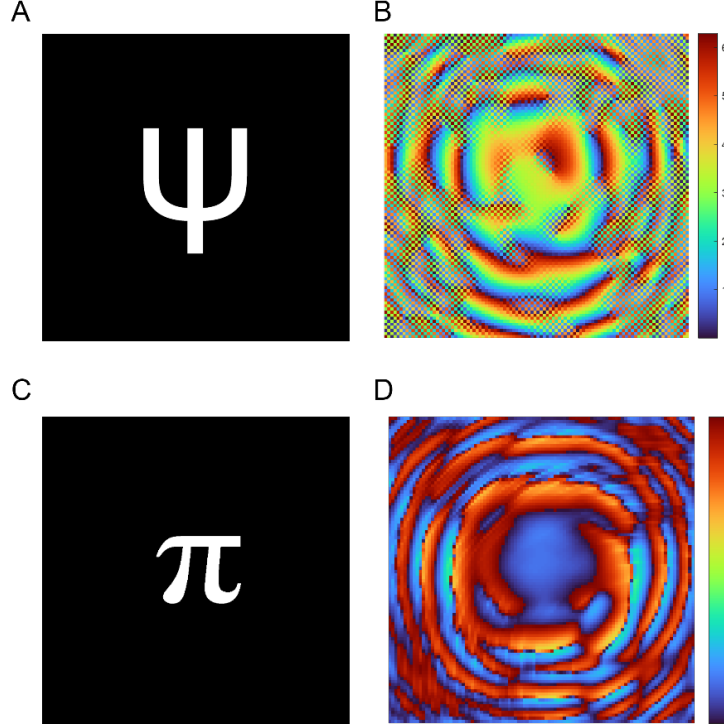


Fig. S7. The target image and the numerically calculated phase profile. A. The image of ψ . **B.** The phase profile for the image ψ . **C.** The image of π . **D.** The phase profile for the image π .

The manipulation of phase at pixel-level can be applied to many other practical applications as well. Another example is the metalens nanolaser. For some applications such as coupling light into fiber, the laser should be focused to match the numerical aperture of the fiber. Different from the conventional laser, this can be easily realized by the metalaser. As mentioned in the main text, the phase profile is purely determined by the geometric phase and thus the rotation angle of the holes in each meta-atom. To focus the emitted laser, we have defined the phase profile following the equation $\phi(x, y) = -\frac{2\pi}{\lambda}(\sqrt{x^2 + y^2 + f^2} - f)$. The corresponding experimental results are summarized in Fig. S21 below. For the designed polarization, the emission is focused to a diffraction-limited spot. By changing the phase profile to another equation of cylindrical lens $\phi(x) = -\frac{2\pi}{\lambda}(\sqrt{x^2 + f^2} - f)$, the emission from the metasurface with designed spin is focused to a line as well (Fig. S22).

Compared with conventional nanolasers or bound state in the continuum (BIC) laser, the metasurface nanolaser can emit the laser beam to arbitrary direction instead of the normal direction. This can be observed in Fig. S24. By applying a deflection phase profile, the high-order vortex emissions with different topological charges have been deflected to different angles away from the normal direction. For the case of vector beams, conventional BICs can also produce vector beams, but their topological charges are quite limited, e.g. ± 1 or ± 2 . With our metasurface, the high-order vector beam (Fig. S23) with large topological charge can be simply achieved by defining a spiral phase profile. For such a case, there is no limitation in topological charge anymore.

In addition, high-dimensional multiplexing of conventional metasurface can also be expected from this new metalaser. Such flexibly designed laser beams shall play essential roles in many practical applications. In the main text, we demonstrated an example of laser hologram. The metalaser actually can be directly coupled with laser systems, waveguides, or other optical components, and then plays an essential role in optical computing, optical sensing, optical communications and bio-medical diagnostics.

Supplementary Note-3: Experimental details

Nanofabrication process. The nanostructures are fabricated within a Si_3N_4 membrane by a combined process of plasma enhanced chemical evaporation deposition (PECVD), electron-beam (E-beam) evaporation, E-beam lithography, lift-off, ICP etching, and spin-coating. Details of the nanofabrication process have been summarized in Methods and are schematically depicted in Fig. S8.

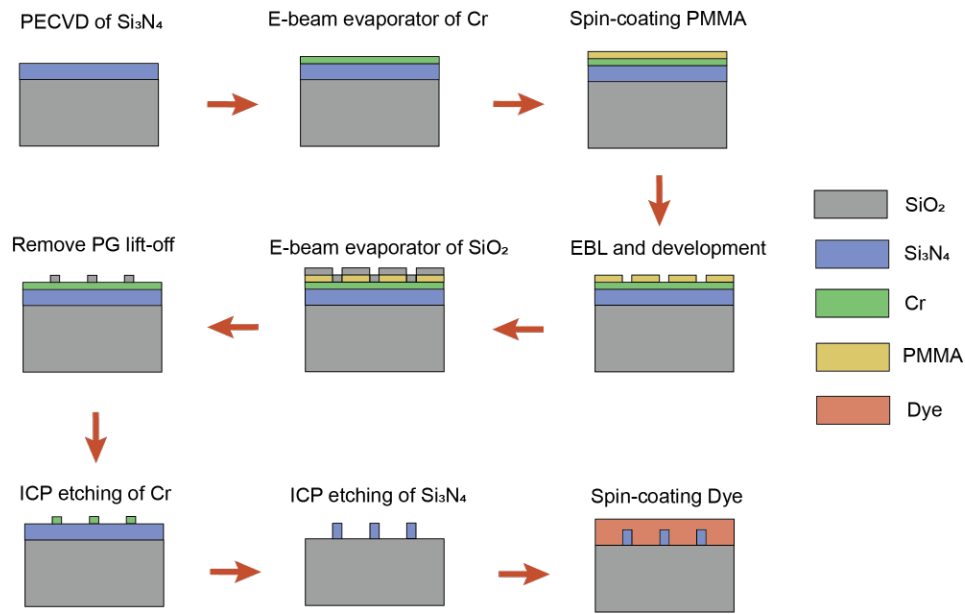


Fig. S8. Fabrication process flow of the metasurfaces.

For the design of metalaser, the precision of hole position plays an essential role in determining the wavefront of nanolaser and its far field radiation. In our system, both the sample size and precision are restricted by our E-beam system. The first one corresponds to the size of writing field. To avoid significant size deviation and misalignment, we kept all the sample size below $180 \times 180 \mu\text{m}^2$. This of course can be improved by using 100 kV E-beam system.

The latter one is related to the step size. Assuming that the center of the nanodisk is at the origin $(0, 0)$, the central coordinates of the eccentric hole as it rotates can be written as $(L\cos\theta, L\sin\theta)$. When the hole rotates by a small angle $\Delta\theta$, the resulting displacements in the x - and y -directions are $\Delta x = L\cos(\theta + \Delta\theta) - L\cos\theta$ and $\Delta y = L\sin(\theta + \Delta\theta) - L\sin\theta$. For small $\Delta\theta$, they can be approximated by (Taylor expansion) $\Delta x \approx -L\Delta\theta \sin\theta$ and $\Delta y \approx L\Delta\theta \cos\theta$.

The constraint of the step size (s) suggests that $\Delta\theta$ must be such that $\max(|\Delta x|, |\Delta y|) \geq s$ (Fig. S9A). Based on this, we can get the angular resolution $\Delta\theta$, which is $\frac{s}{L} \leq \Delta\theta \leq \frac{s\sqrt{2}}{L}$ for different θ . Here, we take the maximum value of $\frac{s\sqrt{2}}{L}$ as the angular resolution, which allows the rotation of the hole to be resolved at any θ . In our fabrication process, the beam step size in E-beam lithography is set to 2 nm. So, we can approximately get an angular resolution of $\sim 2.7^\circ$, which is going to be smaller for larger L . In addition, the subsequent processes (including development, lift-off, etching, etc.) will also affect the effective pixel precision, which is quite difficult to evaluate, thus the actual resolvable pixel size might be a bit larger.

Although the rotation angle of the hole is not varied continuously, the fabricating angular resolution is still sufficient for wavefront shaping of the metalaser. To prove this, we calculate the eigen-fields of cylindrical lens metalaser composed of discrete phases. The numerical apertures are set as 0.2. The focusing efficiency is calculated and plotted as a function of number of phase step within $0-2\pi$ (Fig. S9B). It is clear that the focusing efficiency reaches a saturation value of 0.8 when phase steps are larger than 16. Such a phase step corresponds to an angular resolution of $\leq 22.5^\circ$, which is almost an order of magnitude larger than our experimental resolution. Therefore, the nanofabrication precision won't degrade the wavefront shaping of our metalaser. This is also consistent with the previous reports on metalens.

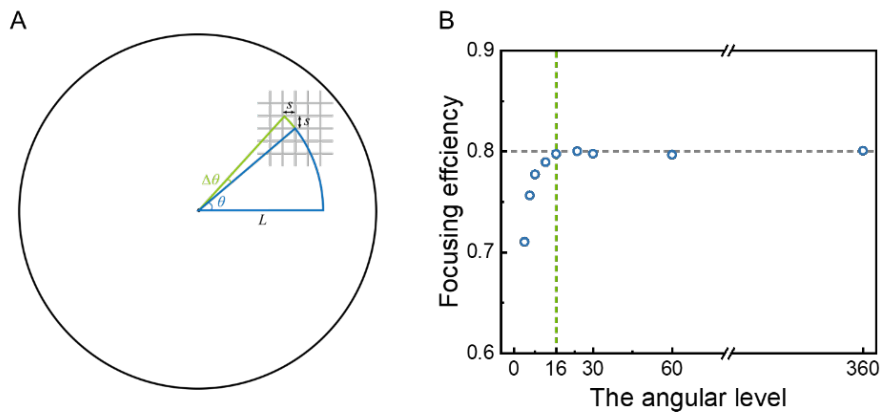


Fig. S9. Effective angular resolution and its impact. **A.** The schematic diagram of angular resolution calculation. **B.** The focusing efficiency as a function of the number of discrete phases within 2π .

Material properties. Optical parameters of Si_3N_4 are measured via spectroscopic ellipsometer (J. A. Woollam, RC2 XI+). The corresponding refractive index and light extinction coefficient are illustrated in Fig. S10A. For the lasing wavelength in the main text (~ 575 nm), Si_3N_4 has a refractive index of about 2.02 and the loss is approximately 0.

In addition, the absorption and photoluminescence (PL) spectra of dye-doped PMMA film have also been recorded and plotted in Fig. S10B. The emission from the dye-doped PMMA film is broad with a peak centered at 573 nm with a full width at half maximum (FWHM) ~ 74 nm. The absorption peak appears at around 520 nm but has value across almost all the spectral range. By increasing the excitation power, amplified stimulated emission (ASE) can also be obtained from the film. The results are shown as a dashed line Fig. S10B. It is obvious that the optical gain occurs at 587 nm with a FWHM of 15 nm, covering the lasing wavelength in the main text very well.

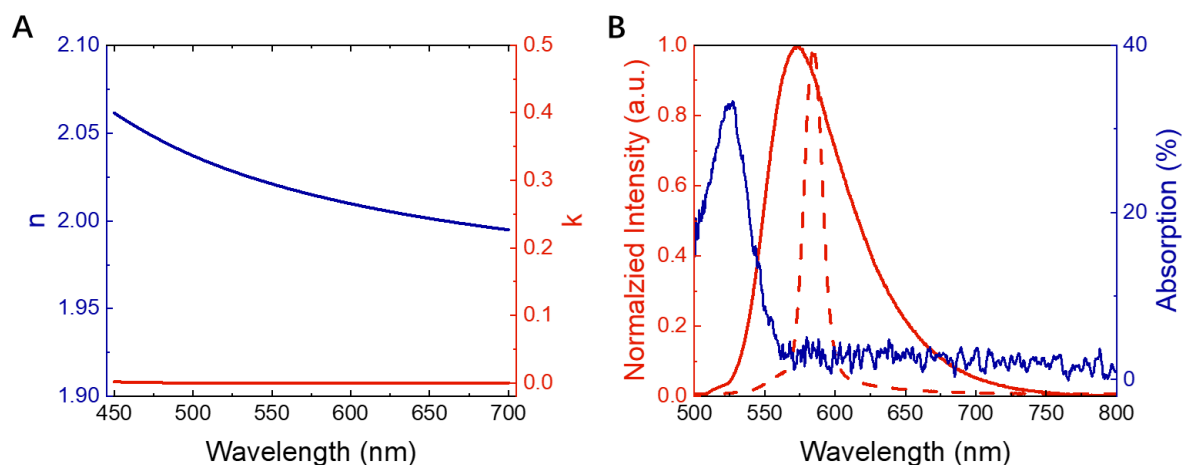


Fig. S10. The properties of Si_3N_4 and dye-doped PMMA films. **A.** Experimentally measured refractive index (n) and light extinction coefficient (k) of the Si_3N_4 membrane. **B.** The experimentally recorded absorption (blue line), PL (red line), and ASE (red dashed line) of the dye-doped PMMA film.

Optical characterization. The schematic diagram of optical setup for measuring the PL spectrum and lasing emission is plotted in Fig. S11. A Nd: YAG nanosecond laser (532 nm, 10 Hz repetition rate) serving as the pump source is normally focused onto the backside of the sample by an objective lens ($5\times$, $\text{NA}=0.12$). The emission light from the sample is collected

by an objective lens, and a long pass filter (550 nm) is utilized to filter out the pumping laser. After passing through the quarter-wave plate, the left-handed (LCP) and right-handed circular polarization (RCP) components are divided into two orthogonal linear polarizations. By rotating the polarizer to -45° and 45° with respect to the quarter-wave plate, the RCP and LCP components of emission from the metasurface can be measured respectively. In experiments, the distance between the metasurface and the collecting objective lens is first adjusted so that the sample can be clearly visualized on the CCD. Then, by adjusting the z -axis position of the metasurface, the emission pattern from the metasurface can be detected by the CCD camera, giving the beam profile of “ π ” and “ ψ ” in experiment. By focusing the emissions into a spectrometer, the emission spectra and laser threshold can be achieved.

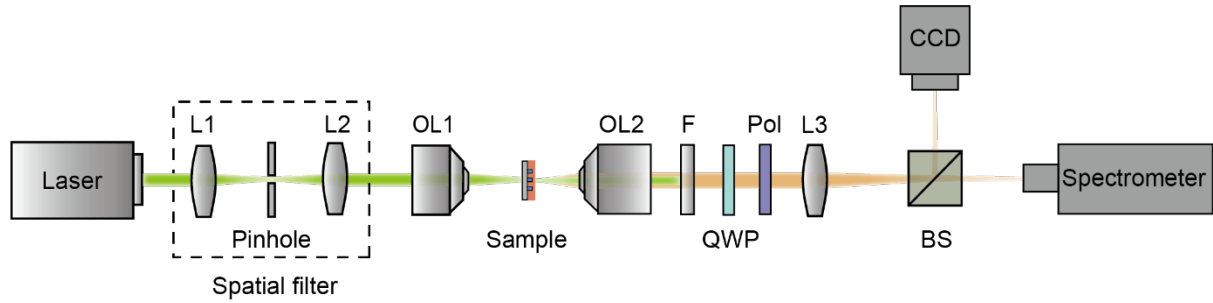


Fig. S11. Optical setup for the lasing experiment.

To characterize the polarization of the emission, a linear polarizer has been inserted between the collective lens and the spectrometer or the CCD camera. By rotating the polarizer, the corresponding intensity or beam profile can be achieved. In the case of far field directionality and vortex emission, the back-focal plane imaging technique has also been added, similar to our previous reports.

Supplementary Note-4: Spatial coherence of metalasers

In the main text, we have mentioned that the coherence of metalaser won't be destroyed with the increase of excitation power. The temporal coherence can be seen from the full linewidth at half maximum (FWHM) of the metalaser in the main text. The FWHM remains at a small value ~ 0.2 nm, demonstrating the high temporal coherence well. Here we show the experimental results about spatial coherence.

The emission from metalaser is focused and collimated. The collimated beam passes through a double-split plate and interferes behind it. The corresponding interference patterns are shown in Fig. S12. By cutting a line at the center, the position-dependent intensity can be plotted. Then the corresponding coherence $V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ has been calculated and shown in Fig. S13. It is obvious to see that the interference patterns are very clear, and the V remains as almost a constant (~ 0.9) above the threshold. Then we can confirm that both the spatial coherence and temporal coherence of the metalaser are well preserved above the laser threshold.

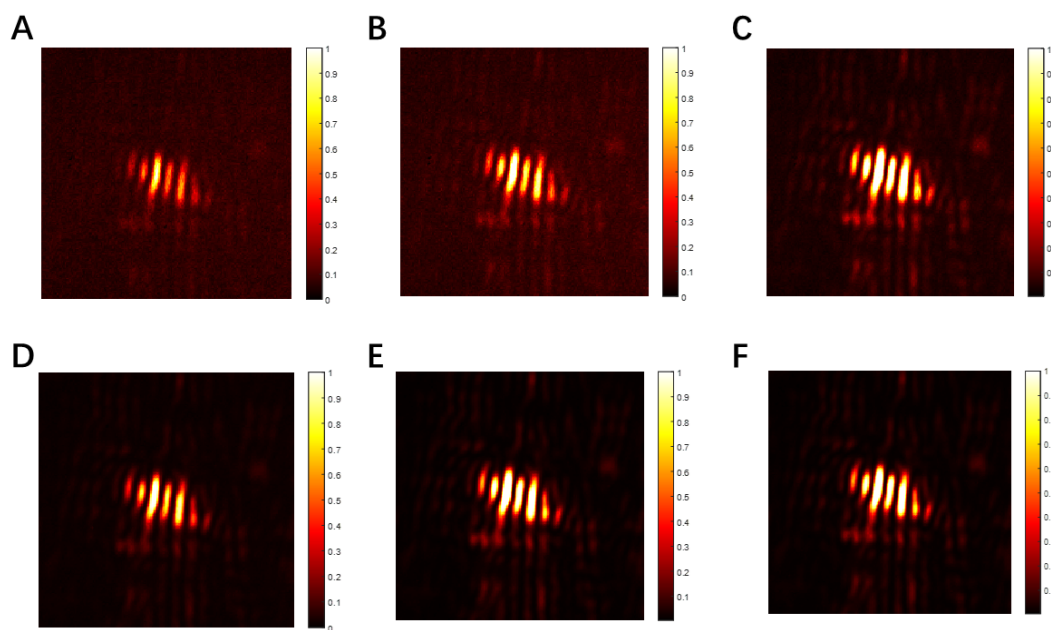


Fig. S12. The interference patterns for different pump fluences. A. $1.25 P_{th}$. B. $1.35 P_{th}$. C. $1.42 P_{th}$. D. $1.49 P_{th}$. E. $1.56 P_{th}$. F. $1.6 P_{th}$.

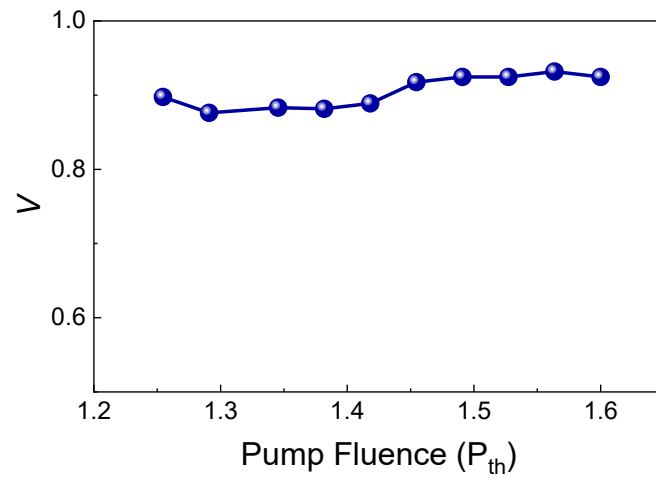


Fig. S13. The spatial coherence V of the metalaser as a function of pump fluence.

Supplementary Note-5: Quasi-BIC microlasers

In this main text, we numerically show that the polarization direction of quasi-BIC laser is determined by the direction of locally induced dipole momenta. This is the fundamental basis for the local geometric phase and the control of laser wavefront. Here we show the experimental results to support the claim in the main text. Fig. S14A shows the top-view SEM image of one metasurface. The detailed structural parameters can be seen from the high-resolution top-view and side-view SEM images (insets in Fig. S14A). Here the rotation angle of the hole is fixed at the angle of $\theta = 150^\circ$ in all nanodisks to verify the dependence of laser emission on the perturbation. Then the laser gain (PM597, Supplementary Note-3) is introduced to the PMMA film and the metasurface is optically excited by a frequency doubled Nd:YAG laser on a homemade microscope system (Methods and Supplementary Note-3). The metasurface produces a broad photoluminescence peak at low pump fluence (inset in Fig. S14B). With the increase of pump power, a narrow peak emerges at 575.7 nm and rapidly dominates the emission spectrum (Fig. S14B). The full width at half maximum (FWHM) of the peak is around 0.16 nm, giving a Q factor of 3500.

Solid dots in Fig. S14C show the integrated intensity as a function of the pump fluence. An obvious “S”-shaped curve can be seen in the log-log plot, indicating the transitions from spontaneous emission to amplification and eventually to gain saturation. Top inset in Fig. S14C is the far field intensity distribution of the metalaser. A laser spot with a divergent angle of 1.66° can be clearly seen. By recording the intensity after a linear polarizer at different rotation angle, the polarization state of the laser beam has been measured and plotted in bottom inset in Fig. S14C. It is obvious that the metalaser is linearly polarized along a direction of $\theta_F = 60^\circ$. Meanwhile, both temporal coherence (linewidth in Fig. S14B) and spatial coherence (interference pattern in the inset in Fig. S14D) are also preserved in the metalaser radiation. All the above observations, associated with the dramatic reduction in FWHM (open squares in Fig. S14C), confirm the onset of lasing action in our metasurface.

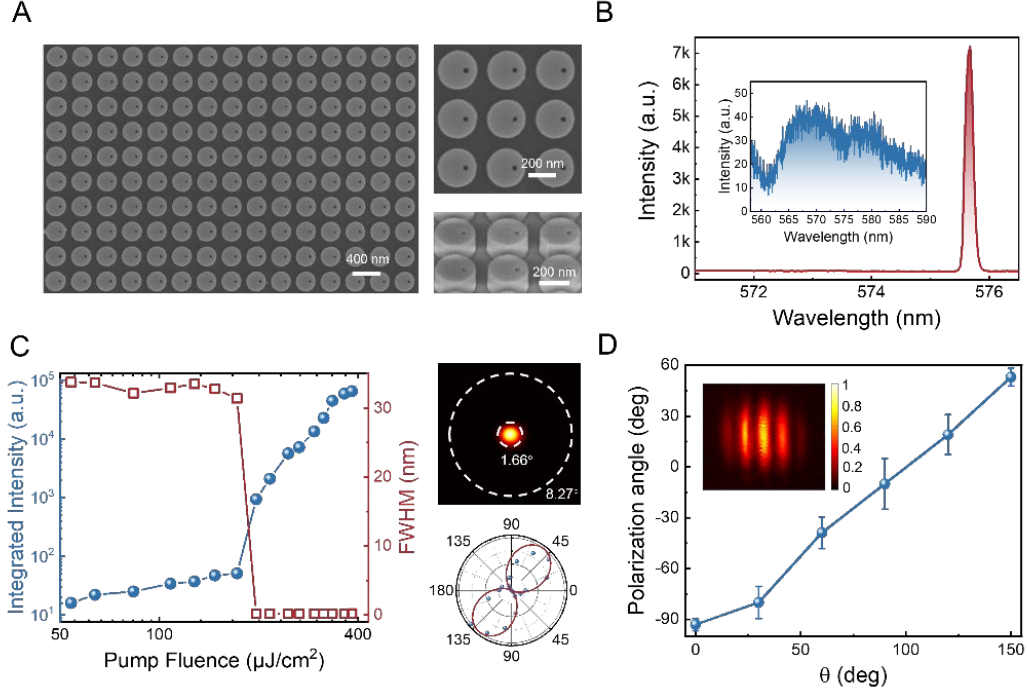


Fig. S14. Characterization of the metalaser. **A.** Top-view SEM image of the Si₃N₄ nanostructures before coating the PMMA film. Insets show the high-resolution top-view and side-view SEM images of Si₃N₄ nanodisks. Here the rotation angle θ of the hole is fixed at $\theta = 0^\circ$. **B.** The experimentally recorded emission spectra from the metasurface under the pump fluence of 265 $\mu\text{J}/\text{cm}^2$. Inset shows the emission spectra under the excitation density of 147 $\mu\text{J}/\text{cm}^2$. **C.** The log-log plot of integrated output intensity (dots) and FWHM (open squares) as a function of pump density. The far-field beam profile and its polarization are depicted as top and bottom insets of the right column. **D.** The recorded polarization direction as a function of the rotation angle θ of the hole. The double-slit interference pattern is shown as an inset to illustrate the spatial coherence of the metalaser.

With a similar process, a series of metasurfaces with different rotation angles θ of the hole have been experimentally fabricated and optically characterized. All the results are summarized in Figs. S15-S20. From the high-resolution scanning electron microscope (SEM) images (Figs. S15A-S20A), we can see the gradual change of rotation angle of the hole in different samples. The angle remains the same in each sample. The corresponding polarization angle of laser emission from the metasurfaces changes with the rotation angle too (Figs. S15B-S20B). The experimental results are summarized in Fig. S14D. The polarization angle linearly changes

with the rotation angle of the hole. All the metalasers are linearly polarized and their polarization directions change with the rotation angle of hole following a relation of $\theta_F \sim \theta - 90^\circ$. These observations are consistent with the above analysis well and demonstrate the impacts of induced local dipole moment on far-field laser polarization. As a consequence, we can expect that the laser wavefront and their radiation characteristics can be tailored if pixel-level geometric phases are assigned to the metalaser.

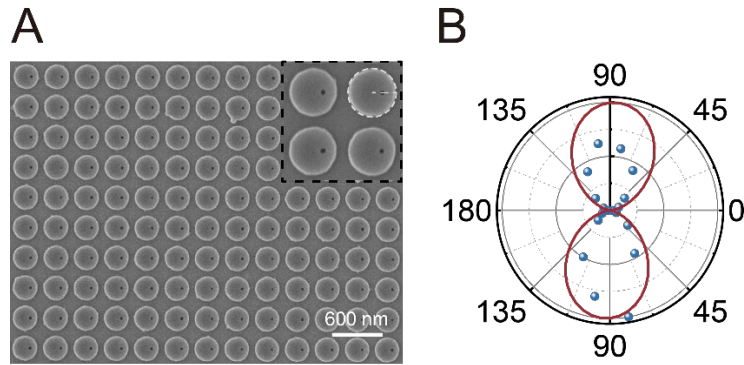


Fig. S15. The top-view SEM image of metalaser with rotation angle of 0 degree and the polarization angle of laser emission.

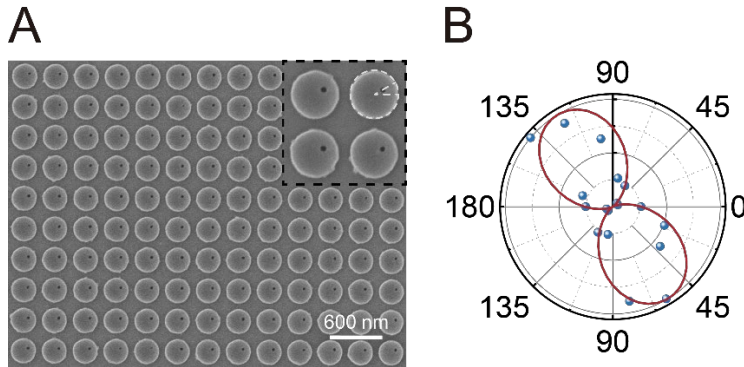


Fig. S16. The top-view SEM image of metalaser with rotation angle of 30 degree and the polarization angle of laser emission.

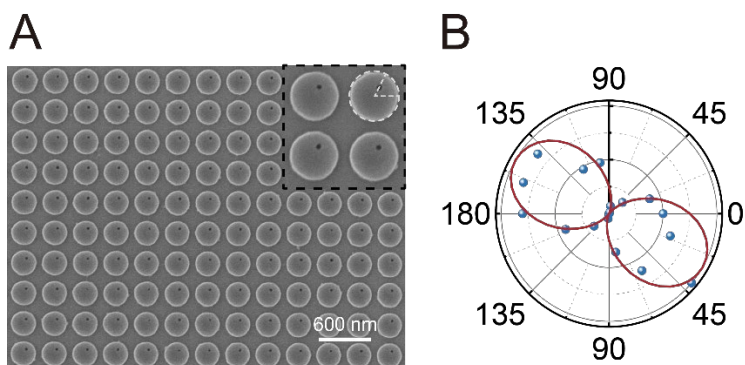


Fig. S17. The top-view SEM image of metalaser with rotation angle of 60 degree and the polarization angle of laser emission.

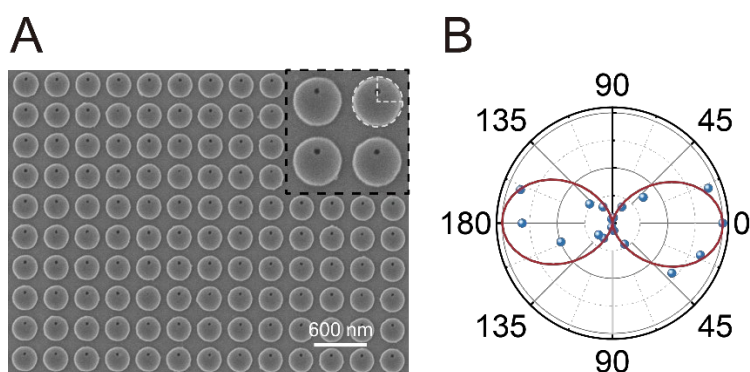


Fig. S18. The top-view SEM image of metalaser with rotation angle of 90 degree and the polarization angle of laser emission.

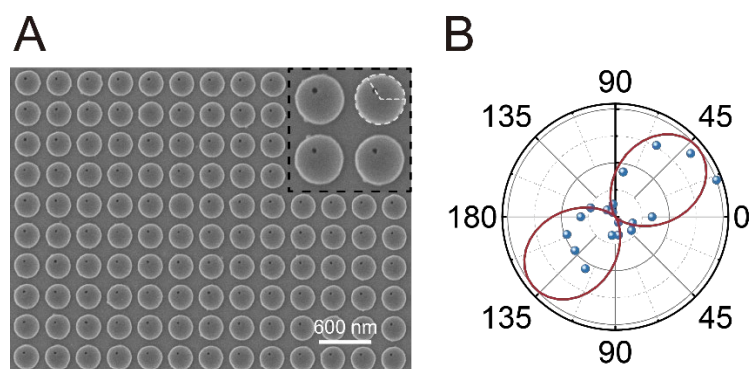


Fig. S19. The top-view SEM image of metalaser with rotation angle of 120 degree and the polarization angle of laser emission.

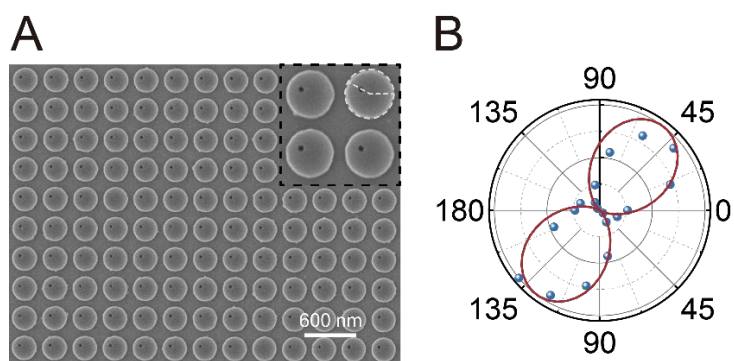


Fig. S20. The top-view SEM image of metalaser with rotation angle of 150 degree and the polarization angle of laser emission.

Supplementary Note-6: Metalasers with different radiation patterns

In the main text, we have shown that the metalasers with different predefined phase profiles can produce quite different radiation patterns. Here we show some experimental details. Fig. S21A shows the top-view microscope image of the metalaser. It is a circular disk with a diameter of 97.2 μm (see the white dashed line). The phase profile is defined following the equation of $\phi(x, y) = \frac{2\pi}{\lambda} (f - \sqrt{f^2 + x^2 + y^2})$. Here (x, y) represents the position in Cartesian coordinates, λ and f are the operation wavelength and focal length of metalens, respectively. Similar to the main text, the geometric phase is also assigned to the RCP waves from the metasurface.

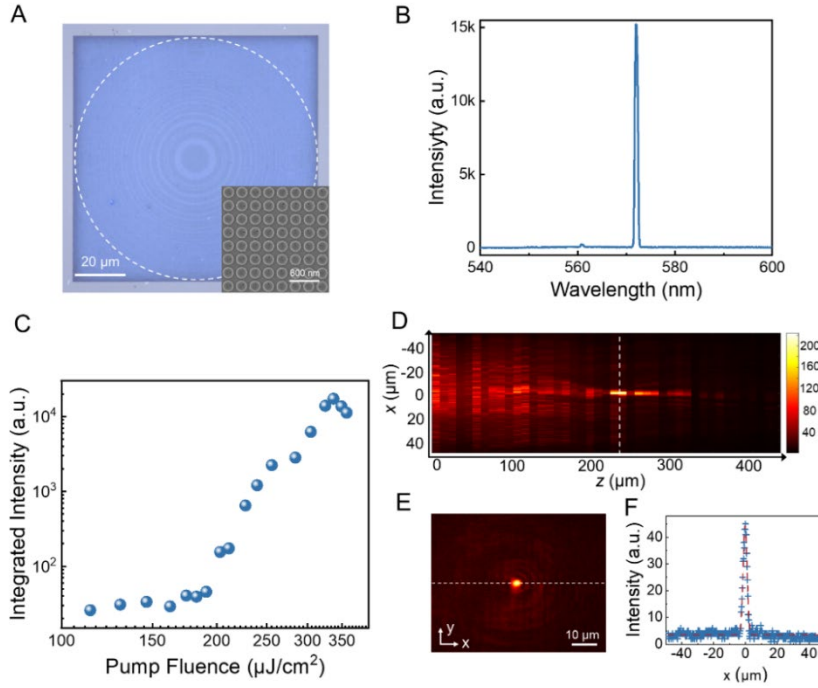


Fig. S21. Metalaser for diffraction-limited laser spot. **A.** Microscope image of the metalaser.

Here the geometric phase is defined following the lens equation $\phi(x, y) = \frac{2\pi}{\lambda} (f - \sqrt{f^2 + x^2 + y^2})$. The diameter and focal length f of metalens are 97.2 μm and 238 μm , giving a numerical aperture (NA) of 0.2. Insert shows the top-view SEM image. **B.** Emission spectrum of the metalaser with a pump fluence of 190 $\mu\text{J}/\text{cm}^2$. **C.** The integrated emission intensity as a function of pump fluence. **D.** RCP radiation profile of the metalaser in x - z plane. **E.** RCP radiation profile in x - y plane at the position of $z \sim 238 \mu\text{m}$. **F.** The corresponding intensity distribution along the diameter of focal spot, marked by dashed line in E.

Then the metalaser is optically pumped and the corresponding emission characteristics have been recorded and analyzed. The results are summarized in Figs. S21B and S21C. With the increase of pump power, both the transition from broad photoluminescence to a single narrow peak and the “S”-shaped intensity curve can be observed, clearly confirming the lasing action in our metasurface. The interesting phenomenon happens in the far field. As depicted in Fig. S21D, the RCP emissions of the metalaser focus on x - z plane and become a bright spot at a position of $z \sim 238 \mu\text{m}$. This is a direct proof that the designed polarized emissions of our metalaser focus directly to a laser spot at far field. Figs. S21E and S21F are the corresponding focal spot in x - y plane and the intensity distribution along its diameter. It is obvious that the nanolaser emission has been directly focused to a diffraction-limited laser spot at far field.

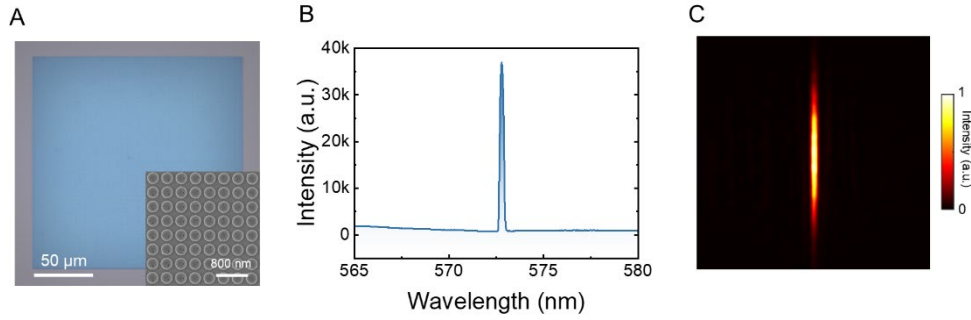


Fig. S22. Metalaser with a phase profile of cylindrical lens. **A.** Top-view microscope image of the metalaser and the corresponding high-resolution SEM image (inset). Here the phase profile of five cylindrical lenses is assigned to the metasurface. The width of each cylindrical lens is $36 \mu\text{m}$ and the focal length is $f = 88.2 \mu\text{m}$. The numerical aperture of metalens is also 0.2. **B** and **C** are the corresponding laser spectrum and the far-field RCP focal line in x - y plane.

One of the most important properties of metalaser is that the laser wavefront and emission patterns can be designed at will. To verify this potential, we have fabricated another metalaser with the phase profile of a cylindrical lens that is defined as $\phi(x) = \frac{2\pi}{\lambda} (f - \sqrt{f^2 + x^2})$. It is clear that the phase along y -direction has not been defined. Fig. S22A shows the corresponding high-resolution SEM image. Then entire metasurface is $180 \mu\text{m} \times 180 \mu\text{m}$ and contains five cylindrical metalenses. The rotational angle of the hole θ varies only in x -direction and remains the same in y -direction. When the metasurface is optically pumped, the corresponding emission spectrum and RCP beam profile are shown in Figs. S22B and S22C. While the metalaser also

produces a single-mode laser at a similar wavelength, the RCP emission from the metalaser now becomes a focal line at a distance of 88.2 μm away from the metalaser. All these experimental results are consistent with conventional cylindrical metalens and demonstrate that the emission pattern can be customized by the assigned phase profile. Then the emission pattern between focal spot and focal line can all be achieved by tailoring the phase equation.

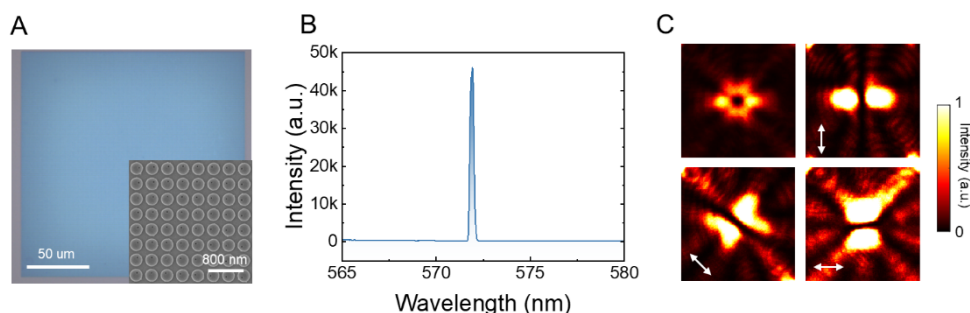


Fig. S23. High-order vectorial emission from metalaser. **A.** Top-view microscope image of the metalaser. The rotation angles of the holes in nanodisks provide a spiral phase profile covering $0 \sim 10\pi$ to RCP components. For the LCP components, the corresponding phase is opposite and becomes $0 \sim -10\pi$. Inset shows the high-resolution SEM image (inset). **B.** The corresponding laser spectrum of the metalaser. **C.** The far-field patterns of the metalaser. An obvious donut with zero intensity at the center can be clearly seen (top left panel). The other three insets show the beam profiles of the donut passing a linear polarizer along 0° , 45° , and 90° , respectively. All the profiles in C match the fifth order vector beam very well. Such a quasi-BIC metalaser is intrinsically different from the vector beams from BIC metasurfaces, whose topological charges are quite restricted to a few numbers.

Our metalaser also has the capability of producing the vector beams. Vector beams can be produced from the BIC metasurfaces. The topological structures in momentum space typically give vector beams at BIC wavelengths. However, the topological charges are typically fixed to small numbers of $l = \pm 1$ or ± 2 . Our metalaser can simply tackle this limitation. One example is shown in Fig. S23. From the top-view SEM image and microscope image, we know that a spiral phase from 0 to 10π has been assigned to the metasurface (Fig. S23A). When the metasurface is optically excited by a Nd:YAG laser, a narrow laser peak has been achieved at around 572 nm (Fig. S23B). The corresponding far-field patterns have been recorded and

shown in Fig. S23C. A donut shape with a zero intensity at the center can be observed at the normal direction. The patterns after passing a linear polarizer of 0° , 45° and 90° have been shown in Fig. S23C too. All the profiles match the 5th order vector beam very well.

The vector beam at the normal direction can be considered as a combination of two vortex beams with opposite topological charges, i.e., $l = 5$ and $l = -5$, respectively. In this sense, if these two beams are separated, two vortex beams with phase singularities can also be achieved. Here we realize this purpose by introducing a deflection phase profile following the equation $\theta_d = \frac{2\pi}{\lambda} \sin(\rho) x$ to the metasurface. Here ρ is the deflection angle and fixed at 10° . In this sense, the overall geometric phase is $\phi(x, y) = \theta_{spiral} + \theta_d$. The top-view microscope image and SEM image are shown in Fig. S24A, and the corresponding phase profile is depicted in Fig. S24B. When the metasurface is optically excited at the center of the metasurface, a narrow peak has also been achieved around 570 nm (Fig. S24C) and two donut beams can be seen at 10° and -10° . The corresponding interference patterns passing a cylindrical lens are also shown in Fig. S24E. It is clear that these two donuts have opposite topological charges, consistent with the design very well. This is because LCP and RCP components experience opposite geometric phases. If one circularly polarized light is deflected to 10° , the light with opposite spin must be deflected to -10° .

We further consider another case that the metasurface is excited by a beam away from the center of the metasurface. As depicted in Fig. S24B, now the emission beams cannot experience the phase singularity. In this sense, the laser emissions are still deflected to 10° and -10° due to the deflection phase profile. The donut beams, however, are degraded to conventional Gaussian beams. As shown in Fig. S24F, such emission beams are high-purity chiral beams. Till now, we know that both high-purity chiral lasing and high-order vortex beams can be achieved from the metalaser too. All the above experimental demonstrations confirm the versatility of the proposed mechanism very well.

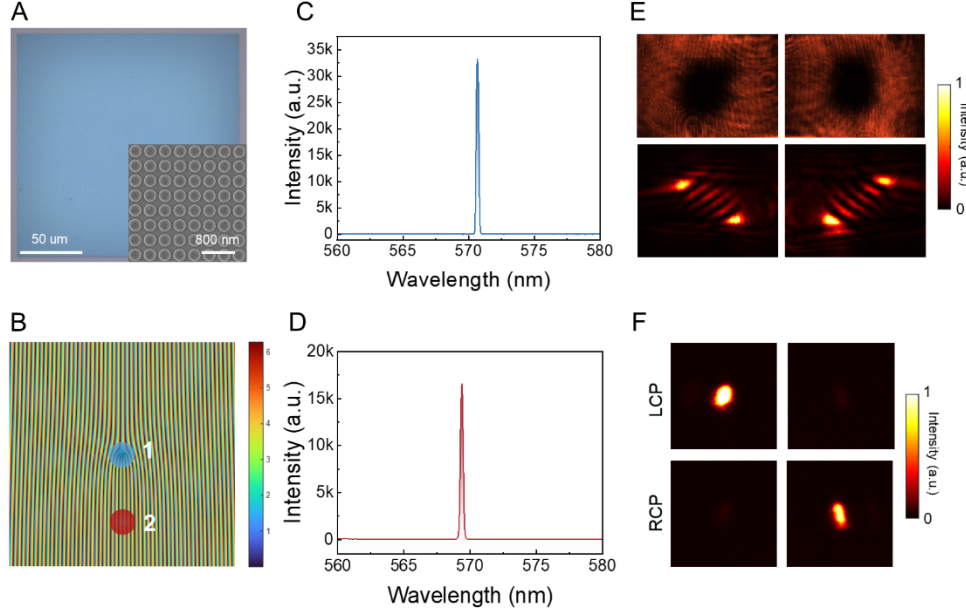


Fig. S24. High-order vortex emission from metalaser. **A.** Top-view microscope image of the metalaser. Phase profiles θ_{OAM} of $0 \sim 10\pi$ and $0 \sim -10\pi$ have been assigned to RCP and LCP components, respectively. Meanwhile, another deflection phase profile following the equation $\theta_d = \frac{2\pi}{\lambda} \sin(\rho) x$ has also been added to the metasurface to deflect RCP and LCP components from the normal direction by an angle of $\rho = 10^\circ$. In this sense, vortex beams with topological charge of $l = \pm 5$ are separated into two opposite directions. **B.** The phase profile and the excitation positions on the metasurface. Position-1 covers the phase singularity and position-2 doesn't cover the phase singularity. **C** and **D** are the spectra of the metalaser recorded from two excitation positions in **B**. **E.** The beam profiles of two donuts at $\pm 10^\circ$ (top row) and their patterns (bottom row) passing a cylindrical lens. Here the beams are generated from excitation position-1. Five dark lines along opposite direction can be seen in the interference patterns, confirming the generation of vortex beams with $l = 5$ and -5 from the metalaser. **F.** LCP and RCP beam profiles recorded from excitation position-2 at 10° (left column) and -10° (right column). Due to the lack of phase singularity, only the deflection phase works and high-purity chiral lasers with near-unity chirality have been achieved. These results demonstrate that both the vortex beams with designed topological charges and high-purity chiral beams can be directly produced at designed angles by a single metalaser.

Supplementary Note-7: Towards programmable metalasers

In the main text, we have demonstrated that the metalaser can produce arbitrarily shaped wavefront and far-field radiation characteristics. Here we provide more results to show that the programmable metalaser in the usual sense is also possible. Similar to spatial light modulators, materials with variable refractive indices are highly desirable for the programmable functionality of metalasers. Here we utilize phase change materials such as Sb_2S_3 and VO_2 and illustrate this potential with several examples. Notably, the phase transition time of phase change materials determines the reconfiguration speed of the programmable metalaser. The time scale of photon-induced phase transition for VO_2 is hundreds of femtoseconds. Specifically, a phase transition time of ~ 100 fs was obtained from measurements⁶ with a pump fluence of 25 mJ/cm^2 . In addition, we have experimentally measured the transition time of Sb_2S_3 from crystalline to amorphous phase under the excitation of femtosecond laser in previous works, which is as short as 800 ns ⁷. These short time scales of photon-induced transition for Sb_2S_3 and VO_2 indicate that the reconfiguration speed of programmable metalasers can be very fast.

(1) *Chiral laser emission with controllable angle*

Highly integrated chiral laser emission is quite essential for many on-chip optical and quantum information processing. In 2022, Zhang et al. exploited chiral quasi-BIC to achieve chiral nanolaser emission in the normal direction⁸. One year later, Chen et al. further utilized the spin-valley locking effect to extend chiral laser emission to tilted angles⁹. In Fig. 3C of the main text, we have shown that both novel functions can be easily realized with our metalaser. Using metalasers, we no longer need to consider either quasi-BIC or spin-valley locking, but only need to design the phase distribution based on the required emission angle.

Below, we show that our metalaser can unlock even more performances. Fig. S25A shows the schematic diagram of the unit cell in our metalaser. It is almost the same as the one in Fig. S24 except that the dielectric Si_3N_4 is replaced with Sb_2S_3 whose refractive index can be optically or electrically tuned. The corresponding phase profile is defined following the equation $\theta_d(x) = 2\pi \sin(\rho) x/\lambda$. By placing dipoles into the metasurface, we simulated the emissions from the Sb_2S_3 metasurface. As solid lines depicted in Fig. S25B, LCP and RCP laser

beams can be achieved at $\pm 15.2^\circ$, respectively. Then the Sb_2S_3 is optically or electrically tuned. The phase change increases the refractive index of Sb_2S_3 from 3.00 to 3.68. As a consequence, the LCP and RCP emissions are tuned to $\pm 18.3^\circ$ (dashed lines in Fig. S25B).

Phase change materials such as Sb_2S_3 have a series of intermediate states. Then their refractive index can be continuously tuned between the amorphous phase to crystalline phase. Using these intermediate refractive indices, we have simulated the angles of chiral emissions and plotted the results in Fig. S25C. It is obvious that the radiation angles of LCP emission continuously increase from 15.2° to 18.3° . The corresponding RCP component also continuously decreases from -15.2° to -18.3° . This is the first time that the chiral emission angle can be continuously controlled. Since the phase change materials can be changed back and forth, the continuous angle control is also reversible.

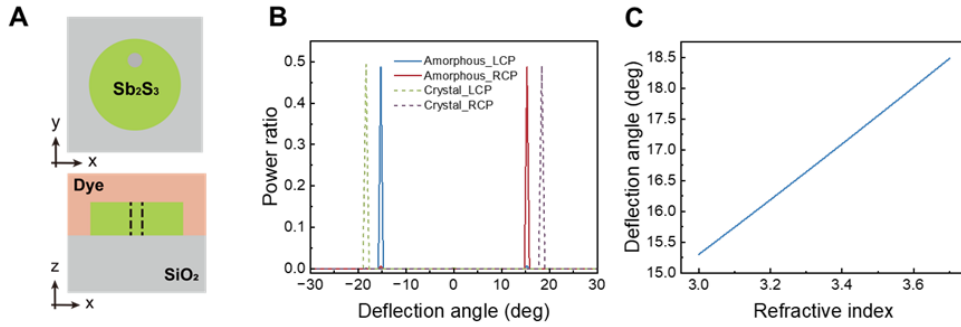


Fig. S25. Tunable angles of chiral emissions. **A.** Schematic of the unit cell of the metalaser composed of Sb_2S_3 and PMMA. **B.** Emission angle of LCP and RCP light before (solid line) and after (dashed line) phase transition. **C.** The emission angle of LCP radiation as a function of intermediate refractive index.

(2) Chiral laser emission with controllable intensity difference

In Fig. S25 and Ref. 9, the intensities of LCP and RCP lasers at opposite tilt angles are always identical. Here we show that their relative intensities can also be tuned with the phase change materials. Fig. S26A shows the schematic of the unit cell. Our metasurface is formed by square-latticed periodic nanostructures (Fig. S26A). The lattice size is $p = 400$ nm. Each unit cell contains two GaP ($n = 3.5$) nanodisks with radius of $R = 150$ nm and thickness of $h = 100$ nm that are separated by a thin layer of VO_2 with thickness of $d = 300$ nm. The entire

nanostructure is embedded in PMMA. Two small holes with radius of $r = 20$ nm are patterned into two nanodisks with polar coordinates of (ρ, θ) and $(\rho, \theta + \delta)$, respectively. Here we keep $\rho = L + r = 70$ nm. As discussed in the main text, the in-plane electric field within the small hole is significantly enhanced. Then the electric fields in two nanodisks are equivalent to two dipoles with an included angle δ (inset in Fig. S26A). Meanwhile, these two dipoles are vertically separated by a fixed distance of $d = 300$ nm.

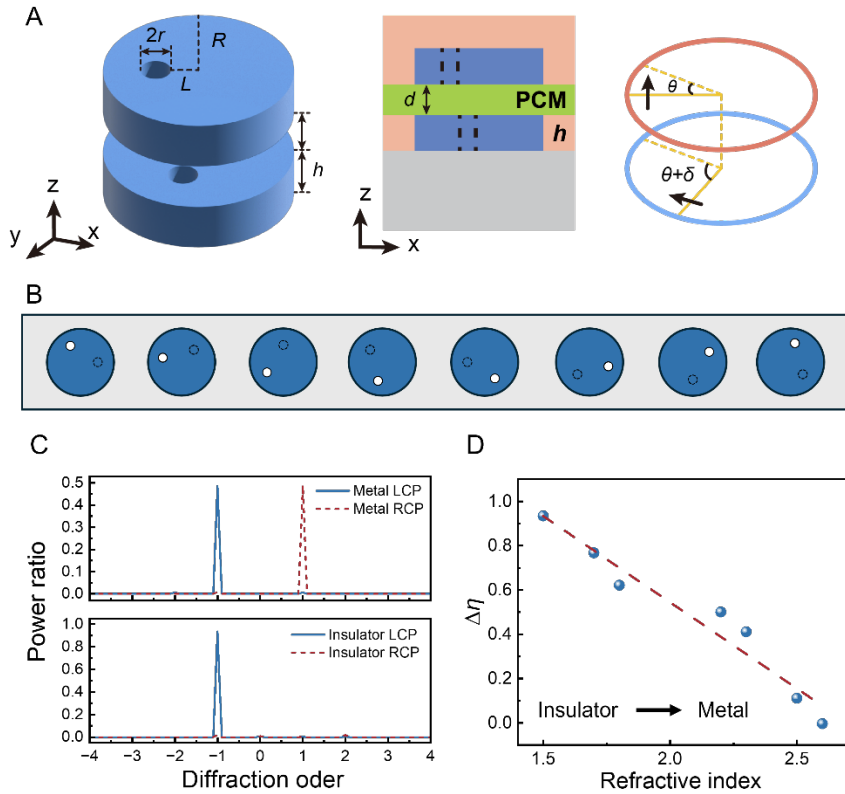


Fig. S26. Tunable relative intensity of chiral emission. **A.** Schematic of the unit cell with all structural parameters. **B.** The top-view of the supercell with a deflection phase profile. The white hole and black circle represent the positions of two holes in top and bottom nanodisks. The included angle is fixed at 120° . **C.** Far-field angular distributions of LCP and RCP emissions before (bottom panel) and after (top panel) phase transition. **D.** Intensity difference between LCP and RCP as a function of index of VO_2 .

Two equivalent dipoles with included angle δ and vertical separation d can break the symmetry between LCP and RCP components following the factor of $\sin(\delta \pm kd)$. Fig. S26B shows the top-view of the supercell with a deflection phase profile. The white hole and black

circle represent the positions of two holes in top and bottom disks. Here we fix the included angle $\delta = 120^\circ$. For the case of insulating phase, we can see that the RCP component is completely cancelled, giving a pure LCP emission (bottom panel in Fig. S26C). The phase transition of VO_2 increases the refractive index and thus changes kd . Since the included angle δ is fixed, the factor $\sin(\delta \pm kd)$ shows that some LCP components are coupled to RCP components. Then RCP emission gradually increases and LCP emission reduces, eventually leading to a balanced LCP and RCP emissions (top panel in Fig. S26C). Fig. S26D summarizes the intensity difference $\Delta\eta = |(I_{LCP} - I_{RCP})/(I_{LCP} + I_{RCP})|$ as a function of phase transition of VO_2 . During the phase transition, it continuously reduces from a factor of 1 to 0, corresponding to the transition from unidirectional emission with a particular spin angular momentum (SAM) to bi-directional laser beams with opposite SAMs. Once again, the refractive index of phase change materials now is close to the refractive index of nanodisk. The dependence can be more linear if we utilize liquid crystals with lower refractive index.

(3) *Controllable OAM nanolasers*

The above mechanism for tunable emission angle and relative intensity is also applicable to OAM nanolaser in Fig. 3D of the main text well. It is almost the same except that a spiral phase profile is introduced. Here we show that the topological charge l of the OAM beam can also be controlled with the phase change material. Fig. S27A shows the overall configuration of the phase plate with $|l| = 6$. It is obvious that the phase plate has been divided into 6 regions with different colors. Each section contains 8 subsections with phases of $0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$ (see schematic in Fig. S27A). Before the phase transition, the radiation from the metasurface corresponds to two OAM beams with $l = 6$ and $l = -6$, respectively.

Then we selectively excite regions with green colors and induce the transition of Sb_2S_3 from amorphous state to partial crystalline state. The increase in the refractive index causes the red shift of the overall band structure of the metasurface, resulting in the resonant wavelength in the amorphous region falling within the bandgap of the phase change region (Fig. S27B). Light in the amorphous region cannot enter the phase change region, thus destroying the long-range interaction between the phase change regions and the rest ones. As a consequence, the emissions in phase change regions are rapidly reduced to ~ 0 . One example is illustrated in Fig.

S27C. Light is widely distributed in all cells before phase transition. After the selective phase transition in different regions, light can only be confined within the unchanged area. The contribution to the total emission from the phase change regions can be neglected. In this sense, the metalaser after selective transition can be considered as a type of metalaser with “1” (amorphous) and “0” (crystalline) states.

With the above information, we can thus simply control the topological charge of metalaser by selectively changing the phase of some regions. For the case that regions with green color are phase-changed, we then numerically calculated the emission profiles from the metalaser. The results are shown in Fig. S27D. The outputs still show a donut-shaped beam with vanished center intensity. But the shape is different and the corresponding phase profiles give topological charges of $l = \pm 3$ now. With a similar process, the topological charge can be tuned to the other number as well. In addition, due to reversibility of Sb_2S_3 , the transition between different topological charges can be also tuned back and forth.

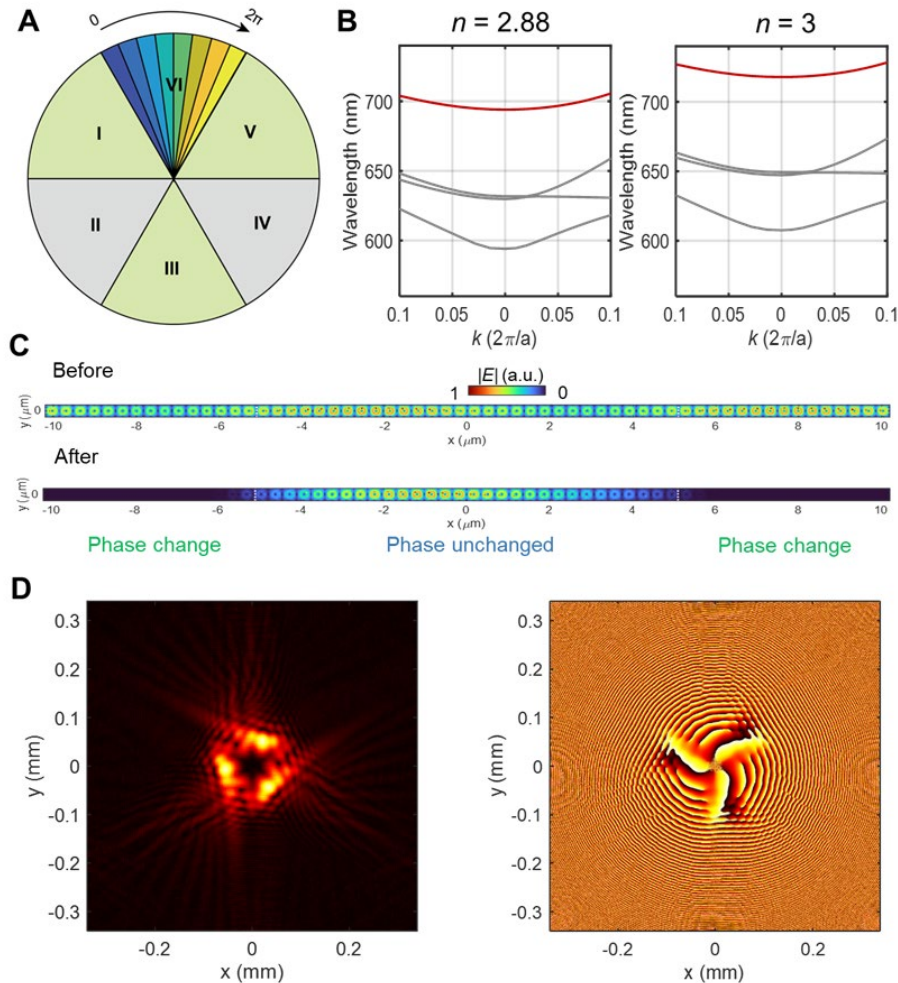


Fig. S27. Tunable topological charge of OAM emissions. **A.** The overall schematic picture of the metalaser phase plate. Phase transition of Sb_2S_3 happens in green-color regions. **B.** The band structures of green-color regions before and after phase transition. **C.** An example to illustrate the field distributions before and after the partial phase transitions. **D.** The calculated far-field emission intensity and its phase profile.

(4) *Programmable metalaser holograms*

In addition to the control of laser emission direction, relative intensity, and topological charge, our metalaser also has the capability of producing programmable metalaser holograms. Here we consider a simple case to realize a real programmable metalaser. Fig. S28A shows the schematic of our meta-atom. It is a GaP nanodisk (with refractive index $n = 3.5$) with two tiny holes with an included angle of $\Delta\theta$. One hole is filled with phase change materials of VO_2 whose refractive index can be tuned from $n = 1.5$ (insulating phase) to $n = 3$ (metallic phase). For the case of metallic phase, its refractive index is almost the same as GaP and the geometric phase is purely determined by the PMMA hole. When VO_2 is in insulating phase, the geometric phase is produced by two holes simultaneously and thus the included angle starts to play an essential role. Fig. S28B shows the overall geometric phase of the meta-atom as a function of angle $\Delta\theta$. It is clear that the geometric phase can have a difference close to π before and after the phase transition when $\Delta\theta$ approaches 360° . As a consequence, the recently developed concept of digital metasurfaces can be applied here to produce many different functions from a single metalaser¹⁰.

Single meta-atom is quite small and relatively difficult to control independently. Here we utilize 2×2 pixels as a supercell with a size around $1 \mu\text{m}$. This size is easy for alignment and to selectively change the phase of VO_2 . Meanwhile, VO_2 can be switched between insulating phase and metallic phase back and forth. Following the above discussion, we have defined four different phase profiles (Fig. S28C) consisting only of geometric phases of 0 and π . Fig. S28D shows the corresponding holographic images. With the same metalaser, different characters of “M E T A” in ancient Greek can be achieved. Due to the reversibility of phase change materials and micrometer sized supercell, all these profiles can be erased and re-written in experiments. There is no fundamental limitation in function number and many other holographic images can

be realized with the same sample too. In this sense, the entire metalaser becomes programmable. Besides holographic images, this digital-metasurface-based concept can also be applied to tunable lens and other functions too. In this sense, we can confirm that our design also has the potential to realize programmable metalasers in common sense.

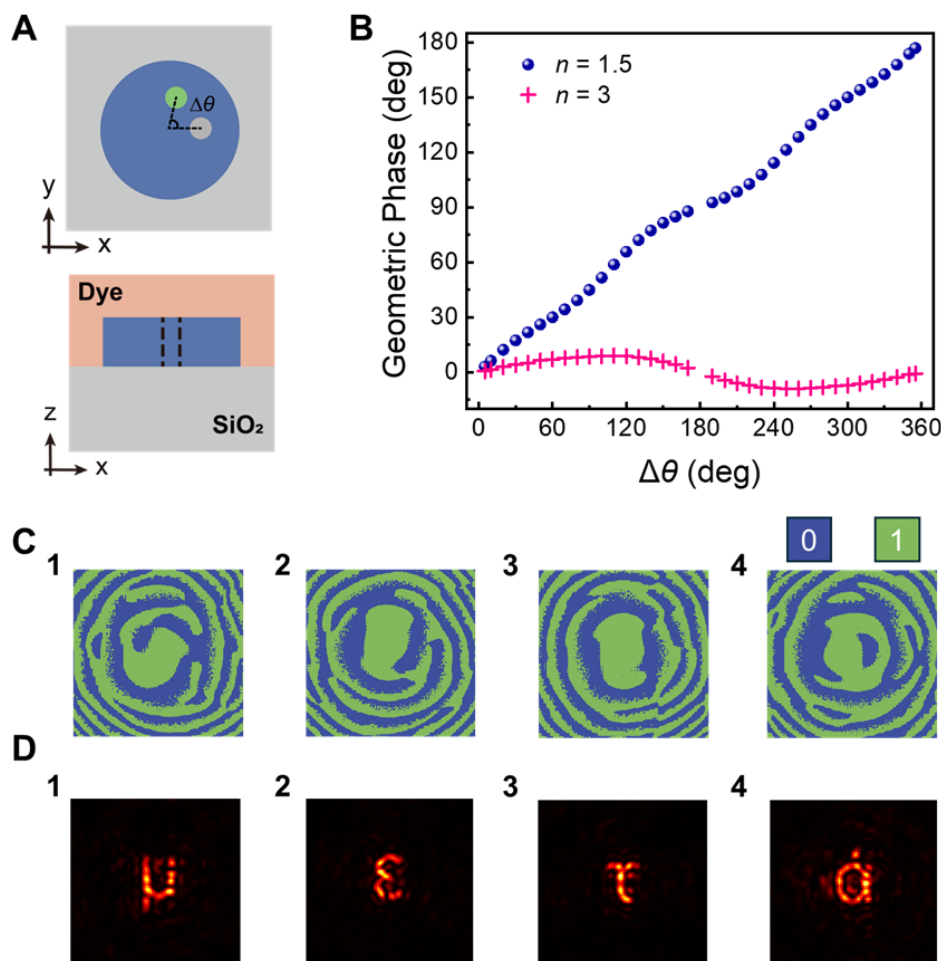


Fig. S28. Programmable metalaser hologram. **A.** The schematic of the meta-atom for programmable metalaser. The entire sample is also patterned with a square lattice. Two holes with PMMA and VO₂ have an included angle $\Delta\theta$ and are marked with different colors. **B.** The difference of geometric phase before ($n = 1.5$) and after ($n = 3.0$) phase transition as a function of the included angle between two holes. **C.** Four phase profiles of digital metalasers defined by the phase switching caused by the phase transition of VO₂. Here the included angle is fixed at 340°. **D.** The corresponding holographic images of “M E T A” in ancient Greek produced by four phase profiles in C. The selection of phase transition in the same metalaser can produce different holographic images, enabling the possibility of programmable emission.

Supplementary Note-8: Metalaser hologram of “ π ”

In the main text, we have shown another holographic image of “ π ”. Here we provide some experimental details. The samples are fabricated and optically characterized with the same processes as Supplementary Note-3. The top-view microscope image is shown in Fig. S29A and the corresponding SEM images are depicted in Fig. S29B. From the high-resolution SEM image, we know that the rotation angle θ varies in each Si_3N_4 nanodisk as the numerical design. As the phase profile of geometric phase is predefined, the corresponding laser emission is customized.

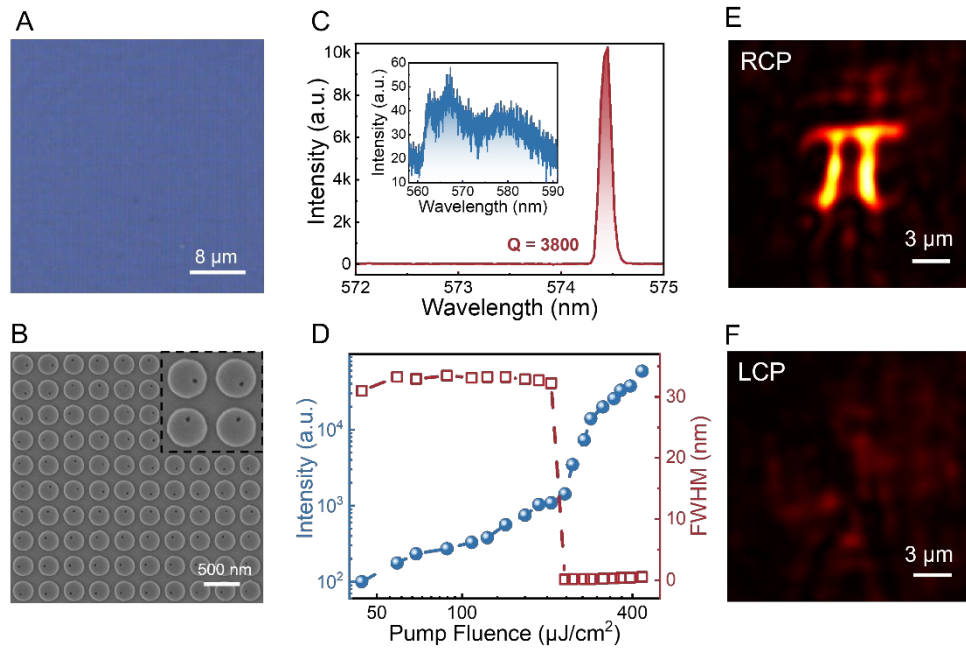


Fig. S29. Experimental demonstration of the holographic image of π . **A.** Microscope image of the Si_3N_4 nanostructures before coating the PMMA film. Here the size of the metasurface is $36 \mu\text{m} \times 36 \mu\text{m}$. **B.** Top-view SEM image of the metasurface. Inset shows the high-resolution SEM image. Recorded emission spectra under a pump fluence of $245 \mu\text{J}/\text{cm}^2$ and $196 \mu\text{J}/\text{cm}^2$ (inset), respectively. **D.** Integrated output intensity (dots) and FWHM (open squares) of emission spectra as a function of pump fluences. **E** and **F** are the experimentally recorded RCP and LCP far-field emission profiles. Here the pump fluence is fixed at $1.5 P_{\text{th}}$.

Similar to the others, here the metasurface also produces a broadband photoluminescence under a low excitation and gives a narrow peak at high pump fluence (Fig. S29C). With the

integration of emission intensity at different pump fluences, a clear threshold can be observed at $\sim 206 \mu\text{J}/\text{cm}^2$. The FWHM at the threshold is about 0.15 nm (Fig. S29D), giving a Q factor of 3800. When the excitation is above laser threshold, obvious laser patterns can be seen at far-field, as shown in Figs. S29E and S29F. Consistent with the design, the RCP emissions produce a clear holographic image of “ π ”, whereas the LCP waves only produce some cluttered spots.

When the excitation is above the laser threshold, we have also recorded the near-field microscope image of the metalaser hologram. The results are plotted in the left panel of Fig. S30. Since the pump laser is a Gaussian beam with maximal intensity at the center, the brightest intensity has also been achieved at the center of the sample. Meanwhile, several ring-like patterns can also be observed. This is caused by the particular phase profile of the metalaser. For a direct comparison, we have numerically calculated the near-field emission pattern from the metasurface by adding a Gaussian amplitude to the phase profile. Obviously, ring-like patterns are also achieved and shown in the right panel of Fig. S30.

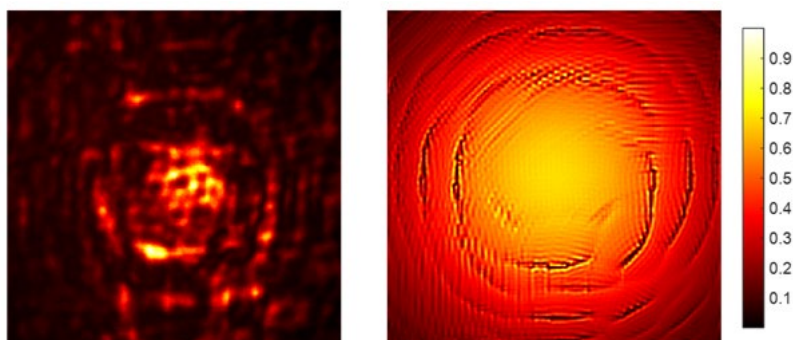


Fig. S30. Experimentally recorded (left) and numerically calculated (right) near-field intensity distributions of the metalaser hologram of “ π ”.

Another important information in Fig. 4C of the main text is that the background noises in different samples are repeatable. This is a direct proof that such non-uniform spots are caused by our design rather than the speckle noise. Here we provide more details to compare the background signals at different positions. Two holographic images of “ π ” are recorded and plotted in Fig. S31. To visual the background signals, the main image of “ π ” is highly saturated. Then we randomly selected ten different lines in horizontal and vertical directions, respectively. These lines are marked as X1-X5 and Y1-Y5. The intensities of background signals from two samples along these 10 lines are plotted together and shown in Fig. S31 below. Similar to the

one in the main text, the relative intensities in two samples are slightly different. But the peak positions of two different samples match one another very well in all lines, further demonstrating the repeatability of the background signals out of the main “ π ” image in two different samples. As a consequence, we have more confidence in the conclusion that the background spots are caused by the design rather than speckle noise.

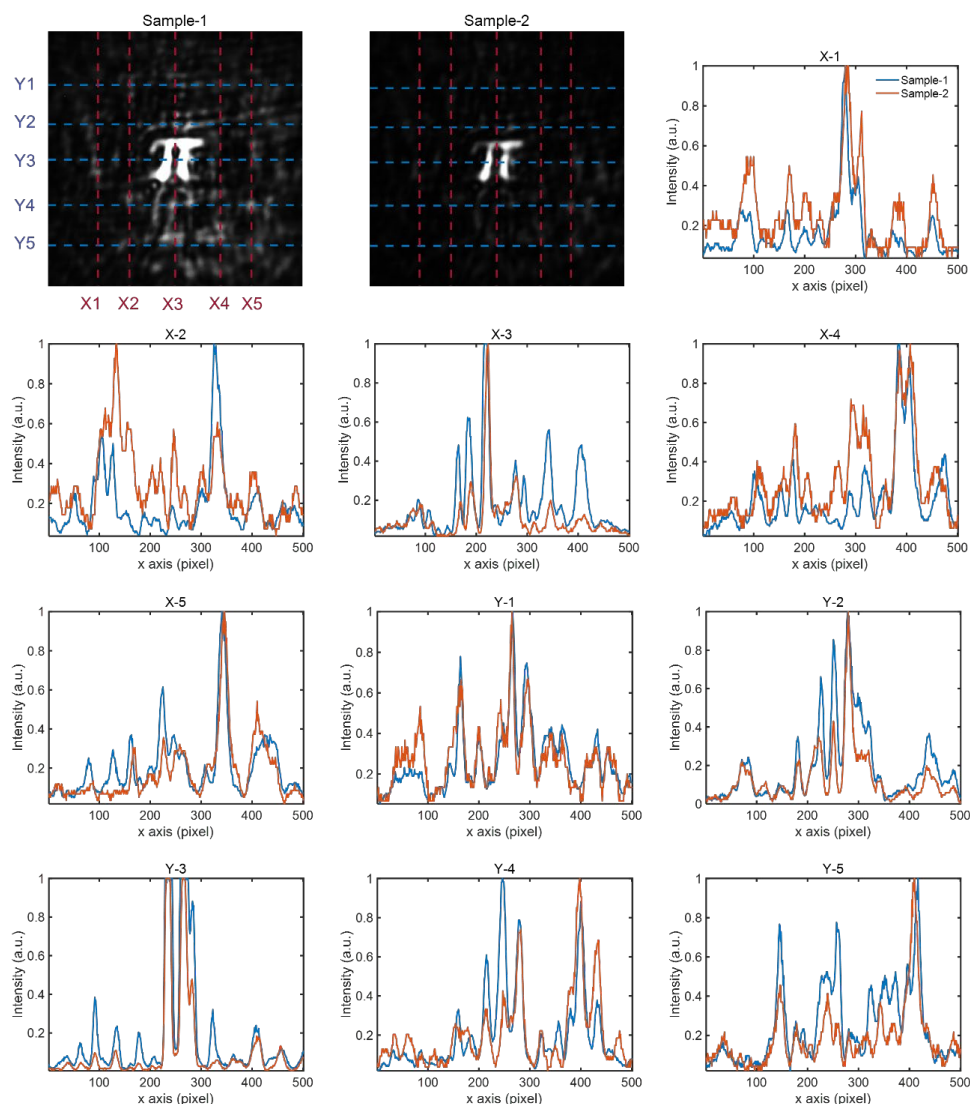


Fig. S31. Two holographic images of “ π ” from two different samples. To magnify the background signals, the main image of “ π ” is largely saturated. Ten lines are randomly selected and marked as X1-X5 and Y1-Y5. The intensity distributions of two different samples along these ten lines are plotted together. All the peak positions match one another very well.

Supplementary Note-9: On non-uniform intensity of holographic images

In the main text, we have demonstrated that the metalaser can reduce the speckle noise without degrading the image quality. However, in experimental observations, there is still obvious inhomogeneity in both the designed image and the background. Below, we will discuss the dominant reason for such inhomogeneity and exclude the possibility of speckle noises. In general, both design and manufacture error also play essential roles and shall affect the homogeneity of the metalaser hologram in experiment. In principle, the relatively low homogeneity in Fig. 2 and Fig. 4 of the main text can be attributed to the following reasons.

(1) Wavelength deviation

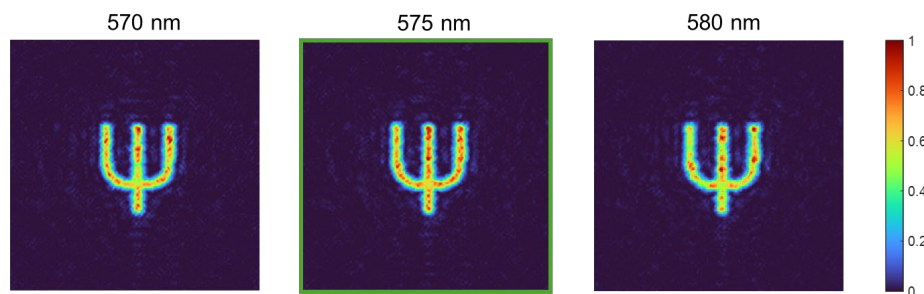


Fig. S32. Impact of wavelength deviation. Calculated holographic images at wavelengths of 570 nm (left), 575 nm (middle), and 580 nm (right). Here the phase profile of the metasurface is designed for the wavelength of 575 nm.

In the phase design process for the holographic image, the light propagations are simulated using the angular spectrum method. The operation wavelength is thus a key parameter of the phase design. Due to slight deviations in dimensions during nanofabrication, the laser wavelength in experiments usually differs by ~ 5 nm from the designed wavelength. Such wavelength deviations will not completely destroy the designed holographic image, but will significantly reduce its uniformity. One example is shown in Fig. S32. The phase distribution of metasurface is optimized for the wavelength of 575 nm. As shown in the central panel of Fig. S32 (also plotted as Fig. 1E of the main text), the holographic image shows relatively good uniformity. When the operating wavelength deviates by 5 nm, as shown in the left and right panels, the intensity variation becomes more significant and approaches the experimental results in Fig. 2 of the main text (with a wavelength deviation of ~ 5 -6 nm). We believe that this

is one of the reasons for the inhomogeneity of experimentally recorded metalaser holograms. Such a wavelength deviation is not a fundamental limitation of the principle and can be technically solved by optimizing the nanofabrication process. For example, a standard deviation in nanofabrication below 0.6 nm has been recently demonstrated¹¹.

(2) Phase deviation

Phase accuracy is another issue that impacts the homogeneity of holographic images. In our experiments, we employ Si_3N_4 and dye-doped PMMA to construct the metalaser. The refractive index difference between them ($n_{\text{Si}_3\text{N}_4} = 2.02$ and $n_{\text{PMMA}} = 1.49$) is only $\Delta n = 0.53$. This index difference is too low to largely confine light within each pixel and fully suppress their proximity interactions, leading to a nonlinear dependence of polarization angle on the hole rotation angle (Fig. 1 of the main text) and inaccurate phases. The inaccurate phases affect the homogeneity greatly. Taking a metasurface with a cylindrical phase as an example, we calculate the eigen-field of this metalens structure. As shown in Fig. S33A, we can see that the RCP component of the emission from the metalens has been focused to the designed focal point, but the phase fluctuation is very significant. For a direct comparison, the emission wavefront of a metalens composed of GaP ($n_{\text{GaP}} = 3.5$) and PMMA has also been calculated. It is obvious that the phase fluctuation becomes much smaller in Fig. S33B.

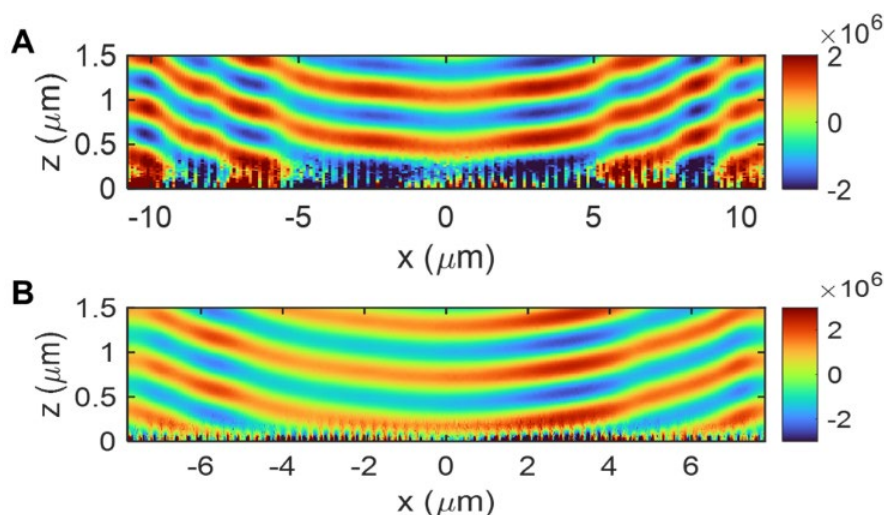


Fig. S33. Impact of phase inaccuracy caused by low-index difference between Si_3N_4 and PMMA. **A.** Calculated radiation field from a metalaser lens composed of Si_3N_4 and PMMA. **B.** Calculated radiation field from a metalaser lens composed of GaP and PMMA.

To clarify the influence of the phase inaccuracies, we added random phase deviations ($\Delta\varphi(x, y)$) to the designed hologram phase distribution. The corresponding holographic images were then calculated and shown in Fig. S34. We found that the uniformity of the hologram drops sharply with the increase of the phase deviation. Phase deviations between 0 and $\sim 0.2\pi$ can produce non-uniform holograms similar to the experimental results. As mentioned above, phase inaccuracies are caused by the small index difference between Si_3N_4 and PMMA. This problem can be alleviated by constructing metalaser holograms using III-V semiconductors with higher refractive index for the visible spectrum, e.g., GaP ($n_{\text{GaP}} \sim 3.5$)-PMMA, GaP-Air or GaN ($n_{\text{GaN}} \sim 2.4\text{-}2.5$)-Air.

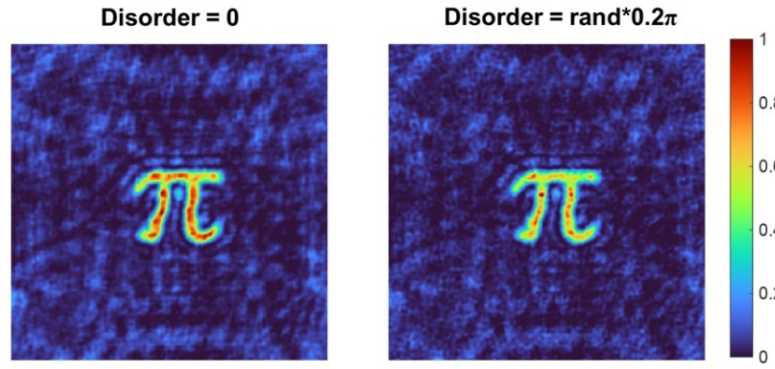


Fig. S34. Impact of the phase deviation. Calculated holographic images with different phase deviations.

(3) Fabrication errors

Besides the wavelength deviation and phase inaccuracy, the manufacture error might also induce intensity fluctuations. According to the referee's comment, we carefully examined the scanning electron microscope (SEM) images of the metasurfaces and indeed found fabrication errors in several samples. One example is shown in Fig. S35A below. This is because we utilized the lift-off process to transfer designed patterns into the SiO_2 layer. The holes are very small and sometimes cannot be completely stripped. In this sense, there will be some phase missing that will affect the device performance. Based on the SEM image, we randomly set the phases of 10% pixels to 0 and calculated the corresponding holographic images. The results without and with defects are plotted together in Fig. S35B and Fig. S35C for a direct comparison. It is clear that the manufacture defects also degrade the image uniformity

significantly. Once again, such manufacturing errors are also not the fundamental limitation and they can be eliminated by directly patterning the Cr layer with a negative resist, e.g. (HSQ).

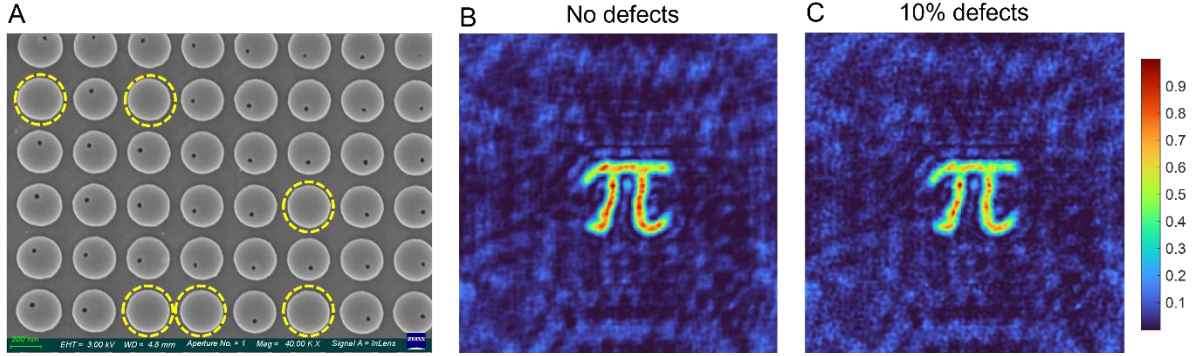


Fig. S35. Impact of fabrication errors. **A.** Top-view SEM image of the Si₃N₄ nanostructures. The missing air holes are marked with yellow dashed circles. **B** and **C** show numerically calculated holographic images without and with defects. Obvious degradation in uniformity can be observed.

From the above analysis, we know that wavelength deviation, phase inaccuracy, and manufacture error can all affect the uniformity of holographic images. None of them are fundamental limitations and all can be improved. Here we discuss the dominant reasons for low-quality holographic images in experiments. One key information can be obtained from Fig. 4C of the main text. In our experiments, we have fabricated two metalasers with the same phase profile of π . When the samples are optically pumped above the threshold, two very similar holographic images have been experimentally achieved at a wavelength of 574.5 nm. To identify the reason for the intensity fluctuations, we focused on the intensity of background signals outside the images of π . The blue and red lines in Fig. 4C of the main text are obtained on the same line- α of the above two holograms. Their intensities are much smaller than the main figure of π and have been normalized for a comparison. More details at other locations can be seen in Fig. S31. We find that all the peaks of two different metalaser holograms surprisingly match one another very well at all these different locations. It can be seen that the background signal in the metalaser hologram is actually not random. In this sense, the wavelength deviation and manufacturing errors caused by the manufacturing process (these effects have considerable randomness in different samples) should have little effects on our

experiments. As a consequence, we can conclude that intensity fluctuations and non-uniformities of holographic images in Fig. 4 of the main text should mainly come from the holographic design and low refractive index difference. The problem in Fig. 2 of the main text is more serious because it is further affected by ~ 5 nm wavelength deviation.

We then plotted the background noise in experiments together with the numerical simulations (Fig. S36). Without considering the phase deviations (orange dashed line), we find that the peaks in the experiment can generally be found in numerical calculations, except that the relative intensities vary. Then we know that our hologram code contributes significantly to the strong background noise in the experiment. Such noise can be numerically eliminated by optimizing the design code. By further considering the phase deviation, we can see that the relative intensities of these peaks are also modified and more closely related to the experimental observations (dash-dotted line). Then we know the small index difference also plays an essential role. This effect, as discussed above, can also be eliminated by using high refractive index semiconductors (such as GaP).

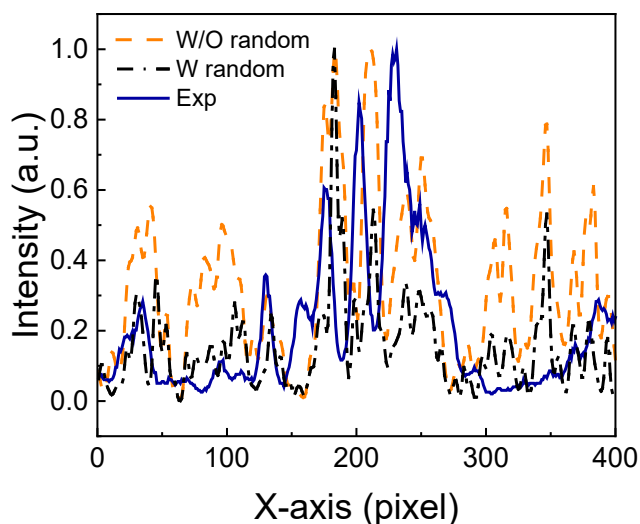


Fig. S36. Comparison between the background noise in experiments (blue solid line) and the numerical simulations with (black dash-dotted line) and without (orange dashed line) random phases. All the intensities are normalized for a direct comparison.

Based on all the above numerical simulations, experimental results, and analysis, we can conclude that the hologram code and the small refractive index difference between Si_3N_4 and dye-doped PMMA are responsible for the obvious intensity fluctuations in our experiments. These non-random spots and fluctuations are not related to the speckles and can be eliminated by optimizing the code and utilizing high index semiconductors.

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