

Metalasers with arbitrarily-shaped wavefront

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Referee #1

(Remarks to the Author)

In this manuscript, the authors realize a novel type of laser based on a resonant silicon nitride metasurface and a dye-doped PMMA film. The design of the metasurface allows manipulation of both dynamic and geometric phase, exploiting nonlocal interaction for lasing and programmable local dipole momenta of meta-atoms for wavefront shaping. This new design is the first demonstration of a laser that directly emits with desired far-field distribution. The authors show that their metalaser can produce speckle reduced holograms thanks to the drastically reduced amplification of non-resonant scattered modes, which are the source of random scattering and diffraction of conventional laser beam shaping of holographic metasurfaces. The authors claim that their metalaser completely solves the long-standing problem of speckle noise in holographic displays.

The design and implementation of the metalaser have been validated through simulations and sufficiently extensive experimental characterizations, which highlight the advantage of the the interplay of local and nonlocal response of the proposed metasurface for novel light manipulation and potential photonics applications. The manuscript is clearly written, the abstract is accessible, and relevant references are properly cited.

My concerns are based on the author's claim of their complete solution to the speckle problem in holographic displays, which I find overstated with the current dataset. This statement is based on the reduction of the speckle contrast when the radiating metalasers are pumped above lasing threshold, motivated by the selective amplification of the desired mode. However, the homogeneity of the holographic patterns, especially in the case of Fig. 2E is surprisingly low, when compared to the simulations of Fig.1E and Fig. S5. This inhomogeneity is also clearly visible in the plots in Fig. 4C and seems to be comparable with what is achievable with standard out-of-cavity holographic films and metasurfaces. In my opinion, the inhomogeneous holograms demonstrated here are still far from what could be a final solution for holographic displays. In addition, there is no particular justification (manufacturing errors, design, etc.) for the significant deviation of the homogeneity of the experimental holograms from the calculated ones.

As a minor issue, the description of the plots in Fig. 4A are mislabeled in the text (panel II and panel III should be inverted).

Referee #2

(Remarks to the Author)

In this work the authors design a metasurface that simultaneously allows geometric phase control and optical gain (resonance) so that the active medium and shaping medium are one and the same. They then use a design approach to create metasurfaces that lase on exotic spatial modes. The paper has a healthy balance of metasurface data and lasing data. making for an informative read. In general it is well written and I notice that very often the authors highlight text to bring it to the attention of the reader (a little unusual). Now the problem is that the main claim appears not to be met. In many places, from the title, abstract through to the main text, the claim is that this is a programmable laser. Well, first, programmable lasers are fairly common. The authors also say that "this is the first time that a laser can directly produce a complex light beam". I suggest the authors read the many works on complex lasers, e.g., see many papers by Nir Davidson's group who has shown very complex patterns in the form of Newton and Einstein as the output, e.g., Optica 8, 880-884 (2021). Similarly, work on semi-conductor lasers: Nature Nanotechnology 15, 125 (2020). But this anyway is moot, because the main issue is that there is no evidence for programmable at all. I read the paper many times and as far as I can tell, the geometric phase comes from the orientation of the holes in the nanodisks, but this is not rewritable. Instead, the SI and Methods make clear that an iterative approach is used to calculate the metasurface details, and that this is repeated for each

pattern needed. There is no "programmable" aspect at all in the usual sense of the word, or as demonstrated in prior lasers with truly programmable devices inside, e.g., spatial light modulators. Thus the challenge of on-demand modes from a small device remains. So what then is the advantage? I guess the gain medium and shaping step have been combined, but in terms of compactness this is not a major step as compared to say Nature Nanotechnology 15, 125 (2020) and other similar works since - you gain very little. Complex modes is also not a major leap - this can almost be taken as a given once you know what phase is needed. As for the speckle free, again the authors need to read the literature, and in particular Nature Reviews Physics volume 1, pages 156–168 (2019) for a good review. A small technical point: the authors repeatedly speak of the wavefront and far-field pattern as if they are two distinct things, but one is the propagated version of the other.

Overall, a nice read but I do not see anything particularly novel here and certainly no evidence for the main claim.

Referee #3

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

Referee #4

(Remarks to the Author)

This paper introduces a significant advancement in laser technology with the development of the "metalaser," which integrates dielectric resonant metasurfaces to achieve programmable emission profiles. By exploiting the interplay between local geometric phase manipulation and nonlocal quasi-BIC modes, while maintaining minimal disruption to the lasing behavior, the authors successfully demonstrated the direct generation of complex beam profiles without the need for external optics. The key contributions of this work are: (1) Programmable wavefront control where the authors utilized the radial direction at the hole inside the Si₃N₄ nanodisk to manipulate the polarization of the radiated light, thereby controlling the local geometric phase of each pixel; (2) Speckle-free holography was achieved by amplifying lasing modes and suppressing scattered waves, leading to improved holographic performance; and (3) Versatile beam shaping, including the generation of high-order vector beams and chiral emissions, was demonstrated experimentally through controlled hologram application. The concept of programmable emission is intriguing. The interplay between nonlocal quasi-BIC modes and local geometric phase manipulation are well-designed, and the paper is generally well-written. However, I would still suggest that the authors address the following issues:

1. The paper introduces geometric phase control through the rotation of off-center holes, which is central to the work. However, the underlying physics, particularly how the geometric phase is precisely controlled, lacks sufficient clarity for a general reader. The authors link the hole rotation (θ) to the far-field polarization angle ($\varphi = \theta - 90^\circ$), but the connection between polarization rotation and the geometric phase should be mathematically derived in an analytical form. This should be explicitly discussed by adding a new section in Methods. Hope an analytical solution can also address the nonlinear relation between the local phase and the hole rotation angle (see Fig. 1B or Fig. S3B) when encoding the phase pattern.
2. Phase resolution is crucial for achieving high-quality wavefront shaping. What is the angular precision of the fabrication process, or are there other factors that could influence this? Additionally, could this precision limitation pose a challenge to scalability for improved wavefront shaping?
3. Scalability and reproducibility are critical for practical applications. In this paper, the largest array is $180\ \mu\text{m} \times 180\ \mu\text{m}$. What limitations might arise when scaling up the array size? Would the pattern quality improve with a larger array? Additionally, while Fig. 4C shows good agreement in the intensity of some dots between the two samples, the authors should provide more comprehensive comparisons of the entire generated patterns across samples with the same hologram to better assess reproducibility.
4. What is the uniformity of the laser intensity across the array? Since this would also impact the far-field pattern, it would be helpful to include some images of the sample plane to better evaluate the lasing performance.
5. Is a single disk cavity sufficient for lasing? If so, the metalaser could be viewed as an array of collectively coupled lasers. In this case, it would be interesting to experimentally characterize the far-field emission intensity and profile for both a single cavity and the collective array and conduct a comparison. For reference, see Science 372, 403-408 (2021), although the underlying physics of laser arrays is different.
6. Real time programming of emission patterns is important to achieve for device applications. See, for example, Nature 612, 246-251 (2022) and PRX 15, 011024 (2025). Is it possible to reconfigure the emission pattern in a real time manner? The authors are encouraged to discuss this based on their metalaser platform.

Version 1:

Reviewer comments:

Referee #1

(Remarks to the Author)

The authors significantly improved the description of the performance limitations and removed the overstated claims about their metalaser holograms. Additionally, new relevant simulations and discussions have been added to the revised Supplementary Information. I suggest accepting the revised manuscript in its current form.

Referee #2

(Remarks to the Author)

Thank you for the comprehensive response to the many questions. My original report was negative because the main claim ("programmable") had no supporting evidence, and its repeated use at pivotal sentences meant that this was the point on which the paper would need to be judged, and unfortunately found wanting. The authors agree and have removed this claim, so now one can judge the novelty and importance based on the revised message. I agree that this is a lovely demonstration and a design concept that is likely to capture the imagination. On the one hand, it is able to produce outputs that would otherwise have required various physical mechanisms to implement, while allowing designer wavefronts to be made into a tiny structure. The "meta" in metalaser is now clearer - not just a metasurface with some gain. The forward look towards programmable with phase transition materials is interesting. I imagine there will be technical hurdles, e.g., VO₂ changes phase with temperature which may be hard to control in a small lasing volume, but nevertheless, it is interesting that there are concepts to move forward with. My only remaining concern is that the authors still keep ambiguous phrases such as "controlled at will" or "tailored at will" which have an element of suggesting 'real-time', even if not said. Any native English speaker would interpret it this way. It would be better to be cautious and rephrase all such statements. Rather than "the geometric phase can be controlled at will" it is more accurate to say "the geometric phase can be designed to be an arbitrary function", or "designed at will". One can only claim what has been shown, not future manifestations that are yet to be verified. There are many instances when such rephrasing would be in order.

Referee #3

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

Referee #4

(Remarks to the Author)

I have read carefully the revised manuscript, the revised supplementary information, and the authors' responses to review comments. I think the authors have done a great job addressing the review comments and concerns raised in the previous round. In particular, the integration of phase-change materials into the meta-laser platform is both elegant and technically compelling. My only remaining question is about the reconfiguration speed of the phase-change materials for programming meta-laser emissions; it would be helpful to specify this in the supplementary information. Overall, I believe the manuscript is very strong and should be accepted with minimal, if any, revisions.

made.

In cases where reviewers are anonymous, credit should be given to 'Anonymous Referee' and the source.

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Response to Referee #1:

General Comment:

In this manuscript, the authors realize a novel type of laser based on a resonant silicon nitride metasurface and a dye-doped PMMA film. The design of the metasurface allows manipulation of both dynamic and geometric phase, exploiting nonlocal interaction for lasing and programmable local dipole momenta of meta-atoms for wavefront shaping. This new design is the first demonstration of a laser that directly emits with desired far-field distribution. The authors show that their metalaser can produce speckle reduced holograms thanks to the drastically reduced amplification of non-resonant scattered modes, which are the source of random scattering and diffraction of conventional laser beam shaping of holographic metasurfaces. The authors claim that their metalaser completely solves the long-standing problem of speckle noise in holographic displays.

The design and implementation of the metalaser have been validated through simulations and sufficiently extensive experimental characterizations, which highlight the advantage of the interplay of local and nonlocal response of the proposed metasurface for novel light manipulation and potential photonics applications. The manuscript is clearly written, the abstract is accessible, and relevant references are properly cited.

Our Response:

We thank Referee #1 for his/her careful reading of the manuscript and valuable suggestions. We are particularly grateful to this referee for the high praise of our study, e.g. “*the first demonstration of a laser that directly emits with desired far-field distribution*” and “*for novel light manipulation and potential photonics applications*”. Based on the suggestions of the referee, we have carefully revised the manuscript and all the comments have been addressed accordingly. We believe the quality of our research has been improved significantly.

Please see the point-to-point responses below for details.

Comment-1:

My concerns are based on the author’s claim of their complete solution to the speckle problem in holographic displays, which I find overstated with the current dataset. This statement is based on the reduction of the speckle contrast when the radiating metalasers are pumped above lasing threshold, motivated by the selective amplification of the desired mode. However, the homogeneity of the holographic patterns, especially in the case of Fig. 2E is surprisingly low, when compared to the simulations of Fig. 1E and Fig. S5. This inhomogeneity is also clearly visible in the plots in Fig. 4C and seems to be comparable with what is achievable with standard out-of-cavity holographic films and metasurfaces. In my opinion, the inhomogeneous holograms demonstrated here are still far from what could be a final solution for holographic displays. In addition, there is no particular justification (manufacturing errors, design, etc.) for the significant deviation of the homogeneity of the experimental holograms from the calculated ones.

Our Response:

We thank the referee for this very careful review and valuable comment. We agree with the referee that the holographic patterns we have currently obtained in our experiments show relatively low homogeneity and are still far from the “final solution” for holographic displays. Based on this valuable comment, we have removed the statements of “completely” and carefully discussed the reasons for the deviation between experimental observations and calculated holographic images.

Our work mainly focuses on the reduction of speckle noise without degrading the image sharpness. As pointed out by the referee, both of the design and manufacture error also play essential roles and shall affect the homogeneity of the metalaser hologram in experiment. In principle, the relatively low homogeneity in Fig. 2 and Fig. 4 can be attributed to the following three reasons.

1. Wavelength deviation

In the phase design process for the holographic image, the light propagations are simulated using the angular spectrum method. The operation wavelength is thus a key parameter of the phase design. Due to slight deviations in dimensions during nanofabrication, the laser wavelength in experiments usually differs by ~ 5 nm from the designed wavelength. Such wavelength deviations will not completely destroy the designed holographic image, but will significantly reduce its uniformity. One example is shown in Fig. R1. The phase distribution of metasurface is optimized for the wavelength of 575 nm. As shown in the central panel of Fig. R1 (also plotted as Fig. 1E of the main text), the holographic image shows relatively good uniformity. When the operating wavelength deviates by 5 nm, as shown in the left and right panels, the intensity variation becomes more significant and approaches the experimental results in Fig. 2 of the main text (with a wavelength deviation of ~ 5 -6 nm). We believe that this is one of the reasons for the inhomogeneity of the experimentally recorded metalaser holograms. **Such a wavelength deviation is not a fundamental limitation of the principle and can be technically solved by optimizing the nanofabrication process.** For example, Harry A. Atwater group at Caltech has demonstrated that the standard deviation in nanofabrication can be kept below 0.6 nanometers (*Nat. Commun.* **14**, 8476 (2023)).

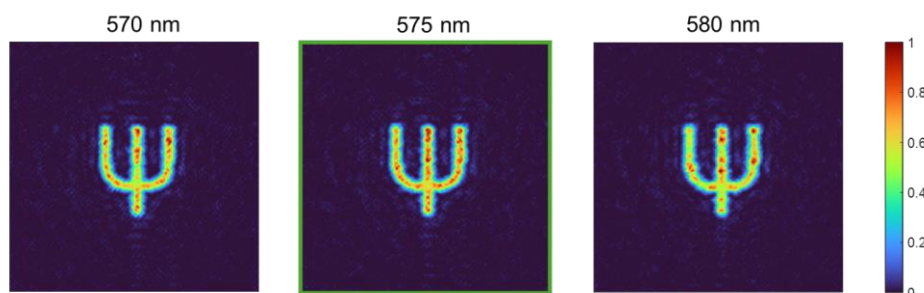


Figure R1 (Figure S32 in Supplementary Information): Impact of wavelength deviation. Calculated holographic images at wavelengths of 570 nm (left), 575 nm (middle), and 580 nm (right). Here the phase profile of the metasurface is designed for the wavelength of 575 nm.

2. Phase deviation

Phase accuracy is another issue that impacts the homogeneity of holographic image. In our experiment, we employ Si_3N_4 and dye-doped PMMA to construct the metalaser. The refractive index difference between them ($n_{\text{Si}_3\text{N}_4} = 2.02$ and $n_{\text{PMMA}} = 1.49$) is only $\Delta n = 0.53$. This index difference is too low to largely confine light within each pixel and fully suppress their proximity interactions, leading to nonlinear dependence of polarization angle on the hole rotation angle (Fig. 1 of the main text) and inaccurate phases. The inaccurate phases affect the homogeneity greatly. Taking a metasurface with a cylindrical phase as an example, we calculate the eigen-field of this metalens structure. As shown in Fig. R2A, we can see that the RCP component of the emission from the metalens has been focused to the designed focal point, but the intensity fluctuation is very significant. For a direct comparison, the emission wavefront of a metalens composed of GaP ($n_{\text{GaP}} = 3.5$) and PMMA has also been calculated. It is obvious that the intensity fluctuation becomes much smaller in Fig. R2B.

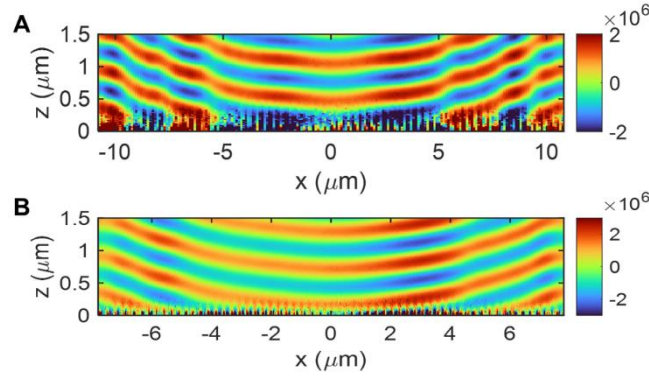


Figure R2 (Figure S33 in Supplementary Information). Impact of phase inaccuracy caused by low-index difference between Si_3N_4 and PMMA. A. Calculated radiation field from a metalaser lens composed of Si_3N_4 and PMMA. **B.** Calculated radiation field from a metalaser lens composed of GaP and PMMA.

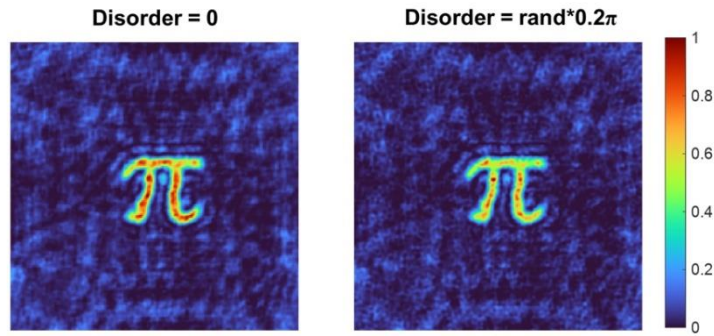


Figure R3 (Figure S34 in Supplementary Information): Impact of the phase deviation. Calculated holographic images with different phase deviations.

As mentioned above, phase inaccuracies are caused by the small difference in between Si_3N_4 and PMMA. **This problem can be alleviated by constructing metalaser hologram using III-V semiconductors with higher refractive index for the visible spectrum, e.g., GaP ($n_{\text{GaP}} \sim 3.5$)-PMMA, GaP-Air or GaN ($n_{\text{GaN}} \sim 2.4$ -2.5)-Air.**

To clarify the influence of the phase inaccuracies, we added random phase deviations ($\Delta\phi(x, y)$) to the designed hologram phase distribution. The corresponding holographic images were then calculated and shown in Fig. R3. We found that the uniformity of the hologram drops sharply with the increase of the phase deviation. Phase deviations between 0 and $\sim 0.2\pi$ can produce non-uniform holograms similar to the experimental results.

3. Fabrication errors

Beside the size deviation and phase inaccuracy, the manufacture error might also induce intensity fluctuations. According to the referee's comment, we carefully examined the scanning electron microscope (SEM) images of the metasurfaces and indeed found fabrication errors in several samples. One example is shown in Fig. R4A below. This is because that we utilized the lift-off process to transfer designed patterns into the SiO₂ layer. The holes are very small and sometimes cannot be completely stripped. In this sense, there will be some phase missing that will affect the device performances.

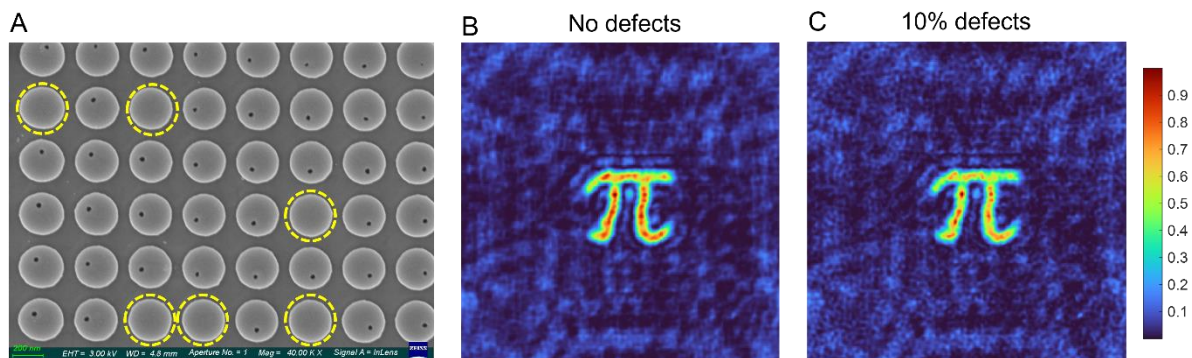


Figure R4 (Figure S35 in Supplementary Information). Impact of manufacture errors. A. Top-view SEM image of the Si₃N₄ nanostructures. The missing air holes are marked with yellow dashed circles. **B** and **C** show the numerically calculated holographic images without and with defects. Obvious degradation in uniformity can be observed.

Based on the SEM image, we randomly set the phases of 10% pixels to 0 and calculated the corresponding holographic image. The results without and with defects are plotted together in Fig. R4B and Fig. R4C for a direct comparison. It is clear that the manufacture defects also degrade the image uniformity significantly. Once again, **such manufacturing errors are also not the fundamental limitation and they can be eliminated by directly patterning the Cr layer with a negative resist, e.g. (HSQ).**

4. Discussions

From the above analysis, we know that wavelength deviation, phase deviation, and manufacture error can all affect the uniformity of holographic images. None of them are fundamental limitations and all can be improved. Below, we discuss the dominant reason to achieve high quality holographic images in experiment.

One key information can be obtained from Fig. 4C of the main text. In our experiment, we have fabricated two metalasers with the same phase profile of π . When the samples are optically pumped above the threshold, two very similar holographic images have been experimentally achieved at a wavelength of 574.5 nm. To identify the reason of intensity fluctuation, we focused on the intensity of background signals outside the images of π . The blue solid line and red dashed line in Fig. 4C of main text are obtained on the same line- α of the above two holograms. Their intensities are much smaller than the main figure of π and have been normalized for a comparison. More details comparison at other locations can be seen in Fig. S31. We find that all the peaks of two different metalaser holograms surprisingly match one another very well at all these different locations. Then we know that the background signal in the metalaser hologram is actually not random. In this sense, the wavelength deviation and manufacturing errors caused by the manufacturing process (these effects have considerable randomness in different samples) should have little effect on our experiments. As a consequence, we can conclude that **intensity fluctuations and non-uniformities of holographic image in Fig. 4 of the main text should mainly come from the holographic design and low refractive index difference**. The problem in Fig. 2 of the main text is more serious because it is further affected by ~ 5 nm wavelength deviation.

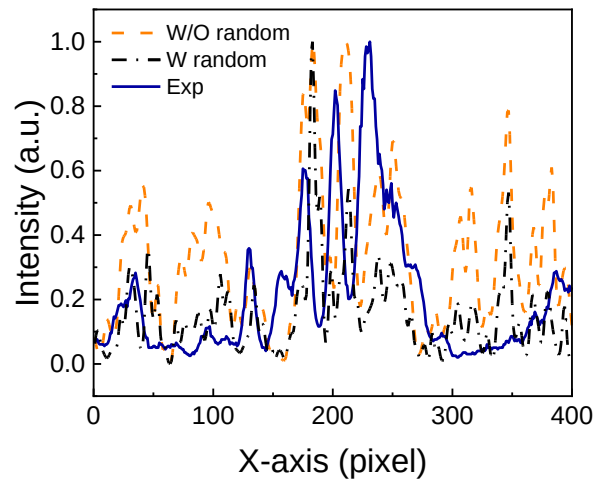


Figure R5 (Figure S36 in Supplementary Information): Comparison between the background noises in experiment (blue solid line) and the numerical simulations (black dash-dotted line) with and without (orange dashed line) the random phase. All the intensities are normalized for a direct comparison.

We then plotted the background noise in experiment together with the numerical simulations (Fig. R5). Without considering the phase deviations (orange dashed line), we find that the peaks in the experiment can generally be found in the numerical calculations, except that the relative intensities vary. Then we know that our hologram code contributes significantly to the strong background noise in the experiment. Such noise can be numerically eliminated by optimizing the design code. By further considering the phase deviation, we can see that the relative intensities of these peaks are also modified and more closely related to the experimental observations (dash-dotted line). Then we know the small index difference also plays an

essential role. This effect, as discussed above, can also be eliminated by using high refractive index semiconductors (such as GaP).

Based on all the above numerical simulation, experimental results, and analysis, we can conclude that **the hologram code and the small refractive index difference between Si₃N₄ and dye-doped PMMA are responsible for the obvious intensity fluctuations in our experiments**. These non-random spots and fluctuations are not related to the speckles, and can be eliminated by optimizing the code and utilizing high index semiconductors.

In the revised manuscript, we have removed the statement and added the above information to **Para-2, Page-7, Para-1, Page-8 and Para-1, Page-9** of the main text.

“Our metalaser has the potential to produce speckle-free holograms without degrading the imaging quality.”

“In the contrast enhanced image, intensity inhomogeneities of " π " and background spots can be observed in panel-I of Fig. 4A. To exclude the possibility of speckle noise, we fabricated a reference metalaser and recorded its holographic image. The intensity along line- α (see Fig. 4A) of two metalaser holograms are plotted in Fig. 4C (more details in Supplementary Note-8). All peaks of holograms generated by two different samples surprisingly match one another well. By comparing with simulations, we found that these inhomogeneities and background spots are more closely related to the hologram design than to speckle, and can be eliminated by optimizing the code and materials (Supplementary Note-9). Combining the above information, we confirm that speckle noise is suppressed in metalaser holograms.”

“With an optimized design, it has the potential to tackle the long-standing challenge of speckle noise in holographic display.”

The detailed information has been added as a new section of **“Supplementary Note-9: Reasons for non-uniform intensity of holographic images”** to supplementary Information.

Comment-2:

As a minor issue, the description of the plots in Fig. 4A are mislabeled in the text (panel II and panel III should be inverted).

Our Response:

We thank the referee for the careful review. We have corrected the label in Fig. 4A of the revised manuscript.

Response to Referee #2:

General Comment:

In this work the authors design a metasurface that simultaneously allows geometric phase control and optical gain (resonance) so that the active medium and shaping medium are one and the same. They then use a design approach to create metasurfaces that lase on exotic spatial modes. The paper has a healthy balance of metasurface data and lasing data, making for an informative read. In general it is well written and I notice that very often the authors highlight text to bring it to the attention of the reader (a little unusual). Now the problem is that the main claim appears not to be met. In many places, from the title, abstract through to the main text, the claim is that this is a programmable laser. Well, first, programmable lasers are fairly common. The authors also say that "this is the first time that a laser can directly produce a complex light beam". I suggest the authors read the many works on complex lasers, e.g., see many papers by Nir Davidson's group who has shown very complex patterns in the form of Newton and Einstein as the output, e.g., *Optica* 8, 880-884 (2021). Similarly, work on semiconductor lasers: *Nature Nanotechnology* 15, 125 (2020). But this anyway is moot, because the main issue is that there is no evidence for programmable at all. I read the paper many times and as far as I can tell, the geometric phase comes from the orientation of the holes in the nanodisks, but this is not rewritable. Instead, the SI and Methods make clear that an iterative approach is used to calculate the metasurface details, and that this is repeated for each pattern needed. There is no "programmable" aspect at all in the usual sense of the word, or as demonstrated in prior lasers with truly programmable devices inside, e.g., spatial light modulators. Thus the challenge of on-demand modes from a small device remains. So what then is the advantage? I guess the gain medium and shaping step have been combined, but in terms of compactness this is not a major step as compared to say *Nature Nanotechnology* 15, 125 (2020) and other similar works since - you gain very little. Complex modes is also not a major leap - this can almost be taken as a given once you know what phase is needed. As for the speckle free, again the authors need to read the literature, and in particular *Nature Reviews Physics* volume 1, pages 156–168 (2019) for a good review. A small technical point: the authors repeatedly speak of the wavefront and far-field pattern as if they are two distinct things, but one is the propagated version of the other.

Overall, a nice read but I do not see anything particularly novel here and certainly no evidence for the main claim.

Our Response:

We thank Referee #2 for his/her careful review of our paper and critical comments. From his/her report, we have realized that the word "programmable" may be misleading, and it has led to some confusion. Following the editor's suggestion, we have removed the word "programmable" and changed the title to "**Metalasers with arbitrarily-shaped wavefront**".

Below, we carefully compare with existing literature and discuss the essential differences with our metalasers and the lasers demonstrated in the two papers referenced by the referee.

1. Differences with the results of Optica 8, 880-884 (2021)

We thank the referee for suggesting this valuable reference of complex laser (*Optica* 8, 880-884 (2021)), which is closely related to this research and has been cited and highlighted as **Ref. 6** in the revised manuscript. At first glance, the complex laser in this reference has very similar capabilities to our metalaser in terms of producing structured light and complex patterns. But there are fundamental differences between their working principles.

In *Optica* 8, 880-884 (2021), the complex laser is composed of a degenerated cavity laser (DCL), an intracavity spatial light modulator (SLM), an intracavity spatial Fourier aperture, and two optical lenses. SLM has the ability to spatially control reflectivity and phase distribution of each pixel. Their functions have been clearly stated in Para-4, Page-2 of the reference. “The control of the reflectivity distribution of the SLM allows control of the intensity distribution of the laser beam in the near field (right inset in Fig. 1), whereas control of the phase distribution of the SLM allows control of the frequency distribution.” Complex patterns of Newton and Einstein are generated by controlling the intensity distribution of over 320,000 independent laser modes by tuning the local reflectivity of the SLM.

Then we can find the following information from *Optica* 8, 880-884 (2021).

- (1) A supercell consisting of 2×2 adjacent pixels correspond to an independent laser mode. In this sense, an SLM with 1272×1024 pixels can spatially multiplex 636×512 modes, resulting in 325,632 independent lasing modes.
- (2) The 325,632 independent lasing modes are employed to produce complex patterns with 325,632 pixels. The intensity of each pixel is independently controlled by the reflectivity of each supercell of the SLM. **This function relies on intensity control rather than phase control.**
- (3) The phase does have important impacts. The introduction of phase to each supercell affects the resonant condition and alters the emission wavelength of each lasing mode, closely related to the coherence of the system.

In our metalaser, the quality (Q) factor of a single pixel is too small to support an independent lasing mode. Instead, all pixels need to work together to produce **a single lasing mode** that is widely distributed throughout the sample. In this sense, the emission profile cannot be simply generated by tuning the intensity at each pixel. Instead, it is closely related to the geometric phases at each pixel. As demonstrated in the main text, many different functionalities such as focused beams, OAM beams, SAM beams, and holograms have been generated directly from the metalasers by introducing different phase distributions to **a single lasing mode** rather than controlling the individual intensities of a large number of spatially multiplexed laser modes.

The exploitation of the phase distribution makes our metalasers even more different. As described in *Optica* 8, 880-884 (2021), the superposition phase of the SLM actually changes the resonant wavelength (frequency). If all the pixels of the SLM correspond to the same laser mode, as in our study, this superimposed phase distribution is detrimental to the resonance and

the corresponding lasing action. In this sense, it is quite challenging and intrinsically different to introduce a phase profile to a single-mode laser without destroying its lasing action. We will discuss this in detail in the next section.

2. Differences with the results of Nature Nanotechnology 15, 125-130 (2020)

We thank the referee again for pointing out this valuable reference (*Nature Nanotechnology* **15**, 125-130 (2020)). This reference, together with *Nature Photonics* **14**, 498-503 (2020), *Nature Communications* **12**, 3787 (2021) and *Nature Photonics* **16**, 359-365 (2022), corresponds to a new research area, i.e., the combination of a metasurface and a single-mode laser. All these references were cited as Refs. 5, 7, 8, 23 (5, 7, 9, 24 in the revised manuscript). The presence of metasurface greatly promoted the laser performances and tailored the laser emissions to high purity LG modes, multiple beams, vortex array, as well as holograms. Basically, such researches can be categorized into two groups. Earlier studies, including the *Nature Nanotechnology* **15**, 125-130 (2020) that pointed out by the referee, simply placed the metasurface outside the laser cavity. Apart from its compactness, it is very similar to a conventional metasurface that is illuminated with an external laser beam close to it. As a consequence, many problems of traditional metasurfaces still exist, such as high speckle noise caused by sub-wavelength pixels when generating complex patterns.

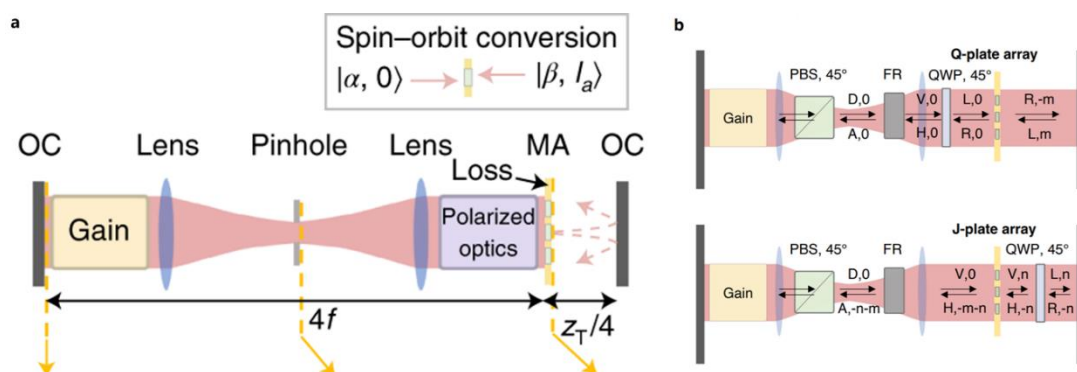


Figure R6. Phase compensation for the case of metasurface inside the cavity. A. Fig.1a of the main text of *Nature Photonics* **16**, 359-365 (2022). B. Fig.S1 of the supplementary materials of *Nature Photonics* **16**, 359-365 (2022).

Recently, A. Forbes and A. Ambrosio et al. presented a unique approach in *Nature Photonics* **16**, 359-365 (2022). Unlike the previous reports, they placed the metasurface phase plate (J-plate or Q-plate) inside the laser cavity and made it participate in optical resonance. Similar to *Optica* **8**, 880-884 (2021), the phase plate locally detunes the resonant condition at different positions of the cavity and alters the resonant wavelength. To preserve the single-mode operation, as pointed out by A. Forbes and A. Ambrosio et al., “**the round-trip condition**” must be satisfied. In this reference, it was fulfilled by employing “**an appropriate intracavity polarization optics**”, which is depicted in Fig. R6B. A polarization beam splitter (PBS), a quarter -wave plate (QWP), and a Faraday rotator (FR) were employed to compensate the phase changing caused by metasurface. Compared with simple combination (such as *Nature Nanotechnology* **15**, 125-130 (2020)) described above, **metasurface inside cavity makes the**

system very complicated. The optical components and their locations should also be carefully adjusted to fit different metasurface such as J-plate and Q-plate (see two panels in Fig. R6B).

Similar to the complex laser in *Optica* **8**, 880-884 (2021), the entire laser resonator in *Nature Photonics* **16**, 359-365 (2022) also has a length around 60 cm. For such a large cavity, the optical components such as PBS, QWP and FR can be easily inserted and precisely adjusted. However, due to size limitations, it is extremely challenging to insert these components into nanolasers, especially non-reciprocal (FR) lasers, which remains a major obstacle in modern integrated optics. In addition, the phase distribution of the Q-plate or J-plate is relatively simple. Their phases usually remain constant within a certain angle range. Considering more complicated functions such as meta-hologram, more optical components are required to compensate the wavefront altered by the metasurface. Of course, this adds even more difficulty to nanolasers.

In our study, we have carefully considered the possibility of introducing a phase profile to a nanolaser while keeping its original resonant conditions. We realized this function by utilizing the interplay of local and nonlocal effects of a quasi-bound state in the continuum (BIC) in our metasurface. The preserved nonlocal interaction allows to maintain a high Q factor at the designed wavelength even when the rotation angle (θ) of the hole in each nanodisk varies. Meanwhile, the local dipole moment causes the emission from each Si₃N₄ nanodisk to acquire a geometric phase of $\phi = \sigma\theta$, where $\sigma = \pm 1$ represent the right-handed circular polarization (RCP) and left-handed circular polarization (LCP) components, respectively. Such geometric phases are independent of the dynamic phase and therefore they do not affect the resonant conditions ($\Phi = 2m\pi$ per round trip). More remarkably, the resonant quasi-BIC mode is widely spread and periodically distributed over a large region. Thus, the dynamical phase in each Si₃N₄ nanodisk is nearly isotropic, and the difference between neighboring nanodisks is fixed at $2m\pi$. In this case, both intensities and propagation phases at the holes are identical regardless of their rotation angles. Therefore, laser wavefront and far-field radiation are determined solely by the geometric phase ϕ assigned to each pixel, and they can be controlled at will.

Through all the above discussions, we know that our metalaser is fundamentally different from all previous reports. Compared with early works such as *Nature Nanotechnology* **15**, 125-130 (2020), we introduced the pixel-level wavefront control capability inside the resonant cavity. The emission of the nanolaser can then be directly tailored to arbitrary shapes without degrading the laser performances, rather than utilizing an external metasurface to modulate. Compared with recent work *Nature Photonics* **16**, 359-365 (2022), our nanolaser has orders of magnitude smaller dimensions but much more powerful functions. Since nanolasers have very few degrees of freedom in wavefront manipulation, more breakthroughs in fundamental physics and technologies have been made to preserve the round-trip condition and to control the emission profiles at will.

3. On the use of the word “programmable”

We agree with the referee that our term “programmable” was not used in the usual sense, and this produced misunderstanding. “Programmable” here refers to the customization of laser

output characteristics at will. In this sense, the new title suggested by the editors is more consistent with the main content of our research. In this section, we would like to further demonstrate that the programmable in the usual sense can also be realized in our metalaser.

Similar to spatial light modulators, materials with viable refractive indices are highly desirable for the functionality of programmable metalaser. Here we utilize the phase change material such as Sb_2S_3 and VO_2 and illustrate this potential with several examples.

(1) Chiral laser emission with controllable angle

Highly integrated chiral laser emission is quite essential for many on-chip optical and quantum information processing. In 2022, Zhang et al. exploited chiral quasi-BIC to achieve chiral nanolaser emission in the normal direction (Ref. 30, *Science* **377**, 1215-1218 (2022)). One year later, Cheng-Wei Qiu's group further utilized the spin-valley locking effect to extend chiral laser emission to tilted angles (Ref. 25, *Nature Materials* **22**, 1065-1070 (2023)). In Fig. 3C of the main text, we have shown that both novel functions can be easily realized with our metalaser. Using metalaser, we no longer need to consider either quasi-BIC or spin-valley locking, but only design the phase distribution based on the required emission angle.

Below, we show that our metalaser can unlock even more performances. Figure R7A shows the schematic diagram of the unit cell our metalaser. It is almost the same as the one in Fig. S24 except that the high-index dielectric Si_3N_4 is replaced with Sb_2S_3 whose refractive index can be optically or electrically tuned. The corresponding phase profile is defined following the equation $\theta = 2\pi \sin(\phi) x / \lambda$. By placing dipoles into the metasurface, we simulated the emissions from the Sb_2S_3 metasurface. As solid lines depicted in Fig. R7B, LCP and RCP laser beams can be achieved at $\pm 15.2^\circ$, respectively. Then the Sb_2S_3 is optically or electrically tuned. The phase change increases the refractive index of Sb_2S_3 from 3.00 to 3.68. As a consequence, the LCP and RCP emissions are tuned to $\pm 18.3^\circ$ (dashed lines in Fig. R7B).

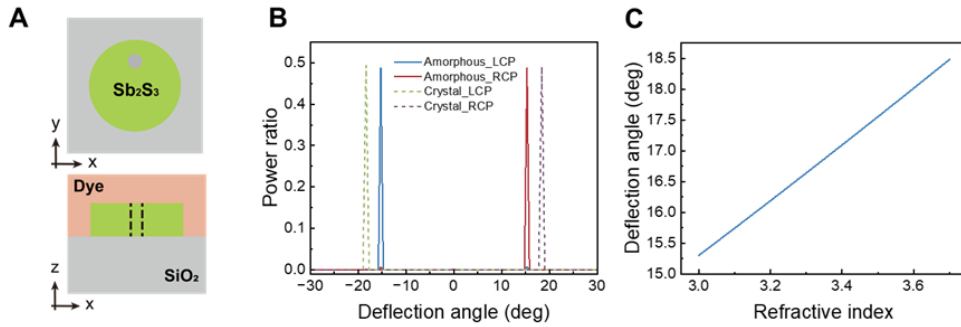


Figure R7 (Figure S25 of Supplementary Information). Tunable angles of chiral emissions. **A.** Schematic of the unit cell of the metalaser composed of Sb_2S_3 and PMMA. **B.** Emission angle of LCP and RCP light before (solid line) and after (dashed line) phase transition. **C.** The emission angle of LCP radiation as a function of intermediate refractive index.

Phase change materials such as Sb_2S_3 have a series of intermediate states. Then their refractive index can be continuously tuned between the amorphous phase to crystalline phase. Using the intermediate refractive indices, we have simulated the angles of chiral emissions and plotted

the results in Fig. R7C. It is obvious that the radiation angles of LCP emission also continuously increase from 15.2° to 18.3° . The corresponding RCP component also continuously decreases from -15.2° to -18.3° . This is the first time that the chiral emission angle can be continuously controlled. Since the phase change materials can be changed back and forth, the continuous angle control is also reversible.

(2) Chiral laser emission with controllable intensity difference

In Fig. R7 and *Nature Materials* **22**, 1065-1070 (2023), the intensities of LCP and RCP lasers at opposite tilt angles are identical. Here we show that their relative intensities can also be tuned with the phase change materials. Figure R8A shows the schematic of the unit cell. Our metasurface is formed by square-latticed periodic nanostructures (Fig. R8A). The lattice size is $p = 400$ nm. Each unit cell contains two GaP ($n = 3.5$) nanodisks with radius of $R = 150$ nm and thickness of $h = 100$ nm that are separated by a thin layer of VO₂ with thickness of $d = 300$ nm. The entire nanostructure is embedded in PMMA. Two small holes with radius of $r = 20$ nm are patterned into two nanodisks with polar coordinates of (ρ, θ) and $(\rho, \theta + \delta)$, respectively. Here we keep $\rho = L + r = 70$ nm. As discussed in the main text, the in-plane electric field within the small hole is significantly enhanced. Then the electric fields in two nanodisks are equivalent to two dipoles with an included angle α (inset in Fig. R8A). Meanwhile, these two dipoles are vertically separated by with a fixed a distance of $d = 300$ nm.

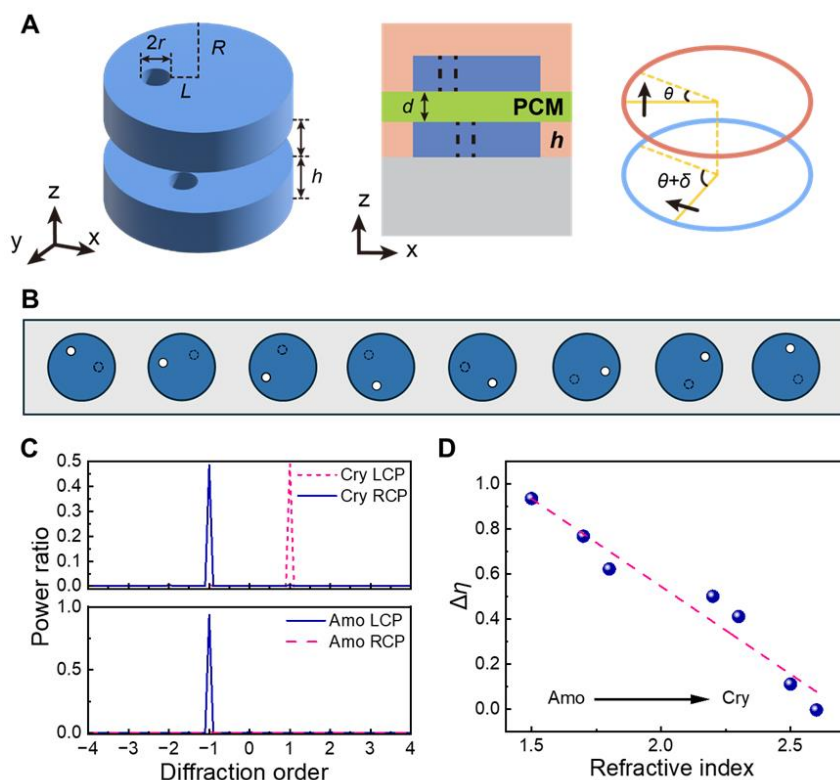


Figure R8 (Figure S26 of Supplementary Information). Tunable relative intensity of chiral emissions. **A.** Schematic picture of the unit cell with all structural parameters. **B.** The top-view of the supercell. The white hole and black circle represent the positions of two holes in top and bottom disks. The included angle is fixed at 120° . **C.** Far field angular distributions of LCP and RCP emissions before (bottom panel) and after (top panel) phase transition. **D.**

Intensity difference between LCP and RCP as a function of index of VO₂.

Two equivalent dipoles with included angle δ and vertical separation d can break the symmetry between LCP and RCP components following the factor of $\sin(\delta \pm kd)$. Figure R8B shows the top-view of the supercell. The white hole and black circle represent the positions of two holes in top and bottom disks. Here we fix the included angle $\delta = 120^\circ$. For the amorphous case, we can see that the RCP component is completely cancelled, giving a pure LCP emission (bottom panel in Fig. R8C). The phase transition of VO₂ increase the refractive index (from $n = 1.5$ to $n = 2.6$) and thus changes kd . Since the included angle δ is fixed, the factor $\sin(\delta \pm kd)$ shows that some LCP components are coupled to RCP components. Then RCP emission gradually increases and LCP emission reduces, eventually leading to a balanced LCP and RCP emissions (top panel in Fig. R8C). Figure R8D summarizes the intensity difference $|(I_{LCP} - I_{RCP})/(I_{LCP} + I_{RCP})|$ as a function of phase transition of VO₂. During the phase transition, it continuously reduces from a factor of 1 to 0, corresponding to the transition from unidirectional emission with a particular spin angular momentum (SAM) to bi-directional laser beams with opposite SAMs. Once again, the refractive index of phase change materials now is close to the refractive index of nanodisk. The dependence can be more linear if we utilize liquid crystals with lower refractive index.

(3) Controllable OAM nanolaser

The above mechanism for tunable emission angle and relative intensity is also applicable to OAM nanolaser in Fig. 3D of the main text well. It is almost that the same except that a spiral phase profile is introduced. Here we show that the topological charge l of the OAM beam can also be controlled with phase change material. Figure R9A shows the overall configuration of the phase plate with $|l| = 6$. It is obvious that the phase plate has been divided into 6 regions with different colors. Each section contains 8 subsections with phases of $0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$ (see schematic in Fig. R9A). Before the phase transition, the radiation from the metasurface corresponds to two OAM beams with $l = 6$ and $l = -6$, respectively.

Then we selectively excite regions with green colors and induce the transition of Sb₂S₃ from amorphous state to partial crystalline state. The increase in the refractive index brings the operating wavelength into the band gap (Fig. R9B). Light in the amorphous region cannot enter the phase change region, thus destroying the long-range interaction between the phase change regions and the rest ones. As a consequence, the emission at the operation wavelength is rapidly reduced to ~ 0 in phase change regions. One example is illustrated in Fig. R9C. Light is widely distributed in all cells before phase transition. After the selective phase transition in different regions, light can only be confined within the unchanged area. The contribution to the total emission from the phase change regions can be neglected. In this sense, the metalaser after selective transition can be considered as a type of metalaser with “1” (amorphous) and “0” (crystalline) states.

With the above information, we can thus simply control the topological charge of metalaser by selectively changing the phase of some regions. For the case that regions with green colors are phase changed, we then numerically calculated the emission profiles from the metalaser. The

results are shown in Fig. R9D. The outputs still show a donut shaped beam with vanished center intensity. But the shape is different and the corresponding phase profiles give topological charges of $l = \pm 3$ now. With a similar process, the topological charge can be tuned to the other number as well. In addition, due to reversibility of Sb_2S_3 , the transition between different topological charges can be also tuned back and forth.

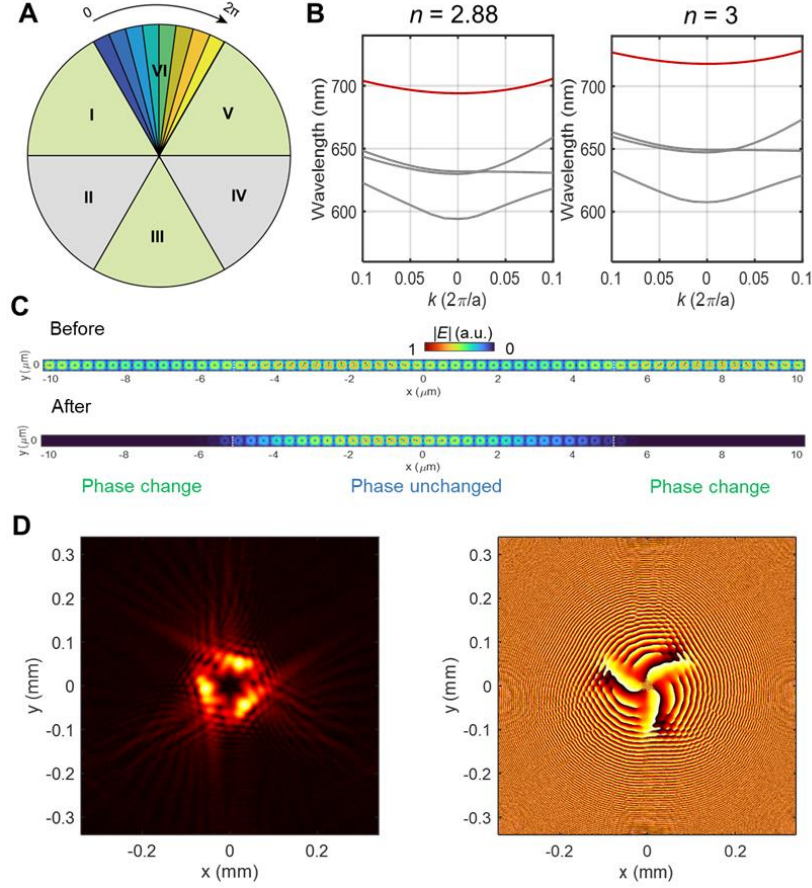


Figure R9 (Figure S27 of Supplementary Information). Tunable topological charge of OAM emissions. **A.** The overall schematic picture of the Sb_2S_3 based metalaser phase plate. Phase transition of Sb_2S_3 happens in regions with green colors. **B.** The band structure of regions with green colors before and after phase transition. The operation wavelength is around 690 nm. **C.** An example to illustrate the field distributions before and after the partial phase transitions. **D.** The simulated far field emission beam and its phase profile.

(4) Programmable metalaser holograms

In addition to the control of laser emission direction, relative intensity, and topological charge, our metalaser also has the capability of producing programmable metalaser holograms. Here we consider a simple case to realize a real programmable metalaser. Figure R10A shows the schematic of our meta-atom. It is a GaP nanodisk (with refractive index $n = 3.5$) with two tiny holes with an included angle of $\Delta\theta$. One hole is filled with phase change materials of VO_2 whose refractive index can be tuned from $n = 1.5$ (amorphous state) to $n = 3$ (crystalline state). For the case of crystalline state VO_2 , its refractive index is almost the same as GaP and the geometric phase is purely determined by the PMMA hole. When VO_2 is in amorphous state, the geometric phase is produced by two holes simultaneously and thus the included angle starts

to play an essential role. Figure R10B shows the overall geometric phase of the meta-atom as a function of angle $\Delta\theta$. It is clear to that the geometric phase can have a difference close to π before and after the phase transition when $\Delta\theta$ approaches 360° . As a consequence, the recently developed concept of digital metasurface can be applied here to produce many different functions from a single metalaser.

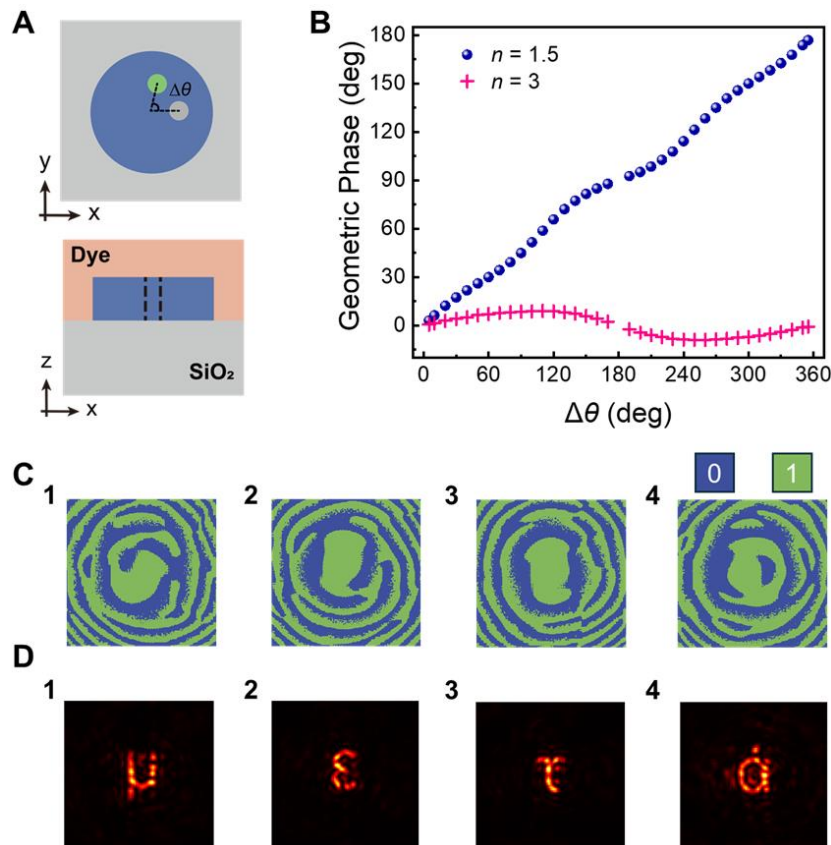


Figure R10 (Figure S28 of Supplementary Information). Programmable metalaser hologram. **A.** The schematic of the meta-atom for programmable metalaser. The entire sample is also periodic patterned with a square lattice. Two holes with PMMA and VO₂ have an included angle $\Delta\theta$ and are marked with different colors. **B.** The difference of geometric phase before ($n = 1.5$) and after ($n = 3.0$) phase transition as a function of the included angle between two holes. **C.** Four phase profiles of digital metalaser defined by the phase switching caused by the phase change and unchanged. Here the included angle is fixed at $\Delta\theta = 340^\circ$. **D.** The corresponding holographic images of “M E T A” in ancient Greek produced by four phase profiles in C. The selection phase transition of the same metalaser can produce different holographic images, enabling the possibility of programmable emission.

Each meta-atom is quite small and relatively difficult to be controlled independently. Here we utilize 2×2 pixels as a supercell with the size around $1 \mu\text{m}$. This size is easy for alignment and selectively change the phase of VO₂. Meanwhile, VO₂ can be switched between states back and forth. Following the above discussion, we have defined four different phase profiles (Fig. R10C) consisting of geometric phases of 0 and π , respectively. Figure R10D shows the corresponding holographic images. With the same metalaser, different characters of “M E T A” in ancient Greek can be achieved. Due to the reversibility of phase change materials and

micrometer sized supercell, all these profiles can be erased and re-written in experiment. There is no fundamental limitation in function number and many other holographic images can be realized with the same sample too. In this sense, the entire metalaser becomes programmable. Beside holographic image, this digital metasurface based concept can also be applied to tunable lens and other functions too. In this sense, we can confirm that our design also has the potential to realize programmable metalasers in the common sense.

In summary, we would like to emphasize here that our metalaser is intrinsically different from the previous demonstrations in *Optica* **8**, 880-884 (2021), *Nature Nanotechnology* **15**, 125-130 (2020) and many other related researches. The realization of complex functionalities in nanolaser is neither a simple combination of metasurface and nanolaser nor a size reduction of conventional complex laser system. Due to the lack of control degree of freedom, fundamental breakthroughs have been made in geometric phase and round-trip condition to simultaneously controlling the emission wavefront and maintaining the round-trip conditions of a single-mode laser.

In the revised manuscript, we have deleted “**programmable**” and changed the title to “**Metalasers with arbitrarily-shaped wavefront**”.

In **Para-1, Page-7** of the revised main text, we also discussed the potential of metalaser in programmable emissions.

“All these results demonstrate that laser wavefronts and far-field radiation can be precisely tailored at will. With the assistance of phase change materials, programmable emission can also be achieved with a single metalaser (Supplementary Note-7).”

The detailed information has been added to a new section of “**Supplementary Note-7: Potential in programmable metalaser**” to Supplementary Information.

Technical Comment:

The authors repeatedly speak of the wavefront and far-field pattern as if they are two distinct things, but one is the propagated version of the other.

Our Response:

We thank the referee for this valuable comment. The referee is correct that the wavefront manipulation and far field pattern are closely related to each other. To avoid potential confusion, we have linked them in **Para-1, Page-4** of the revised main text.

“Therefore, laser wavefront and the corresponding far-field radiation are determined solely by the geometric phase ϕ assigned to each pixel, and they can be controlled at will.”

Response to Referee #3:

General Comment:

I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

Our Response:

We thank this referee for his/her help with the review of our manuscript

Response to Referee #4:

General Comment:

This paper introduces a significant advancement in laser technology with the development of the "metalaaser," which integrates dielectric resonant metasurfaces to achieve programmable emission profiles. By exploiting the interplay between local geometric phase manipulation and nonlocal quasi-BIC modes, while maintaining minimal disruption to the lasing behavior, the authors successfully demonstrated the direct generation of complex beam profiles without the need for external optics. The key contributions of this work are: (1) Programmable wavefront control where the authors utilized the radial direction at the hole inside the Si₃N₄ nanodisk to manipulate the polarization of the radiated light, thereby controlling the local geometric phase of each pixel; (2) Speckle-free holography was achieved by amplifying lasing modes and suppressing scattered waves, leading to improved holographic performance; and (3) Versatile beam shaping, including the generation of high-order vector beams and chiral emissions, was demonstrated experimentally through controlled hologram application. The concept of programmable emission is intriguing. The interplay between nonlocal quasi-BIC modes and local geometric phase manipulation are well-designed, and the paper is generally well-written. However, I would still suggest that the authors address the following issues:

Our Response:

We sincerely thank the referee for taking the time and effort to review our paper and provide such thoughtful and constructive feedback. The reviewer's insights not only highlighted areas for improvement but also guided us to enhance the clarity and depth of our study. Based on the very constructive suggestions, we have carefully revised the manuscript and addressed all the comments accordingly. Below, we provide a detailed response to each of the comments.

Comment-1:

The paper introduces geometric phase control through the rotation of off-center holes, which is central to the work. However, the underlying physics, particularly how the geometric phase is precisely controlled, lacks sufficient clarity for a general reader. The authors link the hole rotation (θ) to the far-field polarization angle ($\varphi = \theta - 90^\circ$), but the connection between polarization rotation and the geometric phase should be mathematically derived in an analytical form. This should be explicitly discussed by adding a new section in Methods. Hope an analytical solution can also address the nonlinear relation between the local phase and the hole rotation angle (see Fig. 1B or Fig. S3B) when encoding the phase pattern.

Our Response:

We greatly appreciate the referee for this valuable suggestion. Following this suggestion, we have analyzed the electric dipole and its connection to the geometric phase, which is consistent with the results of temporal coupled mode theory (TCMT).

In case of the fundamental TE mode (TE₁ mode) in this research, the introduction of hole renders the BIC at Γ point into quasi-BIC, which can be approximate by an in-plane electric dipole

$$\mathbf{P} \approx \int_{V_{res}} \mathbf{E}(\mathbf{r}) d^3\mathbf{r}. \quad (\text{R1})$$

$\mathbf{E}(\mathbf{r})$ indicates the resonant field, and the integral is taken over the volume V_{res} of the unit cell. According to the electric field distribution of BIC, the orientation of this in-plane electric dipole is related to the rotation angle θ of the hole and approximately follows $\mathbf{P} \propto \left(\cos\left(\theta - \frac{\pi}{2}\right), \sin\left(\theta - \frac{\pi}{2}\right) \right)^T$, where the superscript T transposes the row vectors into column vectors. For the resonances at Γ point ($\mathbf{k}_{\parallel} = 0$), the emission also has the same in-plane momentum ($\mathbf{k}_{\parallel} = 0$) governed by the conservation of momentum. So, the polarization azimuthal angle of emission from the quasi-BIC is along the orientation of the in-plane electric dipole $\mathbf{P}$ and can be approximately described by

$$D = \sqrt{\gamma}(\sin \theta, -\cos \theta), \quad (\text{R2})$$

where γ is the decay rate of the quasi-BIC and related to the Q factor. For the metasurface with different rotation angle $\theta(x, y)$ of holes at different spatial positions (x, y) , the polarization angle of resonant emission is varied, i.e., $D(x, y) = \sqrt{\gamma}(\sin \theta(x, y), -\cos \theta(x, y))^T$. By changing from linear polarization (x - y) basis to the helical basis, the emission of resonance follows

$$D_h(x, y) = \sqrt{\frac{\gamma}{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \begin{pmatrix} \sin \theta(x, y) \\ -\cos \theta(x, y) \end{pmatrix} = i \sqrt{\frac{\gamma}{2}} \begin{pmatrix} -e^{i\theta(x, y)} \\ e^{-i\theta(x, y)} \end{pmatrix}. \quad (\text{R3})$$

So, the emission can be formulated as

$$|D\rangle = i \sqrt{\frac{\gamma}{2}} e^{-i\theta(x, y)} |L\rangle - i \sqrt{\frac{\gamma}{2}} e^{i\theta(x, y)} |R\rangle \quad (\text{R4})$$

where $|R\rangle$ and $|L\rangle$ denote the right- and left-handed circularly polarized (RCP and LCP) unit vector. From this formula, we can clearly see that the RCP and LCP components acquire opposite geometric phases, depending on the rotation angles of holes, which can also be understood more phenomenologically from the Poincare sphere picture (top inset in Fig. 1B). The geometric phase is equal to half the solid angle enclosed trajectory of polarization states on the Poincare sphere.

The nonlinear behavior between rotation angle of hole and geometric phase arises from the low refractive index difference. Since Si₃N₄ itself does not emit light, we have to introduce dye into PMMA layer and thus the refractive index difference is only $\Delta n = 0.53$. Such a small index difference prevents the resonant electric field from being completely bound in the nanodisk. As plotted in Fig. R11A, a large portion of electromagnetic field distributes out of the nanodisk. Then the interaction between neighboring nanodisks becomes dramatic and the square-latticed periodic nanostructure affects the field distribution, leading to a square-like distribution of electric fields in the unit cell. Such a field pattern does not match the circular rotation of the hole and results in a nonlinear relationship between the rotation angle of the hole and the polarization angle.

Based on the above result, it is easy to know that the nonlinear relationship can be overcome by replacing Si_3N_4 with GaN or GaP. Since GaN and GaP are direct bandgap semiconductors, the dye-doped PMMA is not necessary any more. With the increase of refractive index difference, we can see that the electric field distribution is more tightly in the nanodisk and closer to a donut. As a consequence, much better linear relationship between the orientation angle of resonant electric dipole and the hole rotation angle can be obtained in Figs. R11B and R11C.

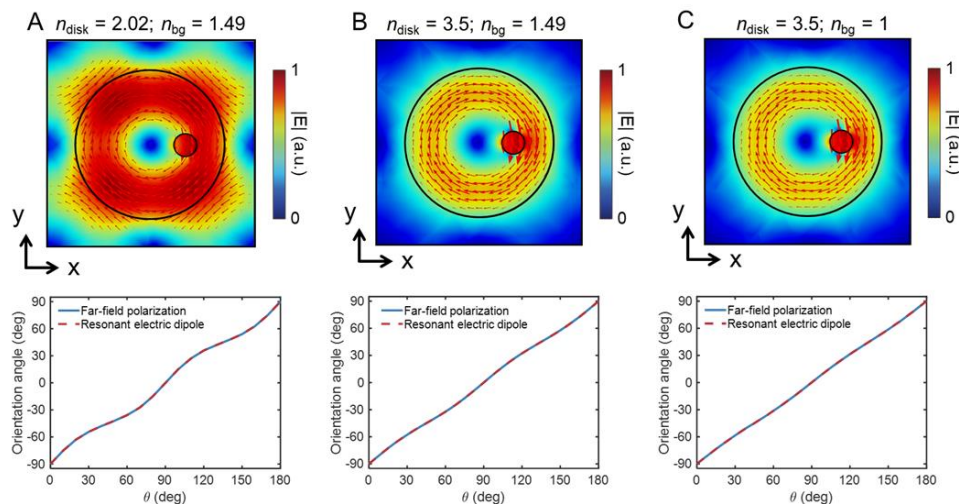


Figure R11 (Figure S4 in the Supplementary Information). The electric field distribution of the quasi-BICs, polarization azimuthal angle of emission and the orientation angle of resonant electric dipole as a function of rotation angle of the hole for materials with $n = 2.02$ in PMMA (A), $n = 3.5$ in PMMA (B), and $n = 3.5$ in air (C).

In the revised manuscript, we have added the above information into **Para-2, Page-4** of the main text.

[“Device performance can be further improved using dielectric with higher refractive index \(Supplementary Note-1\).”](#)

Detailed information has been added as new sections **“Dipole approximation and the corresponding geometric phase”** and **“Supplementary Note-1: Geometric phase”** to Methods and Supplementary Information.

Comment-2:

Phase resolution is crucial for achieving high-quality wavefront shaping. What is the angular precision of the fabrication process, or are there other factors that could influence this? Additionally, could this precision limitation pose a challenge to scalability for improved wavefront shaping?

Our Response:

We thank the referee for the careful review and valuable suggestion. Ideally, the rotation angle of the hole can be continuously varied. However, there is a step size s in E-beam lithography, which is the distance between the steps of the beam as it writes the patterns. This step size determines the minimum angular resolution. Here, we present a simple derivation to evaluate this angular resolution. Assuming that the center of the nanodisk is at the origin $(0, 0)$, the central coordinates of the eccentric hole as it rotates can be written as $(L\cos\theta, L\sin\theta)$. When the hole rotates by a small angle $\Delta\theta$, the resulting displacements in the x - and y -directions are $\Delta x = L\cos(\theta + \Delta\theta) - L\cos\theta$ and $\Delta y = L\sin(\theta + \Delta\theta) - L\sin\theta$. For small $\Delta\theta$, they can be approximated as (by Taylor expansion) $\Delta x \approx -L\Delta\theta \sin\theta$ and $\Delta y \approx L\Delta\theta \cos\theta$. The constraint of the step size suggests that $\Delta\theta$ must be such that $\max(|\Delta x|, |\Delta y|) \geq s$ (Fig. R12A).

Based on this, we can get the angular resolution $\Delta\theta$, which is $\frac{s}{L} \leq \Delta\theta \leq \frac{s\sqrt{2}}{L}$ for different θ .

Here, we take the maximum value of $\frac{s\sqrt{2}}{L}$ as the angular resolution, which allows the rotation of the hole to be resolved at any θ . In our fabrication process, the beam step size in E-beam lithography is set to 2 nm. So, we can approximately get an angular resolution of $\sim 2.7^\circ$, which is going to be smaller for larger L . In addition, the subsequent processes (including development, lift-off, etching, etc.) will also affect the effective pixel precision, which is quite difficult to evaluate, thus the actual resolvable pixel size might be a bit larger.

Although the rotation angle of the hole is not varied continuously, the fabricating angular resolution is still sufficient for wavefront shaping of the metalaser. To prove this, we calculate the eigen-fields of cylindrical lens metalaser composed of discrete phases. The numerical apertures are set as 0.2. The focusing efficiency is calculated and plotted as a function of number of phase step within $0-2\pi$ (Fig. R12B). It is clear that the focusing efficiency reaches a saturation value of 0.8 when phase steps are larger than 16. Such a phase step corresponds to an angular resolution of $\leq 22.5^\circ$, which is almost an order of magnitude larger than our experimental resolution. Therefore, the nanofabrication precision won't degrade the wavefront shaping of our metalaser. This is also consistent with the previous reports of F. Capasso's group.

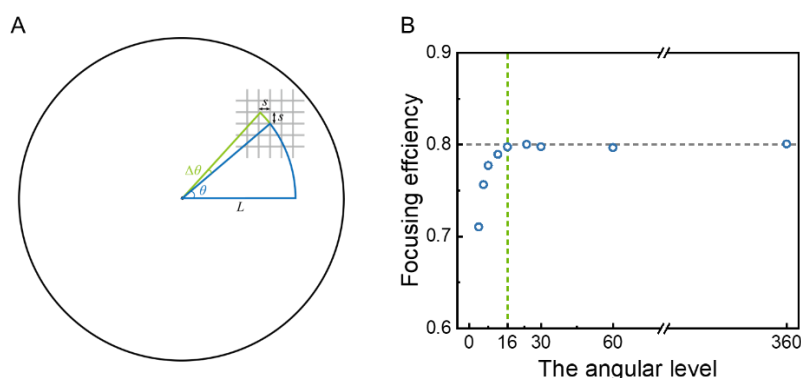


Figure R12 (Figure S9 in Supplementary Information). Effective angular resolution and its impact. A. The schematic diagram of angular resolution calculation. **B.** The focusing efficiency as a function of the number of discrete phases within 2π .

In the revised manuscript, we have added the above information to **Method** section.

[“In our experiment, the beam step size in E-beam lithography is 2 nm, giving an effective angular precision of the hole of 2.7°.”](#)

The detailed discussion has been added to “**Supplementary Note-3: Experimental details**” of the Supplementary Information.

Comment-3:

Scalability and reproducibility are critical for practical applications. In this paper, the largest array is $180\ \mu\text{m} \times 180\ \mu\text{m}$. What limitations might arise when scaling up the array size? Would the pattern quality improve with a larger array? Additionally, while Fig. 4C shows good agreement in the intensity of some dots between the two samples, the authors should provide more comprehensive comparisons of the entire generated patterns across samples with the same hologram to better assess reproducibility.

Our Response:

We highly appreciate the referee for this very careful review and valuable comment. There is actually no fundamental limitation of our metalaser. In principle, both bound state in the continuum (BIC) and quasi-BIC requires an infinite size in at least one dimension to maintain the extremely high Q factors. In this sense, the larger sample is always better. Practically, our sample size is restricted by writing field of our E-beam aligner. Size deviation and misalignment become more dramatic close to the edge of writing fields, restricting the sample size below $180 \times 180\ \mu\text{m}^2$.

In case of lines in Fig. 4C, they are used to show that the background dots from two different samples are actually consistent with each other very well. This indicates that such dots arise from the design of our samples rather than the speckle patterns. The latter one is supposed to be random in different samples. Following the referee’s comment, we have randomly selected ten different lines of two images in horizontal and vertical directions, respectively. Here the main image of “ π ” is saturated to enhance the visibility of background signals. The intensities of background signals along 10 lines from two samples are plotted together and shown in Fig. R13 below. Similar to the one in the main text, the relative intensities in two samples are slightly different. But the peak positions of two different samples match one another very well in all lines. Based on these observations, we have more confidence on the conclusion that the background spots are caused by the design rather than speckle noise.

In the revised manuscript, we have added the above information to **Para-1, Page-8** of the main text.

“The intensity along line- α (see Fig. 4A) of two metalaser holograms are plotted in Fig. 4C ([more details in Supplementary Note-8](#)). [All peaks of holograms generated by two different samples surprisingly match one another well.](#)”

The detailed results have been added to “**Supplementary Note-8: Metalaser hologram of “ π ”**” of the Supplementary Information.

The restriction of sample size is added to **Method** section.

“[Sample size is restricted by the writing field of electron-beam system.](#)”

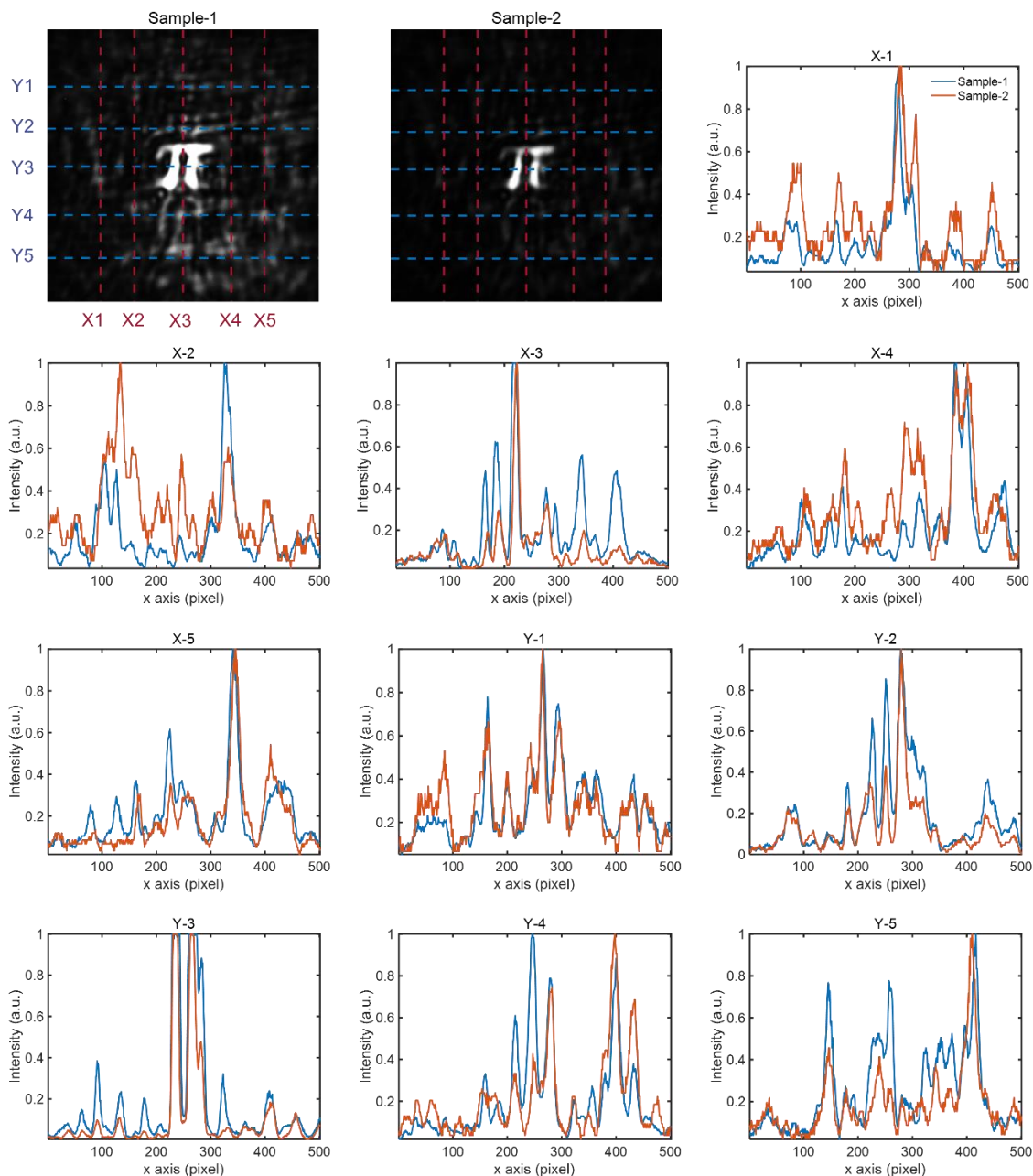


Figure R13 (Figure S31 in Supplementary Information). Two holographic images of “ π ” from two different samples. To magnify the background signals, the main image of “ π ” is largely saturated. Ten lines are randomly selected and marked as X1-X5 and Y1-Y5. The intensity distributions of two different samples along these ten lines are plotted together. All the peak positions match one another very well.

Comment-4:

What is the uniformity of the laser intensity across the array? Since this would also impact the far-field pattern, it would be helpful to include some images of the sample plane to better evaluate the lasing performance.

Our Response:

We thank the referee for this valuable suggestion. Based on this suggestion, we have experimentally recorded the near-field image of the metalaser hologram of “ π ”. Here the focused laser beam is an optimized Gaussian shaped pump beam. The result is depicted in left panel of Fig. R14. In addition to the bright center, a series of ring like patterns can also be observed. This is caused by the phase profile of our metalaser. For a direct comparison, we have numerically simulated the near-field emission pattern of the metasurface. Ring like patterns can indeed be observed in right panel of Fig. R14.

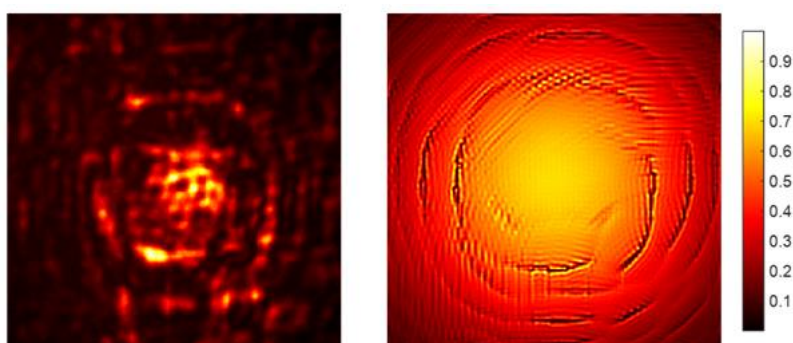


Figure R14 (Figure S30 in Supplementary Information). Experimentally recorded (left) and numerically calculated (right) near-field intensity distributions of metasurface hologram of π .

In the revised manuscript, we have added the above information into “**Supplementary Note-8: Metalaser hologram of “ π ”**” of the Supplementary Information.

Comment-5:

Is a single disk cavity sufficient for lasing? If so, the metalaser could be viewed as an array of collectively coupled lasers. In this case, it would be interesting to experimentally characterize the far-field emission intensity and profile for both a single cavity and the collective array and conduct a comparison. For reference, see Science 372, 403-408 (2021), although the underlying physics of laser arrays is different.

Our Response:

We highly appreciate the referee for this valuable comment and suggestion. For the case of quasi-BIC or BIC, as mentioned above, it requires at least one dimension to be infinitely large. In this sense, only a single nanodisk without considering the periodicity won't be able to achieve the controllability of the metalaser. To verify this point, we have increased the

periodicity to 800 nm and 1600 nm. Since the separation distance is much larger than the lasing wavelength, the interaction between neighboring nanodisks can be neglected. Then each nanodisk resonant like an individual one. Now the Q factor is extremely low because of the small refractive index difference between Si₃N₄ and PMMA. The corresponding experimental results are shown in Figs. R15A and R15B. No lasing action can be achieved with the increase of pump power. And no obvious modulation on the spontaneous emission can be seen at all. The entire curve is almost identical to the emission from a bare PMMA film (Fig. R15C). A possible way to realize a collective array like *Science* **372**, 403-408 (2021) is utilizing the supercavity mode in a single nanoparticle if a higher refractive index dielectric such as GaAs or GaP is utilized.

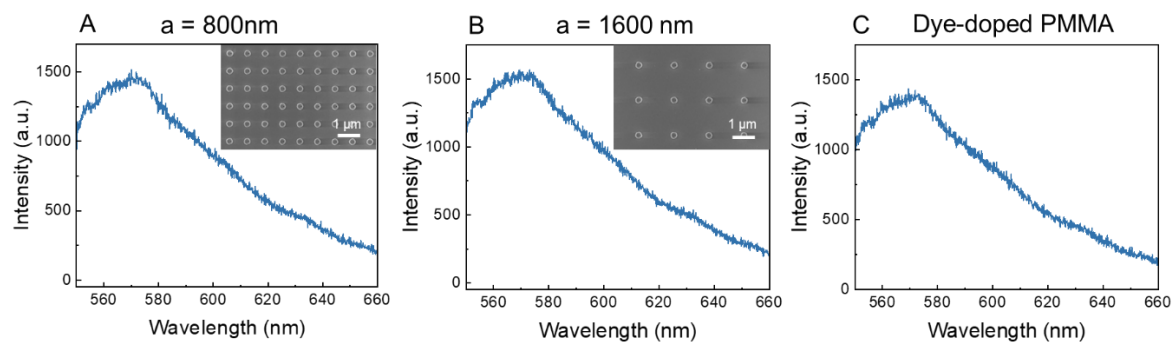


Figure R15. Emission spectrums for the samples with a period of 800 nm (A), 1600 nm (B), and a bare dye-doped PMMA film (C). Insets in (A) and (B) show the top-view SEM images of samples.

In the revised manuscript, we have added this information to **Para-2, Page-2** and **Para-2, Page-9** of the main text.

[“Recent developments in active systems and momentum space manipulation suggest further possibilities to directly control nanolasers.²⁵⁻³¹”](#)

[“Our main physical concept and underlying realizations can be extended to other nanophotonic devices significantly improving their performance in various optics and photonics applications^{1-9,31}.”](#)

The reference *Science* **372**, 403-408 (2021) has been cited as **Ref. 31** in the revised main text.

Comment-6:

Real time programming of emission patterns is important to achieve for device applications. See, for example, *Nature* 612, 246-251 (2022) and PRX 15, 011024 (2025). Is it possible to reconfigure the emission pattern in a real time manner? The authors are encouraged to discuss this based on their metalaser platform.

Our Response:

We thank the referee for this valuable comment. We agree with the referee that real time programming of emission pattern is more important for device applications. Based on the referee's suggestion, we have discussed the potential of real-time programmable metalaser.

Similar to spatial light modulators, materials with viable refractive indices are needed to tune the functionality of metalaser. Here we utilize the phase change material such as Sb_2S_3 and VO_2 and illustrate this potential with several examples.

(1) Chiral laser emission with controllable angle

Highly integrated chiral laser emission is quite essential for many on-chip optical and quantum information processing. In 2022, Zhang et al. exploited chiral quasi-BIC to achieve chiral nanolaser emission in the normal direction (Ref. 30, *Science* **377**, 1215-1218 (2022)). One year later, Cheng-Wei Qiu's group further utilized the spin-valley locking effect to extend chiral laser emission to tilted angles (Ref. 25, *Nature Materials* **22**, 1065-1070 (2023)). In Fig. 3C of the main text, we have shown that both novel functions can be easily realized with our metalaser. Using metalaser, we no longer need to consider various complex physical mechanisms, but only need to design the phase distribution based on angular requirements.

Below, we show that our metalaser can unlock even more performances. Figure R16A shows the schematic diagram of the unit cell our metalaser. It is almost the same as the one in Fig. S24 except that the high-index dielectric Si_3N_4 is replaced with Sb_2S_3 whose refractive index can be optically or electrically tuned. The corresponding phase profile is defined following the equation $\theta = 2\pi \sin(\phi) x/\lambda$. By placing dipoles into the metasurface, we simulated the emissions from the Sb_2S_3 metasurface. As solid lines depicted in Fig. R16B, LCP and RCP laser beams can be achieved at $\pm 15.2^\circ$, respectively. Then the Sb_2S_3 is optically or electrically tuned. The phase change increases the refractive index of Sb_2S_3 from 3.00 to 3.68. As a consequence, the LCP and RCP emissions are tuned to $\pm 18.3^\circ$ (dashed lines in Fig. R16B).

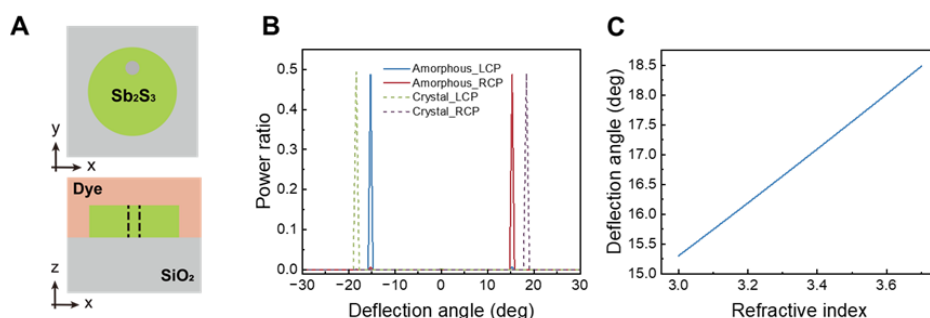


Figure R16 (Figure S25 of Supplementary Information). Tunable angles of chiral emissions. **A.** Schematic of the unit cell of the metalaser composed of Sb_2S_3 and PMMA. **B.** Emission angle of LCP and RCP light before (solid line) and after (dashed line) phase transition. **C.** The emission angle of LCP radiation as a function of intermediate refractive index.

Phase change materials such as Sb_2S_3 have a series of intermediate states. Then their refractive index can be continuously tuned between the amorphous phase to crystalline phase. Using the intermediate refractive indices, we have simulated the angles of chiral emissions and plotted

the results in Fig. R16C. It is obvious that the radiation angles of LCP emission also continuously increase from 15.2° to 18.3° . The corresponding RCP component also continuously decreases from -15.2° to -18.3° . This is the first time that the chiral emission angle can be continuously controlled. Since the phase change materials can be changed back and forth, the continuous angle control is also reversible.

(2) Chiral laser emission with controllable intensity difference

In Fig. R16 and *Nature Materials* **22**, 1065-1070 (2023), the intensities of LCP and RCP lasers at opposite tilt angles are identical. Here we show that their relative intensities can also be tuned with the phase change materials. Figure R17A shows the schematic of the unit cell. Our metasurface is formed by square-latticed periodic nanostructures (Fig. R17A). The lattice size is $p = 400$ nm. Each unit cell contains two GaP ($n = 3.5$) nanodisks with radius of $R = 150$ nm and thickness of $h = 100$ nm that are separated by a thin layer of VO₂ with thickness of $d = 300$ nm. The entire nanostructure is embedded in PMMA. Two small holes with radius of $r = 20$ nm are patterned into two nanodisks with polar coordinates of (ρ, θ) and $(\rho, \theta + \delta)$, respectively. Here we keep $\rho = L + r = 70$ nm. As discussed in the main text, the in-plane electric field within the small hole is significantly enhanced. Then the electric fields in two nanodisks are equivalent to two dipoles with an included angle α (inset in Fig. R17A). Meanwhile, these two dipoles are vertically separated by with a fixed a distance of $d = 300$ nm.

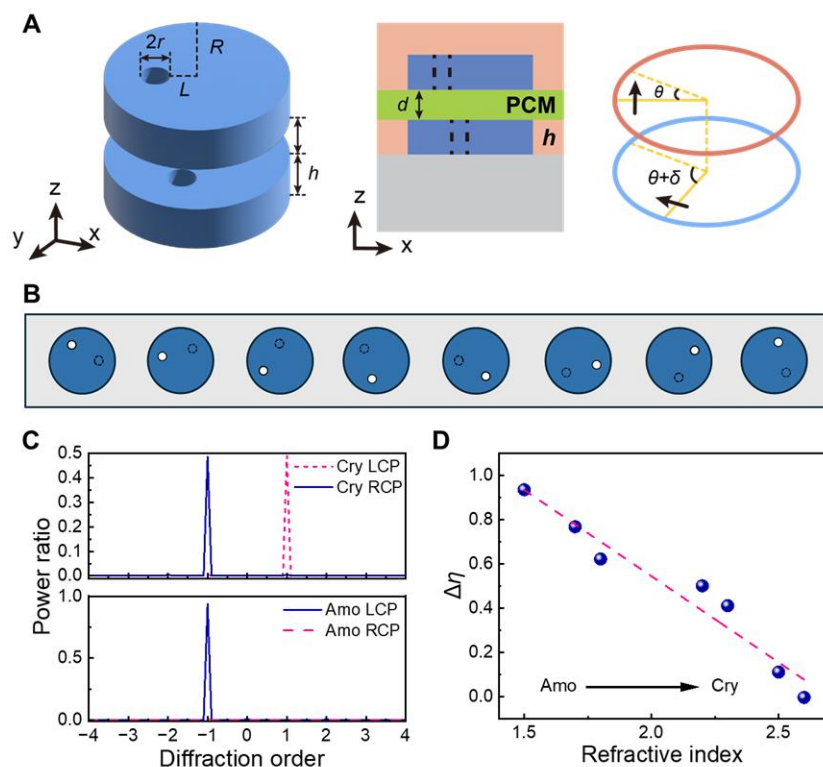


Figure R17 Tunable relative intensity of chiral emissions. **A.** Schematic picture of the unit cell with all structural parameters. **B.** The top-view of the supercell. The white hole and black circle represent the positions of two holes in top and bottom disks. The included angle is fixed at 120° . **C.** Far field angular distributions of LCP and RCP emissions before (bottom panel) and after (top panel) phase transition. **D.** Intensity difference between LCP and RCP as a function of index of VO₂.

Two equivalent dipoles with included angle δ and vertical separation d can break the symmetry between LCP and RCP components following the factor of $\sin(\delta \pm kd)$. Figure R17B shows the top-view of the supercell. The white hole and black circle represent the positions of two holes in top and bottom disks. Here we fix the included angle $\delta = 120^\circ$. For the amorphous case, we can see that the RCP component is completely cancelled, giving a pure LCP emission (bottom panel in Fig. R17C). The phase transition of VO₂ increase the refractive index (from $n = 1.5$ to $n = 2.6$) and thus changes kd . Since the included angle α is fixed, the factor $\sin(\delta \pm kd)$ shows that some LCP components are coupled to RCP components. Then RCP emission gradually increases and LCP emission reduces, eventually leading to a balanced LCP and RCP emissions (top panel in Fig. R17C). Figure R17D summarizes the intensity difference $|(I_{LCP} - I_{RCP})/(I_{LCP} + I_{RCP})|$ as a function of phase transition of VO₂. During the phase transition, it continuously reduces from a factor of 1 to 0, corresponding to the transition from unidirectional emission with a particular spin angular momentum (SAM) to bi-directional laser beams with opposite SAMs. Once again, the refractive index of phase change materials now is close to the refractive index of nanodisk. The dependence can be more linear if we utilize liquid crystals with lower refractive index.

(3) Controllable OAM nanolasers

The above mechanism for tunable emission angle and relative intensity is also applicable to OAM nanolaser in Fig. 3D of the main text well. It is almost that the same except that a spiral phase profile is introduced. Here we show that the topological charge l of the OAM beam can also be controlled with phase change material. Figure R18A shows the overall configuration of the phase plate with $|l| = 6$. It is obvious that the phase plate has been divided into 6 regions with different colors. Each section contains 8 subsections with phases of $0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$ (see schematic in Fig. R18A). Before the phase transition, the radiation from the metasurface corresponds to two OAM beams with $l = 6$ and $l = -6$, respectively.

Then we selectively excite regions with green colors and induce the transition of Sb₂S₃ from amorphous state to partial crystalline state. The increase in the refractive index brings the operating wavelength into the band gap (Fig. R18B). Light in the amorphous region cannot enter the phase change regions, thus destroying the long-range interaction between the phase change region and the rest ones. As a consequence, the emission at the operation wavelength is rapidly reduced to ~ 0 in phase change regions. One example is illustrated in Fig. R18C. Light is widely distributed in all cells before phase transition. After the selective phase transition in different regions, light can only be confined within the unchanged area. The contribution to the total emission from the phase change regions can be neglected. In this sense, the metalaser after selective transition can be considered as a type of metalaser with “1” (amorphous) and “0” (crystalline) states.

With the above information, we can thus simply control the topological charge of metalaser by selectively changing the phase of some regions. For the case that regions with green colors are phase changed, we then numerically calculated the emission profiles from the metalaser. The results are shown in Fig. R18D. The outputs still show a donut shaped beam with vanished

center intensity. But the shape is different and the corresponding phase profiles give topological charges of $l = \pm 3$ now. With a similar process, the topological charge can be tuned to the other number as well. In addition, due to reversibility of Sb_2S_3 , the transition between different topological charges can be also tuned back and forth.

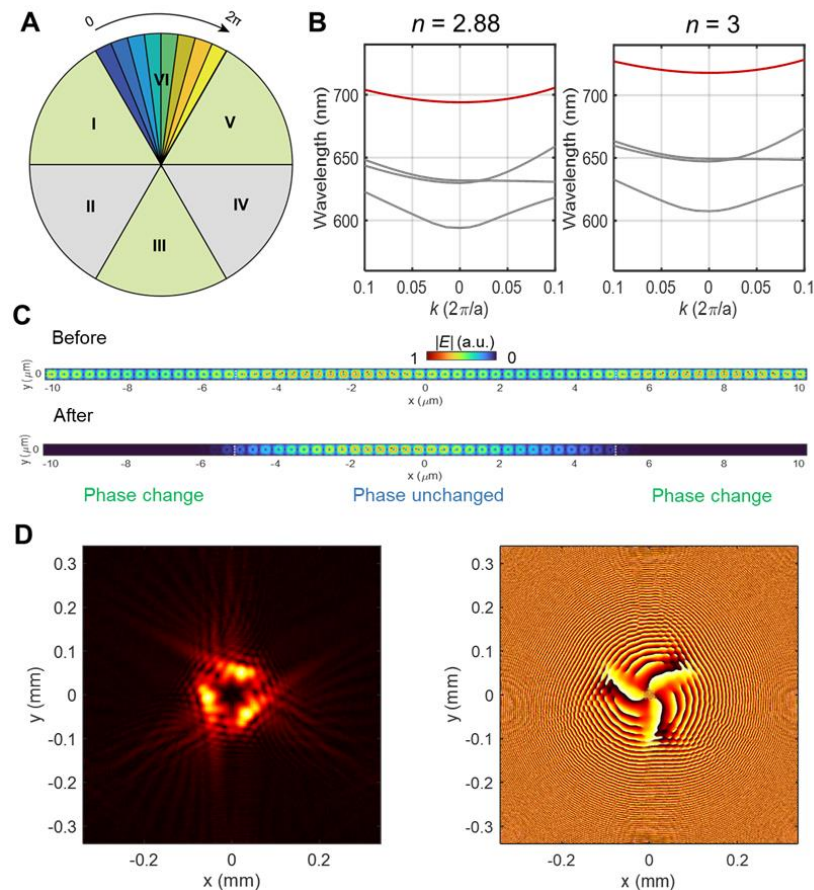


Figure R18 (Figure S27 of Supplementary Information). Tunable topological charge of OAM emissions. **A.** The overall schematic picture of the Sb_2S_3 based metalaser phase plate. Phase transition of Sb_2S_3 happens in green-color regions. **B.** The band structure of green-color regions before and after phase transition. The operation wavelength is around 690 nm. **C.** An example to illustrate the field distributions before and after the partial phase transitions. **D.** The simulated far field emission beam and its phase profile.

(4) Programmable metalaser holograms

In addition to the control of laser emission direction, relative intensity, and topological charge, our metalaser also has the capability of producing programmable metalaser holograms. Here we consider a simple case to realize a real programmable metalaser. Figure S28A shows the schematic of our meta-atom. It is a GaP nanodisk (with refractive index $n = 3.5$) with two tiny holes with an included angle of $\Delta\theta$. One hole is filled with phase change materials of VO_2 whose refractive index can be tuned from $n = 1.5$ (amorphous state) to $n = 3$ (crystalline state). For the case of crystalline state VO_2 , its refractive index is almost the same as GaP and the geometric phase is purely determined by the PMMA hole. When VO_2 is in amorphous state,

the geometric phase is produced by two holes simultaneously and thus the included angle starts to play an essential role. Figure S28B shows the overall geometric phase of the meta-atom as a function of angle $\Delta\theta$. It is clear to that the geometric phase can have a difference close to π before and after the phase transition when $\Delta\theta$ approaches 360° . As a consequence, the recently developed concept of digital metasurface can be applied here to produce many different functions from a single metalaser.

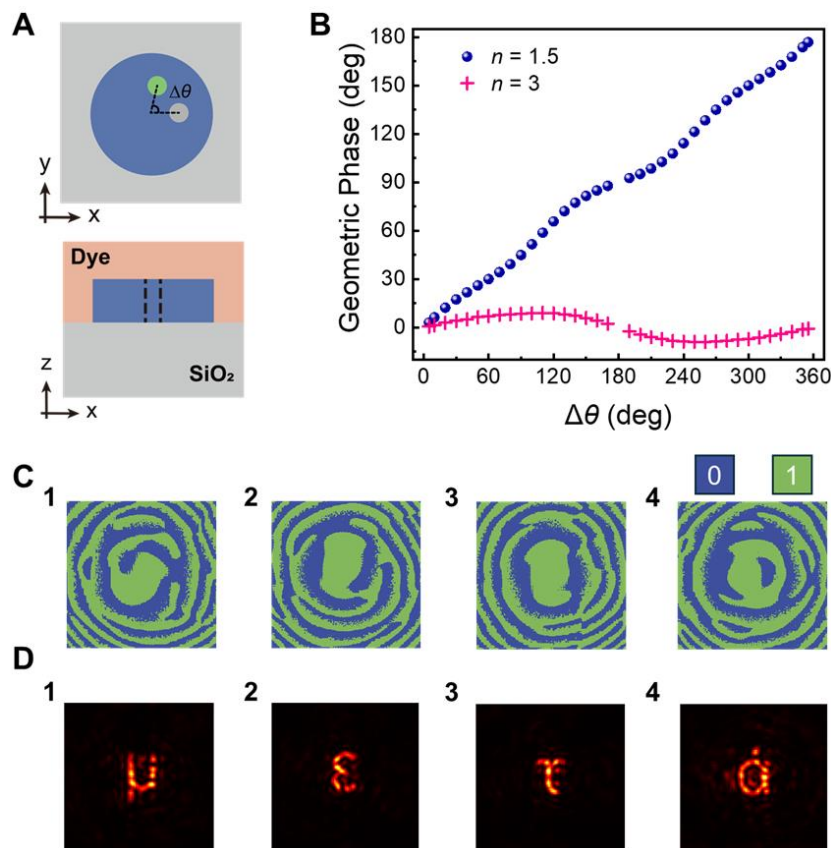


Figure R19 (Figure S28 of Supplementary Information). Programmable metalaser hologram. **A.** The schematic of the meta-atom for programmable metalaser. The entire sample is also periodic patterned with a square lattice. Two holes with PMMA and VO₂ have an included angle $\Delta\theta$ and are marked with different colors. **B.** The difference of geometric phase before ($n = 1.5$) and after ($n = 3.0$) phase transition as a function of the included angle between two holes. **C.** Four phase profiles of digital metalaser defined by the phase switching caused by the phase change and unchanged. Here the included angle is fixed at $\Delta\theta = 340^\circ$. **D.** The corresponding holographic images of “M E T A” in ancient Greek produced by four phase profiles in C. The selection phase transition of the same metalaser can produce different holographic images, enabling the possibility of programmable emission.

Each meta-atom is quite small and relatively difficult to be controlled independently. Here we utilize 2×2 pixels as a supercell with the size around $1 \mu\text{m}$. This size is easy for alignment and selectively change the phase of VO₂. Meanwhile, VO₂ can be switched between states back and forth. Following the above discussion, we have defined four different phase profiles (Fig. R19C) consisting of geometric phases of 0 and π , respectively. Figure R19D shows the

corresponding holographic images. With the same metalaser, different characters of “M E T A” in ancient Greek can be achieved. Due to the reversibility of phase change materials and micrometer sized supercell, all these profiles can be erased and re-written in experiment. There is no fundamental limitation in function number and many other holographic images can be realized with the same sample too. In this sense, the entire metalaser becomes programmable. Beside holographic image, this digital metasurface based concept can also be applied to tunable lens and other functions too. In this sense, we can confirm that our design also has the potential to realize programmable metalasers in the common sense.

In **Para-1, Page-7** of the revised main text, we also discussed the potential of metalaser in programmable emissions.

“All these results demonstrate that laser wavefronts and far-field radiation can be precisely tailored at will. With the assistance of phase change materials, programmable emission can also be achieved with a single metalaser (Supplementary Note-7).”

The detailed information has been added to a new section of “**Supplementary Note-7: Towards programmable emission**” to Supplementary Information.

Response to Referee #1:

The authors significantly improved the description of the performance limitations and removed the overstated claims about their metalaser holograms. Additionally, new relevant simulations and discussions have been added to the revised Supplementary Information. I suggest accepting the revised manuscript in its current form.

Our Response:

We thank Referee #1 for his/her careful review, and the final suggestion of acceptance.

Response to Referee #2:

Thank you for the comprehensive response to the many questions. My original report was negative because the main claim ("programmable") had no supporting evidence, and its repeated use at pivotal sentences meant that this was the point on which the paper would need to be judged, and unfortunately found wanting. The authors agree and have removed this claim, so now one can judge the novelty and importance based on the revised message. I agree that this is a lovely demonstration and a design concept that is likely to capture the imagination. On the one hand, it is able to produce outputs that would otherwise have required various physical mechanisms to implement, while allowing designer wavefronts to be made into a tiny structure. The "meta" in metalaser is now clearer - not just a metasurface with some gain. The forward look towards programmable with phase transition materials is interesting. I imagine there will be technical hurdles, e.g., VO₂ changes phase with temperature which may be hard to control in a small lasing volume, but nevertheless, it is interesting that there are concepts to move forward with. My only remaining concern is that the authors still keep ambiguous phrases such as "controlled at will" or "tailored at will" which have an element of suggesting 'real-time', even if not said. Any native English speaker would interpret it this way. It would be better to be cautious and rephrase all such statements. Rather than "the geometric phase can be controlled at will" it is more accurate to say "the geometric phase can be designed to be an arbitrary function", or "designed at will". One can only claim what has been shown, not future manifestations that are yet to be verified. There are many instances when such rephrasing would be in order.

Our Response:

We sincerely thank Referee #2 for taking time and effort to review our paper and provide very constructive feedback. We have revised the descriptions in the main text as follows:

In the **Abstract**:

"Although polarization, orbital angular momentum, and directivity of nanolasers have been successfully manipulated⁷⁻⁹, neither their laser wavefront nor radiation characteristics can be customized at will."

"As a consequence, the speckle noise becomes negligibly small in our metalaser holograms, providing a viable solution to the speckle noise problem of conventional laser holograms."

In **Para-2, Page-3** of the main text:

"Therefore, laser wavefront and corresponding far-field radiation patterns are determined solely by the geometric phase ϕ assigned to each pixel, and they can be designed at will."

In **Para-1, Page-5** of the main text:

"These results demonstrate that laser wavefronts and far-field radiation can be arbitrarily customized."

In the **Conclusion**:

"With the manipulation of the geometric phase at each pixel, we have shown that the laser emission can have arbitrarily-shaped wavefront."

"Owing to the inherent advantage of our metalaser based on the interplay of local and nonlocal

responses of metasurfaces, it has the potential to resolve the long-standing challenge of speckle noise in holographic displays without the expense of the image quality.”

Response to Referee #3:

I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

Our Response:

We thank Referee #3 for his/her help with the review of our manuscript.

Response to Referee #4:

I have read carefully the revised manuscript, the revised supplementary information, and the authors' responses to review comments. I think the authors have done a great job addressing the review comments and concerns raised in the previous round. In particular, the integration of phase-change materials into the meta-laser platform is both elegant and technically compelling. My only remaining question is about the reconfiguration speed of the phase-change materials for programming meta-laser emissions; it would be helpful to specify this in the supplementary information. Overall, I believe the manuscript is very strong and should be accepted with minimal, if any, revisions.

Our Response:

We thank Referee #4 for his/her careful reading of our manuscript and suggestion of acceptance. In Supplemental Note-7, we have added the comment on the reconfiguration speed of phase-change materials, as follows.

“Notably, the phase transition time of phase change materials determines the reconfiguration speed of the programable metalaser. The time scale of photon-induced phase transition for VO₂ is hundreds of femtoseconds. Specifically, a phase transition time of ~100 fs was obtained from measurements⁶ with a pump fluence of 25 mJ/cm². In addition, we have experimentally measured the transition time of Sb₂S₃ from crystalline to amorphous phase under the excitation of femtosecond laser in previous works, which is as short as 800 ns⁷. These short time scales of photon-induced transition for Sb₂S₃ and VO₂ indicate that the reconfiguration speed of programmable metalasers can be very fast.”