
Supplementary information

Squeaking at soft–rigid frictional interfaces

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1 **Supplementary Materials**

2 **Squeaking at soft–rigid frictional interfaces**

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- 6 Supplementary text
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Supporting Information Text

S1. Fabrication

The samples used in this study are composed of nearly incompressible elastomers and thermoplastic polyurethane (TPU). Below, we outline the fabrication processes employed for each of these two materials.

S1.1. Elastomeric samples. The elastomeric samples used in this study are made of nearly incompressible polyvinylsiloxane (PVS) elastomers: (i) Elite Double 32 from Zhermack with green color and initial shear modulus $\mu \approx 0.32$ MPa (1) for the results presented in Fig. 1 to 4 of the main text; (ii) Smooth-Sil 960 from Smooth-on with platinum color and a shear modulus $\mu \approx 2$ MPa at 100% elongation for the results presented in Fig. 5 of the main text. All samples are cast using a mold, which is designed using Fusion 360 (Autodesk) and 3D printed with Formlab SLA 3D printer. This step-by-step procedure is followed (see Fig. S1):

- **Step 1:** we 3D print the mold, which is the negative of a cuboid with a base that is either flat or patterned with ridges. Note that 1/8" holes are introduced at each corner to facilitate alignment.
- **Step 2:** we laser cut an acrylic base to which we later attach the elastomeric sample (see Step 4). To improve bonding between this plate and the sample, we etch a square region at the center of the base with the dimensions of the sample. This process increases the roughness of the central region by introducing features with size of $\approx 200 \mu\text{m}$. We place alignment holes at each corner of the base.
- **Step 3:** we fill up the mold with uncured elastomer mixture.
- **Step 4:** we close the top of the mold with the acrylic base while the etched region is facing down and we introduce 1/8" pins at each corner to facilitate alignment. We apply pressure on top of the acrylic base to ensure drainage of excess material. Then we keep the mold at room temperature (25°C) for 20 minutes.
- **Step 5:** we remove the acrylic plate from the sample while keeping the rubber sample inside the mold. We spray the top of the sample with a primer (PP Primer, Aron Alpha) to increase the bonding of the glue and let it rest for 3 minutes.
- **Step 6:** we spray the square rough region on the acrylic base with a cyanoacrylate based glue (Gorilla) and immediately bring it in contact with the surface of the sample (still inside the mold) to achieve bonding. We use pins to align the rubber sample and the acrylic plate.
- **Step 7:** we maintain the mold and acrylic plate under compression using a vice for 2 hours, and finally remove the mold.

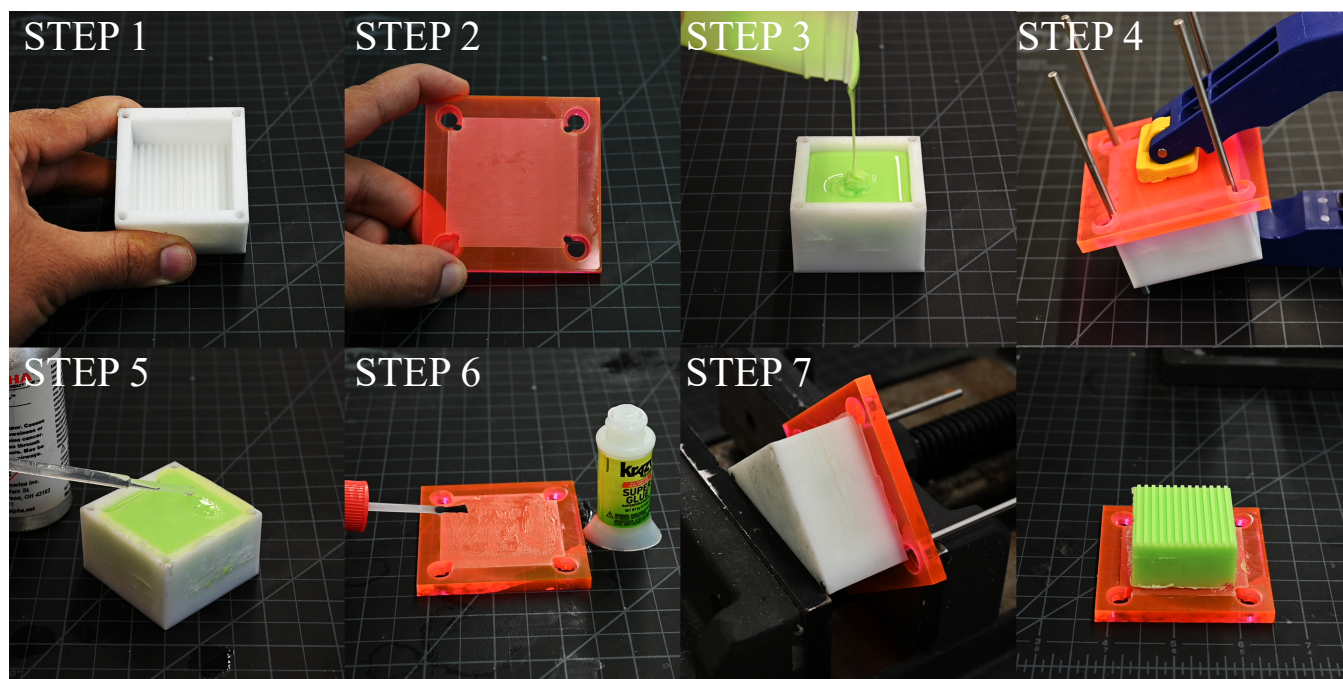


Fig. S1. Fabrication of elastomeric samples. Snapshots of the 7 steps required to fabricate our elastomeric samples.

36 **S1.2. TPU samples.** We also fabricated samples using a thermoplastic polyurethane (TPU), specifically white NinjaTek NinjaFlex
37 85A (Ninjaflex, Manheim, PA), which has a Shore hardness of 85A and a Young's modulus of $E = 12$ MPa. The material was
38 procured in filament form and printed using a fused deposition modeling 3D printer (Bambu Lab X1C) with 100% concentric
39 infill to ensure uniform density. To avoid delamination (a failure mode observed in preliminary samples where filament lines
40 ran parallel to the ridges) the filament paths were oriented perpendicular to the direction of sliding in the experiments.

41 Each TPU print was bonded to an acrylic plate using the same procedure describe in Section [S1.1](#) for the elastomeric samples.
42 To ensure a flat surface, samples were thermally processed on a hotplate (Corning PC-2420D) set to 235°C, a temperature
43 found to produce smooth surfaces. Samples were not placed directly on the hotplate but instead rested on a 40 mm × 60 mm
44 glass slide atop the heated surface. A 500 g weight was applied to the sample, which was heated for 15 seconds. The sample
45 was then removed and cooled for one minute before being returned to the hotplate with the weight reapplied for an additional
46 15 seconds.

47 Because of the thermoplastic nature of TPU, localized heating from mechanical stress and pressure during actuation can
48 soften or partially melt the material. To account for plastic deformation behavior (unlike the fully elastic response of cross-linked
49 elastomers) multiple samples (2–3 per test condition) were prepared for each test.

S2. Wave speed characterization for the samples

In this Section we present the two procedures used to to characterize the shear wave speed in our samples.

S2.1. Wave speed characterization via capillary break tests. We begin by measuring material P-wave, c_P , and Rayleigh-wave, c_R , speeds via capillary break tests on an elastomer block (Elite Double 32 from Zhermack) of dimension $150 \times 150 \times 50$ mm with four acoustic emission sensors (KRNBB-PC) placed in line along the center of its surface with spacing of 15 mm (Fig. S2A). Our tests were similar to those recently reported in (2). A 1 mm diameter glass capillary was broken 15 mm away from the first sensor, leading to the emission of elastic waves (Fig. S2B). The P-wave arrival is identified as the time when the signal amplitude first exceeds the noise level (square markers - Fig. S2C) and c_P is estimated from the time delay between a pair of sensors, Δt_{ij} , as

$$c_P = \frac{\Delta L_{ij}}{\Delta t_{ij}}, \quad [S1]$$

where ΔL_{ij} denotes the distance between the considered pair of sensors. We obtain a mean value $c_P = 1005 \pm 85$ m/s. To identify the Rayleigh-wave arrival we apply a low-pass filter with a 40 kHz cutoff frequency on the collected data in the time window $1000 \leq t \leq 4000$ μ s and locate the first local minimum (triangular markers) and local maximum (circular markers) after the P-wave signal (Fig. S2D). We obtain a mean value $c_R = 21 \pm 1$ m/s. We then calculate the shear wave speed, c_S , from the measured c_P and c_R by solving the following equation (3)

$$0 = 4\sqrt{1 - c_R^2/c_P^2}\sqrt{1 - c_R^2/c_S^2} - (2 - c_R^2/c_S^2)^2, \quad [S2]$$

and find $c_S = 22 \pm 1$ m/s.

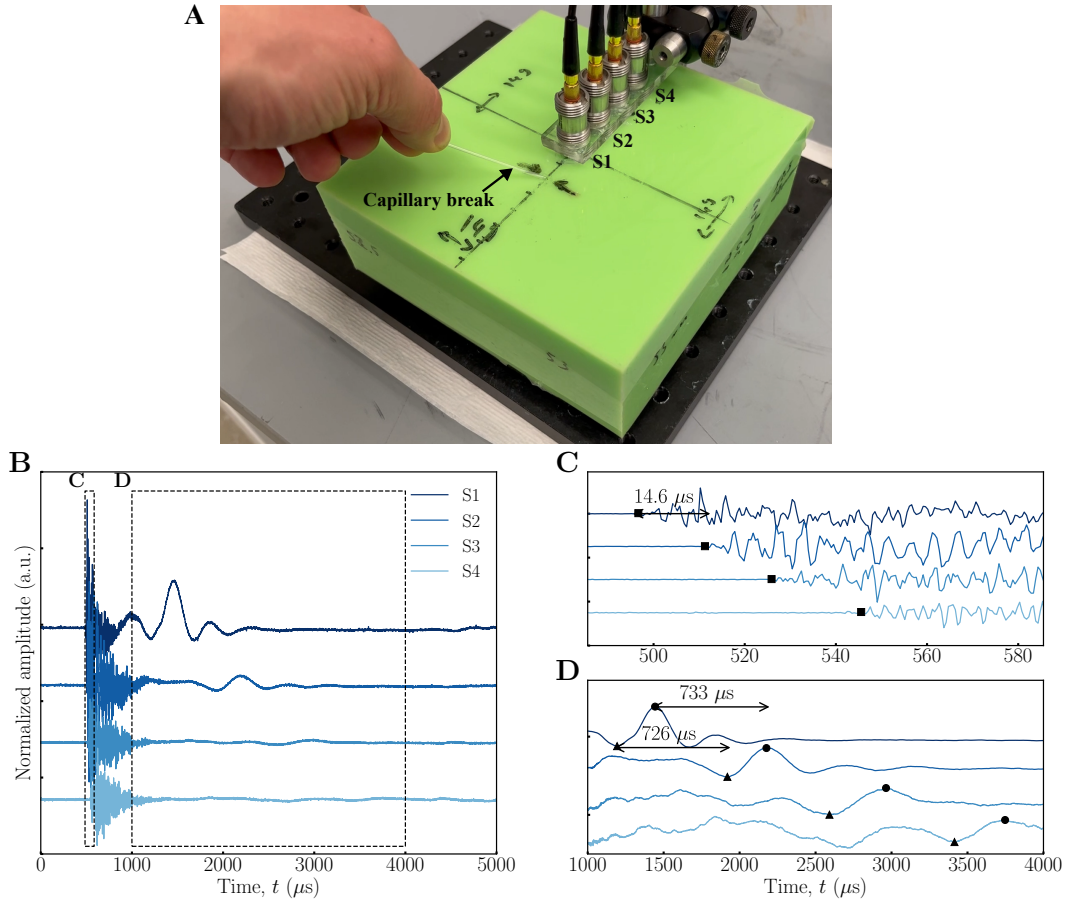


Fig. S2. Wave speed measurements in elastomer sample. (A) Our setup. (B) Raw signal recorded by each acoustic emission sensor during a test. (C) Zoom-in of the signal recorded (with normalized amplitude) in the time window $485 \leq t \leq 585$ μ s. (D) To identify the Rayleigh-wave arrival we apply a low-pass filter with a 40 kHz cutoff frequency on the collected data in the time window $1000 \leq t \leq 4000$ μ s and locate the first local minimum (triangular markers) and local maximum (circular markers) after the P-wave signal.

S2.2. Shear wave speed characterization via dynamic bending tests. While the tests described in Section S2 allow for the characterization of both c_P and c_S , dynamic bending tests can be performed to estimate c_S alone. For these test we fabricate beams with dimensions 30 mm × 25 mm × 7.8 mm for Elite Double 32, Smooth-Sil 960 and 59.6 mm × 10.05 mm × 2 mm for Ninjaflex TPU. Note that the dimensions of the much stiffer TPU sample are chosen to make it relatively more slender and to allow for signatures of the bending resonance in the force signal. All samples are clamped on one edge and excited around their first bending mode using a shaker (see Fig. S3). To determine the resonant frequency of this vibration resonance, we use an impedance head that captures the dynamic force between the sample and the shaker, as well as the acceleration at the vibrated clamp. After manual exploration of the frequency of the first bending mode, a 20-second frequency sweep is performed around this resonance, to measure it precisely as the minimum of the force magnitude. We measure First bending mode frequency $f_b = 1.4 \pm 0.2$ Hz, 3.2 ± 0.2 Hz, and 45 ± 2 Hz for Elite Double 32, Smooth-Sil 960 and Ninjaflex TPU, respectively (see Table S1).

Having determined the frequency associated to the first bending mode, we then use Finite Element (FE) simulations to estimate the Young's modulus of the materials. Specifically, we create 3D models of the beams and perform eigenfrequency analyses where we adjust the value of the Young's modulus until the predicted frequency of the first bending mode matches the experimentally obtained one. Note that in these simulations we assume the Poisson's ratio of Elite Double 32, Smooth-Sil 960 to be $\nu = 0.495$ and that of the TPU to be $\nu = 0.47$. The Poisson's ratio is that indicated by the manufacturer, but its role in our final estimate of shear wave velocity is in any case limited for these almost incompressible materials. Finally, the shear wave speed of each material is estimated as

$$c_s = \sqrt{\frac{E}{2(1+\nu)\rho}}. \quad [S3]$$

As shown in Table S1, we find that $c_S = 21 \pm 2$ m/s, 29 ± 2 m/s and 113 ± 2 m/s for Elite Double 32, Smooth-Sil 960 and Ninjaflex TPU, respectively (see Table S1). Note that the estimated shear wave speed for Elite Double 32 using this bending mode resonance method (21 ± 2 m/s) matches the shear wave speed obtained from the elastomer block surface wave experiment described in the previous Section S2 (22 ± 1 m/s), confirming the validity of this method to estimate the shear wave speed of the materials.

	Elite Double 32	Smooth-Sil 960	Ninjaflex TPU
Beam dimensions (mm ³)	30x25x7.8	30x25.3x7.75	59.6x10.05x2
First bending mode frequency (Hz)	54 ± 1	71.5 ± 1	18.3 ± 0.4
Assumed Poisson's ratio ν	0.495	0.495	0.47
Estimated Young's modulus E (MPa)	1.4 ± 0.2	3.2 ± 0.2	45 ± 2
Density ρ (kg/m ³)	1050	1253	1190
Shear wave speed c_s (m/s)	21 ± 2	29 ± 2	113 ± 5

Table S1. Shear wave speed characterization via dynamic bending tests.

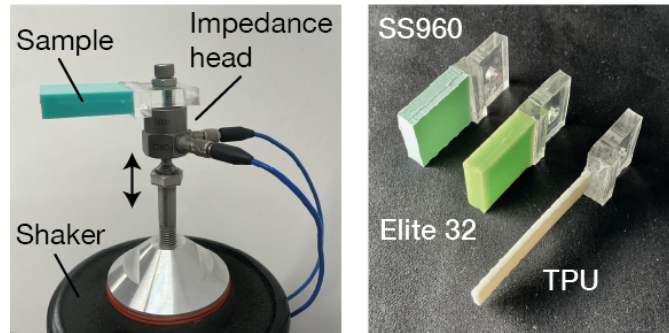


Fig. S3. Shear wave speed characterization via dynamic bending tests Photos of the setup used in the tests and the samples.

S3. Rubber-soled shoes and hands sliding on a nearly-rigid surface

To characterize the behavior of rubber-soled shoes and hands sliding on a nearly rigid surface, we use the experimental setup shown in Fig. S4. It consists of a transparent glass plate measuring 12" × 12" × 0.5", securely clamped to an optical table using an aluminum T-slotted frame. In the shoe tests, a Nike CU3530-100 shoe is worn by an operator who slides it across the glass surface at a velocity of approximately $v_{\text{slide}} \approx 1$ m/s. Similarly, in the hand tests, the operator's hand - cleaned with isopropanol and dried with absorbent tissue - is slid against the glass at a velocity of approximately $v_{\text{slide}} \approx 1$ m/s. We performed two types of experiments with the hands: one where the palm was used (a relatively flat and unstructured surface), and another where the fingers were used, acting as a ridge-like structures. For all experiments, we simultaneously recorded sound data and visualized the frictional interface.

To record the sound, we use a small-diaphragm condenser directional Microphone (NT5, Rode), connected to a dual-channel 48V amplifier (Focusrite Scarlett 2i2). The amplified sound signal is captured using a 12 bit digital oscilloscope (Logic Pro 8 USB Logic Analyzer, Saleae), connected to a computer. The signal acquisition is triggered using an external trigger in the form of a push-button.

To visualize the frictional interface, we rely on a Total Internal Reflection (TIR) optical setup (4–6). More specifically, we use black tape to attach white LED stripe lights (UltraBright™ Industrial Series LED Strip Light, Flexfire LEDs) facing inward to the edges of the glass plate. The LED stripe is connected to a 24V power source. When the light is on and an object makes contact with the glass plate, the contact region is illuminated in white. Beneath the glass plate, we place a first surface flat mirror (200 x 200mm, Enhanced Aluminum, 4-6λ Mirror Edmund Optics) oriented at 45° and directed toward a high-speed camera (Phantom TMX 5010, Vision Research). The camera is synchronized with the microphone through the same external trigger to ensure data acquisition alignment.

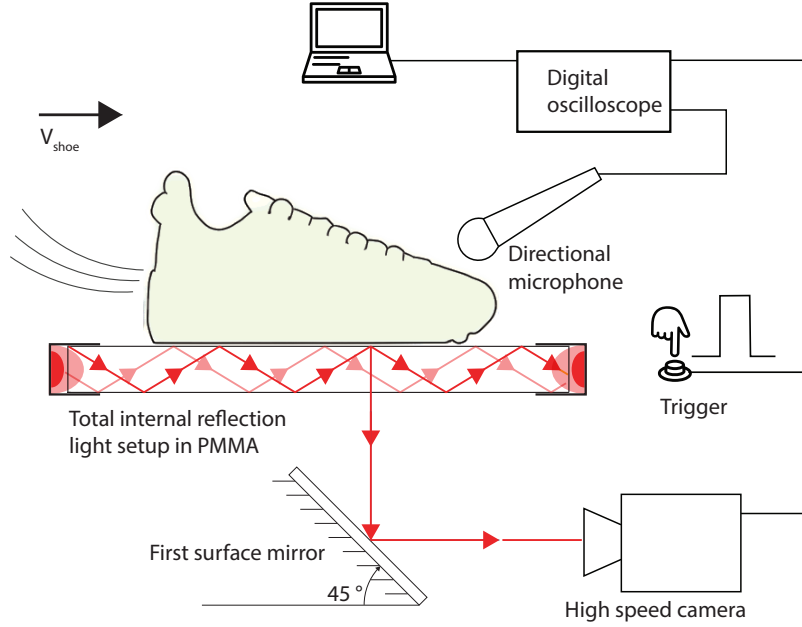


Fig. S4. Experimental setup for testing rubber-soled shoes. Schematic diagram of the test setup used to characterize the behavior of rubber-soled shoes sliding on rigid surface.

S3.1. Results for the rubber-soled shoes. In Fig. S5 we compare sound and contact intensity signals for the shoe experiment presented in Figure 1 of the main text. In Fig. S5A we show the temporal evolution of the recorded sound (black) and evolution as a function of time of the gray intensity values, I_g , for the pixels located along the green line in Figure 1D of the main text. The repetition period of opening pulses, $\Delta t = 0.207 \pm 0.001$ ms, is determined from the local minima in the temporal evolution of contact intensity. The related measurement error of 0.001 ms reflects the standard deviation of the measured Δt . In Fig. S5B we show the spectrum of the temporal signals shown in panel A with dashed red lines indicating the fundamental frequency f_0 and its harmonics. The uncertainty of the frequency measurement is determined by the length of the temporal signal from which the spectrum is computed. For a time window of 10 ms the resulting measurement error is 50 Hz.

S3.2. Results for the human hand. In Figs. S6 and S7 we present results for a palm and a finger of a human hand sliding on the glass plate at $v_{\text{slide}} \approx 1$ m/s. For both the palm and the finger, opening slip pulses are observed and propagate at $c_{\text{pulse}} = 74 \pm 6$ m/s. These opening pulses are directly visible in the high-speed snapshots of Fig. S6, where their propagation is highlighted by dashed purple lines. Importantly we find that the palm - a broad and unstructured surface - produces pulses at irregular time intervals (Fig. S7A), which interact and result in broadband acoustic emission without a dominant frequency (Fig. S7B). In contrast, the finger - which can be seen as a single, narrow ridge- generates more regularly timed pulses (Fig. S7D) and produces a strong acoustic peak at $f_0 = 9$ kHz with harmonics at 18 kHz (Fig. S7E). Furthermore, for the rubber-soled shoes, we observe a strong correlation between the recorded sound and the propagation of the opening pulses. In particular, for the fingers the fundamental frequency of the recorded sound matches the pulse repetition rate (Fig. S7F). While qualitative, these observations support the idea in the presence of ridges acting as waveguides for the opening pulses, a consistent repetition frequency of pulse initiation occurs, resulting in the familiar squeaking sound.

S4. Cuboid samples sliding on a nearly-rigid surface

To characterize the processes occurring at interfaces between our cuboid samples and an acrylic plate, we use the experimental setup shown in Fig. S8. For these tests, we keep the samples fixed in space and slide the acrylic plate. More specifically, we fix

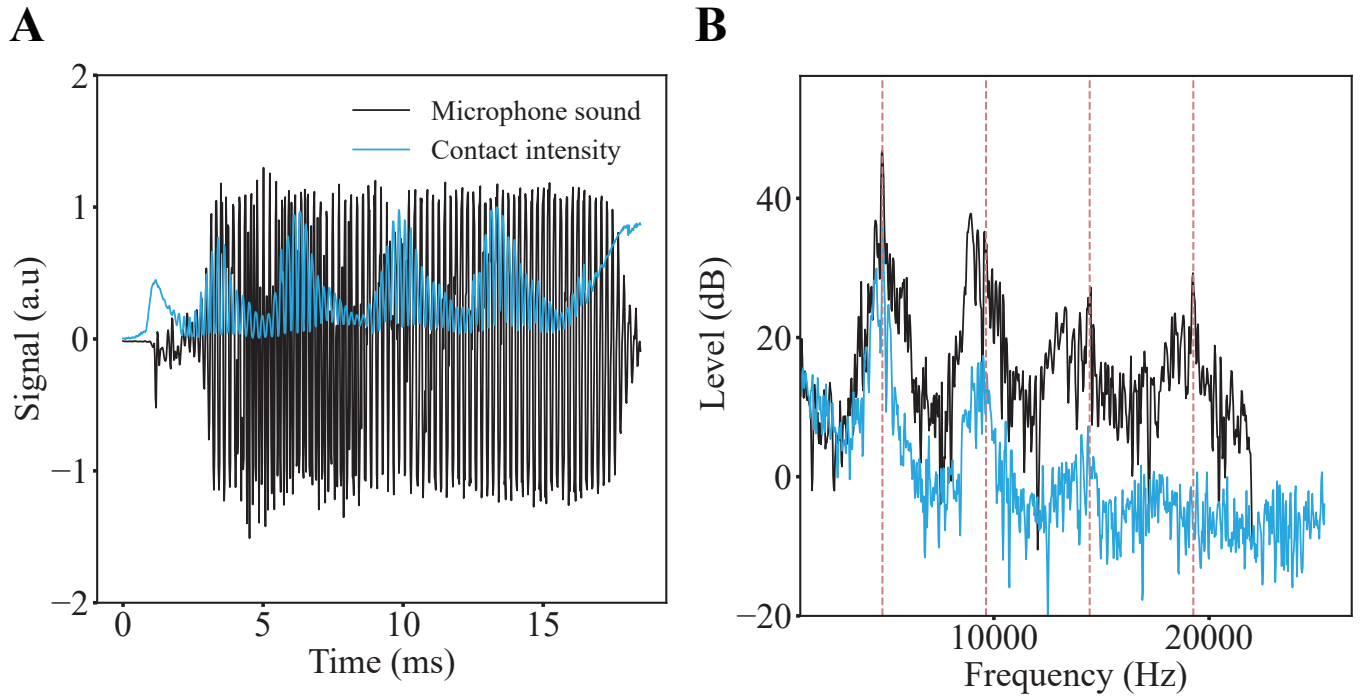


Fig. S5. Comparing sound and contact intensity signals for the shoe experiment in Figure 1 of the main text. (A) Temporal evolution of the recorded sound (black) and evolution as a function of time of the gray intensity values, I_g , for the pixels located along the green line in Figure 1D of the main text. (B) Spectrum of the temporal signals shown in panel A. Dashed red lines indicate the fundamental frequency f_0 and its harmonics.

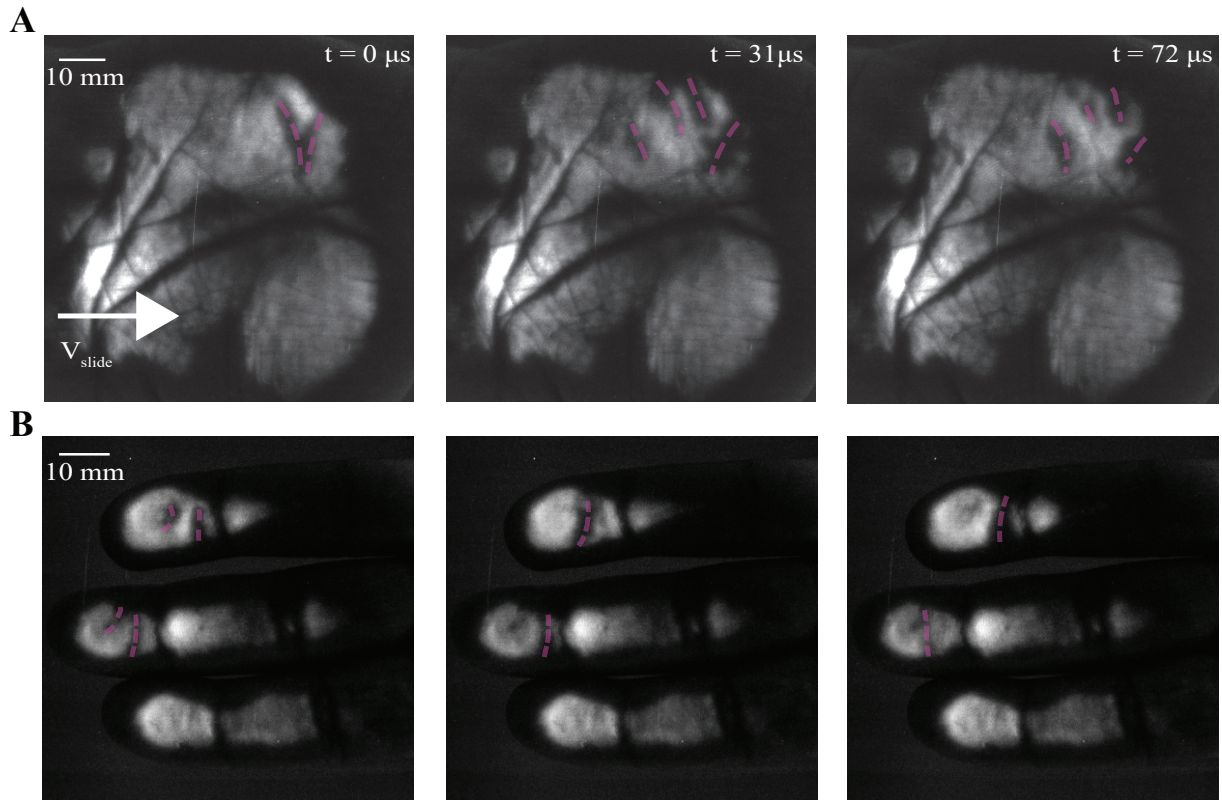


Fig. S6. Snapshots of the frictional interface during sliding of (A) a palm and (B) 3 fingers at $t = 0, 31$ ms, and 72 ms, sliding at $V_{\text{slide}} \approx 1$ m/s. Dashed purple lines highlight fast opening pulses recorded at $97,000$ fps.

134 the samples on a lab jack (Newport 271 lab jack) and move the lab jack to apply the desired normal stress (see Fig. S15).

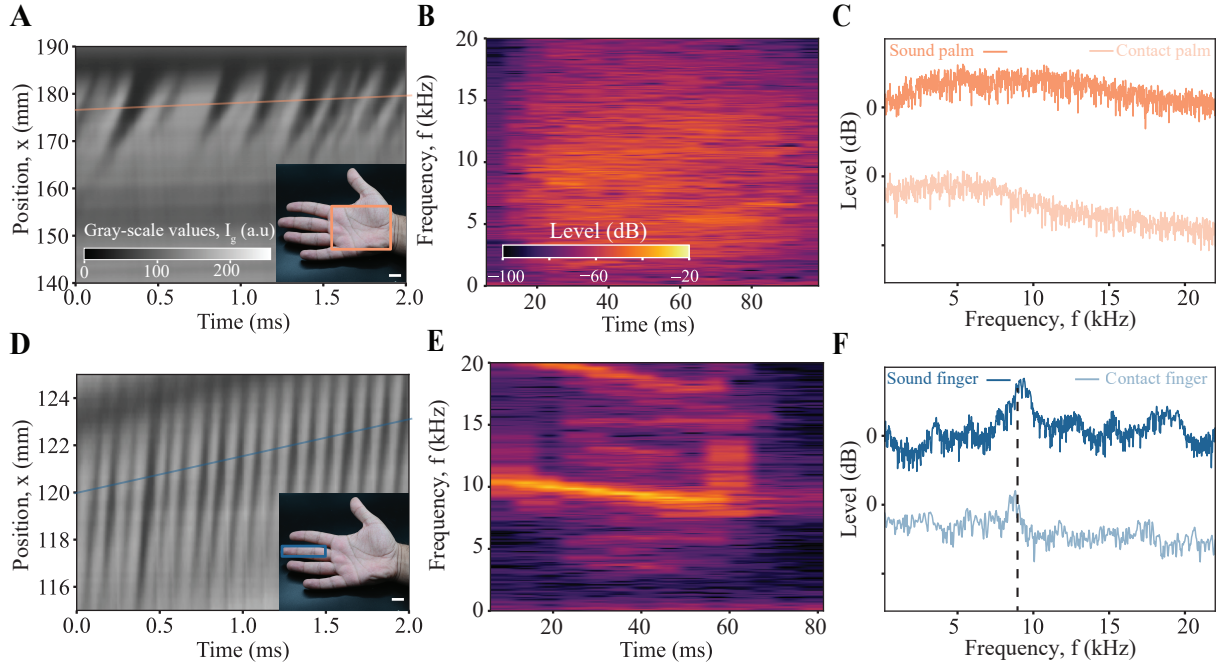


Fig. S7. Sliding a hand and fingers across a glass plate. (A) Contact spatio-temporal maps showing the evolution of gray-scale intensity I_g during sliding of a palm. (B) Spectrograms of the corresponding sound recordings. For the palm, broadband acoustic emission is observed without a dominant frequency. (C) FFTs of both the recorded sound and I_g along the orange lines in panels (A). The palm shows no dominant peak. (D) Contact spatio-temporal maps showing the evolution of gray-scale intensity I_g during sliding of three fingers. (E) Spectrograms of the corresponding sound recordings. The fingers exhibit a clear spectral peak at $f_0 = 9$ kHz. (F) FFTs of both the recorded sound and I_g along the blue lines in panels (D). The palm shows no dominant peak. The fingers exhibit a clear spectral peak at $f_0 = 9$ kHz.

We use an acrylic plate of $12" \times 24" \times 1/2"$ in dimension mounted on 2 cylindrical rails (1200 mm in length, 19.05 mm in diameter) using linear ball bearings bolted on the acrylic plate. The rails are supported by 6 quad T-slotted frames (3 on each side), fixed to an optical table. On one side of the plate we attach two timing belts and use them to connect to a weight of mass m_w . We use a 1000 N electromagnet to suspend the weight. As soon as the electromagnet is turned off, the weight is released and accelerates towards the floor, which in turn drives the plate forward, with a constant driving force $m_w g = 160$ N. In between the plate and the weight, we installed a rotating axis, supplied with timing belt pulleys to guide the timing belt. At one end of the rotating axis, we place a rotary optical encoder (Trinamic TMCS-28-6.35-10k-AT-01) connected to a 12 bit digital oscilloscope (Logic Pro 8 USB Logic Analyzer, Saleae), which allows us to measure the speed of the plate. To trigger the camera and the digital oscilloscope, we use a limit switch at the rear of the plate, which sends a pulse when the plate first moves from its rest position, once the electromagnet is turned off.

As for the case of the rubber-soled shoes we use total internal reflection to visualize the frictional interface and a microphone to record the sound (using the same setup described in Section S3).

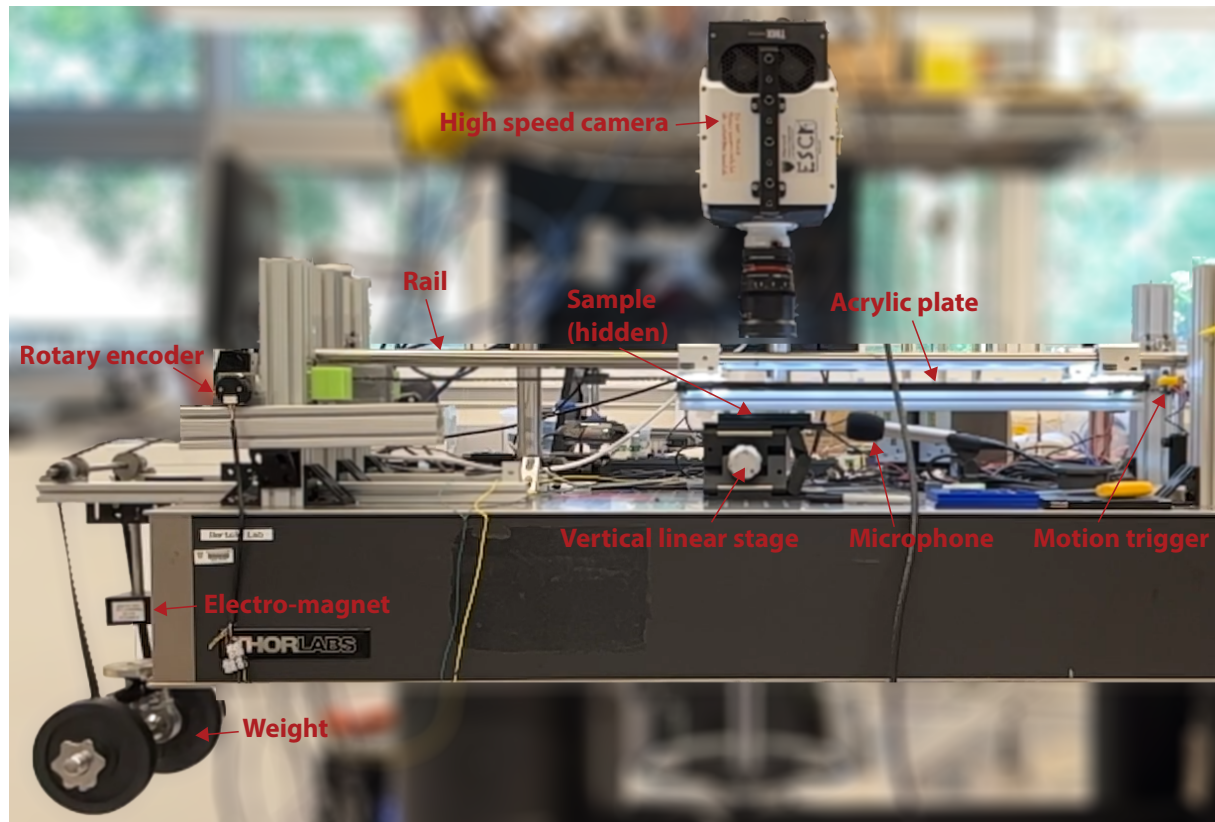
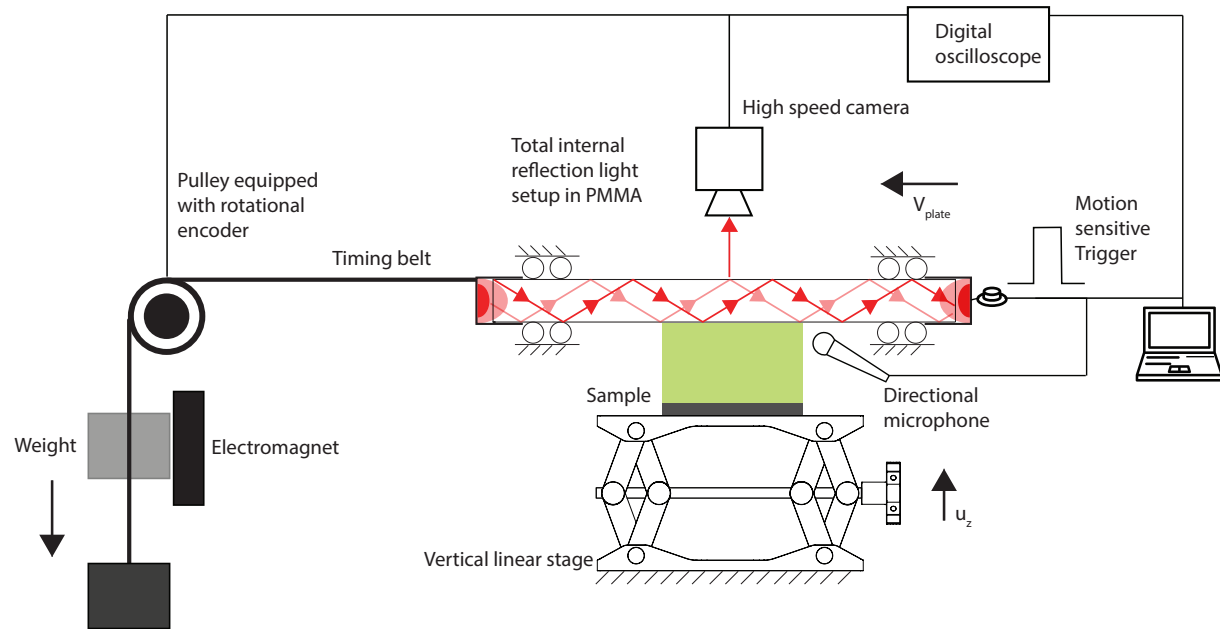


Fig. S8. Experimental setup for testing cuboid elastomeric samples. Schematic diagram (top) and picture (bottom) of the test setup used to characterize the behavior of cuboid elastomeric samples sliding on a rigid surface.

S4.1. Frequency Selection Mechanism for Pulse Nucleation. To gain deeper insight into the mechanism responsible for the regular-frequency nucleation of pulses in samples with interfacial ridges, we first examine a patterned specimen with dimensions $L \times W \times H = 40 \times 40 \times 5$ mm, subjected to a normal load of 35 kPa (Fig. S9). Sliding causes the trailing edge to lift and oscillate locally, periodically modulating contact pressure and momentarily detaching from the substrate (Figs. S9B and C and Video S8). These oscillations have a period of approximately $T \approx 0.5$ ms, corresponding to a well-defined flapping frequency of $f_{\text{flap}} = 2100 \pm 100$ Hz (Fig. S9D). Importantly, the trailing-edge oscillations induce the nucleation of opening pulses that initiate at the rear and propagate toward the leading edge (Figs. S9B and C). Initially, these opening pulses are generated irregularly and reinforce the trailing-edge flapping upon reflecting from the leading edge. However, for $t > 0.13$ s, the system attains a steady state in which each oscillation of the trailing edge gives rise to a new opening pulse (Figs. S9E and F). Consequently, the opening pulses occur at the same frequency as the trailing-edge oscillations, $f_{\text{pulse}} = f_{\text{flap}} = 2080 \pm 100$ Hz. This synchronization demonstrates that periodic contact-pressure modulation by the trailing-edge oscillation governs the tempo of pulse nucleation.

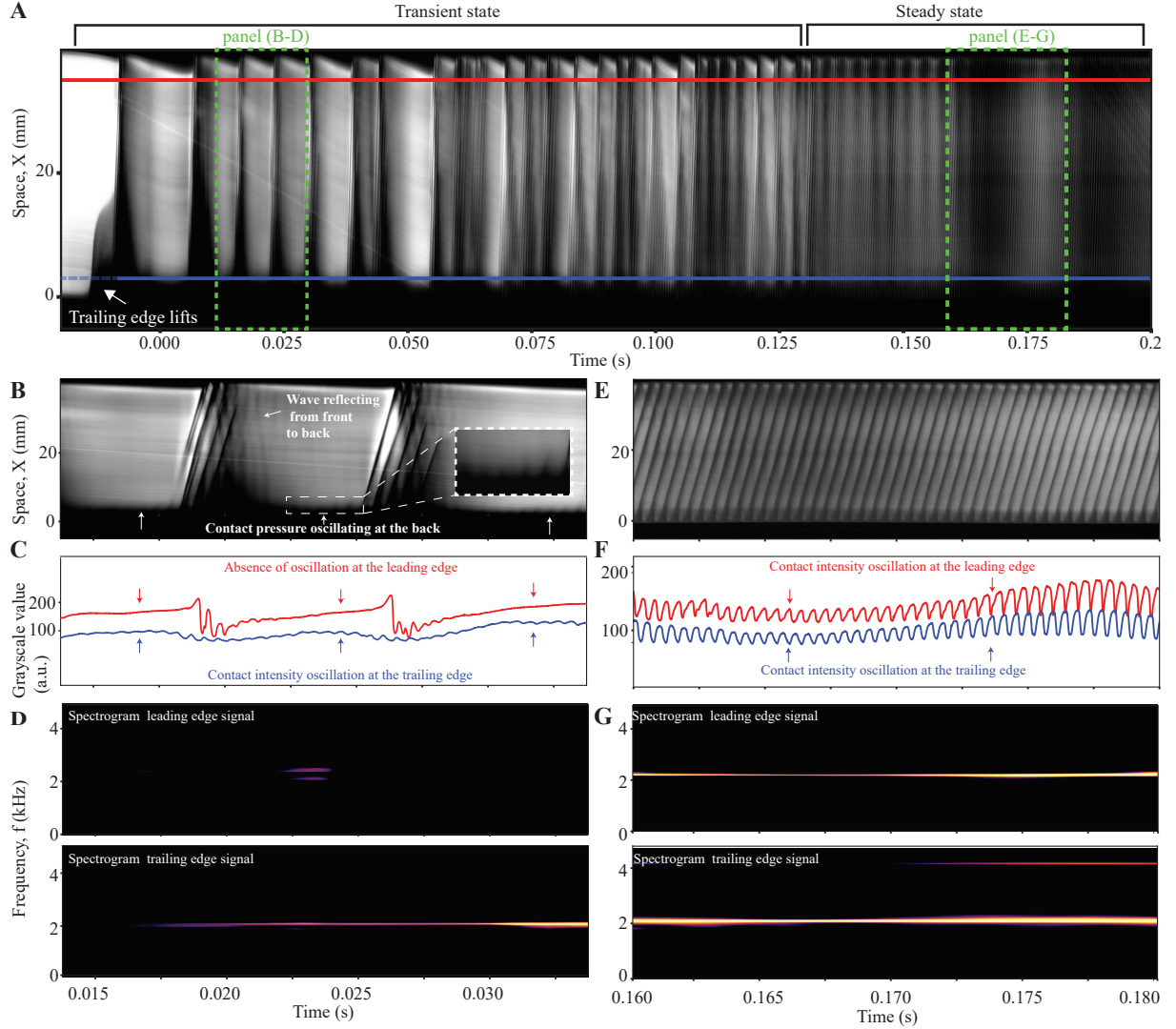


Fig. S9. Trailing-edge oscillations and opening pulses. Typical experiment on a patterned sample of dimensions $L \times W \times H = 40 \times 40 \times 5$ mm under a normal load of 35 kPa. (A) Space-time diagram showing that as sliding begins, the trailing edge lift and oscillate locally, periodically modulating contact pressure and momentarily detaching from the substrate. These trailing-edge oscillations induce the nucleation of opening pulses that initiate at the rear and propagate toward the leading edge. (B) Zoomed-In space-time diagram ($t \in [0.015, 0.033]$ s). In this time interval the opening pulses are generated irregularly and reinforce the trailing-edge flapping upon reflecting from the leading edge. (C) Evolution of the grayscale intensity extracted at location defined in (A): These signals are a proxy for the contact pressure. We observe the presence of oscillations at the trailing edge (blue) with a period $\Delta t \approx 0.5$ ms and their absence at the leading edge (red), beyond the opening pulses signature. (D) Spectrograms of the intensity signals in (C): Trailing edge spectrogram shows a well-defined frequency $f = 2083 \pm 100$ Hz which corresponds to the contact pressure oscillation. Leading edge spectrogram shows no dominant frequency. (E) Zoomed-In space-time diagram ($t \in [0.16, 0.18]$ s). In this time interval the system attains a steady state in which each oscillation of the trailing edge nucleates an opening pulse. (F) Evolution of the grayscale intensity extracted at location defined in (A). We observe the presence of oscillations at the trailing and leading edges (blue and red respectively) with a period $\Delta t \approx 0.5$ ms beyond the opening pulses signature. (G) Spectrograms of the intensity signals in (F) showing a well-defined frequency $f = 2137 \pm 100$ Hz for both the trailing edge and leading edge signals, similar to flapping frequency measured at the trailing edge of the sample in the absence of opening pulses.

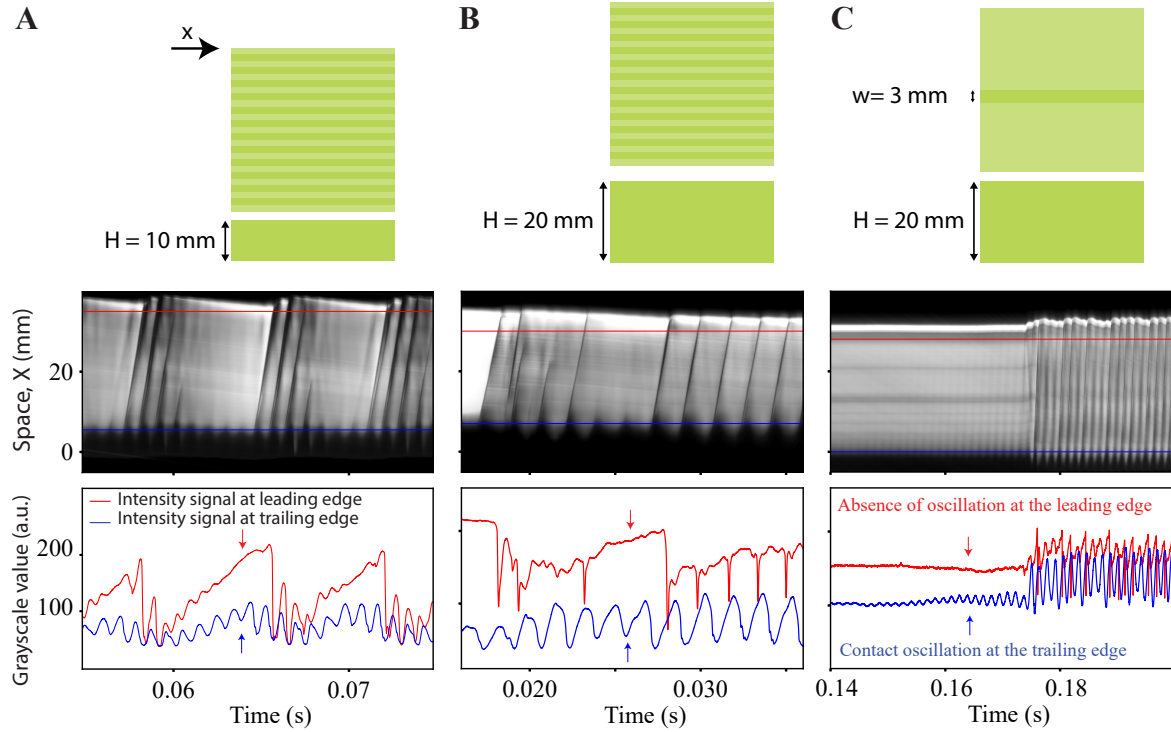


Fig. S10. Trailing-edge oscillations set the frequency of opening pulse nucleation. (A-B) Patterned samples of $L \times W = 40 \times 40$ mm with $H \in \{5, 10, 20\}$ under a normal loading of 35 kPa. Their respective space-time diagrams show a coupling between opening pulses and the flapping of the trailing edge that modulates contact pressure. The contact intensity signals further demonstrate the existence of the trailing edge oscillation whose frequency corresponds to that of the opening pulses measured in steady-state. (C) Sample with a single ridge of 3 mm, under a normal loading of 35 kPa. The space-time diagram shows that as the sample slides, its lifted trailing edge starts to oscillate before the prior nucleation of opening pulses. The frequency of oscillation is measured at $f = 1/\Delta t = 1/0.0016 = 620$ Hz. Opening pulses are generated after $t > 0.175$ s at the same frequency as the flapping of the trailing edge.

The mechanism identified in Fig. S9 is robust across different sample geometries. In patterned specimens with identical in-plane dimensions ($L \times W = 40 \times 40$ mm) but varying heights ($H = 10$ and 20 mm), the trailing edge consistently begins to oscillate regularly, eventually nucleating irregularly opening pulses. These opening pulses later emerge regularly at the same frequency as the trailing-edge oscillations (Figs. S10A–B). Interestingly, for the sample containing a single ridge of width 3 mm, the trailing edge initially oscillates spontaneously for some time without nucleating any opening pulses (Fig. S10C). At $t = 0.17$ s, opening pulses begin to be generated periodically at the same frequency as the trailing-edge oscillation ($f \approx 620$ Hz), with no observable transient. This transition is accompanied by an increase in the amplitude of the trailing-edge oscillations. These observations demonstrate that the trailing-edge oscillations themselves govern the timing of opening pulse generation, provided the frictional interface behaves as a one-dimensional waveguide.

Taken together, these experimental observations demonstrate that trailing-edge flapping dictates the timing of opening-pulse nucleation when one-dimensional waveguides formed by the ridges are present. To uncover the origin of this flapping frequency, we computed the natural frequencies of the deformed block using the Finite Element Method. Specifically, in the simulations we considered two-dimensional models featuring a one-dimensional interface. The models were first subjected to compression and subsequently to shear, while a portion of the rear interface was left free to lift. The natural frequencies were then calculated (Fig. S11B; see Section S6 for details of the numerical procedure). The analysis reveals that the first eigenmode is a shear mode whose frequency scales as $f \propto c_s/2H$ ($c_s = \sqrt{G/\rho}$ being the shear wave velocity of the material), consistent with the frequency of pulse generation reported in Fig. 4 of the main text.

We also note that the samples with a flat frictional interface exhibit trailing-edge flapping during sliding (Figs. S19, S25). However, these oscillations lose coherence once opening pulses begin to nucleate. In this case, the feedback between the propagating pulses and the bulk motion does not lock the system dynamics into a single mode, as in the case of patterned frictional interfaces, but instead excites multiple bulk modes in the sample, preventing sustained synchronization of the trailing-edge oscillations.

S4.1.1. Elastodynamic frictional simulations. The results shown in Figs. S9–S11 indicate that periodic modulation of the contact pressure by trailing-edge flapping promotes the nucleation of opening pulses at the flapping frequency, which closely matches the fundamental shear mode of the block. To further validate this observation, we performed finite element simulations incorporating frictional contact (see Section S6 for details of the numerical procedure). In these simulations, we model two-dimensional viscoelastic blocks with a one-dimensional frictional interface governed by Coulomb friction. The block is initially compressed and subsequently sheared by imposing a velocity boundary condition on its upper surface.

Simulation results for a block of height $H = 0.02$ m are shown in Fig. S12. They confirm the experimental observations and

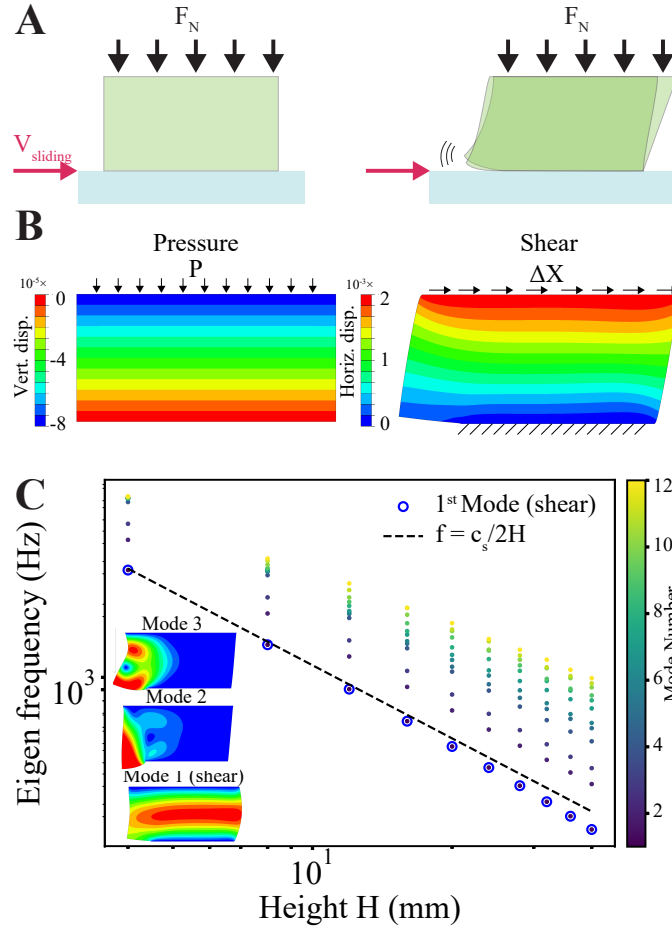


Fig. S11. Eigenfrequency analysis. (A) Illustration of sample lifting and flapping. (B) 2D block simulation comprises three steps: normal loading, then shearing, then finally linear perturbation (see Section S6 for details). (C) The first mode corresponds to the first shear mode for all considered models and the associated frequency scales closely as $f \propto 24/2H$.

reveal an initial transient phase followed by a steady-state characterized by the propagation of slip-opening pulses synchronized with the macroscopic “flapping” motion of the trailing edge. Specifically, the trailing edge first loses contact at $t = 1.5$ ms. As the applied shear increases due to the velocity boundary condition, the noncontact region expands to $x \approx 6$ mm (Figs. S12B–C). At $t = 4$ ms, the first opening pulse nucleates and propagates along the entire frictional interface. The pulse produces local interfacial opening (Fig. S12B) accompanied by high local slip velocity (Fig. S12C). Then, at $t \approx 10$ ms, the system reaches a steady state characterized by a macroscopic flapping motion of the trailing edge (Fig. S12A). Consequently, the front of the noncontact region oscillates periodically between $x \approx 0$ and $x \approx 10$ mm (Figs. S12B–C). The period of these oscillations is $T = 1.59 \pm 0.01$ ms (Fig. S12D) and corresponds to a frequency of $f_{\text{flap}} = 629 \pm 5$ Hz (Fig. S12E), which closely matches the experimentally measured frequency of 620 Hz for a block with the same dimensions. Interestingly, we find that each oscillation of the trailing edge initiates a packet of opening pulses that propagate along the interface at approximately the shear-wave speed, $c_{\text{pulse}} \approx 1.2 c_s$ (Figs. S12B–C). In full agreement with our experimental observations, also in simulations we find that $f_{\text{pulse}} \approx f_{\text{flap}} = 629$ Hz (Figs. S12D and E). This frequency is approximately 11% higher than the analytically predicted shear eigenfrequency (Eq. S19), $f_{\text{theory}} = 565$ Hz, calculated using the instantaneous Young’s modulus E_0 and shear-wave speed $c_s = 22.6$ m/s. The moderately higher frequency observed in the simulations is attributed to the intermittent contact boundary condition, which effectively introduces a stiffening effect.

To validate the experimentally observed frequency scaling, $f \propto c_s/2H$, we varied the block height in the elastodynamic frictional simulations over the range $H = 0.005$ – 0.04 m, while keeping the length fixed at $L = 0.04$ m. As shown in Fig. S13, the simulations exhibit behavior analogous to that reported in Fig. S12 for $H = 0.02$ m. Importantly, these results confirm the experimentally observed scaling relationship $f \propto c_s/2H$ (see Fig. S14).

In summary, our experiments, linear eigenmode analysis, and frictional finite element simulations collectively demonstrate that periodic modulation due to trailing-edge flapping promotes the nucleation of opening pulses at the flapping frequency, which closely matches the fundamental shear mode of the block. As shown in Fig. S14A, for all geometries, the numerically predicted and experimentally measured flapping and pulse frequencies collapse onto a single identity line ($f_{\text{pulse}} = f_{\text{flap}}$). Furthermore, both experimental and simulated frequencies exhibit the same height-dependent scaling as the first shear eigenmode of the

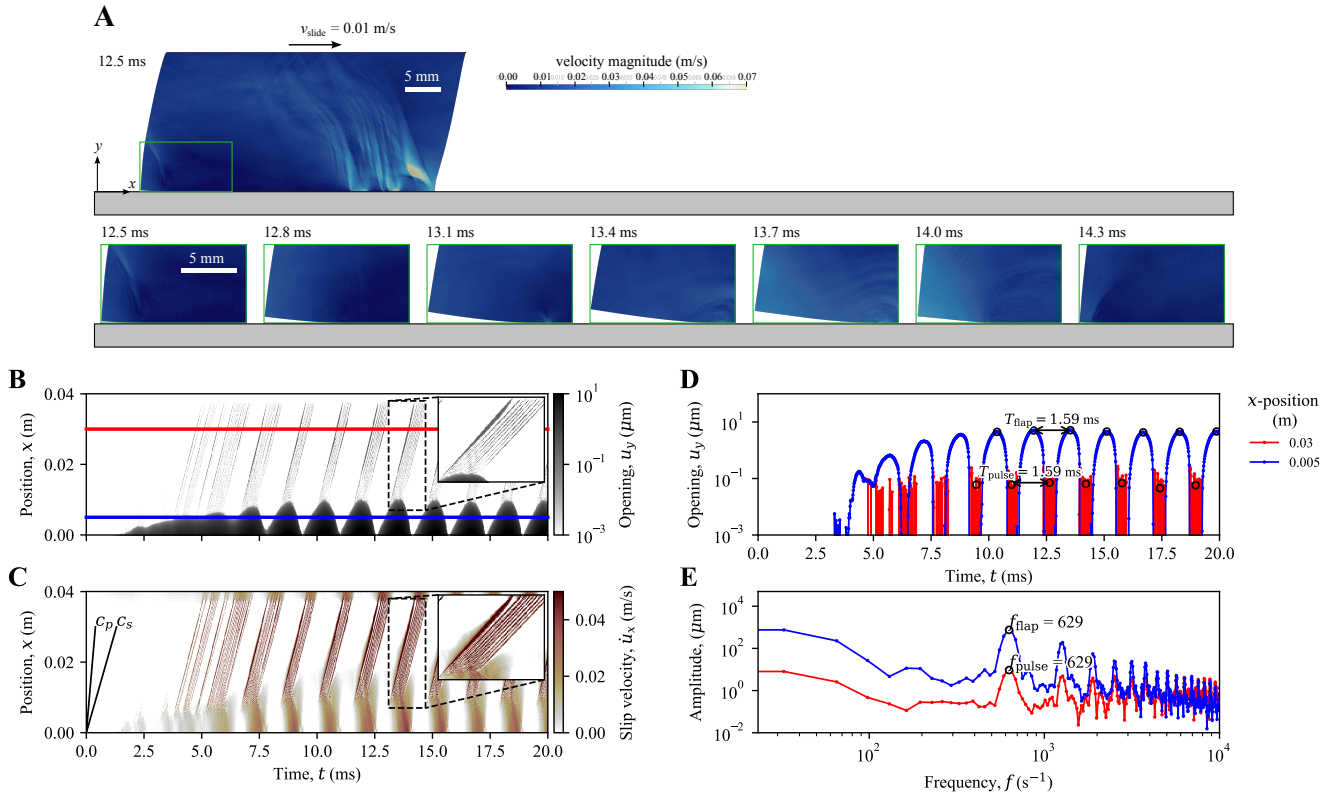


Fig. S12. Elastodynamic frictional simulation results for a block of height $H = 0.02 \text{ m}$ sliding at velocity $v_{slide} = 0.01 \text{ m/s}$ on a rigid substrate subjected to Coulomb friction. (A) Full field velocity magnitude, $||\dot{u}||$, at $t = 12.5 \text{ ms}$ (top), zoom-in at the trailing edge during $t \in [12.5, 14.3] \text{ ms}$ (bottom). (B-C) Space-time plots of interfacial opening (B) and slip-velocity (C). Inset show that opening pulses occur in packets in this simulation. (D) Temporal evolution of interfacial opening u_y at selected x -positions. Tracking local extrema of u_y at the trailing edge ($x = 0.005 \text{ m}$) for $t > 9 \text{ ms}$ yields a flapping period $T_{flap} = 1.59 \pm 0.01 \text{ ms}$. Tracking slip pulse packets in the contacting region ($x = 0.03 \text{ m}$) yields a period $T_{pulse} = 1.59 \pm 0.03 \text{ ms}$. (E) Spectrum of u_y at selected x -positions shows corresponding peaks flapping and pulse frequencies of $f_{flap} = f_{pulse} = 629 \pm 5 \text{ Hz}$.

212 block ($f \propto c_s/2H$; Fig. S14B).

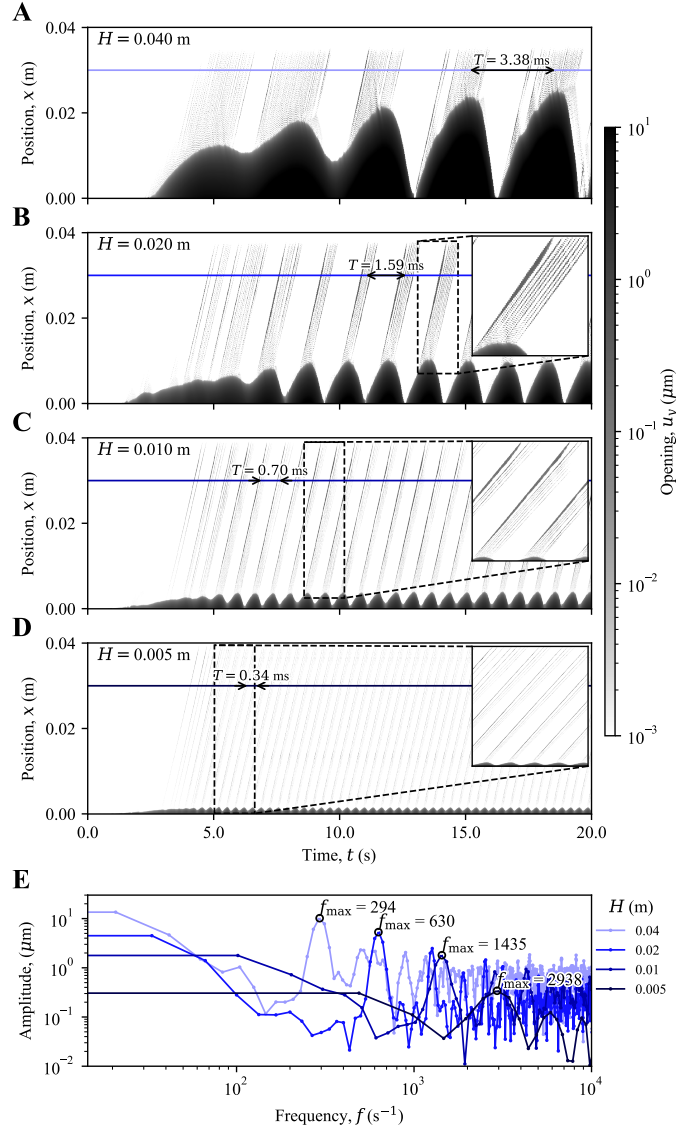


Fig. S13. Elastodynamic frictional simulations results for block of height $H = 0.005 - 0.04$ m. (A-D) Space-time plots of interfacial opening for $H = 0.005 - 0.04$ m. (E) Spectrogram of interfacial opening computed from temporal signal at $x = 0.03$ m (blue lines in panels A-D).

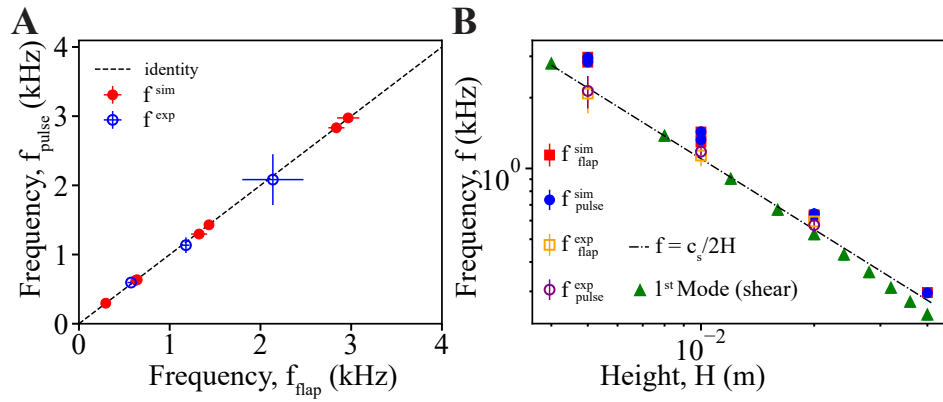


Fig. S14. Opening pulse frequency is set by the trailing edge frequency. (A) Frequency of opening pulse generation matches closely the frequency of the flapping of the trailing edge, measured both experimentally and from 2D friction simulations. (B) Frequency of flapping and frequency of pulse generation measured from the 2D friction simulations and experimentally closely match the first eigen mode computed via 2D linear perturbation analysis, which corresponds to the first shear mode. All scale closely to the proposed scaling $f = c_{\text{pulse}}/2H$.

S4.2. Additional results for Elite Double 32 cuboid samples.

S4.2.1. Normal stress measurements on cuboid elastomeric samples. To control the normal force on the cuboid elastomeric samples, we use a vertical linear stage (Newport 271). We position the vertical linear stage under the acrylic plate (see Fig. S8) and adjust its vertical height so that the sample is barely touching the plate. Once the reference position is found, we move the vertical linear stage to apply a compressive force to the sample while monitoring the force with a load cell (500 N). Finally, we divide the measured force by the contact area (in the absence of sliding) to compute the normal stress.

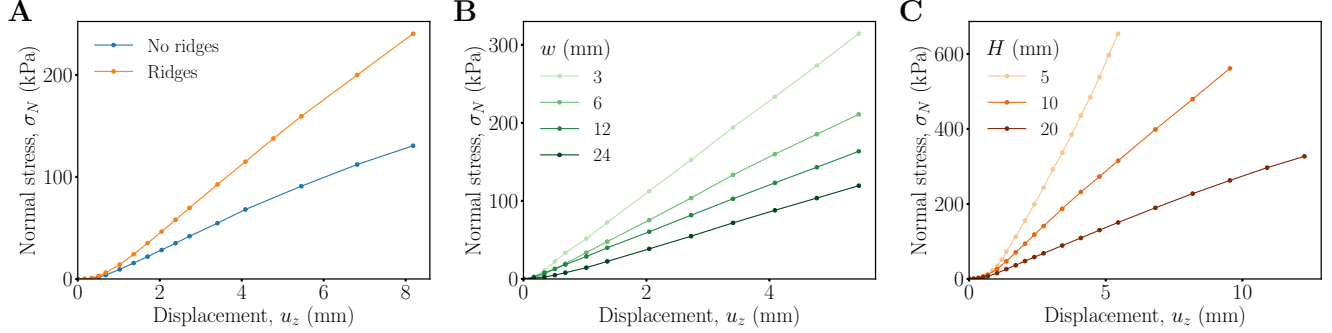


Fig. S15. Normal stress measurements on cuboid elastomeric samples. (A)-(C) Evolution of the normal stress as a function of the displacement of the linear stage from the reference configuration for samples with (A) bulk of size $40 \times 40 \times 20$ mm; one sample has a flat sliding surface and the other one has a sliding surface featuring an array of parallel ridges with height $h = 1.5$ mm, width $w = 1.5$ mm, and center-to-center distance $s = 3$ mm; (B) bulk of size $40 \times 40 \times 20$ mm and one single ridge of different width $w = [3, 6, 12, 24]$ mm and height $h = 1.5$ mm.; (C) bulk of size $40 \times 40 \times H$ mm with height $H = [5, 10, 20]$ mm and with sliding surface featuring an array of parallel ridges with $h = 1.5$ mm, $w = 1.5$ mm, and $s = 3$ mm.

S4.2.2. Effect of normal stress σ_N on the fundamental frequency f_0 . While Fig. 4 of the main text explores the effect of block height H on the fundamental frequency f_0 , Fig. S16 examines how normal stress σ_N influences f_0 for the patterned rubber sample shown in Fig. 2 of the main text. We find that f_0 is only minimally affected by σ_N within the range of normal stresses considered.

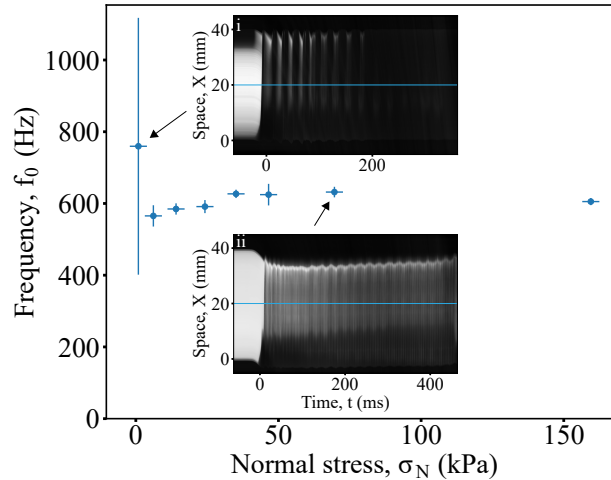


Fig. S16. Effect of normal stress σ_N on fundamental frequency f_0 . Fundamental frequency f_0 as function of the normal stress σ_N for a patterned rubber sample of $L \times W \times H = 40 \times 40 \times 20$ mm bulk dimensions. We observe that the fundamental frequency of the sound f_0 is minimally affected by σ_N for the considered range. Inset (i) represents the contact spatio-temporal map for $\sigma_N = 1$ kPa. The lack of contrast does not allow for an accurate measurement of the the fundamental frequency f_0 which translates into a larger uncertainty. Inset (ii) represents the contact spatio-temporal map for $\sigma_N = 70$ kPa. Here, the image's contrast is strong enough to resolve the opening pulses features, which translates into a narrower uncertainty in the fundamental frequency f_0 .

S4.2.3. Effect of normal stress σ_N and bulk height H on the critical sliding speed v_{slide}^{cr} . To investigate the effect of σ_N and H on the critical velocity, v_{slide}^{cr} , above which opening pulses are generated consistently, we focus on rubber cuboid samples of size $40 \times 40 \times H$ mm, with $H \in [5, 20]$ mm, and ridges of height and width $h = w = 1.5$ mm on their frictional interface (corresponding to the samples of Fig. 4 in the main text).

Fig. S17 shows the spatio-temporal evolution of contact for the sample with $H = 13.5$ mm. Opening pulses begin to emerge at $t = 0$ s for a sliding velocity of 0.15 m/s, but during the first 110 seconds they are generated only sporadically. When the

sliding velocity reaches a critical value of $v_{\text{slide}}^{\text{cr}} = 0.54 \pm 0.01$ m/s, a steady state is established in which pulses are generated periodically at a constant frequency f_0 , resulting in an audible squeaking sound. Note that for samples without ridges, such a steady-state regime is never reached (see Fig. S19), and therefore $v_{\text{slide}}^{\text{cr}}$ cannot be defined.

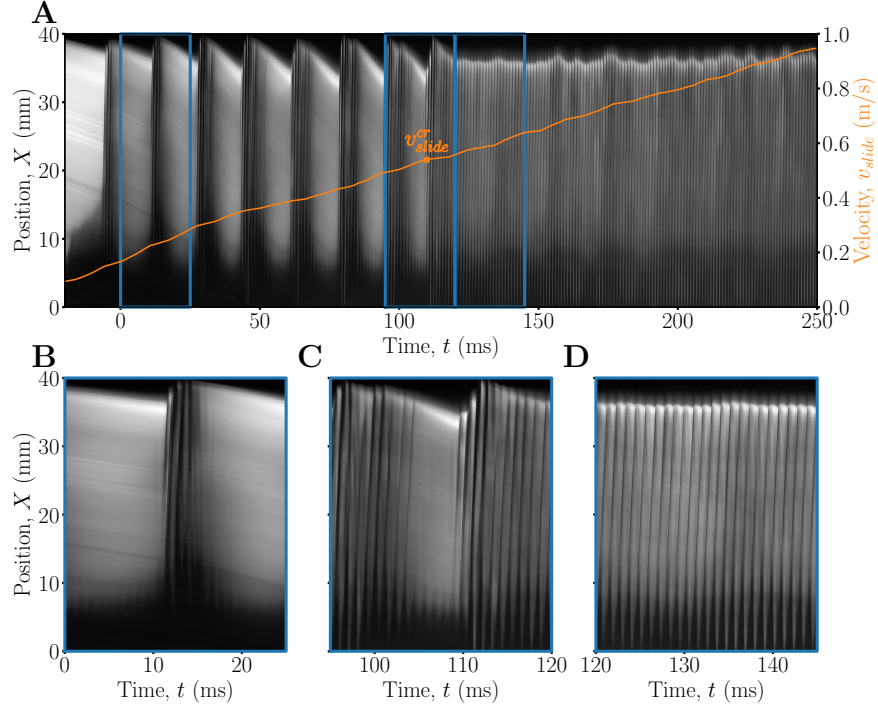


Fig. S17. Contact spatio-temporal map for a sample with ridges and $H = 13.5$ mm. Spatio-temporal evolution of contact for a rubber cuboid samples of size $40 \times 40 \times 13.5$ mm and with ridges of size $h = w = 1.5$ mm at its frictional interface. **(A)** Full spatio-temporal evolution of contact during the experiment. **(B-D)** Three close-up views.

In Fig. S18A, we report $v_{\text{slide}}^{\text{cr}}$ as a function of the normal stress σ_N for samples of different heights, H . We find that $v_{\text{slide}}^{\text{cr}}$ is only weakly dependent on σ_N , but varies significantly with H . Note that the uncertainty in σ_N , ± 3 kPa, is estimated based on the precision with which we can control the vertical linear stage position (see Fig. S15). The error in $v_{\text{slide}}^{\text{cr}}$ is calculated based on the standard deviation of multiple measurements. Where only a single measurement was carried out, then an average error is provided. The uncertainty of the velocity measurement is determined by the signal-to-noise ratio of the rotation encoder measurement and the subsequently applied 100Hz low-pass filter. Hence, the resulting measurement error is 0.01 m/s.

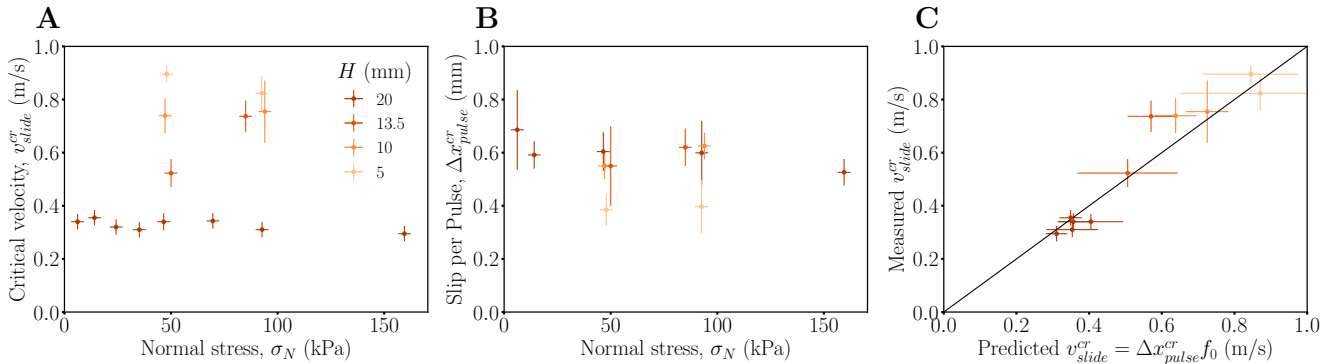


Fig. S18. Scaling for $v_{\text{slide}}^{\text{cr}}$. Data is reported for cuboid samples of size $40 \times 40 \times H$ mm with varying bulk height, $H \in [5, 20]$ mm, and with ridges of size $h = w = 1.5$ mm at their interface. **(A)** Critical velocity $v_{\text{slide}}^{\text{cr}}$, as a function of the applied normal stress, σ_N . **(B)** Slip achieved through a single slip pulse, $\Delta x_{\text{pulse}}^{\text{cr}}$, in the vicinity of $v_{\text{slide}}^{\text{cr}}$ as a function of σ_N . **(C)** $v_{\text{slide}}^{\text{cr}}$ vs $\Delta x_{\text{pulse}}^{\text{cr}} f_0$.

Next, we quantify the slip associated with a single pulse $\Delta x_{\text{pulse}}^{\text{cr}}$ in the vicinity of $v_{\text{slide}}^{\text{cr}}$ for all tests by tracking material points at the frictional interface as in Fig. 2C-D of the main text. In Fig. S18B, we report $\Delta x_{\text{pulse}}^{\text{cr}}$ as a function of the normal stress σ_N for samples of different heights, H . Note that the uncertainty in $\Delta x_{\text{pulse}}^{\text{cr}}$ is calculated based on the standard deviation over multiple measurements and on multiple repetitions of the same experiments. Finally, in Fig. S18D, we plot $v_{\text{slide}}^{\text{cr}}$ against

$\Delta x_{\text{pulse}}^{cr} f_0$, and observe that the experimental data closely follow the scaling

$$v_{\text{slide}}^{cr} = \Delta x_{\text{pulse}}^{cr} f_0, \quad [\text{S4}]$$

for all samples with ridges at their interface, regardless of H and σ_N .

Note that Eq. (S4) can be derived from a global slip-rate balance, as the global sliding velocity v_{slide} , measured from the rail position, must match the average local velocity at the frictional interface:

$$v_{\text{slide}} = \frac{\Delta x(t + \Delta t) - \Delta x(t)}{\Delta t}, \quad [\text{S5}]$$

where Δx is the net displacement of material points over the time interval Δt . Note that a material point on the frictional interfaces alternates between perfect stick periods and contactless slipping, with slip events occurring asynchronously at different locations (see Fig. 2C-D of the main text). Assuming that N slip events occur during Δt , it follows that

$$\Delta x(t + \Delta t) - \Delta x(t) = \sum_{i=1}^N \Delta x_{\text{pulse},i} \quad [\text{S6}]$$

and Eq. (S5) can be rewritten as

$$v_{\text{slide}} = \frac{\sum_{i=1}^N \Delta x_{\text{pulse},i}}{\Delta t}. \quad [\text{S7}]$$

Near the critical sliding velocity v_{slide}^{cr} , Eq. (S7) can be further simplified by assuming that each pulse produces a constant displacement $\Delta x_{\text{pulse}}^{cr}$. Introducing this approximation, we recover Eq. (S4):

$$v_{\text{slide}}^{cr} = \frac{N \Delta x_{\text{pulse}}^{cr}}{\Delta t} = \Delta x_{\text{pulse}}^{cr} f_0, \quad [\text{S8}]$$

where the pulse repetition frequency is defined as $f_0 = N/\Delta t$.

S4.2.4. Effect of opening pulses on the friction force.. To investigate the effect of opening pulses on the friction force, we conduct experiments on a sample without ridges under a normal stress $\sigma_N = 54$ kPa. The contact spatio-temporal map of Fig. S19A shows that opening pulses are nucleated repeatedly starting at $t > 215$ ms. Transient episodes of opening pulses are observed earlier (i.e. for $t \in (125, 160)$), but they are not sustained. In Fig. S19B we report the evolution of the sliding velocity (red line) during the experiment, measured by the rotational encoder (see Fig. S8). We find that opening pulses are nucleated for $v_{\text{slide}} > 0.29 \pm 0.03$ m/s. The frictional force, F_{fric} , during the test can be computed by imposing force balance

$$F_{\text{fric}} = F_d - M \frac{dv_{\text{slide}}}{dt} - F_{\text{fric rail}} \quad [\text{S9}]$$

where $M \approx 20.8$ kg is the total accelerating mass, $F_d \approx 160$ N is the driving force, $F_{\text{fric rail}} \approx 56$ N is the frictional force provided by the rails where the plate is attached and a 40Hz low-pass filter is applied when computing the acceleration. The evolution of F_{fric} (green line) reveals that the transition from steady sliding to the opening pulse regime is associated with a reduction in friction force and an increase in acceleration.

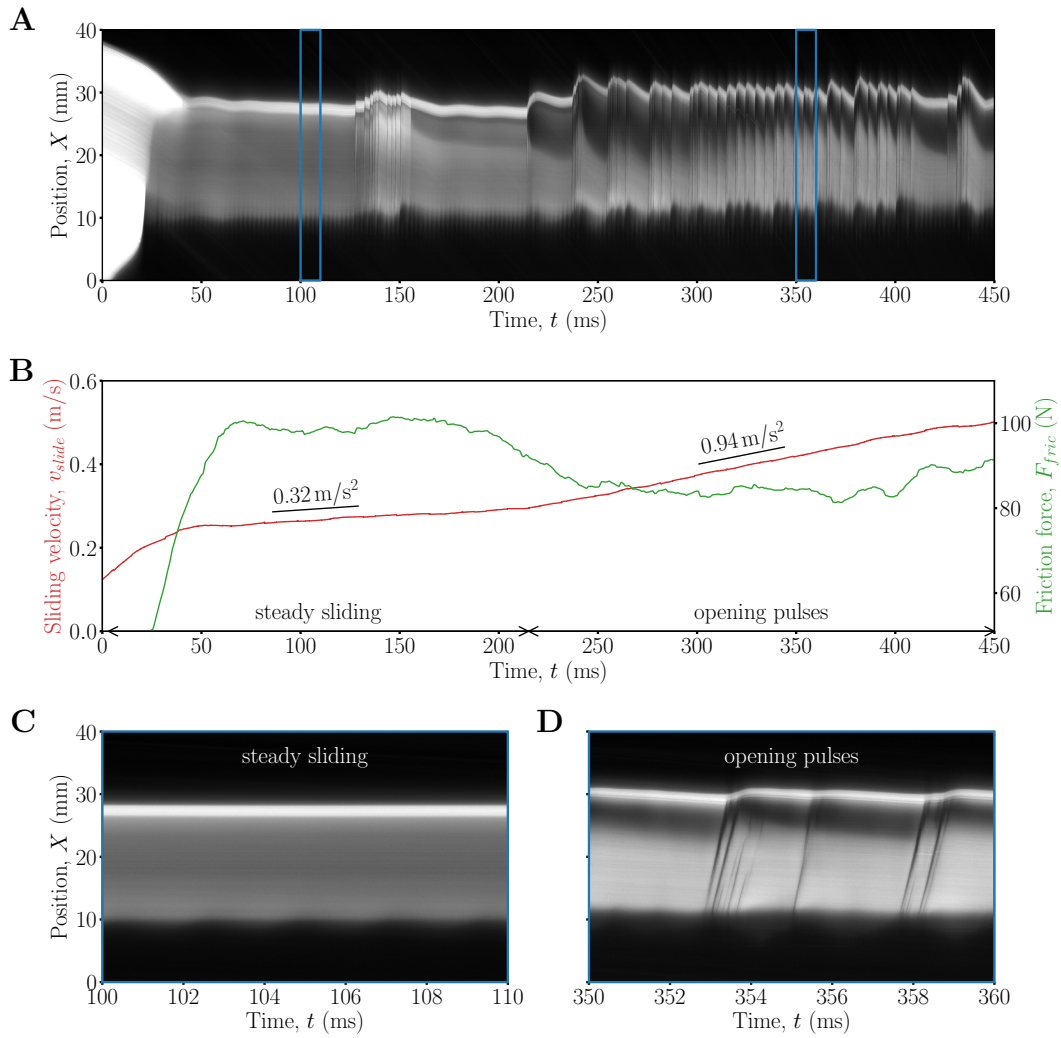


Fig. S19. Effect of opening pulses on the friction force. (A) Contact spatio-temporal map for a flat VPS sample. (B) Evolution of the sliding velocity (red line) during the experiment, measured by the rotational encoder (see Fig. S8). (C) Close up of contact spatio-temporal map during steady sliding. (D) Close up of contact spatio-temporal map during opening pulse regime.

S4.2.5. Effect of ridge orientation and ridge curvature. To explore the effect of ridge orientation, we conducted additional experiments using a cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring an array of parallel ridges with height $h = 1.5$ mm, width $w = 1.5$ mm and center-to-center distance $s = 3$ mm. In these tests, the ridges were oriented at 45° and 90° with respect to the sliding direction. In Fig. S20 we report the spatio-temporal evolution of contact along representative lines on the samples (indicated by green lines in the snapshots on the left). For an angle of 0 degrees between the ridges and the direction of sliding, which is the reference case described in Fig. 2 of the main text, we measure the period of generation of these opening pulses as $\Delta T = 1.63$ ms (Fig. S20A). This corresponds to a repetition frequency $f_0 = 1/\Delta T = 614$ Hz. When the sample is oriented with an angle of 45 degrees between the ridges and the direction of motion, we observe opening pulses traveling along the ridges. This is confirmed from the space-time diagram constructed in the direction of motion which shows curved trajectories of the opening slip pulses (Fig. S20B), whereas they are straight in the space-time diagram constructed along the direction of the ridges (Fig. S20C). We measured the period of generation of these opening pulses and find a period $\Delta T = 1.65$ ms which corresponds to a repetition frequency $f_0 = 606$ Hz. When the sample is oriented with an angle of 90 degrees, between the ridges and the direction of motion, we observe that the ridges bend and exhibit a line contact with the acrylic plate (Fig. S20D). Further, we observe no opening pulses.

Next, we investigate how the sliding angle influences the critical velocity v_{slide}^{cr} . As shown in Fig. S21, sliding at 45° results in higher critical velocities (Fig. S21A) and larger slip per pulse (Fig. S21B) compared to the reference case of 0° , for a given normal stress σ_N . Furthermore, the data at 45° no longer follow the scaling law predicted by Eq. (S4) (Fig. S21C). It is worth noting that at a sliding angle of 90° , no opening pulses are observed; as a result, neither v_{slide}^{cr} nor $\Delta x_{\text{pulse}}^{cr}$ can be defined.

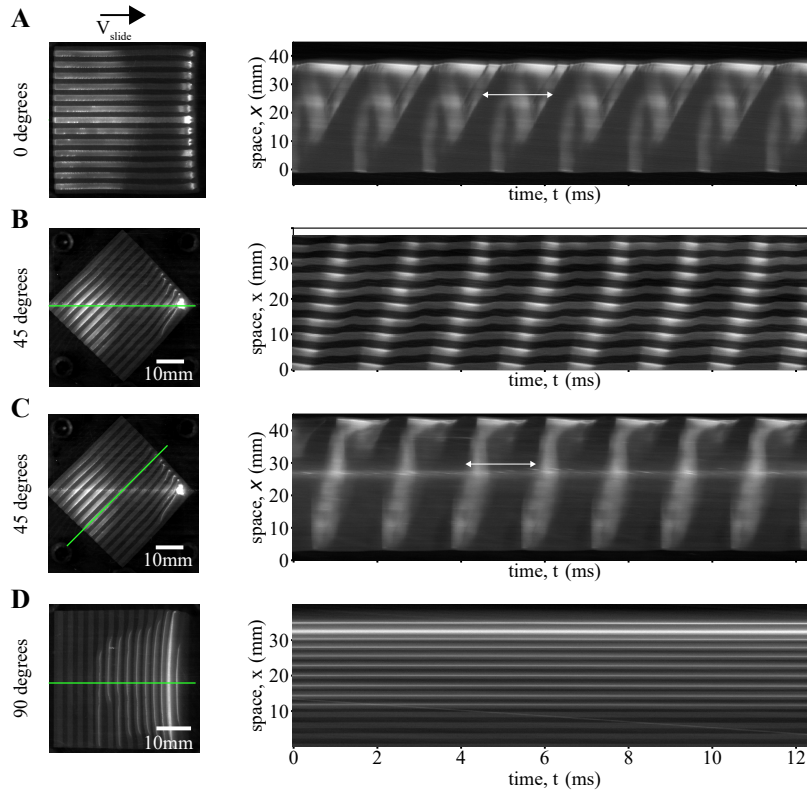


Fig. S20. Effect of ridge orientation with respect to the direction of sliding. (A) Contact spatio-temporal map for an angle of 0 degrees between the ridges and the direction of sliding. (B)-(C) Contact spatio-temporal map for an angle of 45 degrees between the ridges and the direction of sliding (B) in the direction of motion and (C) along the direction of the ridges. (D) Contact spatio-temporal map for an angle of 90 degrees between the ridges and the direction of sliding.

To explore the effect of the shape of the ridges, we conducted experiments on cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring a single curved ridge (Fig. S22A). Snapshots of the frictional interface at different times show the propagation of 2 opening pulses along the curved ridge (indicated by the orange arrows in Fig. S22B). Further, the spatio-temporal evolution of contact along the straight part of the ridge (indicated by the purple dashed line in Fig. S22B) shows a repetition period $\Delta T = 1.61$ ms which corresponds to a repetition frequency $f_0 = 621$ Hz (Fig. S22C). This experiment shows that opening pulses travel along thin ridges even when curved.

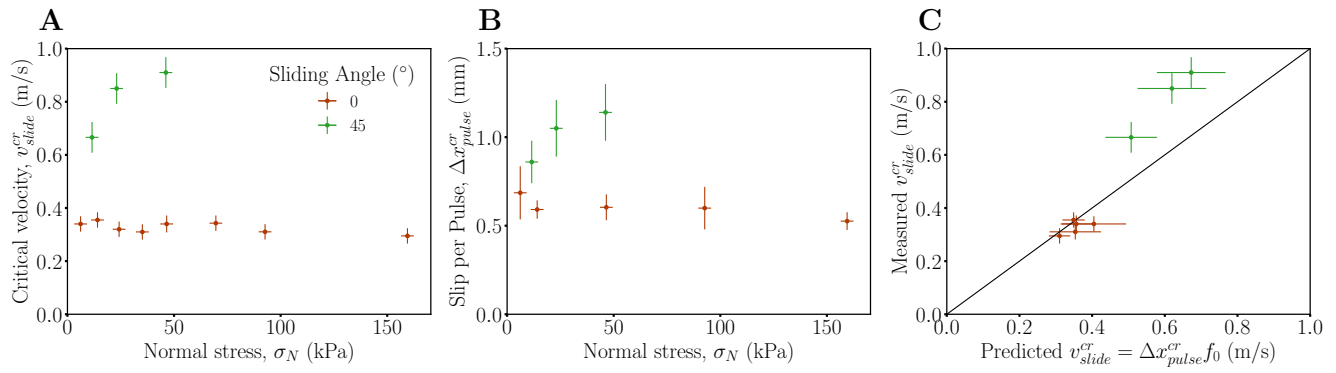


Fig. S21. Scaling for v_{slide}^{cr} . Data is reported for cuboid samples of size $40 \times 40 \times 20$ mm and with ridges of size $h = w = 1.5$ mm at their interface. Two different directions of sliding are considered. **(A)** Critical velocity v_{slide}^{cr} as a function of the applied normal stress, σ_N . **(B)** Slip achieved through a single slip pulse, Δx_{pulse}^{cr} , in the vicinity of v_{slide}^{cr} as a function of σ_N . **(C)** v_{slide}^{cr} vs $\Delta x_{pulse}^{cr} f_0$.

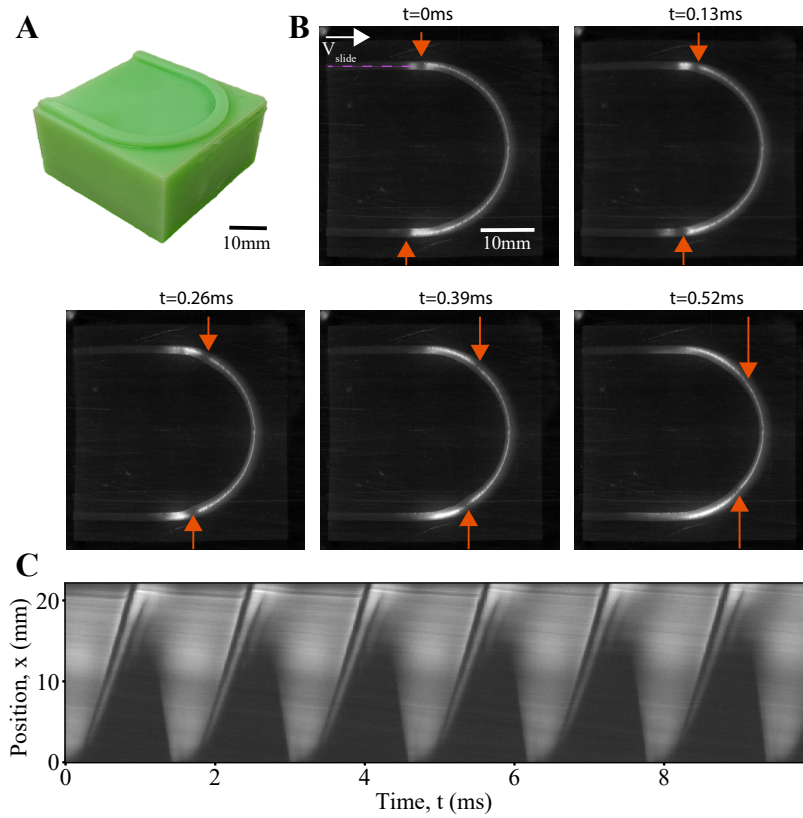


Fig. S22. Effect of ridge curvature. (A) A cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring a single curved ridge. (B) Snapshots of the frictional interface at times $t = 0, 0.13, 0.26, 0.39$ and 0.52 ms, show the propagation of 2 opening pulses (indicated by the orange arrows in each snapshot) along the curved ridge. (C) Spatio-temporal evolution of contact along the straight part of the ridge (indicated by the purple dashed line in panel B).

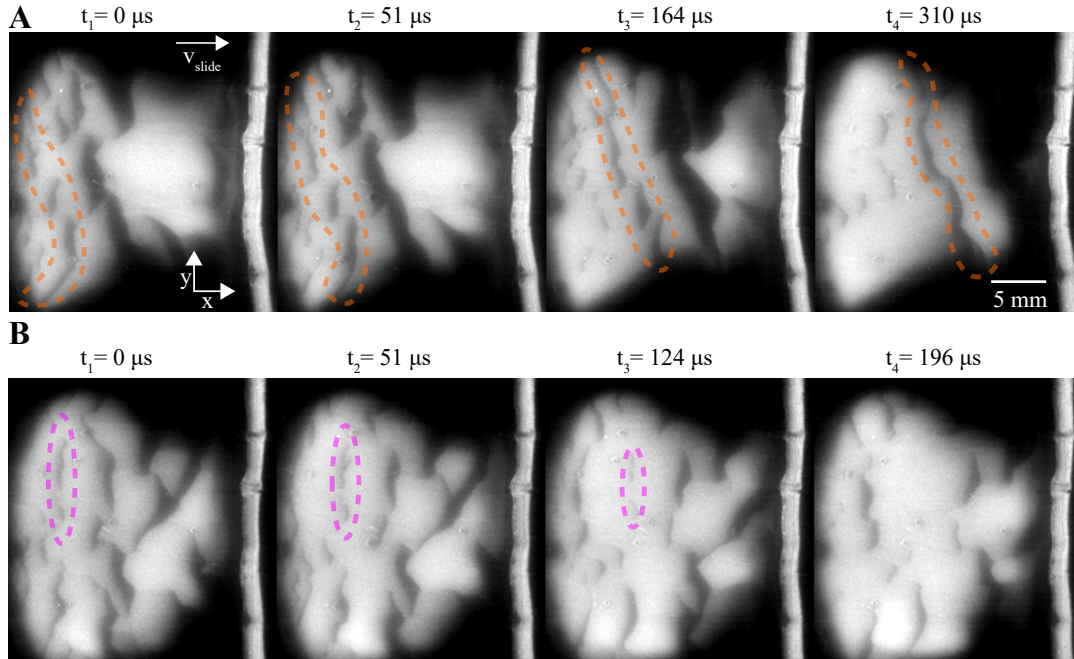


Fig. S23. Evolution of opening pulses on flat surfaces: Coalescence and vanishing. Here we present additional results for a cuboid sample of size $40 \times 40 \times 20$ mm with a flat sliding surface. **(A)** These snapshots of the frictional interface at different times shows the growth and coalescence of a group of small opening pulses (highlighted by the orange dashed line). **(B)** These snapshots of the frictional interface at different times show a group of slip opening pulses (highlighted by the purple dashed line) propagating from left to right, decreasing in size and then suddenly disappearing at $t = 196 \mu s$.

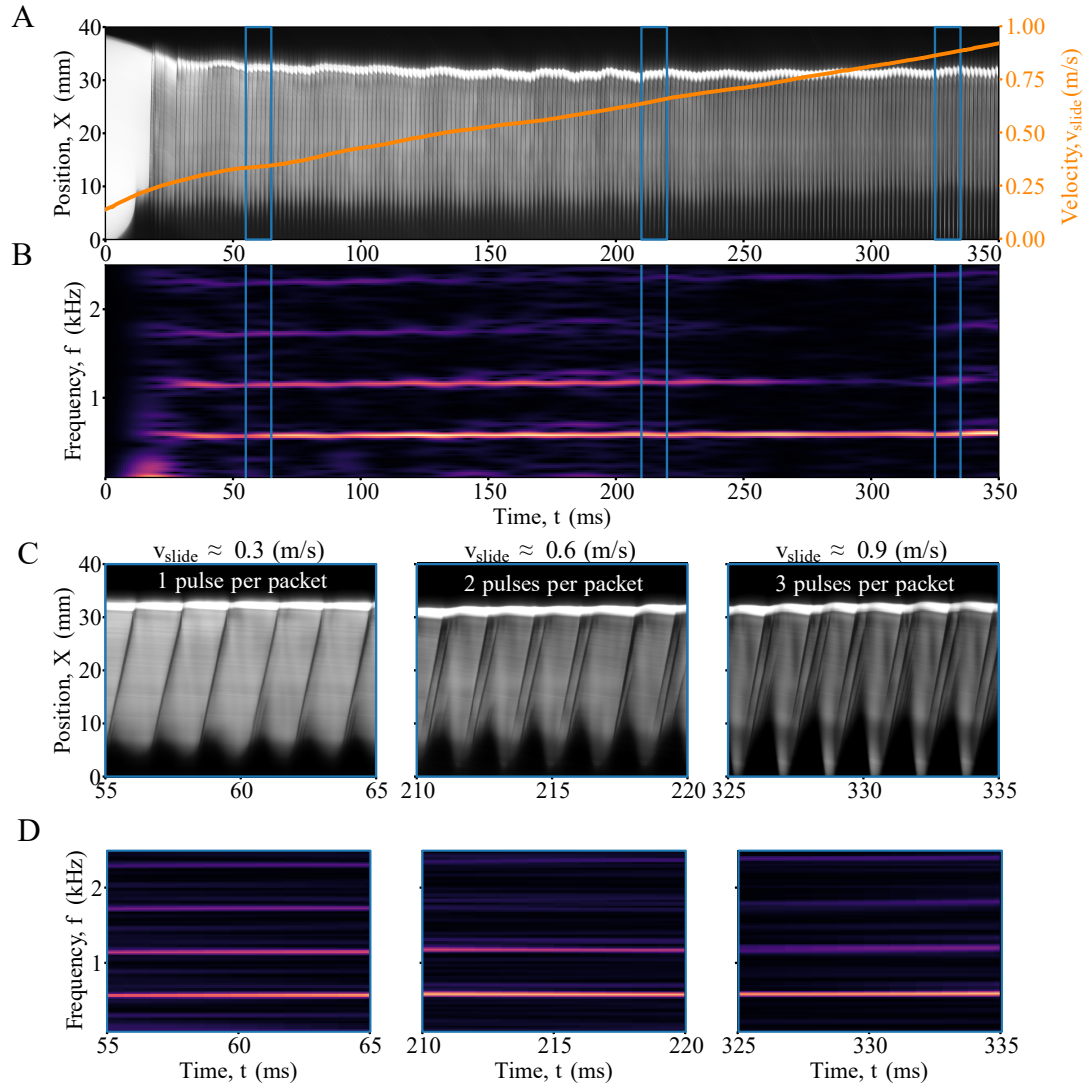


Fig. S24. Effect of sliding velocity on opening slip pulses. (A) Full spatio-temporal evolution of contact during the experiment shown in Fig. 2F of the main text, alongside the evolution of the plate's sliding velocity, v_{slide} . (B) Spectrogram calculated from the spatio-temporal contact evolution in panel (A) at position $X = 20$ mm. (C) Zoomed-in view of the spatio-temporal contact map for $v_{\text{slide}} \approx 0.3$, 0.6 , and 0.9 m/s. These results demonstrate that, although the sliding velocity increases over time during the experiment, the slip opening pulses continue to be generated at a consistent fundamental frequency. Rather than increasing the frequency of events, an additional timescale arises to accommodate the higher slip rate. Specifically, with each slip event, a series of opening pulse packets is generated, allowing for faster slip without altering the primary pulse frequency. (D) Close-up of the spectrogram for $v_{\text{slide}} \approx 0.3$, 0.6 , and 0.9 m/s. These plots confirm that the pulse generation repetition frequency remains constant as the sliding velocity increases.

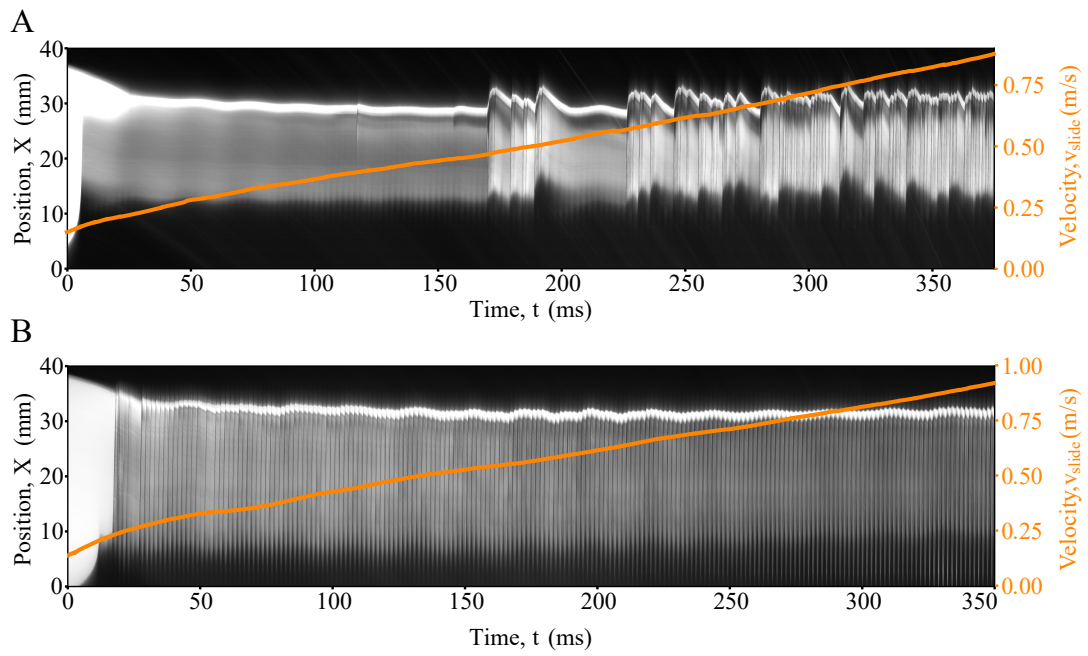


Fig. S25. Evolution of the plate's sliding velocity. (A)-(B) Full spatio-temporal evolution of contact during the experiment shown in (A) Fig. 2E and (B) Fig. 2F of the main text, alongside the evolution of the plate's sliding velocity, v_{slide} .

S4.3. Additional results for Smooth-on Smooth-Sil 960 cuboid samples. In Fig. 5A of the main text, we show that elastomeric blocks made of Smooth-Sil 960 behave similarly to those made of Zhermack Elite Double 32 when sliding on a smooth, rigid surface under dry conditions. In particular, we find that in the presence of thin ridges on the frictional interfaces, opening pulses are periodically triggered, leading to stick-slip oscillations and squeaking. In the absence of ridges, however, pulse initiation becomes aperiodic, resulting in broadband noise. These results are further supported by experimental snapshots of the frictional interface during sliding shown in Fig. S26. For both the patterned and the flat samples, opening pulses are observed propagating at 37 ± 2 m/s. For the flat sample, the pulses are disordered, while in the patterned sample, they are regularly spaced with a frequency $f_0 = 890 \pm 120$ Hz (uncertainty corresponds to the full width at half maximum).

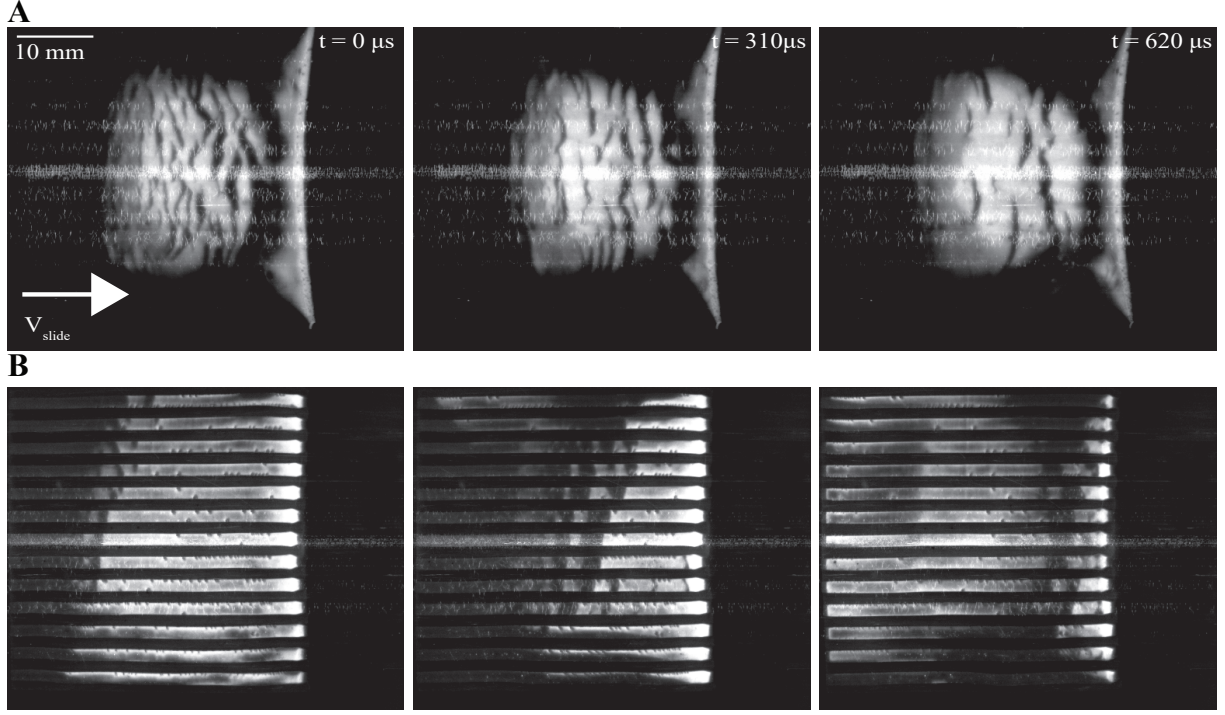


Fig. S26. Evolution of opening pulses on flat and patterned Smooth-Sil 960 samples. (A)-(B) Snapshots of the frictional interface recorded at 97,000 frames per second reveal the presence of opening pulses for both (A) flat and (B) a patterned frictional interfaces. (A) For the flat sample, pulses are disordered. (B) For the patterned sample, pulses are regularly spaced.

Next, we investigate how the squeaking frequency of the patterned Smooth-Sil 960 samples depend on their height H . Towards this end, we manufactured Smooth-Sil 960 patterned blocks with fixed bulk dimensions $L = W = 40$ mm and height $H = 5, 10, 20$ mm. For each height, three sliding experiments were performed and dominant squeaking frequency f_0 was extracted as the first peak in the FFT of the recorded sound signal. The results shown in Fig. S27 indicate that, identically to the patterned Zhermack Elite Double 32 samples, the squeaking frequency of the Smooth-Sil 960 samples also varies with height as

$$f_0 = \frac{c_s}{2H}, \quad [\text{S10}]$$

where $c_s = \sqrt{G/\rho} \approx 29$ m/s is the shear wave speed propagation in the bulk of Smooth-Sil 960, as reported in Table S1.

S4.4. Additional results for TPU cuboid samples. Fig. 5B in the main text demonstrates that blocks fabricated from TPU respond similarly to the elastomeric ones when sliding over a smooth, rigid surface in dry conditions. The presence of thin ridges along the frictional interface consistently triggers opening pulses, giving rise to audible squeaking. As shown in Fig. S28B, these pulses are regularly spaced with a dominant frequency $f_0 = 840 \pm 120$ Hz (uncertainty corresponds to the full width at half maximum). When these ridges are absent, however, the pulses are disordered (Fig. S28A) and their triggering becomes irregular, producing a more broadband noise spectrum. These observations support the conclusion that a structured frictional interface featuring 1D waveguides is required to give rise to a dominant squeaking frequency, and show that the mechanism persists in a stiffer thermoplastic material with no crosslinking, with an elastic modulus approximately one order of magnitude higher than the elastomers studied in Figs. 2, 4, and 5A.

Next, to evaluate whether the frequency–height scaling observed in elastomeric blocks also applies to TPU, we conducted experiments on TPU samples with ridged surfaces and heights ranging from $H \in [5, 20]$ mm. In Fig. S29, we report the first peak in the FFT of the recorded sound signal for each sample. As with the elastomeric blocks, we observe an inverse relationship between the fundamental frequency and H . However, the measured fundamental frequencies are significantly lower

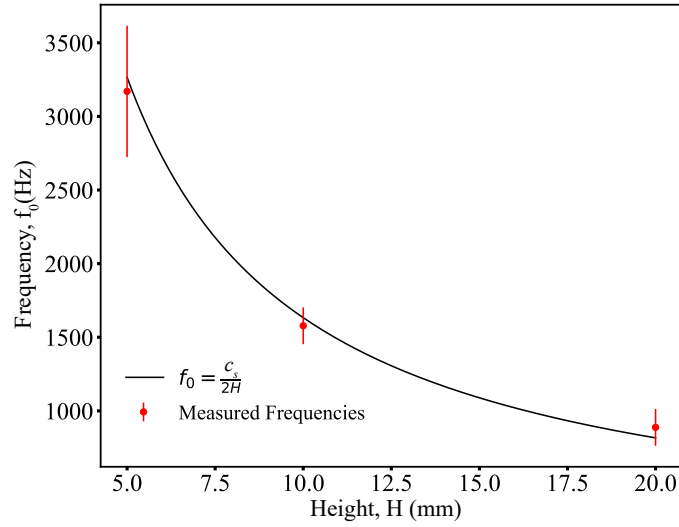


Fig. S27. Measured squeaking frequency f_0 as a function of block height H for Smooth-Sil 960 samples with ridges.

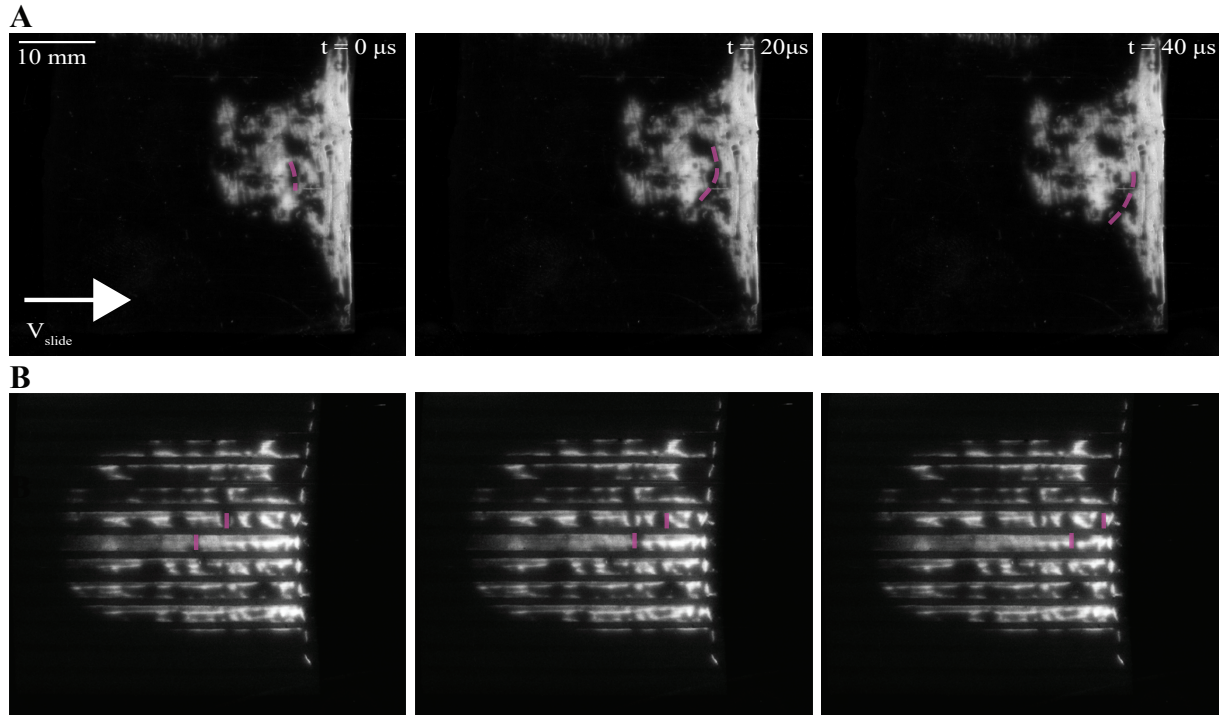


Fig. S28. Opening pulse dynamics in TPU samples. (A) Snapshots of the frictional interface for the flat TPU sample reveal unstructured and spatially disordered opening pulses (see dashed purple lines). (B) For the patterned sample, pulses propagate along the ridge direction. These qualitative observations are consistent with the frequency spectra and spatio-temporal contact maps shown in Fig. 5 of the main text.

than those predicted by Eq. (S10) (black line in Fig. S29). Instead, the data follow the scaling:

$$f_0 = \frac{c_{\text{pulse}}}{2H}, \quad [\text{S11}]$$

where $c_{\text{pulse}} = 41 \pm 6$ m/s is the median slip pulse speed determined from space-time diagrams (see inset in Fig. S29).

To understand the discrepancy between the measured squeaking frequencies and the predictions based on the bulk shear wave speed (Fig. S29), we investigated the state of the frictional interface under different levels of normal stress. Unlike the elastomeric samples discussed in Figs. 2 and 4, sliding TPU against glass leaves visible traces on the glass surface (Fig. S30A). These traces appear even at low normal stress ($\sigma_N = 10$ kPa) and become significantly more pronounced at higher normal stress ($\sigma_N = 100$ kPa). To quantify this effect, we measured the number of white pixels in the abraded region and found a marked increase with increasing normal stress (Fig. S30B). Notably, these marks could only be removed by wiping the glass with isopropanol, suggesting local melting of the TPU. This interpretation aligns with previous tribological studies, which have

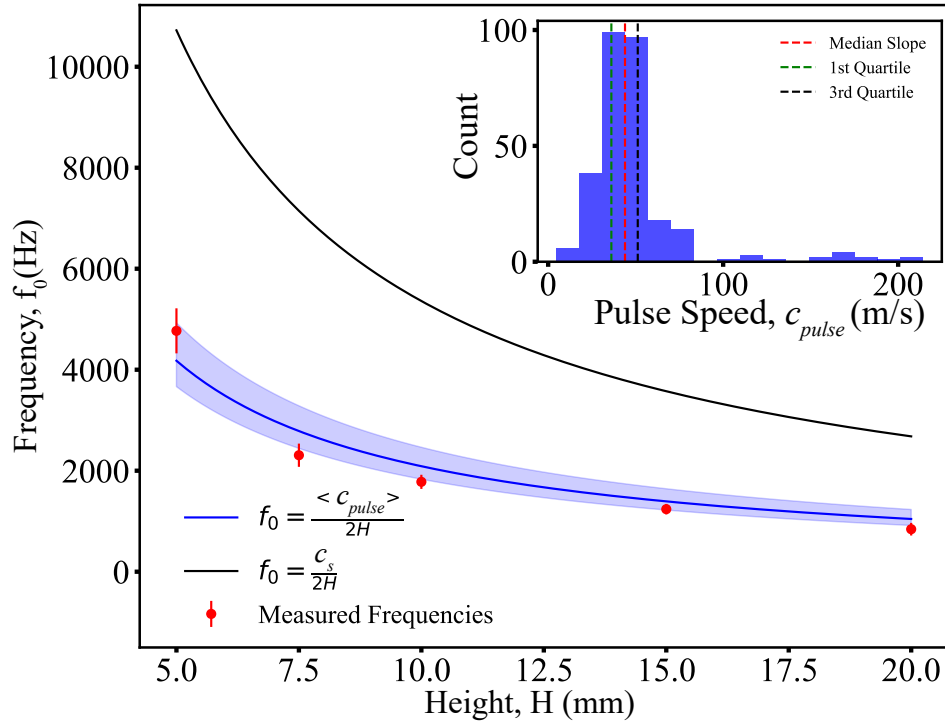


Fig. S29. Measured squeaking frequency f_0 as a function of block height H for TPU samples with ridges. Uncertainty corresponds to the full width at half maximum of the peak. The prediction $f_0 = C_s/2H$, where $C_s = 113$ m/s is the bulk shear wave speed, overestimates the measured frequencies. The shaded blue region corresponds to the prediction $f_0 = C_{\text{pulse}}/2H$, where $C_{\text{pulse}} = 41 \pm 6$ m/s is the median slip pulse speed measured from space-time diagrams. The bounds represent the first and third quartiles of the pulse speed distribution. In the inset we show distribution of measured pulse speeds, with vertical lines indicating the first quartile, median, and third quartile.

shown that TPU can experience friction-induced softening or melting under moderate normal stress and sliding conditions (7–9). These findings indicate that the TPU near the interface becomes significantly softer than the bulk material during sliding, which in turn explains the lower squeaking frequencies observed in our experiments compared to those predicted using the bulk shear wave speed.

This normal stress-dependent surface modification correlates with a marked change in slip pulse dynamics. At low normal stress, pulses propagate at 150 ± 19 m/s, while at high normal stress they slow to 53 ± 6 m/s. This sharp decrease in pulse speed suggests that interfacial softening alters the local wave propagation properties, potentially explaining the breakdown of frequency scaling based on the bulk shear modulus.

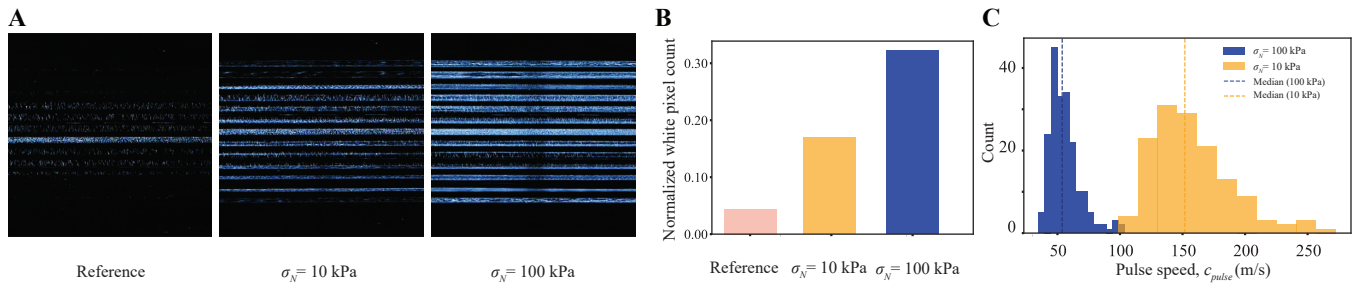


Fig. S30. Normal stress-dependent interface modification in TPU samples. (A) Images of the moving glass plate after sliding TPU samples at increasing nominal normal stress: reference (no sliding), $\sigma_N = 10$ kPa, and $\sigma_N = 100$ kPa. The intensity and extent of the abraded region increases significantly with normal stress. (B) Quantification of the abrasion by normalized white pixel count in the abraded area. (C) Corresponding slip pulse speeds measured from space-time diagrams under the same normal stress conditions. Pulse propagation slows from 150 ± 19 m/s at $\sigma_N = 10$ kPa to 53 ± 6 m/s at $\sigma_N = 100$ kPa, consistent with normal stress-induced interfacial softening.

S5. Derivation of shear eigenfrequencies

We assume an isotropic linear elastic layer of height H and infinite lateral dimensions. Wave propagation in this layer is governed by the elasto-dynamic wave equation

$$\rho \ddot{u}_j = (\lambda + G)u_{k,kj} + Gu_{j,ll} , \quad [S12]$$

where λ and G are the Lamé constants, ρ the medium density and u_j the component along coordinate axis j of the displacement field.

We introduce a coordinate system, x, y, z , such that the considered layer lateral dimensions are along x and y and its height is along z . The general solution for plane shear waves propagating along the z direction and polarized along x direction can be written as

$$u_x(z, t) = Ae^{i\omega t} \sin(kz) + Be^{i\omega t} \cos(kz), \quad [S13]$$

where A and B are two constants to be determined, ω is the angular frequency, t the time, $i = \sqrt{-1}$ and $k = \omega/c_s$ is the wavenumber for the shear waves with $c_s = \sqrt{G/\rho}$ being the shear wave velocity.

We now assume clamped boundary conditions at $z = 0$ and $z = H$,

$$u_x(0, t) = 0, \quad u_x(H, t) = 0, \quad \forall t. \quad [S14]$$

The first boundary condition yields

$$B = 0, \quad [S15]$$

while the second one leads to

$$\sin(kH) = 0. \quad [S16]$$

Eq. (S16) is satisfied for

$$k = k_n = n \frac{\pi}{H}, \quad \text{with } n = \pm 1, \pm 2, \pm 3 \dots \quad [S17]$$

The lowest natural frequency of these planar shear waves can then be calculated from

$$k_1 = \frac{\pi}{H} = \frac{2\pi f}{c_s}, \quad [S18]$$

since $\omega_n = c_s k_n$. It follows that

$$f = \frac{c_s}{2H}. \quad [S19]$$

Note that the shape of the n -th shear mode is given by

$$u_x(z, t) = Ae^{i\omega_n t} \sin(k_n z). \quad [S20]$$

S6. Finite Element simulations

Eigenfrequency analysis. To determine the natural modes of the preloaded block, we performed two-dimensional plane-strain finite-element simulations in Abaqus/Standard using a custom Python script (code used for all simulations is available at https://github.com/gabrielealbertini/elastodynamics_friction_squeaking.git)

The model consists of a rectangular block of length $L = 40$ mm and height H , discretized with quadratic quadrilateral hybrid elements (Abaqus element type: CPE8H) of characteristic size $H/50$. The material was modeled as a nearly incompressible Neo-Hookean solid with $E = 1.6$ MPa and density $\rho = 1060$ kg m⁻³.

A vertical partition was introduced at $x_p = H/2$ to define two bottom regions: a *free* left segment ($0 \leq x < x_p$) and a *sticking* right segment ($x_p \leq x \leq L$). All static steps were solved with geometric nonlinearity enabled (`nlggeom=ON`) to retain the finite prestrain and contact-lifting effects prior to the eigenvalue analysis.

The simulation sequence comprised three steps:

1. **Normal preload:** A uniform pressure $P = 10$ kPa was applied on the top surface ($y = H$), while the bottom boundary ($y = 0$) was constrained in the normal direction only ($u_2 = 0$) and free tangentially. The top edge was free to move tangentially.
2. **Shear preload:** The normal roller condition on the bottom was released. The right portion of the bottom ($x_p \leq x \leq L$, $y = 0$) was clamped ($u_1 = u_2 = 0$), while the left portion ($0 \leq x < x_p$) was traction-free, allowing local lift-off. The top boundary was displaced laterally by $\Delta X = H/10$ under the same normal pressure P , generating a preloaded sheared state.
3. **Eigenmode analysis:** A Lanczos eigensolver was used to compute the small-amplitude vibration modes about the deformed configuration obtained in Step 2. Twelve eigenpairs were extracted.

The first eigenmode was consistently a shear mode localized near the trailing edge ($x \approx 0$), with frequency scaling as

$$f_1 \simeq \frac{c_s}{2H}, \quad c_s = \sqrt{G/\rho},$$

matching the experimentally measured flapping and opening-pulse frequencies. This confirms that the trailing-edge oscillation observed experimentally corresponds to the fundamental shear resonance of the block.

Elastodynamic simulations. To investigate the frequency selection mechanism for pulse nucleation, we perform Finite Element simulations on a 2D plane-strain model with a 1D frictional interface, as our experimental results indicate that, in the presence of ridges, the frictional interface effectively behaves as one-dimensional. In our simulations, we consider an elastomer block of length $L = 0.04$ m and height $H = 0.005 - 0.04$ m in frictional contact along the x -axis with a rigid substrate. The models are discretized using a uniform structured mesh of bilinear quadrilateral elements, each with an edge length of $H/400$ and four quadrature points.

The response of the elastomeric block is modeled as that of a viscoelastic solid. Specifically, we employ the *Standard Linear Solid* (SLS) model, which represents the material behavior using a spring in parallel with a Maxwell element (a spring and dashpot connected in series). For the Maxwell element, we set the viscosity to $\eta = 64$ Pa · s and the stiffness to $E_V = 0.64$ MPa, while the elastic spring in parallel has a stiffness of $E_\infty = 0.96$ MPa. At long times ($t \rightarrow \infty$), the material response approaches the equilibrium modulus E_∞ . Conversely, the instantaneous modulus is $E_0 = E_\infty + E_V$. With these parameters, the resulting shear wave speed is

$$c_s = \sqrt{\frac{E_0}{\rho 2(1 + \nu)}} = 22.6 \text{ m/s}, \quad [\text{S21}]$$

when we use $\rho = 1050 \text{ kg/m}^3$ and $\nu = 0.45$. This value is consistent with our experimental measurements for Zhermack Elite Double 32 (see Section S2).

At the frictional interface, we adopt a Coulomb friction law, such that the instantaneous frictional strength τ_{fric} is given by

$$\tau_{\text{fric}} = \mu \sigma_n, \quad [\text{S22}]$$

where σ_n denotes the normal pressure at the interface and μ is the friction coefficient. In all simulations, we use $\mu = 2$.

In all simulations, the initial condition consists of a state of uniform uniaxial compressive stress $\sigma_{yy} = 600$ Pa, while all other stress components are zero $\sigma_{xx} = \sigma_{xy} = 0$. The upper boundary of the block is then driven at a velocity v_{slide} along the x -direction while being constrained in the y -direction. The imposed velocity increases linearly from rest to its final value $v_{\text{slide}} = H/2$ m/s over a ramp time of 1 ms. At the two lateral sides of the block (that is, at $x = 0$ and $x = L$) we apply a free boundary condition. Small strains are assumed, and time integration is performed with an explicit central-difference scheme in a predictor–corrector form. The time step is chosen as $\Delta t = 0.1 \Delta t_{\text{cr}}$, where $\Delta t_{\text{cr}} = \Delta x/c_p$ is the critical time step satisfying the Courant-Friedrichs-Lewy condition.

A traction-at-split-node technique is used to apply the tractions at the frictional interface, allowing independent tractions to be enforced across coincident nodes along the interface. The traction vector τ^{t+1} is computed by a forward Lagrange multiplier method (10), which uses a prediction procedure to pre-compute the slip-rate for the next time step. At frictional interfaces between elastic solids of dissimilar stiffness, $\sigma_n(x, t)$ may vary in space and time, resulting in corresponding fluctuations in τ_{fric} . This coupling can render the problem ill-posed, as studied analytically by Adams and co-workers (11–13). To make the problem well-posed, it is common practice (14–16) to apply the Prakash–Clifton regularization (17, 18), which accounts for the finite adaptation time required for frictional resistance to adjust to changes in normal stress. As such, in the simulations the effective frictional strength evolves according to (17, 18)

$$\frac{d\tau_{\text{fric}}}{dt} = \frac{1}{t^*} [\mu \sigma_n(t) - \tau_{\text{fric}}(t)] \quad [\text{S23}]$$

where t^* is the regularization timescale (in all simulations we use $t^* = 1$ ms). This treatment renders τ_{fric} a history-dependent state variable rather than an instantaneous quantity.

The Finite Element code used for all simulations is available at https://github.com/gabrielealbertini/elastodynamics_friction_squeaking.git and uses the Akantu open-source Finite Element library (<https://akantu.ch>). Results from the Finite Element simulations are shown in Section S4.

429 **Video S1. Squeaking of a shoe and a human hand.** *Part 1:* To explore the origin of squeaking, we slide a commercially available
430 basketball shoe (Nike CU3503-100) across a glass plate at a velocity of $v_{\text{slide}} \approx 1$ m/s while simultaneously recording sound and
431 visualizing the contact interface. *Part 2:* To examine the role of geometry, we slide a human palm—a broad, two-dimensional
432 interface—and three fingers—each acting as natural macroscopic ridges—across a glass plate at $v_{\text{slide}} \approx 1$ m/s, again capturing
433 both sound and contact evolution.

434 **Video S2. Sliding flat and patterned samples.** *Part 1:* Experimental setup. *Part 2:* Sliding across an acrylic plate of an elastomeric
435 sample with flat surface. In our experiments we simultaneously capture the sound and visualize the contact. *Part 3:* Sliding
436 across an acrylic plate of an elastomeric sample with a surface featuring an array of parallel ridges. In our experiments we
437 simultaneously capture the sound and visualize the contact.

438 **Video S3. Initiation triggered by electric discharge.** Electric discharge as a mechanism for nucleation of opening pulses. This is
439 observed more frequently in the case of flat samples under high normal stress.

440 **Video S4. Squeaking at desired frequencies.** We exploit the bulk's height to design six patterned samples that squeak at
441 frequencies corresponding to musical notes and play a song (part of the Darth Vader's theme song from the Star Wars saga) by
442 sliding these blocks by hand over a glass plate.

443 **Video S5. Evidence of backward-propagating opening pulses traveling at longitudinal wave speed of the material.** Sliding of an elastomeric
444 sample with flat surface. We observe the simultaneous propagation of a opening pulse which travels forward at a speed ≈ 24
445 m/s, and an opening pulse traveling backward at a speed ≈ 600 m/s.

446 **Video S6. Effect of ridge orientation with respect to the direction of sliding.** Sliding of an elastomeric sample with a patterned surface
447 with the ridges oriented at 45° and 90° with respect to the sliding direction.

448 **Video S7. Sliding flat and patterned samples made of different materials.** *Part 1:* Sliding of elastomeric samples made from Smooth-
449 On Smooth-Sil 960 across an acrylic plate, comparing flat and patterned surfaces. *Part 2:* Sliding of 3D-printed thermoplastic
450 samples across an acrylic plate, also comparing flat and patterned surfaces.

451 **Video S8. Mode lock-in mechanism.** Experiments and elastodynamic finite element simulations show local oscillations of the
452 sample's trailing-edge that modulate contact-pressure and facilitate in-phase opening pulse generation.

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