

Squeaking at soft–rigid frictional interfaces

Corresponding Author: Professor Katia Bertoldi

Version 1:

Reviewer comments:

Referee #1

(Remarks to the Author)

Djellouli et al. present 'soft-on-hard' squeaking friction experiments in which both sound is recorded and interface dynamics are visualized. The authors demonstrate that the fundamental frequency of the squeaking sound that is generated when a basketball shoe or rubber sample slides over glass or pmma is equal to the frequency with which opening pulses (pulses of contact area reduction) travel accross the interface. Furthermore, the authors show that the frequency with which opening pulses, which can be guided by ridges on the sliding surface, occur, is controlled by the first shear mode of the sample. As the first shear mode is set by the height of the sample, the authors demonstrate control over the squeaking frequency by changing the sample height.

What is novel and exciting, to the best of my knowledge, is the detail with which the authors managed to reveal the stick-slip process and its connection to sound, culminating in control over the frequency.

Weaknesses:

1. The explanation of the stick-slip induced squeaking sound -the relation between sound, stick-slip and elastic modes of the slider, is not completely new, as Martyn Shorten and Xia Xi already knew in 2006 (see ref 1 of the draft): "The herringbone structures of the shoe outsole are induced to vibrate at their low-order natural frequencies by stick-slip contact with the surface,". What I miss in the draft is a more complete literature review; the current introduction and reference list seems to hide the fact that the explanation behind the phenomenon was already there. It would be good to sharply define what was already known and what new insight the present experiments provide. Although it is not my area of expertise, I believe there is a lot of previous work on stick slip of soft materials. Searching 'stick slip squeaking rubber' on google scholar the first hit I find includes high velocity (0.4 m/s) rubber on glass experiments. The authors should carefully review the literature that is focussed on systems similar to the system they study and then sharply define what the new insight in their work is relative to the literature. In the present draft, most of the references are more distantly related to the presented work.

2. Related to the first point, it seems like understanding based control over squeaking sounds -for example in basketball shoes- is already achieved by the manufacturers of such shoes. In fact, according to ref 1 in the draft, shoe manufacturers make sure the squeaking sound is there as it reassures basketball players. This makes me believe that the statements in the paper on 'new design space for friction and energy dissipation' are exaggerated.

Advice:

The experiments and results are beautiful and the paper reads well. The novelty of the insight and the implications are questionable. I am inclined to recommend rejection of the paper for nature. However, if the authors have good arguments to refute the criticism above I would be happy to review the draft again. Finally, I would recommend sending the draft to an expert in the field of rubber stick slip squeaking. Such an expert could be of more help in placing the work in context and judging its novelty.

Referee #2

(Remarks to the Author)

The manuscript entitled "Squeaking at soft–rigid frictional interfaces" by A. Djellouli et al presents an experimental study about the squeaking sound emitted by a rubber-like block when sliding against a rigid body. The authors use an experimental setup allowing to image the contacts at the interface between the two solids at a high frequency and to record

the emitted sound. The novelty lies in the study of a soft sliding body against a rigid one at high sliding velocity, while low sliding velocities have been studied in the literature. The study is phenomenological in nature. It reports experimental measurements that allow hypothesizing a possible mechanism underlying the squeaking noise.

The key results presented in this manuscript are:

- The squeaking of a basketball shoe is due to the propagation of slip pulses at the soft-rigid interface. The frequency of the acoustic emissions is consistent with the frequency of the slip pulses generation.
- For a model solid block, the squeaking noise only occurs if one or more ridges are present at the interface. For blocks with ridges at the contacting surface, the slip pulses have a narrow distribution of orientation, length, and width, while the distributions are wider for blocks with a smooth contacting surface.
- In the case of the solid block with multiple ridges, it is shown that the squeaking frequency is equal to the natural frequency associated with the first shear mode for an elastic block.
- The conclusion of the study is that the ridges act as waveguides for the slip pulses.

The text is generally well written and structured, and the research conducted in this work is of high quality. The abstract is honest and reflects the work performed in this study. Some values of measured quantities, such as fundamental frequencies and critical sliding velocities, should be provided with error bars. The methodology for measuring them should be detailed.

The conclusion of this study seems unclear to me (see below for detailed comments). Some of the qualitative results need further investigation or at least discussion, and the experimental choices made limit the generality of the results obtained. Finally, I understand that the study is not focused on the physics of slip pulses, but an understanding of why and how these pulses are generated along smooth or ridge interfaces is lacking. In general, I think the results presented by the authors are interesting however, I do not believe that they are of general interest to the scientific community, or that there are significant breakthroughs about frictional interfaces. I do believe that phenomenological studies can sometimes lead to major breakthroughs, but in this case, the results go some way toward explaining the squeaking phenomenon. This is an interesting application case, but it does not reveal any new conceptual advances in the physics of frictional interfaces.

My comments are as follows:

- To my understanding, the conclusion about the mechanism responsible for the selection of the squeaking frequency is unclear. The authors mention two mechanisms for this, but only one of them is retained in the conclusion. A first result is that the squeaking frequency corresponds to the frequency of occurrence of slip pulses along the interface (in the case of the shoe and for blocks with ridges in Figs. 2D&2F). A second result is that the squeaking frequency corresponds to the natural frequency for a solid block. What is the relationship between the frequency of occurrence of slip pulses and the natural frequency of the block? Do these two results agree or are they contradictory?
- The fact that slip pulses propagate along a soft/rigid interface is well known, both theoretically and experimentally, as reflected in the references cited by the authors. The novel observation of this study is that for an interface with ridges, the slip pulses are generated with a well-defined frequency. The role of the ridges on the regularity of pulse generation is not addressed in the manuscript, while it seems to be essential for the squeaking process. Does the number of wrinkles affect the spread of the frequency values?
- What determines the sliding velocity beyond which squeaking sound is emitted ($v \sim 0.2$ m/s) and how does this quantity vary with the experimental parameters? The sliding velocity of the soft solid is not measured in this study. How is it expected to affect the results?
- Why does the presence of ridges determine the orientation of slip pulses (Figs. 2G-J)? Do the measurements in Figs. 2I-J reflect the variation in the orientation of the pulses?
- What determines the critical width of a single ridge for generating an audible squeaking sound? Why adding more ridges makes the acoustic signal better defined?
- Why did the authors choose a ridge orientation parallel to the sliding direction, whereas for shoes it is perpendicular? How does this affect the results?

Version 2:

Reviewer comments:

Referee #1

(Remarks to the Author)

The authors have provided a clear response to the comments brought forward by both referees, and the manuscript is revised accordingly. As far as I can judge, the revised manuscript contains sufficiently novel and important findings to warrant publication in nature: the authors explain in detail how squeak emerges and how it can be tuned.

In the current draft (and thanks to the author's response) it becomes more clear how this work is different from what has been previously reported in the literature.

It is known that soft on hard interfaces like those presented by the authors display stick-slip friction. It is also known that during slip, relative displacement is accommodated through the propagation of cracks. Furthermore, previous experiments have shown that regularization of the stick-slip behavior is achieved by introducing ridges at the interface, as the authors do in the present manuscript (Audry et al. 'Slip dynamics at a patterned rubber/glass interface during stick-slip motions' is a relevant paper that the authors should consider referring to).

Several new aspects are presented in the manuscript, as far as I can judge. The velocities explored in the manuscript are higher than those typically found in the literature. Squeaking sound only emerges at these higher velocities (you can notice this if you try to make your shoe squeak). The authors show that the propagation speed of the slip pulses is very high, equal to the bulk shear wave speed of the material. The authors convincingly show that the first shear mode of the rubber block is the key parameter controlling the squeaking frequency. This resonance of the block is 'activated' by the slip pulses; a phenomenon the authors describe as 'mode lock-in'. The authors show that, regardless of the sliding velocity, the first shear mode of the block is active, causing slip pulses to be launched with a frequency equal to that of the block's first shear mode. The only thing that changes with increasing sliding velocity is that more slip pulses are included in a single period of the blocks oscillation to accommodate larger displacement per unit time. The revised explanation on how the ridges function as wave guides is also nice.

In the revised abstract, the authors distinguish between stick-slip of rigid bodies and friction of extended interfaces, claiming that at extended interfaces slip is accommodated by the propagation of cracks and slip pulses. This is certainly true for some systems, for example the rubber on glass studied here. However, there are also many extended interfaces that slide without the occurrence of slip pulses. Therefore, I think the authors should simply add the word 'can': slip can be accommodated by....

About the critical velocity for emergence of squeaking: could this simply be set by the typical displacement achieved through a single slip pulse, multiplied by the first shear mode frequency of the block? $600 \text{ Hz} \times 1 \text{ mm} = 0.6 \text{ m/s}$. Would the angle of the ridges with respect to the sliding distance then affect the critical velocity?

It would also be interesting if the authors can discuss broader implications; does a break squeak through the same mechanism?

I wonder if the current title of the draft is optimal. Is it the rapid slip pulses that cause the squeaking, or is it the structuring of the slip pulses with the period of the block resonance that cause the squeaking?

Referee #2

(Remarks to the Author)

I would like to thank the authors for their detailed responses to my questions. This is a high-quality study that deserves to be published. However, the opinion I expressed in my first review has not changed. In my view, this manuscript does not represent a significant scientific advance that would justify its publication in the esteemed journal Nature.

In this study, we learn that the squeaking sound produced by a soft solid rubbing against a rigid one is due to the propagation of slip pulses along the contacting interface. The central result is that, for reasons not addressed in the study, when there are ridges on the surface of the soft solid, the nucleation frequency of these pulses is equal to the fundamental frequency of the rubber. This is not the case when the rubber surface is smooth. This let me think that the observation of the mode lock-in is specific to this particular system and does not contribute to our understanding of friction dynamics. A large number of observations and characterizations of the slip pulses are provided by the authors, much of it in the Supplementary Materials. All these observations are interpreted within the specific framework of the system studied (sound emitted by shoes or a block with ridges), but they do not represent a breakthrough in the field of rubber friction, neither in solid friction in general, nor in the dynamics of slip pulses.

As explained by the authors in response to reviewer 1's comments, the question of the origin of the squeaking sound has not yet been resolved. It is indeed interesting to find the causes of this sound and to explain the frequency selection mechanism. However, the exercise is rather technical and I don't think it is of interest to the scientific community as a whole. For these reasons, I recommend that the paper be rejected.

Version 3:

Reviewer comments:

Referee #1

(Remarks to the Author)

The authors have addressed my remaining concerns and have further strengthened their manuscript, I recommend publication in nature.

I noticed two typos while reading the manuscript:

two times 'the' in the caption of fig.2 under (G)-(H)

equation reference in Fig. 4 (d) caption does not work

Referee #2

(Remarks to the Author)

I thank the authors for clarifying the scope of their observations and the impact of their findings on public interest. In response to the referee's comments, the authors generalized the results to other soft-on-hard systems with different materials and discussed the relevance of their study for a broad audience. Regarding novelty, the authors argue that they contribute new fundamental knowledge about friction and raise the question of transitioning from a 1D to a 2D system, which may have been overlooked in the literature. Most of the literature on frictional ruptures relates to fault mechanics. In this case, however, soft-on-hard systems driven at high velocities are not relevant. Previous studies have focused on hard solids driven at low speeds, including homogeneous and bimaterial systems. I don't believe the transition from 1D to 2D observed in this study is relevant to traditional frictional systems studied within the framework of fault mechanics. For example, Latour et al. (1) drive a soft solid on a hard substrate and observe single ruptures crossing the entire interface instead of a train of ruptures, as in the present study.

At each stage of the review process, I asked the authors a question that they never responded to: why does the natural frequency of the solid influence the nucleation frequency of the pulses? This is the central mechanism of the problem, yet it is not addressed in the manuscript; it is only mentioned as an observation. Sound is emitted due to (i) the nucleation of slip pulses and (ii) their propagation over a long distance. Although we understand that ridges allow propagation by acting as a waveguide, the question of why nucleation is controlled by the proper frequency of the solid remains. For this reason, I consider the study to be too specific. The authors explain the mechanisms behind their observations but do not provide an overview of the phenomenon. This study does not provide an overall understanding of squeaking noise generation.

Furthermore, I am not in favor of this article format, in which a second article is written in the supplementary materials, which are extended with each round of reviews. I believe this article deserves to be published in a specialized journal as a long article that includes all the measurements and conclusions in the main text.

I therefore maintain my opinion that this article should be rejected. However, if the editor decides otherwise, I have a few minor comments on the text.

- p.2, "The fact that this same frequency f_0 appears in the recorded sound, in the repetition rate of the pulses, and in the spectrum of I_g suggests a causal relationship...". To my understanding, the repetition rate of the pulses and the spectrum of I_g are equivalent, representing merely a different presentation of the same information.

- Fig. 2D, Δx_{pulse} appears on the figure but is not mentioned in the text.

- In the experiment with the fingers, the authors state that one finger is equivalent to one ridge. However, one finger is notably thick, measuring over 15mm. Why do we expect a sound to be emitted at a well-defined frequency?

- p.4, "In the presence of sufficiently thin ridges, the opening pulses are all perpendicular to the ridges and exclusively stimulate this mode leading to a mode lock-in condition". In the case of the shoe or the finger, the pulses are not perpendicular to the ridges (Fig. 2), yet the mode lock-in condition appears to be in effect. This sentence should be clarified.

- Fig. 4D is not discussed in the main text, nor is the reason for presenting it or the conclusions drawn from it. I question the value of keeping it.

1 Latour, S. et al. Ultrafast ultrasonic imaging of dynamic sliding friction in soft solids: The slow slip and the super-shear regimes. EPL (Europhysics Letters) 96, 59003 (2011).

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Response to the Referees

Squeaking at soft–rigid frictional interfaces

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Response to Referee #1

Djellouli et al. present 'soft-on-hard' squeaking friction experiments in which both sound is recorded and interface dynamics are visualized. The authors demonstrate that the fundamental frequency of the squeaking sound that is generated when a basketball shoe or rubber sample slides over glass or pmma is equal to the frequency with which opening pulses (pulses of contact area reduction) travel across the interface. Furthermore, the authors show that the frequency with which opening pulses, which can be guided by ridges on the sliding surface, occur, is controlled by the first shear mode of the sample. As the first shear mode is set by the height of the sample, the authors demonstrate control over the squeaking frequency by changing the sample height. What is novel and exciting, to the best of my knowledge, is the detail with which the authors managed to reveal the stick-slip process and its connection to sound, culminating in control over the frequency..

We thank the Referee for the thorough review and the in-depth comments that helped us improve the quality of the manuscript. We appreciate their recognition of the novel aspects of our work, particularly the connection between the stick-slip process and sound generation

Comment 1A

The explanation of the stick-slip induced squeaking sound -the relation between sound, stick-slip and elastic modes of the slider, is not completely new, as Martyn Shorten and Xia Xi already knew in 2006 (see ref 1 of the draft): "The herringbone structures of the shoe outsole are induced to vibrate at their low-order natural frequencies by stick-slip contact with the surface"

As the reviewer correctly pointed out, Martyn Shorten and Xia Xi explored the relationship between high-pitched "squeaks" and stick-slip behavior. Their findings were summarized in a two-page conference abstract (1), where they concluded:

"Based on our limited observations, the best explanation of the squeaks that we can offer is as follows: The herringbone structures of the shoe outsole are induced to vibrate at their low-order natural frequencies by stick-slip contact with the surface. Their occurrence is limited to low normal force conditions by the non-linear force-dependence of the friction coefficient and by heavy damping at higher normal forces."

Our results indicate that while the hypothesis proposed by Shorten and Xi is intuitive and merits recognition, it is fundamentally incorrect. Although the herringbone structures of the shoe outsole play a crucial role, the squeaking frequency is not a natural frequency of the herringbone patterns, as Shorten and Xi suggested. Instead, it corresponds to the repetition frequency of rapid opening slip pulses. Their frequency is set by the lowest shear mode of the bulk material, as demonstrated by Fig. 4 of the manuscript. We also show that squeaking frequency is independent of the normal load σ_N , as we found it be unaffected for $\sigma_N \in [1, 160]$ kPa.

Surprisingly, and to the best of our knowledge, no other studies have been published to validate this hypothesis. To clarify this, we have added the following sentence to the abstract:

"Although squeaking at soft-rigid frictional interfaces has been linked to stick-slip oscillations (1–5), a comprehensive understanding of the underlying mechanisms remains lacking."

To address this gap, we conducted a series of experiments in which we simultaneously recorded sound and visualized the contact interface to validate the hypothesis. Our results show that stick-slip is caused by opening slip pulses traveling at sonic speed, generated during the sliding of rubber on a hard surface. When thin ridges, acting as waveguides, are present, these pulses are periodically initiated, leading to a squeaking sound with the same fundamental frequency. In contrast, without ridges, initiation is aperiodic, resulting in broadband noise rather than squeaking. Additionally, we found that for a given soft material, the frequency of squeaking is determined solely by the bulk height of the sliding block and corresponds to the natural frequency of its lowest shear mode. As such, our findings not only confirm the relationship between stick-slip and squeaking but also uncover the underlying mechanism behind it.

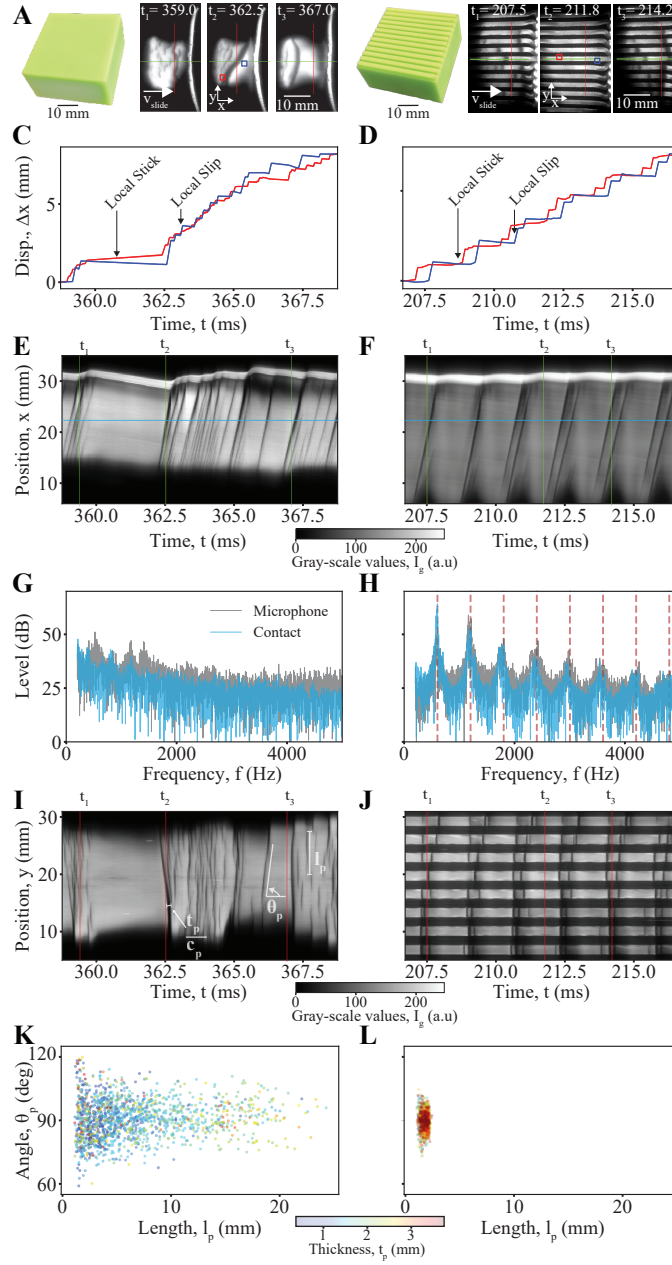


Fig. R1. Squeaking of flat and patterned elastomer cuboid samples. (A)-(B) Snapshots of the frictional interface at three different times for sample with a sliding surface that is (A) flat and (B) feature an array of parallel ridges. (C)-(D) Temporal evolution of the displacement with respect to the acrylic plate of two material points along the frictional interface (indicated by the red and blue circular markers in panels A and B). (E)-(F) Contact spatio-temporal maps showing the evolution as a function of time of the gray-scale values, I_g , for the pixels located along the green lines in panels A and B, respectively. (G)-(H) Spectrum of the recorded sound (grey line) and I_g along the blue line in panels E and F, for the sample with (G) a flat sliding surface and (H) a sliding surface featuring an array of ridges. (I)-(J) Contact spatio-temporal maps showing the evolution as a function of time of I_g for the pixels located along the red lines in panels A and B, respectively. (K)-(L) Measured pulse length, l_p , angle, θ_p , and thickness, t_p (indicated by the marker color), for the sample with (K) a flat sliding surface and (L) a sliding surface featuring an array of ridges. l_p , θ_p , and t_p are defined in panel G.

It is also important to note that the stick-slip behavior observed at the soft-rigid interface significantly differs from that of a classical spring-block model. First, slips occur asynchronously at different locations rather than uniformly across the interface. Second, while in the classical spring-block model the frequency of stick-slip events typically increases with higher sliding rates, our experiments show that as sliding velocity increases, slip pulses are generated at the same fundamental frequency. This is achieved through a train of opening pulse packets produced during each slip event, allowing faster slipping without altering the

primary pulse frequency (see Fig. R3 (AD: and updated video S2)).

To highlight these interesting aspects of our work, we have modified Fig. 2 (reported here as Fig. R1, for completeness) (AD: , updated the supplementary information video S2) and added the following text on pages 2 and 3

"The sliding of our elastomeric blocks is governed by the propagation of opening slip pulses. By monitoring the displacements of material points along the frictional interface, we observe that, at high speeds, the interface alternates between periods of perfect sticking and contactless slipping, with slip occurrences happening asynchronously at different locations (Fig. 2C and D). For the flat samples, the slip amplitude fluctuates significantly both over time and across the surface (Fig. 2C). In contrast, introducing an array of ridges regularizes this behavior, minimizing the positional variation at the interface and resulting in a more uniform slip amplitude and frequency (Fig. 2D)."

and

"In a classical spring-block model, the frequency of stick-slip events typically increases to accommodate higher sliding rates (6). However, our experiments reveal a distinctly different behavior. As the sliding velocity rises with time, slip pulses continue to be generated at the same fundamental frequency. Instead of increasing the event frequency, an additional timescale emerges to support the higher slip rate. Specifically, with each slip event, a train of opening pulse packets is produced (Fig. 2F for $t > 210$ ms and Supplementary Information Fig. S13 and video S2), enabling faster slip without changing the primary pulse frequency."

Comment 1B

What I miss in the draft is a more complete literature review; the current introduction and reference list seems to hide the fact that the explanation behind the phenomenon was already there. It would be good to sharply define what was already known and what new insight the present experiments provide. Although it is not my area of expertise, I believe there is a lot of previous work on stick slip of soft materials. Searching 'stick slip squeaking rubber' on google scholar the first hit I find includes high velocity (0.4 m/s) rubber on glass experiments.

As mentioned in our response to Comment 1A, although squeaking at soft-rigid frictional interfaces has been associated with stick-slip oscillations (1–5), the underlying mechanism behind stick-slip and audible squeaking at such interfaces remains poorly understood. Our paper addresses this gap by uncovering the mechanism driving this phenomenon. To better highlight the contribution of our work, we have completely rewritten the abstract and introduction, which now read as follows:

"Squeaking is a constant companion in various aspects of our daily lives, whether we are sliding rubber-soled shoes across hardwood floors (7), scraping chalk on a blackboard (8), engaging the brakes on a bicycle (9), or walking with a hip replacement (10, 11). When two rigid bodies slide over each other, squeaking is widely understood to result from self-excited stick-slip oscillations, triggered by a decrease in the friction coefficient with increasing slip velocity (12–16). However, sliding of extended interfaces presents a different scenario, where the motion of a large block is mediated by the propagation of cracks (17–21) and slip pulses (22–26) rather than simple block movement. This distinction is especially pronounced for the sliding of a soft body on a rigid one, where large deformations and material mismatch can lead to complete detachment via opening slip pulses (27–33). Surprisingly, the propagation of these pulses has primarily been investigated in the context of slow sliding (23, 31, 34–39), where pulses are also slow and squeaking is typically absent. Although squeaking at soft-rigid frictional interfaces has been linked to stick-slip oscillations (1–5), a comprehensive understanding of the underlying mechanisms remains lacking. Here, we experimentally investigate the dynamics occurring at soft-rigid interfaces for sliding velocities large enough to trigger squeaking. Using high-speed optical measurements of contact dynamics at the frictional interface, coupled with acoustic analysis, we observe that opening pulses propagate at the shear wave speed of the soft material. When thin ridges act as waveguides, these pulses are periodically triggered, leading to stick-slip oscillations and squeaking. In the absence of ridges, however, the pulse initiation is aperiodic, resulting in broadband noise. Additionally, we find that for a given soft material, the squeaking frequency depends solely on the height of the sliding block and matches the natural frequency of its lowest shear mode. These insights have significant implications, enabling the design of surfaces with geometric features that either prevent squeaking during interaction with rigid surfaces or intentionally generate squeaking at specific frequencies."

and

"More than 50 years ago, Schallamach observed that sliding between rubber and hard materials could occur either as uniform frictional sliding or as propagating waves of detachment, now referred to as Schallamach waves (31). These waves fully open the interface, nucleated by a surface buckling instability and propagate at speeds significantly slower than the shear wave speed. Although extensively studied (23, 34–42), Schallamach waves are not associated with squeaking due to their low frequencies, despite common misconceptions.

Simultaneously, researchers – primarily driven by studies of earthquake dynamics – began to numerically and theoretically investigate rapid slip pulses generated during sliding of bi-material interfaces (22). Solutions have been developed for local separation waves at interfaces that cannot support tension (28), and it has been proposed

that dynamic reductions in compressive normal traction could enable slip pulses to travel at approximately the shear wave speed of the softer material (29, 30, 43, 44). On an atomic scale, self-healing cracks propagating at the shear wave speed have also been predicted (18), facilitating slip via interfacial separation, similar to opening pulses. Recent observations (32) and theoretical models have similarly noted opening pulses traveling at sonic speeds along soft-rigid interfaces (18, 28, 43–46). Although these rapid slip pulses generated during sliding resemble Schallamach waves, they are fundamentally different. The Schallamach waves are overdamped and primarily influenced by adhesion and viscosity (35, 37, 39), whereas rapid sliding is inertial and the opening pulses are governed by elasticity (28, 29, 43, 44).

Building on these insights, our experimental investigation reveals that at soft–rigid frictional interfaces squeaking is, in fact, analogous to an earthquake. At higher sliding velocities ($v_{slide} \sim 1$ m/s), the squeaking sounds are driven by opening slip pulses propagating at the shear wave speed of the soft material that are regularized by the presence of thin ridges. We establish a direct link between the regular propagation of these pulses and the generation of squeaking sounds. "

Comment 1C

Related to the first point, it seems like understanding based control over squeaking sounds -for example in basketball shoes- is already achieved by the manufacturers of such shoes. In fact, according to ref 1 in the draft, shoe manufacturers make sure the squeaking sound is there as it reassures basketball players. This makes me believe that the statements in the paper on 'new design space for friction and energy dissipation' are exaggerated.

As outlined in our response to Comment 1A and in the revised abstract, while previous studies have linked squeaking at soft-rigid frictional interfaces to stick-slip oscillations (1–5), none have provided a comprehensive understanding of the underlying mechanism. Furthermore, discussions with BASF, one of the largest global suppliers of footwear materials, confirm that even within the industry, this knowledge remains unavailable. Instead, a trial-and-error approach is predominantly used to produce squeaking.

The manuscript entitled “Squeaking at soft–rigid frictional interfaces” by A. Djellouli et al presents an experimental study about the squeaking sound emitted by a rubber-like block when sliding against a rigid body. The authors use an experimental setup allowing to image the contacts at the interface between the two solids at a high frequency and to record the emitted sound. The novelty lies in the study of a soft sliding body against a rigid one at high sliding velocity, while low sliding velocities have been studied in the literature. The study is phenomenological in nature. It reports experimental measurements that allow hypothesizing a possible mechanism underlying the squeaking noise.

The key results presented in this manuscript are:

- The squeaking of a basketball shoe is due to the propagation of slip pulses at the soft-rigid interface. The frequency of the acoustic emissions is consistent with the frequency of the slip pulses generation.
- For a model solid block, the squeaking noise only occurs if one or more ridges are present at the interface. For blocks with ridges at the contacting surface, the slip pulses have a narrow distribution of orientation, length, and width, while the distributions are wider for blocks with a smooth contacting surface.
- In the case of the solid block with multiple ridges, it is shown that the squeaking frequency is equal to the natural frequency associated with the first shear mode for an elastic block.
- The conclusion of the study is that the ridges act as waveguides for the slip pulses.

The text is generally well written and structured, and the research conducted in this work is of high quality. The abstract is honest and reflects the work performed in this study. Some values of measured quantities, such as fundamental frequencies and critical sliding velocities, should be provided with error bars. The methodology for measuring them should be detailed.

We thank the reviewer for their thoughtful and detailed feedback. We appreciate their positive comments on the structure, quality, and clarity of our work, and we are glad that the key results of the study came across effectively. We now provide error bars for measured quantities and detail the methodology for measuring them in SI.

Comment 2A

To my understanding, the conclusion about the mechanism responsible for the selection of the squeaking frequency is unclear. The authors mention two mechanisms for this, but only one of them is retained in the conclusion. A first result is that the squeaking frequency corresponds to the frequency of occurrence of slip pulses along the interface (in the case of the shoe and for blocs with ridges in Figs. 2D&2F). A second result is that the squeaking frequency corresponds to the natural frequency for a solid block. What is the relationship between the frequency of occurrence of slip pulses and the natural frequency of the block? Do these two results agree or are they contradictory?

As the reviewer correctly pointed out, the paper discusses three key frequencies: (i) the frequency of slip pulse occurrence along the interface, (ii) the squeaking frequency, and (iii) the frequency associated with the lowest shear mode of the elastomeric block. Importantly, these frequencies are closely interconnected. Our results indicate that the squeaking frequency matches the repetition rate of the slip pulses (see Fig. R1), and this repetition rate is determined by the frequency corresponding to the lowest shear mode of the rubber sample. To clarify this, we have added the following text to the discussion of Fig. 4:

"These findings suggest that the squeaking frequency, which corresponds to the repetition rate of slip pulses along the ridges, is determined by the lowest shear mode frequency of the sample."

Comment 2B

The fact that slip pulses propagate along a soft/rigid interface is well known, both theoretically and experimentally, as reflected in the references cited by the authors. The novel observation of this study is that for an interface with ridges, the slip pulses are generated with a well-defined frequency. The role of the ridges on the regularity of pulse generation is not addressed in the manuscript, while it seems to be essential for the squeaking process. Does the number of wrinkles affect the spread of the frequency values?

The reviewer correctly points out that the propagation of slip pulses along soft/rigid interfaces is well-documented, both theoretically and experimentally. As we discussed in our response to Comment 1B, we have revised the abstract and introduction to clearly convey this foundational knowledge.

Building upon this understanding, our experimental findings reveal that at higher sliding velocities ($v_{slide} \sim 1$ m/s), squeaking is driven by opening slip pulses traveling at the shear wave speed of the soft material. These pulses are regularized by the presence of thin ridges, which play a key role in controlling the propagation. Our results demonstrate a clear connection between the regularity of these pulses and the generation of squeaking sounds.

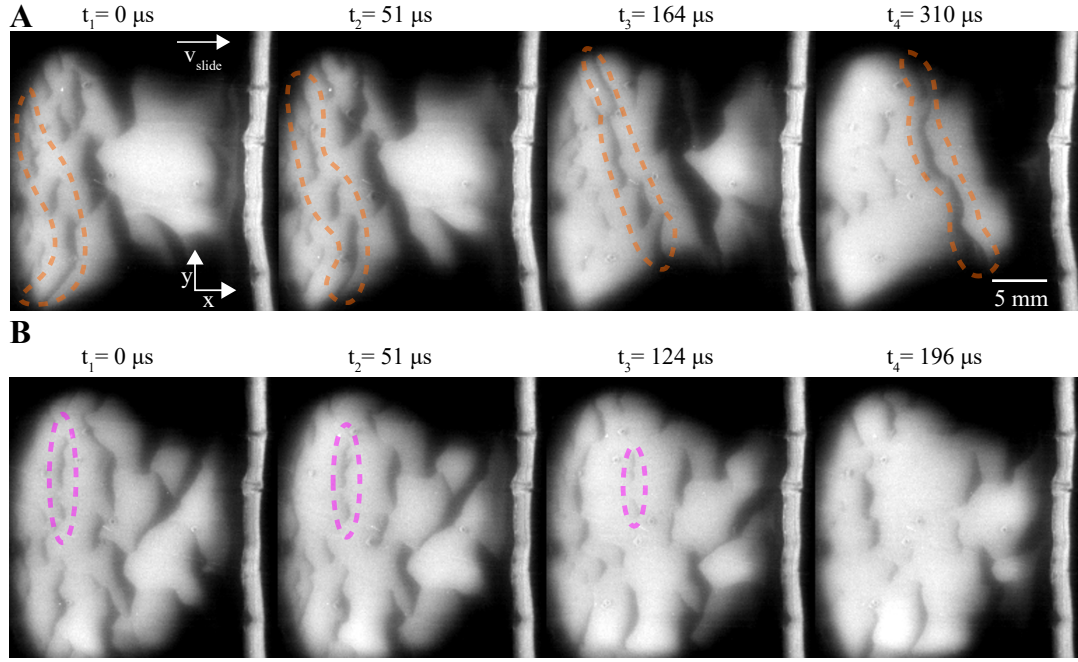


Fig. R2. Evolution of opening pulses on flat surfaces: Coalescence and vanishing. Here we present additional results for a cuboid sample of size $40 \times 40 \times 20$ mm with a flat sliding surface. **(A)** These snapshots of the frictional interface at different times shows the growth and coalescence of a group of small opening pulses (highlighted by the orange dashed line). **(B)** These snapshots of the frictional interface at different times show a group of slip opening pulses (highlighted by the purple dashed line) propagating from left to right, decreasing in size and then suddenly disappearing at $t = 196 \mu\text{s}$.

Our results specifically show that thin ridges act as waveguides for the opening pulses, ensuring unidirectional propagation and preventing the complex interactions between neighboring pulses that occur in their absence (see Fig. R2). Ultimately, these waveguides lead to squeaking at a highly discernible frequency, corresponding to the lowest shear mode of vibration of the rubber. This occurs due to a mode lock-in condition, a phenomenon observed in various other friction-related problems (47).

To clarify this point, we added Fig. R2 to Supplementary Information (where it appears as Fig. S12) and revised the main text on page 4 as follows:

"As such, these results suggest that thin enough ridges act as wave guides for the opening pulses. These ridges ensure the unidirectional propagation of the pulses, prevent complex interactions between neighboring pulses observed in their absence (see Supplementary Information Fig. S12) and ultimately result in the occurrence of squeaking with a highly discernible frequency. In the presence of sufficiently thin ridges, the opening pulses are all perpendicular to the ridges and exclusively stimulate this mode leading to a mode lock-in condition (47). Conversely, in the absence of thin ridges, the diverse orientation of the pulses trigger various vibration modes, preventing mode lock-in and effectively suppressing squeaking. "

Finally, regarding the effect of the number of wrinkles on the frequency spread, our system behaves differently from classical spring-block models, where the frequency of stick-slip events increases with sliding velocity (6). In our case, as the sliding velocity increases, slip pulses are generated at a constant fundamental frequency. Instead of increasing the pulse frequency, an additional timescale emerges, allowing for higher slip rates while maintaining the primary pulse frequency. Each slip event produces a packet of opening pulses, and we observe that the number of pulses per packet increases with sliding velocity (see Fig. R3).

We added Fig. R3 to the Supplementary Material as Fig. S13(AD: , updated Video S2) and clarified this in the main text on page 3:

"In a classical spring-block model, the frequency of stick-slip events typically increases to accommodate higher sliding rates (6). However, our experiments reveal a distinctly different behavior. As the sliding velocity rises with time, slip pulses continue to be generated at the same fundamental frequency. Instead of increasing the event frequency, an additional timescale emerges to support the higher slip rate. Specifically, with each slip event, a train of opening pulse packets is produced (Fig. R1F for $t > 210$ ms and Supplementary Information Fig. S13), enabling faster slip without changing the primary pulse frequency."

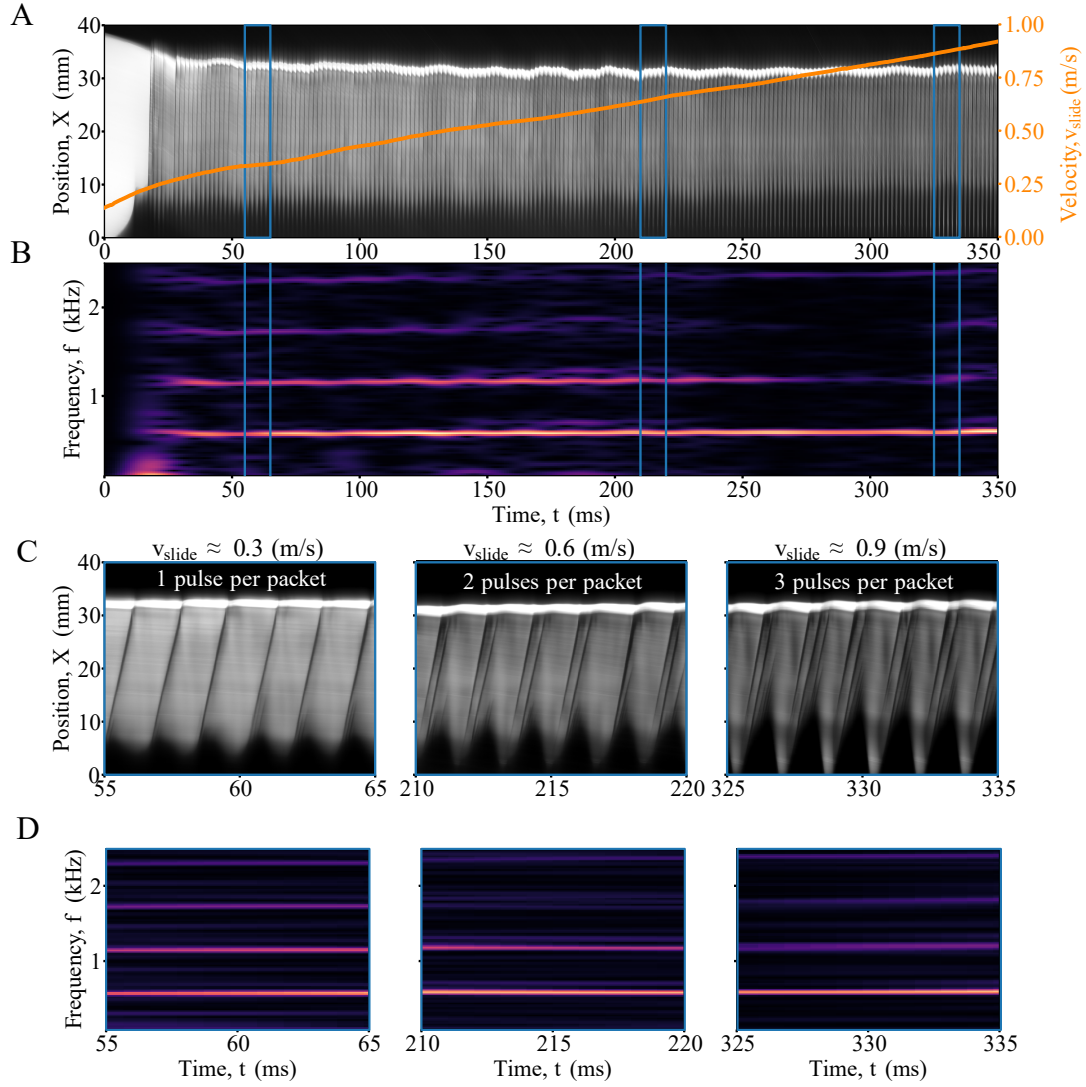


Fig. R3. Effect of sliding velocity on opening slip pulses. (A) Full spatio-temporal evolution of contact during the experiment shown in Fig. 2F of the main text, alongside the evolution of the plate's sliding velocity, v_{slide} . (B) Spectrogram calculated from the spatio-temporal contact evolution in panel (A) at position $X = 20$ mm. (C) Zoomed-in view of the spatio-temporal contact map for $v_{slide} \approx 0.3, 0.6$, and 0.9 m/s. These results demonstrate that, although the sliding velocity increases over time during the experiment, the slip opening pulses continue to be generated at a consistent fundamental frequency. Rather than increasing the frequency of events, an additional timescale arises to accommodate the higher slip rate. Specifically, with each slip event, a series of opening pulse packets is generated, allowing for faster slip without altering the primary pulse frequency. (D) Close-up of the spectrogram for $v_{slide} \approx 0.3, 0.6$, and 0.9 m/s. These plots confirm that the pulse generation repetition frequency remains constant as the sliding velocity increases.

Comment 2C

What determines the sliding velocity beyond which squeaking sound is emitted ($v \sim 0.2$ m/s) and how does this quantity vary with the experimental parameters? The sliding velocity of the soft solid is not measured in this study. How is it expected to affect the results?

We appreciate the reviewer for raising this question. The critical sliding speed, v_{slide}^{cr} , marks the transition between two sliding regimes: when $v_{slide} < v_{slide}^{cr}$, the interface slides uniformly, whereas for $v_{slide} > v_{slide}^{cr}$, opening slip pulses are initiated. In this study, we examined how v_{slide}^{cr} varies with normal load and interface geometry. Specifically, we conducted experiments on the samples described in Figure 2 of the main text, systematically varying the applied normal stress (σ_N) and measuring the critical velocity for pulse nucleation. The results are detailed in Fig. S7 of the Supplementary Information and reproduced here in Fig. R4 for completeness. We find that for a flat surface with no ridges, the critical velocity decreases with increasing normal stress. In contrast, for a surface with ridges, the critical velocity remains approximately constant at $v_{slide}^{cr} \approx 0.2$ m/s. Explaining the dependence of the critical velocity on both σ_N and geometry is a challenging problem that requires further investigation. It relates to the meta-stability between the two sliding regimes, which has been shown to be theoretically complex and ill-posed even in the simple case of 1D interfaces, in similar systems involving steady frictional sliding between dissimilar

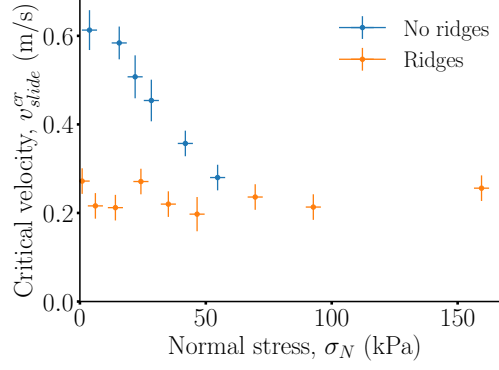


Fig. R4. Critical sliding speed v_{slide}^{cr} above which opening pulses are nucleated. Below v_{slide}^{cr} the entire interface slides in a homogeneous manner. Above v_{slide}^{cr} opening pulses are nucleated. We carried out experiments on the samples described in Figure 2 of the main text where we systematically vary the normal stress, σ_N , and measure the critical velocity, v_{slide}^{cr} , above which opening pulses are nucleated. We observe that for a sample with no ridges on the frictional surface the critical velocity decreases with increasing normal stress, while for a sample with ridges the critical velocity is approximately constant $v_{slide}^{cr} \approx 0.2$ m/s. The uncertainty on σ_N is estimated based on the precision with which we can control the vertical linear stage position. The error on v_{slide}^{cr} is calculated based on the standard deviation over multiple measurements. Where only a single measurement was carried out, then an average error is provided. The uncertainty of the velocity measurement is determined by the signal to noise ratio of the rotation encoder measurement and the subsequently applied 100Hz low-pass filter. Hence, the resulting measurement error is 0.01 m/s.

elastic solids with Coulomb friction (29, 30, 48, 49).

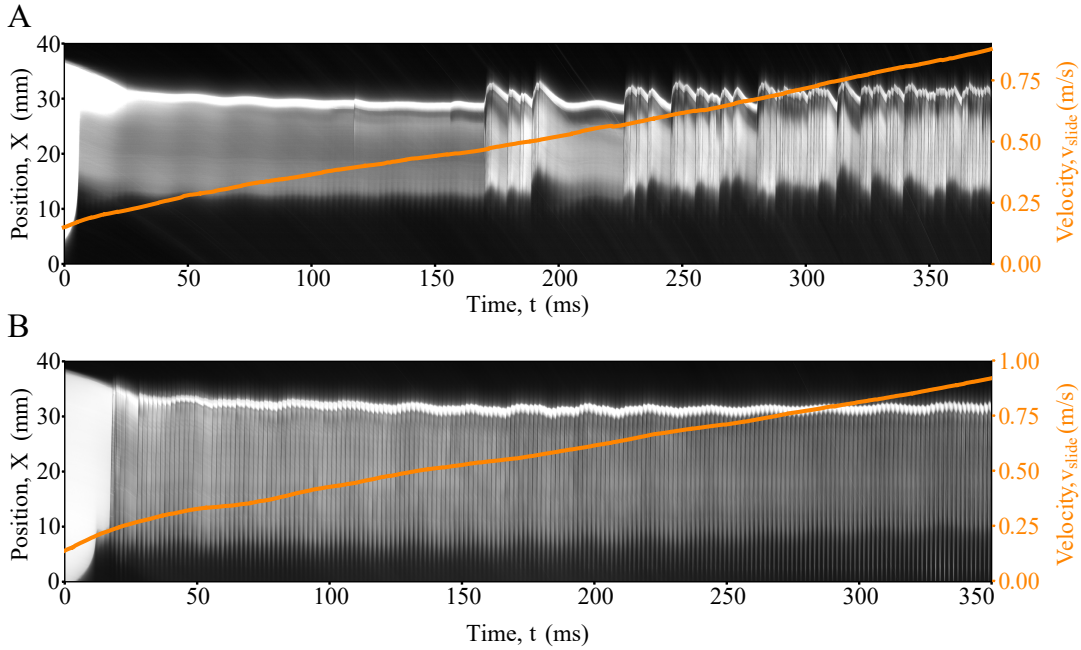


Fig. R5. Evolution of the plate's sliding velocity. (A)-(B) Full spatio-temporal evolution of contact during the experiment shown in (A) Fig. 2E and (B) Fig. 2F of the main text, alongside the evolution of the plate's sliding velocity, v_{slide} .

Finally, regarding the reviewer's question on the sliding velocity of the soft solid, we would like to clarify that the soft sample remains fixed in the laboratory frame during the experiments. The acrylic plate is driven by a constant force applied via a dropped weight, and its displacement and velocity are measured using a rotary encoder (see Fig. S3 in the Supplementary Information for further details). In Fig. R5 (which is also included in the revised Supplementary Materials as Fig. S14), we present the evolution of the plate's sliding velocity, v_{slide} , for the two experiments shown in Fig. 2 of the main text. Additionally, we measured the displacement of material points at the frictional interface (Fig. R6 (AD: and Video S2)). These measurements indicate that slip events occur asynchronously at different locations, though the net displacement of the block in the laboratory frame averages to zero.

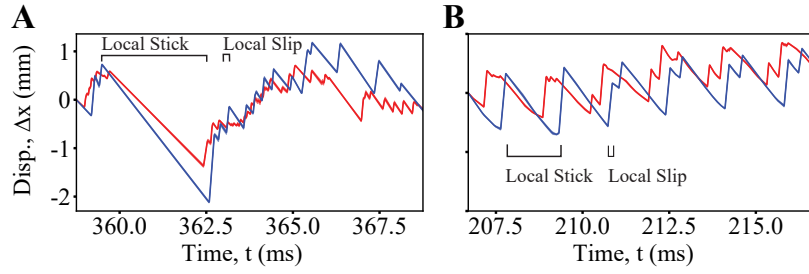


Fig. R6. Displacement of material points in the laboratory frame. (A) Displacement of two material points at the frictional interface of the flat sample shown in Fig. 2C of the main text. We observe a sequence of local sticks states followed by local slips. These events occur asynchronously in different locations and there is no clear period over time. The amplitude of slip differs depending on the location and time. The average net displacement is close to zero. (B) Displacement of two material points at the frictional interface of the patterned sample shown in Fig. 2D of the main text. We observe a sequence of local sticks states followed by local slips. These events still occur asynchronously, but they now display a clear period over time. The amplitude of the slip at a given location is more regular.

Comment 2D

Why does the presence of ridges determine the orientation of slip pulses (Figs. 2G-J)? Do the measurements in Figs. 2I-J reflect the variation in y of the orientation of the pulses?

To better understand how ridges influence the orientation of slip pulses, let's first examine the case of the flat cuboid sample. As shown in Fig. 2 of the main text, multiple opening slip pulses nucleate simultaneously at the rear of the contact, propagating predominantly in the direction of motion while expanding in the y direction. Due to asymmetries in the loading conditions, these elongated pulses tend to rotate and merge with neighboring pulses (see Fig. R2), leading to complex front shapes. This behavior can be quantified by measuring the length, width, and angle of the pulses. For a flat interface, we observe a broad distribution of both length and angles (Fig. R1I).

In contrast, for a sample with a sliding surface featuring an array of parallel ridges, the nucleation and transverse growth of the pulses are restricted by the ridge width, leading to more compact pulses. These pulses are more stable and exhibit less rotation when excited (see Fig. R1F and J). This increased stability is reflected in the narrower distribution of pulse length (l_p) and orientation angle (θ_p) shown in Fig. R1L.

Lastly, we would like to emphasize that the measurements of θ_p in Figs. 2K-L do capture the variation in the y -direction of the pulse orientations, as schematically illustrated in Fig. R7.

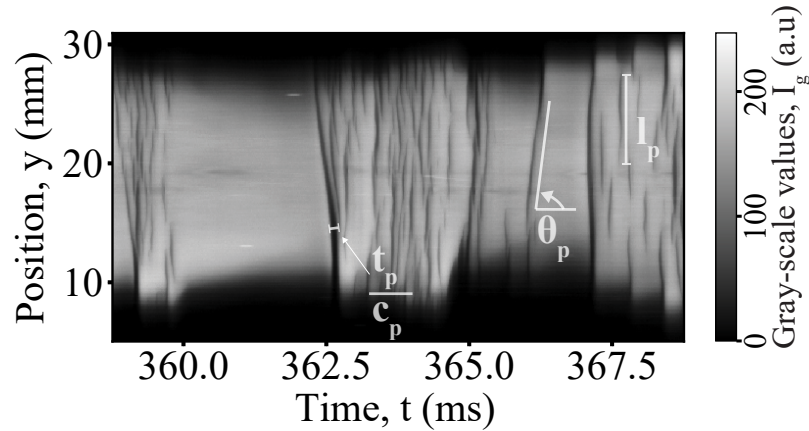


Fig. R7. Characterization of the pulses Schematic representation of pulse characteristics, showing the length (l_p), orientation angle (θ_p), and thickness (t_p) of the pulses.

Comment 2E

What determines the critical width of a single ridge for generating an audible squeaking sound? Why adding more ridges makes the acoustic signal better defined?

The critical width below which a squeaking sound with a well-defined frequency is generated marks the transition from a quasi-1D to a 2D frictional interface. As shown in Fig. 3 of the main text, we identify a critical width of approximately 12 mm, though the transition is gradual rather than abrupt.

The key dimensionless parameter governing this transition is the ratio of pulse length, l_p , to ridge width, w . When this ratio is close to one, the interface behaves as quasi-1D. In contrast, when the ratio is significantly less than one, the interface transitions to 2D, where the pulses become disordered, vary in length and width, coalesce into complex front shapes, and

occasionally vanish abruptly.

Finally, we would like to highlight that the results in Fig. 2F and Fig. 3 show that the number of ridges does not influence the repetition frequency. However, increasing the number of ridges enhances the clarity of the acoustic signal, as pulses along different ridges tend to synchronize due to their coupling to the same bulk material. This synchronization leads to an accumulation of coherent power, making the signal more distinct from background noise.

Comment 2F

Why did the authors choose a ridge orientation parallel to the sliding direction, whereas for shoes it is perpendicular? How does this affect the results?

To address this comment, we carried out additional experiments where the ridges were oriented at 45° and 90° with respect to the sliding direction ([AD: \(see supplementary information video S6\)](#)). These additional experiments were conducted using a cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring an array of parallel ridges with height $h = 1.5$ mm, width $w = 1.5$ mm and center-to-center distance $s = 3$ mm. In Fig. R8, we report the spatio-temporal evolution of contact along representative lines on the samples (indicated by green lines in the snapshots on the left). When the sample is oriented with an angle of 45 degrees between the ridges and the direction of motion, we observe opening pulses traveling along the ridges. This is confirmed from the space-time diagram constructed in the direction of motion (Fig. R8B) which shows curved trajectories of the opening slip pulses, whereas they are straight in the space-time diagram constructed along the direction of the ridges (Fig. R8C). We measured the period of generation of these opening pulses and find a period $\Delta T = 1.65$ ms which corresponds to a repetition frequency $f_0 = 606$ Hz. By contrast, when the sample is oriented with an angle of 90 degrees, between the ridges and the direction of motion, we observe that the ridges bend and exhibit a line contact with the acrylic plate (Fig. R8D). Further, we observe no opening pulses.

Additionally, to explore the effect of the shape of the ridges, we conducted new experiments on cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring a single curved ridge. The snapshots of the frictional interface reported in Fig. R9 show the propagation of two opening pulses (indicated by the orange arrows in each snapshot) along the curved ridge, indicating that opening pulses travel along thin ridges even when curved.

These new findings have been incorporated into the revised Supplementary Materials as Figs. S10 and S11.

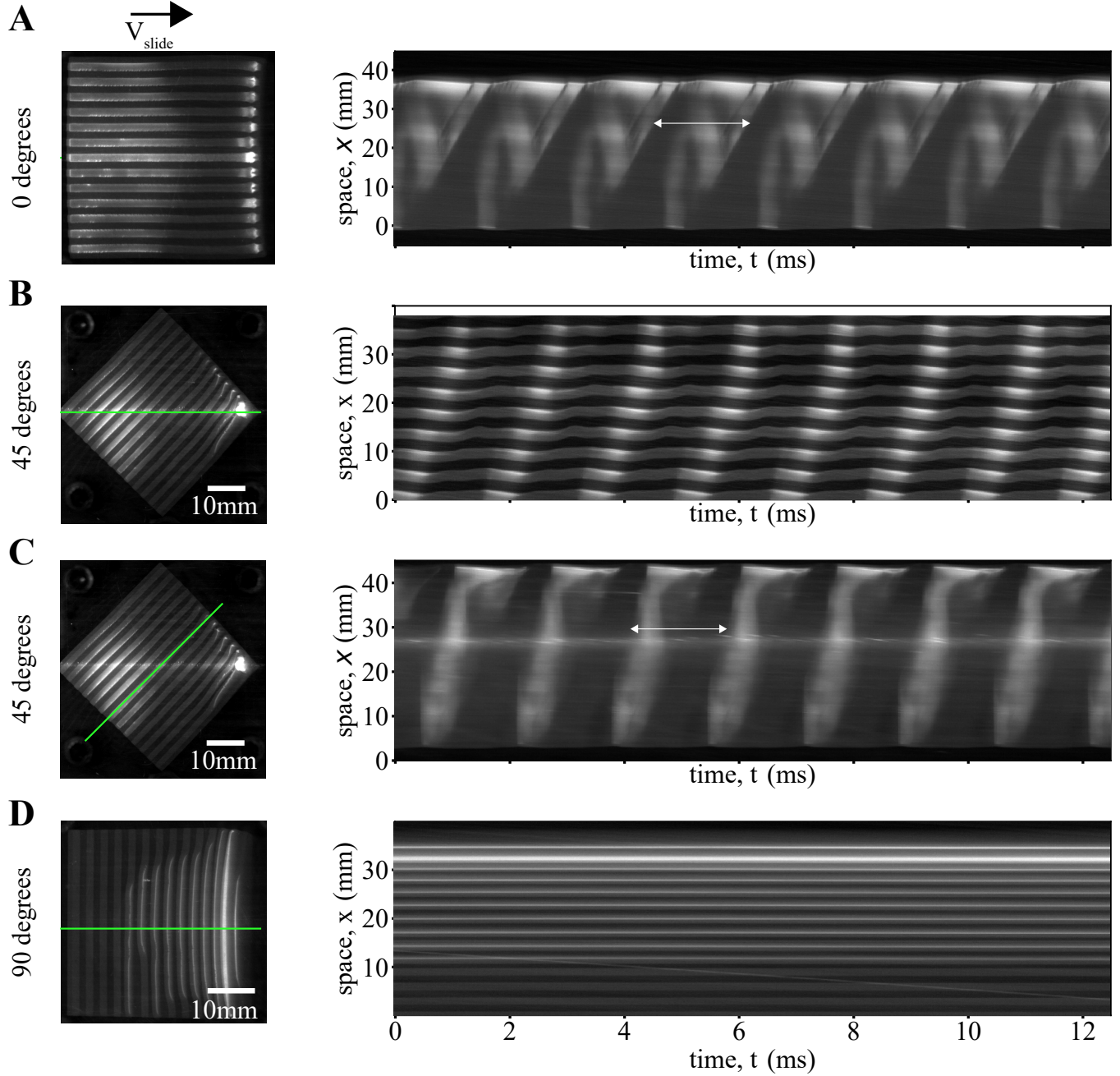


Fig. R8. Effect of ridge orientation with respect to the direction of sliding. To explore the effect of ridge orientation, we conducted additional experiments using a cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring an array of parallel ridges with height $h = 1.5$ mm, width $w = 1.5$ mm and center-to-center distance $s = 3$ mm. In these tests the ridges were oriented at 45° and 90° with respect to the sliding direction. Here, we report the spatio-temporal evolution of contact along representative lines on the samples (indicated by green lines in the snapshots on the left). **(A)** For an angle of 0 degrees between the ridges and the direction of sliding, which is the reference case described in Fig. 2 of the main text, we measure the period of generation of these opening pulses as $\Delta T = 1.63$ ms. This corresponds to a repetition frequency $f_0 = 1/\Delta T = 614$ Hz. **(B)-(C)** When the sample is oriented with an angle of 45 degrees between the ridges and the direction of motion, we observe opening pulses traveling along the ridges. This is confirmed from the space-time diagram constructed in the direction of motion which shows curved trajectories of the opening slip pulses (Panel **(B)**), whereas they are straight in the space-time diagram constructed along the direction of the ridges (Panel **(C)**). We measured the the period of generation of these opening pulses and find a period $\Delta T = 1.65$ ms which corresponds to a repetition frequency $f_0 = 606$ Hz. **(D)** When the sample is oriented with an angle of 90 degrees, between the ridges and the direction of motion, we observe that the ridges bend and exhibit a line contact with the acrylic plate. Further, we observe no opening pulses.

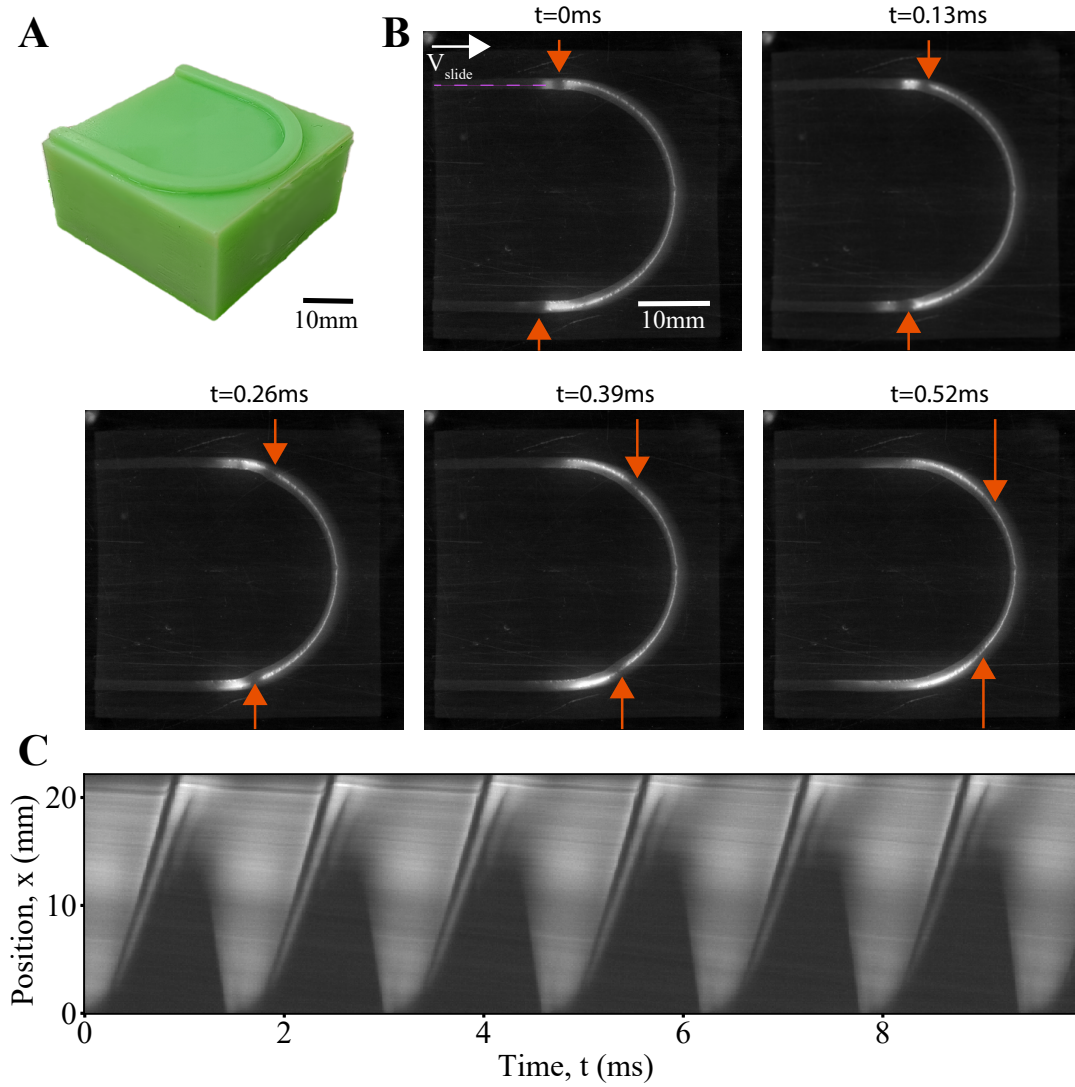


Fig. R9. Effect of ridge curvature. (A) To explore the effect of the shape of the ridges, we conducted experiments on cuboid sample of size $40 \times 40 \times 20$ mm with a sliding surface featuring a single curved ridge. (B) Snapshots of the frictional interface at times $t = 0, 0.13, 0.26, 0.39$ and 0.52 ms, show the propagation of 2 opening pulses (indicated by the orange arrows in each snapshot) along the curved ridge. (C) Spatio-temporal evolution of contact along the straight part of the ridge (indicated by the purple dashed line in panel B). This plot a repetition period $\Delta T = 1.61$ ms which corresponds to a repetition frequency $f_0 = 621$ Hz. This experiment shows that opening pulses travel along thin ridges even when curved.

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Response to the Referees

Squeaking at soft–rigid frictional interfaces

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Response to Referee #1

The authors have provided a clear response to the comments brought forward by both referees, and the manuscript is revised accordingly. As far as I can judge, the revised manuscript contains sufficiently novel and important findings to warrant publication in nature: the authors explain in detail how squeak emerges and how it can be tuned.

In the current draft (and thanks to the author's response) it becomes more clear how this work is different from what has been previously reported in the literature.

It is known that soft on hard interfaces like those presented by the authors display stick-slip friction. It is also known that during slip, relative displacement is accommodated through the propagation of cracks. Furthermore, previous experiments have shown that regularization of the stick-slip behavior is achieved by introducing ridges at the interface, as the authors do in the present manuscript (Audry et al. 'Slip dynamics at a patterned rubber/glass interface during stick-slip motions' is a relevant paper that the authors should consider referring to).

Several new aspects are presented in the manuscript, as far as I can judge. The velocities explored in the manuscript are higher than those typically found in the literature. Squeaking sound only emerges at these higher velocities (you can notice this if you try to make your shoe squeak). The authors show that the propagation speed of the slip pulses is very high, equal to the bulk shear wave speed of the material. The authors convincingly show that the first shear mode of the rubber block is the key parameter controlling the squeaking frequency. This resonance of the block is 'activated' by the slip pulses; a phenomenon the authors describe as 'mode lock-in'. The authors show that, regardless of the sliding velocity, the first shear mode of the block is active, causing slip pulses to be launched with a frequency equal to that of the block's first shear mode. The only thing that changes with increasing sliding velocity is that more slip pulses are included in a single period of the blocks oscillation to accommodate larger displacement per unit time. The revised explanation on how the ridges function as wave guides is also nice.

We thank the Referee for their thorough review and appreciate their recognition of the novel aspects of our work. We also thank the Referee for highlighting the relevant study by [Audry et al. \(1\)](#), which examines the influence of surface patterning on stick-slip dynamics at rubber/glass interfaces. That work focuses on low sliding velocities (0.005–1 mm/s), where squeaking is absent, and observes the propagation of relatively slow interfacial cracks (approximately 100 mm/s) – a regime that is distinct from the high-speed, squeaking behavior explored in our study.

Given the relevance of this previous work, we have updated the abstract to include a citation to it:

"To date, the propagation of these pulses has primarily been investigated in the context of slow sliding ([1–9](#)), where pulses are also slow and squeaking is typically absent."

Similarly, we included this citation within the introduction:

"Although extensively studied ([1, 3–12](#)), Schallamach waves are not associated with squeaking due to their low frequencies, despite common misconceptions."

Comment 1A

In the revised abstract, the authors distinguish between stick-slip of rigid bodies and friction of extended interfaces, claiming that at extended interfaces slip is accommodated by the propagation of cracks and slip pulses. This is certainly true for some systems, for example the rubber on glass studied here. However, there are also many extended interfaces that slide without the occurrence of slip pulses. Therefore, I think the authors should simply add the word 'can': slip can be accommodated by...

We thank the reviewer for their comment, which we agree with. To clarify this point, we have modified the abstract to read as follows.

"However, sliding of extended interfaces presents a different scenario, where the motion of a large block [can be](#) mediated by the propagation of cracks ([13–17](#)) and slip pulses ([6, 18–21](#)) rather than simple block movement."

Comment 1B

About the critical velocity for emergence of squeaking: could this simply be set by the typical displacement achieved through a single slip pulse, multiplied by the first shear mode frequency of the block? $600 \text{ Hz times } 1 \text{ mm} = 0.6 \text{ m/s}$. Would the angle of the ridges with respect to the sliding distance then affect the critical velocity?

We thank the Referee for their insightful comment and for proposing the following scaling for the critical sliding velocity $v_{\text{slide}}^{\text{cr}}$ for the emergence of squeaking (see Fig. R1):

$$v_{\text{slide}}^{\text{cr}} = \Delta x_{\text{pulse}}^{\text{cr}} f_0, \quad [\text{R1}]$$

where $\Delta x_{\text{pulse}}^{\text{cr}}$ is the slip associated with a single pulse in the vicinity of $v_{\text{slide}}^{\text{cr}}$, and f_0 is the frequency of the first shear mode of the block.

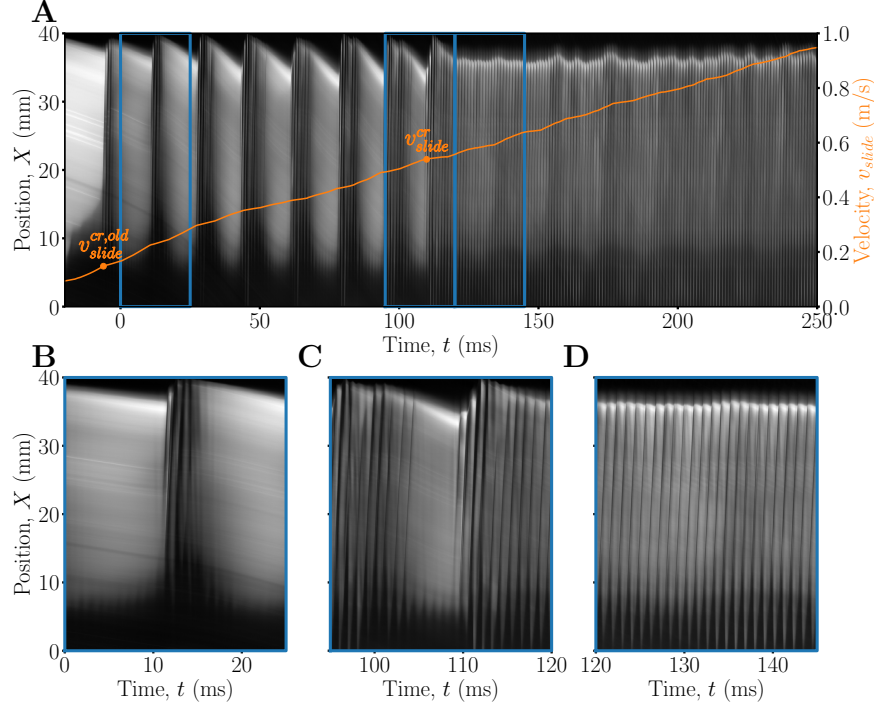


Fig. R1. Contact spatio-temporal map for a sample with ridges and $H = 13.5 \text{ mm}$. Spatio-temporal evolution of contact for a rubber cuboid samples of size $40 \times 40 \times 13.5 \text{ mm}$ and with ridges of size $h = w = 1.5 \text{ mm}$ at its frictional interface. **(A)** Full spatio-temporal evolution of contact during the experiment. **(B-D)** Three close-up views.

To verify the proposed scaling (Eq. (R2)), we focus on the experimental results obtained for rubber cuboid samples of size $40 \times 40 \times H \text{ mm}$, with $H \in [5, 20] \text{ mm}$, and ridges of height and width $h = w = 1.5 \text{ mm}$ on their frictional interface. Fig. R1 shows the spatio-temporal evolution of contact for the sample with $H = 13.5 \text{ mm}$. We begin by noting that in the original submission, we defined the critical sliding velocity as the velocity at which the first opening pulse nucleates (denoted as $v_{\text{slide}}^{\text{cr,old}}$ in Fig. R1A). However, using this definition, the data do not follow the scaling predicted by Eq. (R2).

Prompted by the Referee's suggestion, we revised our definition of $v_{\text{slide}}^{\text{cr}}$ as the velocity at which a steady-state regime emerges, characterized by the periodic generation of pulses at frequency f_0 , which gives rise to audible squeaking (Fig. R1A). We believe this revised definition better captures the physical onset of squeaking and is more appropriate for assessing the proposed scaling.

In Fig. R2A, we report $v_{\text{slide}}^{\text{cr}}$ (as newly defined) as a function of the normal stress σ_N for samples of different heights, H . We find that $v_{\text{slide}}^{\text{cr}}$ is only weakly dependent on σ_N , but varies significantly with H . Next, we quantify the slip associated with a single pulse $\Delta x_{\text{pulse}}^{\text{cr}}$ in the vicinity of $v_{\text{slide}}^{\text{cr}}$ for all tests (Fig. R2B). Finally, in Fig. R2C, we plot $v_{\text{slide}}^{\text{cr}}$ against $\Delta x_{\text{pulse}}^{\text{cr}} f_0$, and observe that the experimental data closely follow the proposed scaling.

To highlight this important point, we added Figs. R1 and R2 and the following text to Section S2.4.1 of the revised version of Supporting Information:

"

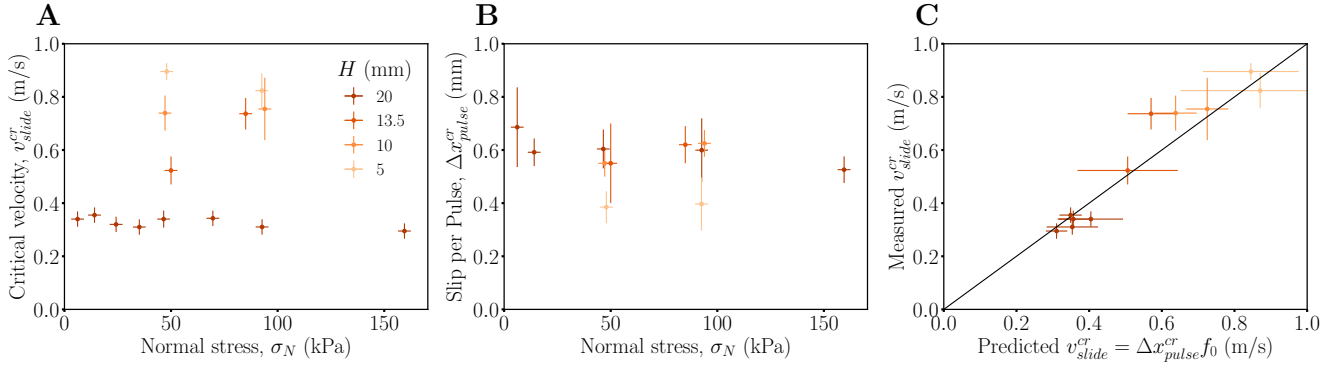


Fig. R2. Scaling for $v_{\text{slide}}^{\text{cr}}$. Data is reported for cuboid samples of size $40 \times 40 \times H$ mm with varying bulk height, $H \in [5, 20]$ mm, and with ridges of size $h = w = 1.5$ mm at their interface. **(A)** Critical velocity $v_{\text{slide}}^{\text{cr}}$, as a function of the applied normal stress, σ_N . **(B)** Slip achieved through a single slip pulse, $\Delta x_{\text{pulse}}^{\text{cr}}$, in the vicinity of $v_{\text{slide}}^{\text{cr}}$ as a function of σ_N . **(C)** $v_{\text{slide}}^{\text{cr}}$ vs $\Delta x_{\text{pulse}}^{\text{cr}} f_0$.

Effect of normal stress σ_N and bulk height H on the critical sliding speed $v_{\text{slide}}^{\text{cr}}$. To investigate the effect of σ_N and H on the critical velocity, $v_{\text{slide}}^{\text{cr}}$, above which opening pulses are generated consistently, we focus on rubber cuboid samples of size $40 \times 40 \times H$ mm, with $H \in [5, 20]$ mm, and ridges of height and width $h = w = 1.5$ mm on their frictional interface (corresponding to the samples of Fig. 4 in the main text).

Fig. R1 shows the spatio-temporal evolution of contact for the sample with $H = 13.5$ mm. Opening pulses begin to emerge at $t = 0$ s for a sliding velocity of 0.15 m/s, but during the first 110 seconds they are generated only sporadically. When the sliding velocity reaches a critical value of $v_{\text{slide}}^{\text{cr}} = 0.54 \pm 0.01$ m/s, a steady state is established in which pulses are generated periodically at a constant frequency f_0 , resulting in an audible squeaking sound. Note that for samples without ridges, such a steady-state regime is never reached (see Fig. S13), and therefore $v_{\text{slide}}^{\text{cr}}$ cannot be defined.

In Fig. R2A, we report $v_{\text{slide}}^{\text{cr}}$ as a function of the normal stress σ_N for samples of different heights, H . We find that $v_{\text{slide}}^{\text{cr}}$ is only weakly dependent on σ_N , but varies significantly with H . Note that the uncertainty in σ_N , ± 3 kPa, is estimated based on the precision with which we can control the vertical linear stage position (see Fig. S9). The error in $v_{\text{slide}}^{\text{cr}}$ is calculated based on the standard deviation of multiple measurements. Where only a single measurement was carried out, then an average error is provided. The uncertainty of the velocity measurement is determined by the signal-to-noise ratio of the rotation encoder measurement and the subsequently applied 100Hz low-pass filter. Hence, the resulting measurement error is 0.01 m/s.

Next, we quantify the slip associated with a single pulse $\Delta x_{\text{pulse}}^{\text{cr}}$ in the vicinity of $v_{\text{slide}}^{\text{cr}}$ for all tests by tracking material points at the frictional interface as in Fig. 2C-D of the main text. In Fig. R2B, we report $\Delta x_{\text{pulse}}^{\text{cr}}$ as a function of the normal stress σ_N for samples of different heights, H . Note that the uncertainty in $\Delta x_{\text{pulse}}^{\text{cr}}$ is calculated based on the standard deviation over multiple measurements and on multiple repetitions of the same experiments. Finally, in Fig. R2D, we plot $v_{\text{slide}}^{\text{cr}}$ against $\Delta x_{\text{pulse}}^{\text{cr}} f_0$, and observe that the experimental data closely follow the scaling

$$v_{\text{slide}}^{\text{cr}} = \Delta x_{\text{pulse}}^{\text{cr}} f_0, \quad [\text{R2}]$$

for all samples with ridges at their interface, regardless of H and σ_N .

Note that Eq. (R2) can be derived from a global slip balance, as the global sliding velocity v_{slide} , measured from the rail position, must match the average local velocity at the frictional interface:

$$v_{\text{slide}} = \frac{\Delta x(t + \Delta t) - \Delta x(t)}{\Delta t}, \quad [\text{R3}]$$

where Δx is the net displacement of material points over the time interval Δt . Note that a material point on the frictional interfaces alternates between perfect stick periods and contactless slipping, with slip events occurring asynchronously at different locations (see Fig. 2C-D of the main text). Assuming that N slip events occur during Δt , it follows that

$$\Delta x(t + \Delta t) - \Delta x(t) = \sum_{i=1}^N \Delta x_{\text{pulse},i} \quad [\text{R4}]$$

and Eq. (R3) can be rewritten as

$$v_{\text{slide}} = \frac{\sum_{i=1}^N \Delta x_{\text{pulse},i}}{\Delta t}. \quad [\text{R5}]$$

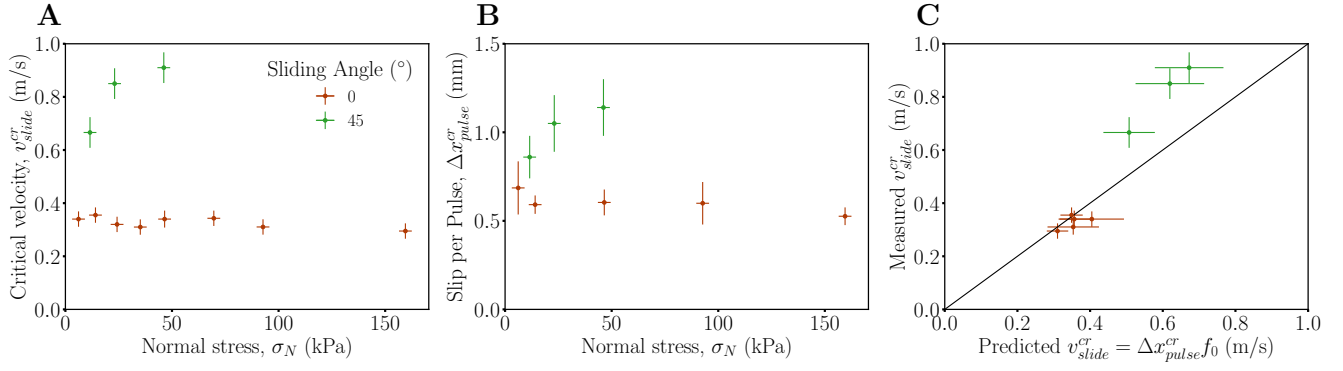


Fig. R3. Scaling for v_{slide}^{cr} . Data is reported for cuboid samples of size $40 \times 40 \times 20$ mm and with ridges of size $h = w = 1.5$ mm at their interface. Two different directions of sliding are considered. **(A)** Critical velocity v_{slide}^{cr} , as a function of the applied normal stress, σ_N . **(B)** Slip achieved through a single slip pulse, Δx_{pulse}^{cr} , in the vicinity of v_{slide}^{cr} as a function of σ_N . **(C)** v_{slide}^{cr} vs $\Delta x_{pulse}^{cr} f_0$.

Near the critical sliding velocity v_{slide}^{cr} , Eq. (R5) can be further simplified by assuming that each pulse produces a constant displacement Δx_{pulse}^{cr} . Introducing this approximation, we recover Eq. (R2):

$$v_{slide}^{cr} = \frac{N \Delta x_{pulse}^{cr}}{\Delta t} = \Delta x_{pulse}^{cr} f_0, \quad [R6]$$

where the pulse repetition frequency is defined as $f_0 = N/\Delta t$. "

Furthermore we have added Fig. R2C to Fig 4 of the main text and added the following text around it

"

However, for samples without ridges, this scaling does not apply since f_0 is not well defined. "

Regarding the influence of ridge orientation on v_{slide}^{cr} , we find that sliding at 45° results in higher critical velocities (Fig. R3A) and larger slip per pulse (Fig. R3B) compared to the reference case of 0° , for a given normal stress σ_N . Furthermore, the data at 45° no longer follow the scaling law predicted by Eq. (R2) (Fig. R3C). It is worth noting that at a sliding angle of 90° , no opening pulses are observed; as a result, neither v_{slide}^{cr} nor Δx_{pulse}^{cr} can be defined.

Fig. R3 has been included into the revised version of Supporting Information (where it appears as Fig. S15) along with the following text:

" Next, we investigate how the sliding angle influences the critical velocity v_{slide}^{cr} . As shown in Fig. R3, sliding at 45° results in higher critical velocities (Fig. R3A) and larger slip per pulse (Fig. R3B) compared to the reference case of 0° , for a given normal stress σ_N . Furthermore, the data at 45° no longer follow the scaling law predicted by Eq. (R2) (Fig. R3C). It is worth noting that at a sliding angle of 90° , no opening pulses are observed; as a result, neither v_{slide}^{cr} nor Δx_{pulse}^{cr} can be defined. "

Comment 1C

It would also be interesting if the authors can discuss broader implications; does a break squeak through the same mechanism?

Motivated by the reviewer's suggestion, we made a substantial effort to emphasize the broader implications of our work. As detailed in our response to the comment raised by Reviewer 2, we expanded the manuscript to include new experimental results that extend our findings beyond the original material system (Zhermack Elite Double 32), providing further evidence that the underlying physical mechanisms are robust and broadly applicable. In particular, we conducted additional experiments on cast elastomeric blocks made of Smooth-Sil 960 (elastic modulus ≈ 4 MPa) and on 3D-printed TPU blocks (elastic modulus ≈ 40 MPa). Furthermore, we significantly revised both the Abstract and Conclusions to better highlight the novelty and broader relevance of this study.

Comment 1D

I wonder if the current title of the draft is optimal. Is it the rapid slip pulses that cause the squeaking, or is it the structuring of the slip pulses with the period of the block resonance that cause the squeaking?

We thank the referee for their comment on the current title: "Rapid opening slip pulses cause squeaking in structured soft frictional interfaces". Our results indicate that it is the pattern at the frictional interface that brings stability to the slip pulses by acting as waveguides. This pulse stability in space and time synchronizes with the block eigen-frequency, giving rise to squeaking with well-defined note. However, the block eigen-frequency alone is not sufficient to bring stability to the pulses - the pattern is key.

Including this multifaceted observation in an 80-characters title is not feasible. Therefore, we revert to the original title: "Squeaking at soft-rigid frictional interfaces".

I would like to thank the authors for their detailed responses to my questions. This is a high-quality study that deserves to be published. However, the opinion I expressed in my first review has not changed. In my view, this manuscript does not represent a significant scientific advance that would justify its publication in the esteemed journal Nature.

In this study, we learn that the squeaking sound produced by a soft solid rubbing against a rigid one is due to the propagation of slip pulses along the contacting interface. The central result is that, for reasons not addressed in the study, when there are ridges on the surface of the soft solid, the nucleation frequency of these pulses is equal to the fundamental frequency of the rubber. This is not the case when the rubber surface is smooth. This let me think that the observation of the mode lock-in is specific to this particular system and does not contribute to our understanding of friction dynamics. A large number of observations and characterizations of the slip pulses are provided by the authors, much of it in the Supplementary Materials. All these observations are interpreted within the specific framework of the system studied (sound emitted by shoes or a block with ridges), but they do not represent a breakthrough in the field of rubber friction, neither in solid friction in general, nor in the dynamics of slip pulses.

As explained by the authors in response to reviewer 1's comments, the question of the origin of the squeaking sound has not yet been resolved. It is indeed interesting to find the causes of this sound and to explain the frequency selection mechanism. However, the exercise is rather technical and I don't think it is of interest to the scientific community as a whole. For these reasons, I recommend that the paper be rejected.

We thank the referee for carefully reading our manuscript and for their thoughtful summary of our findings. We also appreciate their positive assessment that the study is of high quality and merits publication. However, we note that the referee raises three main concerns: (1) that the observed behavior may be specific to the particular system considered; (2) that the work does not constitute a breakthrough in the field of solid friction; and (3) that the phenomenon of squeaking lacks broader scientific interest. Below, we address each of these points in detail.

1. Generality of the observed behaviors. The reviewer raised the valid point that the observed behavior might appear specific to our particular experimental setup. To robustly address this concern, we have substantially extended our experimental framework beyond the original material system (Zhermack Elite Double 32 with an elastic modulus of approximately 1 MPa) and provided further evidence that the underlying physical mechanisms are broadly applicable. Specifically, we performed additional experiments on cast elastomeric blocks made of Smooth-Sil 960 (with an elastic modulus of approximately 4 MPa) and on 3D-printed TPU blocks (with an elastic modulus of approximately 40 MPa). These new results demonstrate that across materials spanning nearly two orders of magnitude in stiffness and differing chemistries and thus different friction properties, the introduction of thin surface ridges - which act as waveguides for opening pulses - consistently results in a repeated initiation of pulses at a characteristic frequency that leads to squeaking.

These findings are summarized in Fig. 5 (also reported here as Fig. R4 for completeness) and discussed in detail on pages 4-5 of the main text and Supporting Information Section S2.2, S2.3.2, S2.4.2, S2.4.3.

"While this study primarily focused on blocks made of Zhermack Elite Double 32, the observed phenomena are not limited to this material. To demonstrate the broader applicability of our findings, in Fig. R4 we present experimental results for cast elastomeric blocks made of Smooth-Sil 960 (with $E \approx 4$ MPa) and 3D-printed TPU blocks (with $E \approx 40$ MPa). In all cases, we observe the propagation of opening slip pulses along the interface (see Video S7). Similar to the Zhermack samples, flat blocks exhibit pulses that vary in shape and spacing, resulting in broadband acoustic emissions without a dominant frequency (Figs. R4A.i and R4B.i). In contrast, patterned surfaces (Figs. R4A.ii and R4B.ii) produce regularly spaced pulses and distinct squeaking frequencies (Figs. R4A.iii and R4B.iii; see Supplementary Information Section S2.4.2-S3 for details), confirming the critical role of surface ridges in regulating squeaking behavior.

To further underscore the broader applicability of our findings, we demonstrated similar frictional dynamics with human hand. We found that sliding three fingers - each functioning as a macroscopic natural ridge - across a glass surface generates regularly spaced opening pulses, resulting in a squeaking sound with a well-defined fundamental frequency. In contrast, when the palm - a broader and flatter surface - is slid across the same substrate, fast opening pulses still occur but are generated irregularly. As a result, the emitted sound lacks a dominant frequency and instead exhibits a broadband spectral profile. These additional results are presented in Video S1, Fig. S7 (also reported here as Fig. R5 for completeness) detailed in Supporting Information Section 2.3.2, and are summarized in the following text added to page 2 of the main manuscript:

Beyond engineered surfaces, biological frictional interfaces exhibit similar dynamics. For example, human fingers - each acting as a macroscopic natural ridge - sliding on glass, can generate regularly spaced opening pulses with a distinct squeaking frequency, while flatter surfaces like the palm produce irregular pulses and broadband acoustic signatures (Figs. S6-S7, Video S1 and Supplementary Information Section S2.3.2).

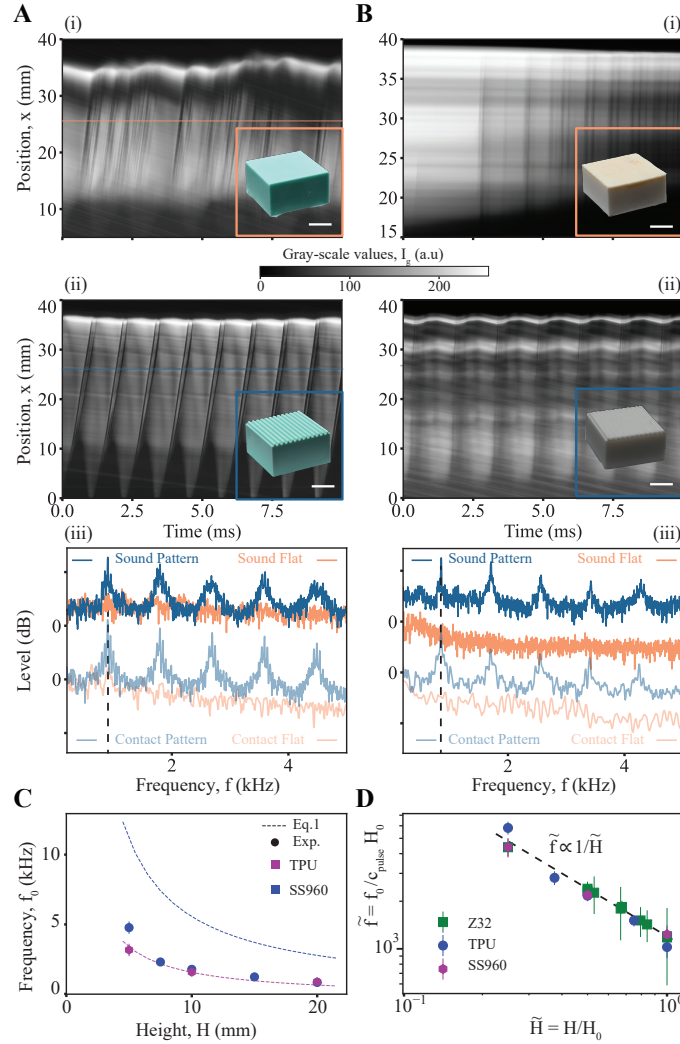


Fig. R4. Squeaking across materials. (A)-(B) Experimental results for (A) cast elastomeric blocks made of Smooth-Sil 960, and (B) 3D-printed TPU blocks. (i)-(ii) Contact spatio-temporal maps showing the evolution as a function of time of the gray-scale values, I_g , for sliding surfaces that are (i) flat and (ii) feature ridges. (iii) Spectrum of the recorded sound and I_g along the blue and orange lines in panels (i) and (ii), for the sample with (i) a flat sliding surface and (ii) a sliding surface featuring an array of ridges. (C) Fundamental frequency of the squeaking sound, f_0 , plotted as a function of H . (D) Data collapse of the fundamental frequencies measured for all three materials onto a single master curve by plotting the reduced frequency $\tilde{f} = f_0 / (c_{\text{pulse}} H_0)$ against the normalized height $\tilde{H} = H / H_0$, where $H_0 = 20$ mm. All data align with $\tilde{f} = 1/(2\tilde{H})$.

To conclude, this consistency across biological and engineered materials strongly refutes the notion of system specificity and highlights the fundamental and broadly applicable nature of our findings.

2. Novelty. We respectfully but firmly disagree with the referee's assessment that our results lack novelty, as we believe they offer new and meaningful insights into frictional behavior.

High-speed detachment fronts and slip pulses have been documented across diverse systems, from earthquake faults (22, 23) to hard solids and rock (24, 25) and soft materials like gels and rubbers (26, 27). However, the vast majority of studies examining frictional rupture mechanisms have focused on effectively 1D interfaces. While some experiments have explored 2D interfaces (28–30), these were typically slow processes or lacked visualization of the propagating fronts, limiting insights into dynamic behavior.

Our study is distinctive as it addresses high-speed frictional sliding along 2D interfaces. We demonstrate that, in the absence of guiding mechanisms, a flat 2D interface exhibits chaotic behavior due to the coexistence and nonlinear interaction of multiple modes. Perhaps expected, but has not been experimentally nor theoretically resolved. By introducing ridges as waveguides, we show that a single mode is selected, leading to synchronized pulse propagation across an array of such structures—a behavior reminiscent of 1D systems but emergent in a 2D context. This finding reveals a fundamental principle: structural features can dictate mode selection and synchronization in frictional systems, bridging the gap between 1D and 2D dynamics. This insight is a clear step forward in understanding how dimensionality influences frictional rupture, with implications for both natural (e.g., fault mechanics) and engineered systems. Notably, our results also prompt a re-evaluation of the generality of a large body of past and

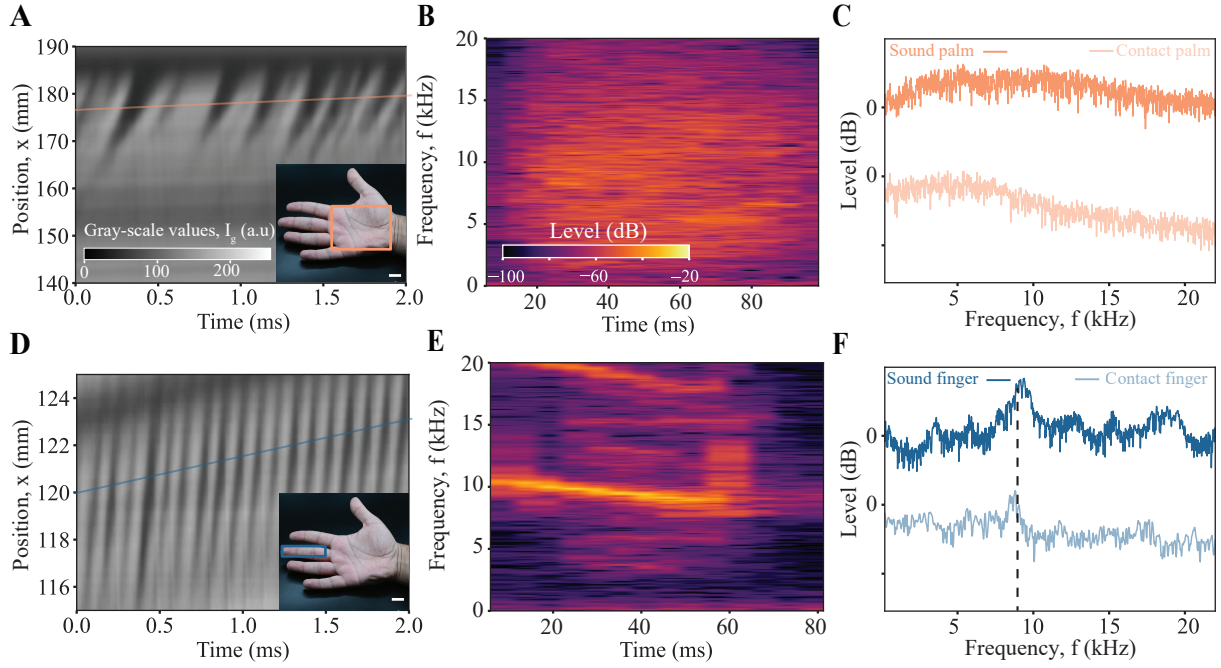


Fig. R5. Sliding a hand and fingers across a glass plate. (A) Contact spatio-temporal maps showing the evolution of gray-scale intensity I_g during sliding of a palm. (B) Spectrograms of the corresponding sound recordings. For the palm, broadband acoustic emission is observed without a dominant frequency. (C) FFTs of both the recorded sound and I_g along the orange lines in panels (A). The palm shows no dominant peak. (D) Contact spatio-temporal maps showing the evolution of gray-scale intensity I_g during sliding of three fingers. (E) Spectrograms of the corresponding sound recordings. The fingers exhibit a clear spectral peak at $f_0 = 9$ kHz. (F) FFTs of both the recorded sound and I_g along the blue lines in panels (D). The palm shows no dominant peak. The fingers exhibit a clear spectral peak at $f_0 = 9$ kHz.

recent work that has relied on 1D interface models. However, we refrain from emphasizing this critique here, as it is not the appropriate context for such a discussion.

Our observation of super-fast (non-Schallamach) slip pulses on soft-hard interfaces represents another significant advance. The existence and importance of super-fast slip pulses at soft-hard interfaces, distinct from the well-known slow Schallamach waves, have received minimal attention in previous literature (only briefly reported only recently in an arXiv publication by Rubinstein and co-authors (31)). Given the critical relevance of these high-speed frictional dynamics to practical systems such as tire-road contacts and braking systems, our identification and detailed characterization of these pulses represent a significant contribution to friction science, both fundamentally and practically. Beyond soft matter, the work Lykotrafitis and Rosakis (24) highlights the importance of high-speed pulses in frictional sliding in very rigid laboratory faults. Our extension to soft-hard interfaces offers a general perspective on these important frictional dynamics. The referee's dismissal of this finding as system-specific overlooks its potential to inform broader theories of rapid friction and pulse propagation.

To emphasize the novelty of our study, we have substantially revised the Abstract, Introduction, and Conclusions, which now read as follows:

" Squeaking is a constant companion in various aspects of our daily lives, whether we are sliding rubber-soled shoes across hardwood floors (32), scraping chalk on a blackboard (33), engaging the brakes on a bicycle (34), or walking with a hip replacement (35, 36). When two rigid bodies slide over each other, squeaking is widely understood to result from self-excited stick-slip oscillations, triggered by a decrease in the friction coefficient with increasing slip velocity (37–41). However, sliding of extended interfaces presents a different scenario, where the motion of a large block **can be** mediated by the propagation of cracks (13, 14, 16, 17, 42) and slip pulses (6, 18–21, 43) rather than simple block movement. This distinction is especially pronounced for the sliding of a soft body on a rigid one, where large deformations and material mismatch can lead to complete detachment via opening slip pulses (2, 31, 44–47). To date, the propagation of these pulses has primarily been investigated in the context of slow sliding (1–9), where pulses are also slow and squeaking is typically absent. Although squeaking at soft-rigid frictional interfaces has been linked to stick-slip oscillations (48–52), a comprehensive understanding of the underlying mechanisms remains lacking.

In this study, we experimentally investigate soft-rigid interfaces subjected to sliding velocities that produce squeaking. Through high-speed imaging and acoustic analysis, we demonstrate that opening pulses propagate at approximately the shear wave speed of the soft material and mediate local slip across a broad range of materials. In flat samples, these pulses are irregular, generating broadband acoustic

emissions. In contrast, when thin ridges are introduced on the surface of the soft material, they act as waveguides, leading to a consistent repetition frequency that corresponds to the first shear mode of the sliding block, and ultimately resulting in squeaking at that characteristic frequency. These findings uncover a structure-driven mechanism that stabilizes rupture in bimaterial friction. By suppressing competing modes, geometric confinement enforces a dimensional reduction, transforming disordered two-dimensional dynamics into coherent trains of one-dimensional pulses. This structural stabilization provides novel insights into frictional rupture processes, from engineered surfaces to geological faults. "

and

" More than 50 years ago, Schallamach observed that sliding between rubber and hard materials could occur either as uniform frictional sliding or as propagating waves of detachment, now referred to as Schallamach waves (2). These waves fully open the interface, are nucleated by a surface buckling instability, and propagate at speeds significantly slower than the shear wave speed (1, 3–12). However, contrary to common misconceptions, Schallamach waves are not responsible for squeaking, as their frequencies are too low.

At the same time, motivated largely by studies of earthquake dynamics, researchers began to numerically and theoretically investigate rapid slip pulses generated during sliding along bimaterial interfaces (18). For one-dimensional interfaces, it was proposed that the bimaterial mismatch can lead to the occurrence of slip pulses propagating at approximately the shear wave speed of the softer material regardless of friction law (14, 46, 47, 53). These predictions have since been experimentally confirmed (20, 43, 54).

A soft solid sliding against a rigid substrate represents the limiting case of a bimaterial interface. For such systems, numerical simulations of one-dimensional interfaces have predicted that intersonic slip pulses can evolve into opening pulses, leading to complete local separation of the interface and enabling localized slip (55) – a behavior later confirmed in preliminary experiments (31). Although these rapid opening pulses may resemble Schallamach waves, they are fundamentally different. Schallamach waves are overdamped opening pulses dominated by adhesion and viscous effects (4, 7, 9), whereas rapid opening pulses are governed by elasticity and inertia (55), thus, representing a candidate mechanism for the generation of sound. "

and finally,

"In summary, our experimental investigation shows that opening pulses, which mediate local slip, propagate at approximately sonic speeds along soft–rigid frictional interfaces across a variety of materials. Flat two-dimensional interfaces exhibit chaotic behavior as the pulses are incoherently initiated, and interact in complex ways. By introducing ridges as waveguides, a single mode is selected, leading to synchronized pulse propagation across them—a behavior reminiscent of 1D systems but emergent in a 2D context. This finding reveals a fundamental principle: structural features can dictate mode selection and synchronization in frictional systems, bridging the gap between 1D and 2D dynamics.

While this study primarily focused on straight ridges and sliding aligned with the ridge direction, squeaking could be further controlled by altering the sliding orientation or incorporating more intricate surface patterns (see Supplementary Information Figs. S14, S16 and Video S6). Although our focus was acoustic emission, the data suggest that these pulses also reduce frictional force (see Supplementary Information Fig. S13), pointing to new strategies for friction modulation through geometry. Finally, the solitary-like nature of the pulses and the possible role of backward-propagating opening waves (see Video S5) raise new questions about the fundamental dynamics of elastodynamic friction. Together, these findings provide a framework for designing tunable frictional interfaces and call for a re-examination of long-standing one-dimensional rupture models. Further, they open new directions for controlling bimaterial friction in engineered systems and offer new insights into earthquake dynamics."

Our results collectively challenge the referee's assertion that we learn "nothing new about friction." By elucidating the role of 2D geometry and structural guidance in mode selection, and by documenting super-fast pulses in a novel material context, our work advances the field beyond the current focus on 1D or slow 2D systems. We believe these contributions are of sufficient generality and significance to merit publication in *Nature*.

3. The relevance and interest of squeaking. The referee claims that understanding squeaking is merely a technical exercise without broader scientific interest. We respectfully disagree. Squeaking is a pervasive phenomenon, widely recognized and intuitively intriguing to both scientific communities and the public alike, as underscored by recent popular science coverage (e.g., NYT, 2017 (56)). Our work clarifies the longstanding mystery of squeaking by revealing its underlying frictional mechanism. Contrary to previous, widely disseminated misconceptions (such as those described in the aforementioned NYT article), our experiments rigorously demonstrate that squeaking emerges

from rapid opening slip pulses whose frequency is dictated by the fundamental shear resonance of the soft sliding object. This explanation provides not only a scientifically robust answer to a commonly encountered yet poorly understood phenomenon but also showcases the intricate interplay between geometry and friction dynamics. We acknowledge that the referee may not share our view on the intrinsic interest of squeaking. However, the combination of its practical relevance, public fascination, and scientific depth, suggests it is a worthy topic for Nature's readership.

In summary, we firmly believe our manuscript presents clear, significant advances that deepen and broaden our understanding of friction dynamics and the origin of squeaking. The new mechanisms, generality, and implications of our results firmly justify publication in Nature. We appreciate the referee's thoughtful critique and hope that, in light of the new results and clarifications, they may view the contribution in a different light. We are grateful for their engagement and for the opportunity to further improve the clarity and impact of our work.

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Response to the Referees

Squeaking at soft–rigid frictional interfaces

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Response to Referee #2

I thank the authors for clarifying the scope of their observations and the impact of their findings on public interest. In response to the referee's comments, the authors generalized the results to other soft-on-hard systems with different materials and discussed the relevance of their study for a broad audience.

Regarding novelty, the authors argue that they contribute new fundamental knowledge about friction and raise the question of transitioning from a 1D to a 2D system, which may have been overlooked in the literature. Most of the literature on frictional ruptures relates to fault mechanics. In this case, however, soft-on-hard systems driven at high velocities are not relevant. Previous studies have focused on hard solids driven at low speeds, including homogeneous and bimaterial systems. I don't believe the transition from 1D to 2D observed in this study is relevant to traditional frictional systems studied within the framework of fault mechanics. For example, Latour et al. (1) drive a soft solid on a hard substrate and observe single ruptures crossing the entire interface instead of a train of ruptures, as in the present study.

We respectfully note that the referee's description of Latour et al. (1) is incomplete and inaccurate. Both our study and Latour et al. involve soft–hard interfaces, but the systems are fundamentally different. Latour's gels are driven at millimeters per second; our experiments probe sliding at meters per second. Their interface is disordered— involving high roughness (sandpaper) and granular particles— and therefore the resulting rupture dynamics is not comparable. In our work we observe detachment pulses, sometimes single, sometimes in trains, whose behavior and scaling we explain, and whose confinement produces mode-locking that leads to coherent squeaking. Latour et al., in contrast, report a broad range of rupture behaviors—slow and fast cracks, partial ruptures, and distributed depinning—because they do not impose a geometry that forces the system to pick a mode. Their own results show that only some ruptures span the interface; many arrest partway, and multiple fronts often appear within a single stick–slip cycle. On sandpaper, sliding proceeds through numerous localized depinning events. Rather than a single, system-spanning rupture, Latour present a disordered phenomenology that actually reinforces our central point: geometric confinement is what reduces disordered 2D dynamics into coherent 1D trains. More generally, while our experiments differ from traditional laboratory faults, our results highlight how geometry can control rupture mode selection, a principle that is broadly relevant to frictional rupture physics, including geophysical settings where confinement or heterogeneity will enforce similar constraints.

Comment 2A

At each stage of the review process, I asked the authors a question that they never responded to: why does the natural frequency of the solid influence the nucleation frequency of the pulses? This is the central mechanism of the problem, yet it is not addressed in the manuscript; it is only mentioned as an observation. Sound is emitted due to (i) the nucleation of slip pulses and (ii) their propagation over a long distance. Although we understand that ridges allow propagation by acting as a waveguide, the question of why nucleation is controlled by the proper frequency of the solid remains. For this reason, I consider the study to be too specific.

We thank the reviewer for highlighting this central question once again. To address it, we carefully analyzed the experimental data, conducted eigenfrequency analyses of the deformed blocks, and performed elastodynamic frictional finite element simulations. Together, these experimental observations and numerical results demonstrate that trailing-edge oscillations govern the timing of opening-pulse nucleation when one-dimensional waveguides are formed by the ridges.

To highlight these new observations, we have revised the text on page 3 of the main manuscript to read as follows

"We first focus on the evolution of contact along the horizontal lines and find that for both samples [the trailing edge lifts and oscillates locally, periodically modulating contact pressure \(see Supplementary Information Section S4.1 for details\)](#), creating pressure weakening regions and facilitating the initiation of opening pulses at the rear which propagate in the direction of sliding at a typical speed of $c_{\text{pulse}} = 24 \pm 1$ m/s (Figs. 2E and F). Note that, in full agreement with analytical predictions (2), c_{pulse} closely matches the independently measured speed for shear waves propagating in the bulk of the considered rubber, $c_s = 22 \pm 1$ m/s (see Supplementary Information Section S2.1

for details). In the case of the flat sample, these pulses appear heterogeneously distributed in time and space. In contrast, for the sample with ridges, the pulse separation $\Delta t \approx 1.65 \pm 0.02$ ms coincides with the period of the trailing-edge oscillations and yields a repetition frequency of $f_0 \approx 606 \pm 7$ Hz, which we find to be minimally affected by normal stress (see Supplementary Information Fig. S16). Consequently, no distinct peak is evident in the spectrum of I_g at a representative position X for the flat sample (Fig. 2G), whereas a clear fundamental frequency $f_0 = 600 \pm 8$ Hz, accompanied by its harmonics, is observed for the sample with ridges (Fig. 2H). Importantly, the spectrum of I_g at X aligns closely with the sound captured during the experiments for both samples, confirming the intrinsic connection between squeaking and the propagation of opening pulses. "

We have also updated Figure 4 (reproduced here as Fig. R1 for completeness) and added the following paragraph on page 4

"To gain deeper insight into the mechanism governing the frequency of pulse nucleation in samples with interfacial ridges, we begin by monitoring the frequency of the trailing-edge oscillations, f_{flap} . As shown in Fig. R1D, we find that in samples with ridges, for all considered values of H , the opening pulses occur at the same frequency as the trailing-edge oscillations (i.e., $f_{\text{pulse}} = f_{\text{flap}}$). This synchronization is observed also in elastodynamic frictional finite-element simulations (solid markers in Fig. R1D - see Supporting Information, Sections S4-S6 for details). Next, we compute the natural frequencies of the preloaded and sheared block with the trailing edge free to lift using the finite element method (see Supporting Information, Sections S4-S6 for details). The first eigenmode is a shear mode whose frequency scales as predicted by Eq. (1) (see Fig. R1E). This mode produces a periodic modulation of the contact pressure localized at the trailing edge, thereby setting the tempo for the nucleation of opening pulses. Finally, samples with flat frictional interfaces also exhibit self-excited trailing-edge flapping during sliding; however, once opening pulses begin to nucleate, these oscillations lose coherence. In such unconfined geometries, the feedback between the propagating pulses and the bulk motion fails to lock the dynamics into a single shear mode and instead excites multiple bulk modes."

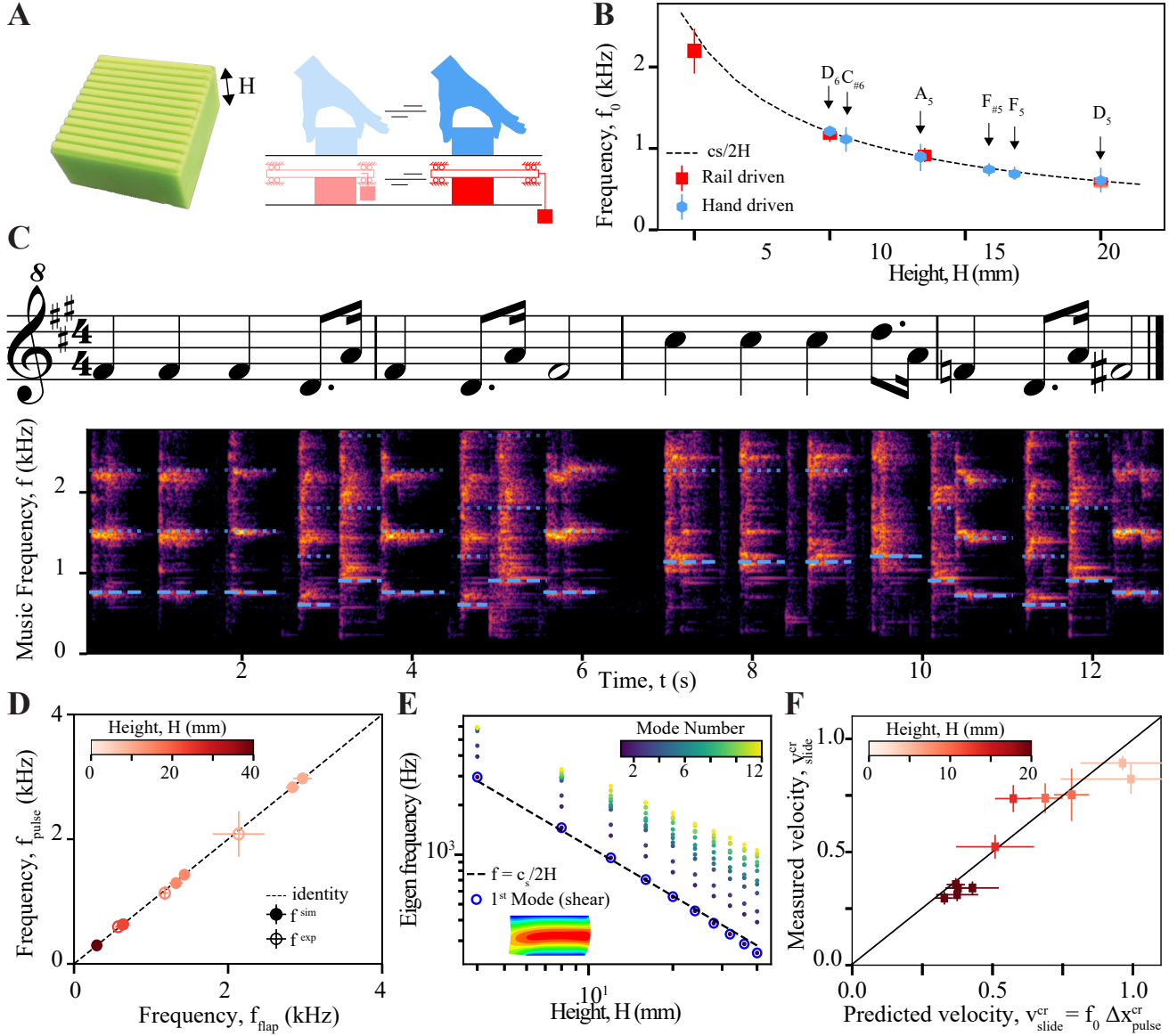


Fig. R1. The height of the samples controls the frequency of squeaking. (A) Cuboid samples of height, H with sliding surface featuring an array of parallel ridges are slid on a PMMA plate by hand (blue) or via a rail driven setup (red). (B) Fundamental frequency of squeaking sound, f_0 , as function of H . The measured frequency follows the scaling prediction $f_0 = c_s/2H$. Measured fundamental frequency for blocks of height $H = 20$ mm and length $L \in [40, 6080]$ mm show that block's length has no influence on squeaking frequency. By varying H , we can design six blocks, each producing a different musical note. (C) Partial transcription of the music score of The Imperial March composed by John Williams for the Star Wars saga (top) with corresponding spectrogram (bottom), played by moving blocks of the corresponding musical notes over rigid substrates (see Video S4). Dashed blue lines represent the target frequency for each note. (D) Pulse frequency (f_{pulse}) vs. trailing-edge oscillation frequency (f_{flap}). Data from both experiments (f^{exp}) and elastodynamic frictional finite element simulations (f^{sim}) are reported. (E) Numerically predicted eigen-frequencies for the preloaded block as a function of the block height H . The first eigenmode, identified as a shear mode (blue circles and inset), scales as $f = c_s/2H$ (dashed line). (F) Measured critical sliding velocity $v_{\text{slide}}^{\text{cr}}$ as a function of $f_0 \Delta x_{\text{pulse}}^{\text{cr}}$, where $\Delta x_{\text{pulse}}^{\text{cr}}$ denotes the experimentally determined slip per pulse near $v_{\text{slide}}^{\text{cr}}$. Marker colors indicate the value of H , and error bars represent experimental uncertainty. The black line shows the predicted scaling according to Eq. (2).

Finally, we have added the following Section to Supporting Information

Frequency Selection Mechanism for Pulse Nucleation To gain deeper insight into the mechanism responsible for the regular-frequency nucleation of pulses in samples with interfacial ridges, we first examine a patterned specimen with dimensions $L \times W \times H = 40 \times 40 \times 5$ mm, subjected to a normal load of 35 kPa (Fig. R2). Sliding causes the trailing edge to lift and oscillate locally, periodically modulating contact pressure and momentarily detaching from the substrate (Figs. R2B and C). These oscillations have a period of approximately $T \approx 0.5$ ms, corresponding to a well-defined flapping frequency of $f_{\text{flap}} = 2100 \pm 100$ Hz (Fig. R2D). Importantly, the trailing-edge oscillations induce the nucleation of opening pulses that initiate at the rear and propagate toward the leading edge (Figs. R2B and C). Initially, these opening pulses are generated irregularly and reinforce the trailing-edge flapping upon reflecting from the leading edge. However, for $t > 0.13$ s, the system attains a steady state in which each oscillation of the trailing edge gives rise to a new opening pulse (Figs. R2E and F). Consequently, the opening pulses occur at the same frequency as the trailing-edge oscillations, $f_{\text{pulse}} = f_{\text{flap}} = 2080 \pm 100$ Hz. This synchronization demonstrates that periodic contact-pressure modulation by the trailing-edge oscillation governs the tempo of pulse nucleation.

The mechanism identified in Fig. R2 is robust across different sample geometries. In patterned specimens with identical in-plane dimensions ($L \times W = 40 \times 40$ mm) but varying heights ($H = 10$ and 20 mm), the trailing edge consistently begins to oscillate regularly, eventually nucleating irregularly opening pulses. These opening pulses later emerge regularly at the same frequency as the trailing-edge oscillations (Figs. R3A–B). Interestingly, for the sample containing a single ridge of width 3 mm, the trailing edge initially oscillates spontaneously for some time without nucleating any opening pulses (Fig. R3C). At $t = 0.17$ s, opening pulses begin to be generated periodically at the same frequency as the trailing-edge oscillation ($f \approx 620$ Hz), with no observable transient. This transition is accompanied by an increase in the amplitude of the trailing-edge oscillations. These observations demonstrate that the trailing-edge oscillations themselves govern the timing of opening pulse generation, provided the frictional interface behaves as a one-dimensional waveguide.

Taken together, these experimental observations demonstrate that trailing-edge flapping dictates the timing of opening-pulse nucleation when one-dimensional waveguides formed by the ridges are present. To uncover the origin of this flapping frequency, we computed the natural frequencies of the deformed block using the Finite Element Method. Specifically, in the simulations we considered two-dimensional models featuring a one-dimensional interface. The models were first subjected to compression and subsequently to shear, while a portion of the rear interface was left free to lift. The natural frequencies were then calculated (Fig. R4B; see Section S6 for details of the numerical procedure). The analysis reveals that the first eigenmode is a shear mode whose frequency scales as $f \propto c_s/2H$ ($c_s = \sqrt{G/\rho}$ being the shear wave velocity of the material), consistent with the frequency of pulse generation reported in Fig. 4 of the main text.

We also note that the samples with a flat frictional interface exhibit trailing-edge flapping during sliding (Figs. S19, S25). However, these oscillations lose coherence once opening pulses begin to nucleate. In this case, the feedback between the propagating pulses and the bulk motion does not lock the system dynamics into a single mode, as in the case of patterned frictional interfaces, but instead excites multiple bulk modes in the sample, preventing sustained synchronization of the trailing-edge oscillations.

Elastodynamic frictional simulations . The results shown in Figs. R2–R4 indicate that periodic modulation of the contact pressure by trailing-edge flapping promotes the nucleation of opening pulses at the flapping frequency, which closely matches the fundamental shear mode of the block. To further validate this observation, we performed finite element simulations incorporating frictional contact (see Section S6 for details of the numerical procedure). In these simulations, we model two-dimensional viscoelastic blocks with a one-dimensional frictional interface governed by Coulomb friction. The block is initially compressed and subsequently sheared by imposing a velocity boundary condition on its upper surface.

Simulation results for a block of height $H = 0.02$ m are shown in Fig. R5. They confirm the experimental observations and reveal an initial transient phase followed by a steady-state characterized by the propagation of slip-opening pulses synchronized with the macroscopic “flapping” motion of the trailing edge. Specifically, the trailing edge first loses contact at $t = 1.5$ ms. As the applied shear increases due to the velocity boundary condition, the noncontact region expands to $x \approx 6$ mm (Figs. R5B–C). At $t = 4$ ms, the first opening pulse nucleates and propagates along the entire frictional interface. The pulse produces local interfacial opening (Fig. R5B) accompanied by high local slip velocity (Fig. R5C). Then, at $t \approx 10$ ms, the system reaches a steady state characterized by a macroscopic flapping motion of the trailing edge (Fig. R5A). Consequently, the front of the noncontact region oscillates periodically between $x \approx 0$ and $x \approx 10$ mm (Figs. R5B–C). The period of these oscillations is $T = 1.59 \pm 0.01$ ms (Fig. R5D) and corresponds to a frequency of $f_{\text{flap}} = 629 \pm 5$ Hz (Fig. R5E), which closely matches the experimentally measured frequency of 620 Hz for a block with the same dimensions. Interestingly, we find that each oscillation of the trailing edge initiates a packet of opening pulses that propagate along the interface at approximately the shear-wave speed, $c_{\text{pulse}} \approx 1.2 c_s$ (Figs. R5B–C). In full agreement with our experimental observations, also in simulations we find that $f_{\text{pulse}} \approx f_{\text{flap}} = 629$ Hz (Figs. R5D and E). This frequency is approximately 11% higher than the analytically

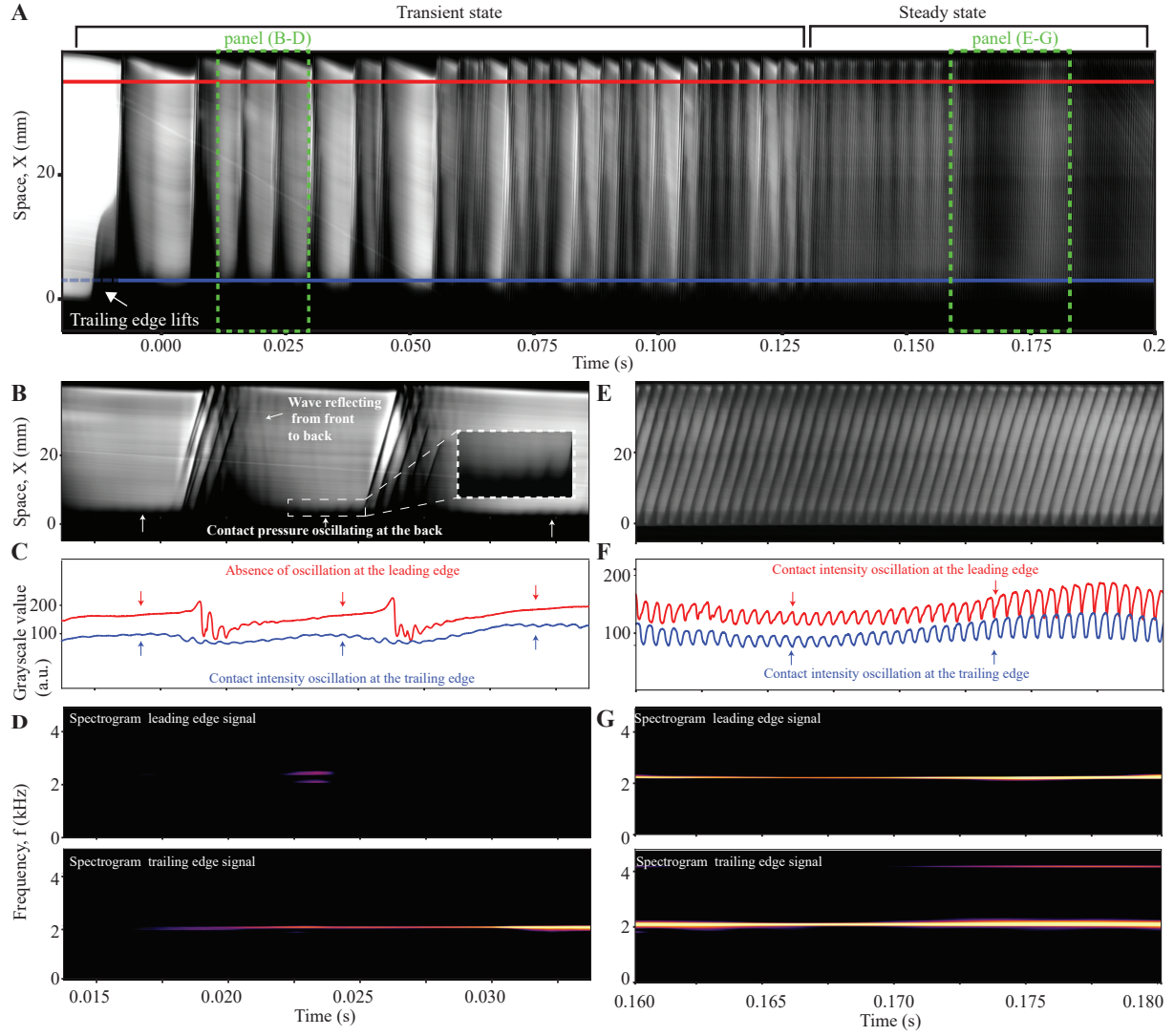


Fig. R2. Trailing-edge oscillations and opening pulses. Typical experiment on a patterned sample of dimensions $L \times W \times H = 40 \times 40 \times 5$ mm under a normal load of 35 kPa. **(A)** Space-time diagram showing that as sliding begins, the trailing edge lift and oscillate locally, periodically modulating contact pressure and momentarily detaching from the substrate. These trailing-edge oscillations induce the nucleation of opening pulses that initiate at the rear and propagate toward the leading edge. **(B)** Zoomed-In space-time diagram ($t \in [0.015, 0.033]$ s). In this time interval the opening pulses are generated irregularly and reinforce the trailing-edge flapping upon reflecting from the leading edge. **(C)** Evolution of the grayscale intensity extracted at location defined in (A): These signals are a proxy for the contact pressure. We observe the presence of oscillations at the trailing edge (blue) with a period $\Delta t \approx 0.5$ ms and their absence at the leading edge (red), beyond the opening pulses signature. **(D)** Spectrograms of the intensity signals in (C): Trailing edge spectrogram shows a well-defined frequency $f = 2083 \pm 100$ Hz which corresponds to the contact pressure oscillation. Leading edge spectrogram shows no dominant frequency. **(E)** Zoomed-In space-time diagram ($t \in [0.16, 0.18]$ s). In this time interval the system attains a steady state in which each oscillation of the trailing edge nucleates an opening pulse. **(F)** Evolution of the grayscale intensity extracted at location defined in (A). We observe the presence of oscillations at the trailing and leading edges (blue and red respectively) with a period $\Delta t \approx 0.5$ ms beyond the opening pulses signature. **(G)** Spectrograms of the intensity signals in (F) showing a well-defined frequency $f = 2137 \pm 100$ Hz for both the trailing edge and leading edge signals, similar to flapping frequency measured at the trailing edge of the sample in the absence of opening pulses.

predicted shear eigenfrequency (Eq. S19), $f_{theory} = 565$ Hz, calculated using the instantaneous Young's modulus E_0 and shear-wave speed $c_s = 22.6$ m/s. The moderately higher frequency observed in the simulations is attributed to the contact boundary condition, which effectively introduces a stiffening effect.

To validate the experimentally observed frequency scaling, $f \propto c_s/2H$, we varied the block height in the elastodynamic frictional simulations over the range $H = 0.005\text{--}0.04$ m, while keeping the length fixed at $L = 0.04$ m. As shown in Fig. R6, the simulations exhibit behavior analogous to that reported in Fig. R5 for $H = 0.02$ m. Importantly, these results confirm the experimentally observed scaling relationship $f \propto c_s/2H$ (see Fig. R7).

In summary, our experiments, linear eigenmode analysis, and frictional finite element simulations collectively demonstrate that periodic modulation due to trailing-edge flapping promotes the nucleation of opening pulses at the flapping frequency, which closely matches the fundamental shear mode of the block. As shown in Fig. R7A, for all geometries, the numerically predicted and experimentally measured flapping and pulse frequencies collapse onto

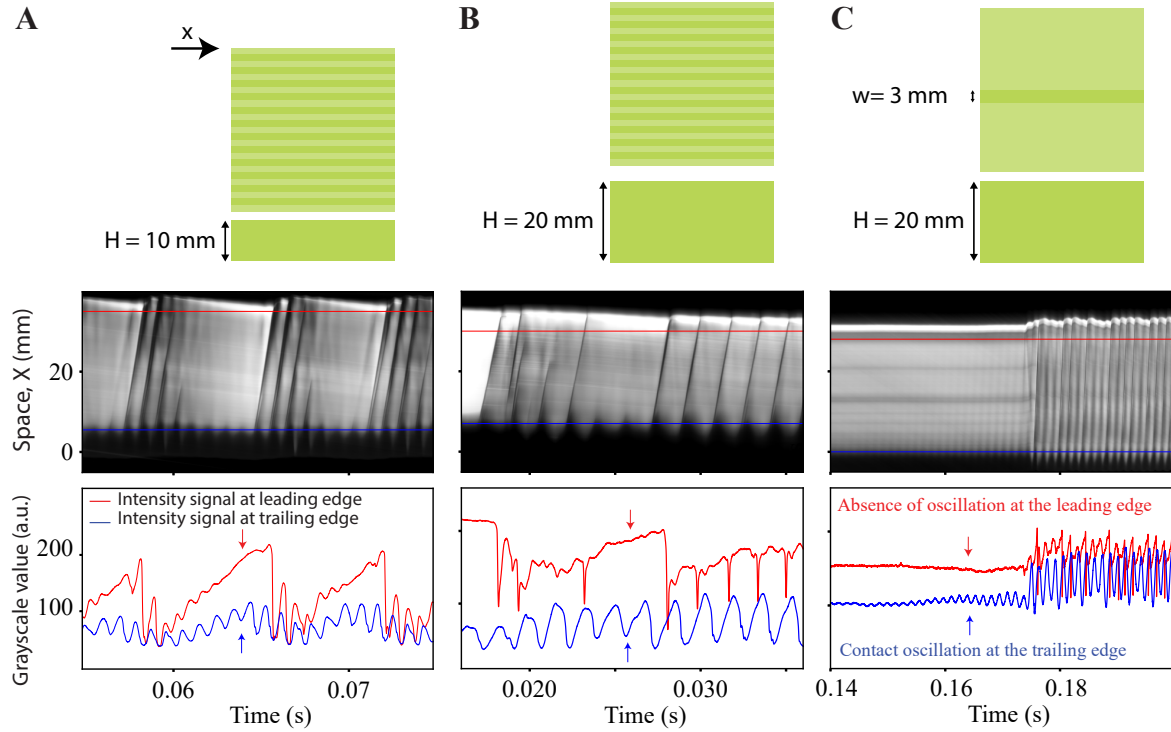


Fig. R3. Trailing-edge oscillations set the frequency of opening pulse nucleation. (A-B) Patterned samples of $L \times W = 40 \times 40$ mm with $H \in 5, 10, 20$ under a normal loading of 35 kPa. Their respective space-time diagrams show a coupling between opening pulses and the flapping of the trailing edge that modulates contact pressure. The contact intensity signals further demonstrate the existence of the trailing edge oscillation whose frequency corresponds to that of the opening pulses measured in steady-state. (C) Sample with a single ridge of 3 mm, under a normal loading of 35 kPa. The space-time diagram shows that as the sample slides, its lifted trailing edge starts to oscillate before the prior nucleation of opening pulses. The frequency of oscillation is measured at $f = 1/\Delta t = 1/0.0016 = 620$ Hz. Opening pulses are generated after $t > 0.175$ s at the same frequency as the flapping of the trailing edge.

a single identity line ($f_{\text{pulse}} = f_{\text{flap}}$). Furthermore, both experimental and simulated frequencies exhibit the same height-dependent scaling as the first shear eigenmode of the block ($f \propto c_s/2H$; Fig. R7B).

Finite Element simulations

Eigenfrequency analysis. To determine the natural modes of the preloaded block, we performed two-dimensional plane-strain finite-element simulations in Abaqus/Standard using a custom Python script (code used for all simulations is available at https://github.com/gabrielealbertini/elastodynamics_friction_squeaking.git)

The model consists of a rectangular block of length $L = 40$ mm and height H , discretized with quadratic quadrilateral hybrid elements (Abaqus element type: CPE8H) of characteristic size $H/50$. The material was modeled as a nearly incompressible Neo-Hookean solid with $E = 1.6$ MPa and density $\rho = 1060$ kg m $^{-3}$.

A vertical partition was introduced at $x_p = H/2$ to define two bottom regions: a *free* left segment ($0 \leq x < x_p$) and a *sticking* right segment ($x_p \leq x \leq L$). All static steps were solved with geometric nonlinearity enabled (`nlgeom=ON`) to retain the finite prestrain and contact-lifting effects prior to the eigenvalue analysis.

The simulation sequence comprised three steps:

1. **Normal preload:** A uniform pressure $P = 10$ kPa was applied on the top surface ($y = H$), while the bottom boundary ($y = 0$) was constrained in the normal direction only ($u_2 = 0$) and free tangentially. The top edge was free to move tangentially.
2. **Shear preload:** The normal roller condition on the bottom was released. The right portion of the bottom ($x_p \leq x \leq L$, $y = 0$) was clamped ($u_1 = u_2 = 0$), while the left portion ($0 \leq x < x_p$) was traction-free, allowing local lift-off. The top boundary was displaced laterally by $\Delta X = H/10$ under the same normal pressure P , generating a preloaded sheared state.
3. **Eigenmode analysis:** A Lanczos eigensolver was used to compute the small-amplitude vibration modes about the deformed configuration obtained in Step 2. Twelve eigenpairs were extracted.

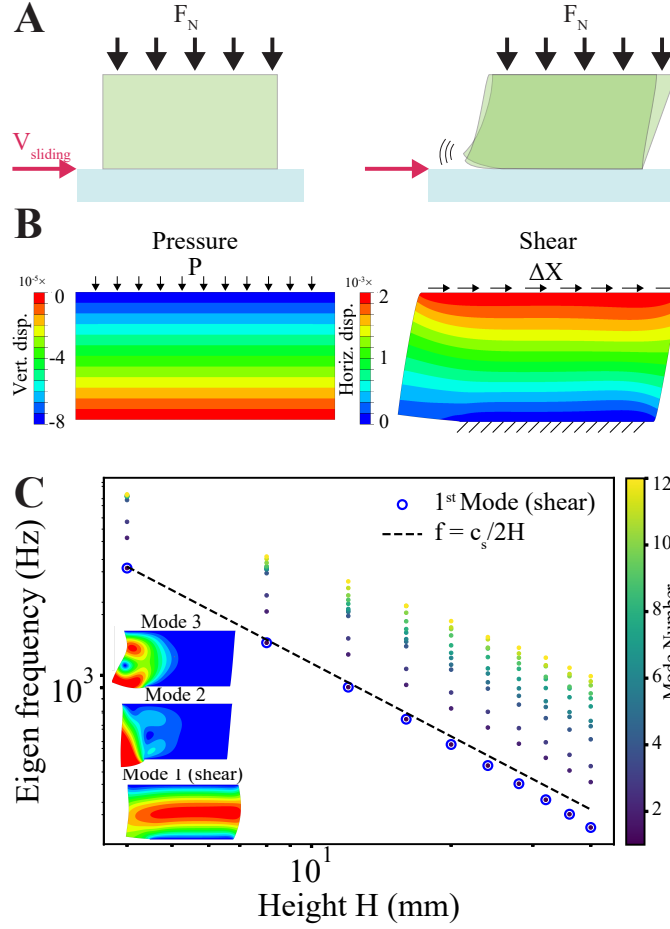


Fig. R4. Eigenfrequency analysis. (A) Illustration of sample lifting and flapping. (B) 2D block simulation comprises three steps: normal loading, then shearing, then finally linear perturbation (see Section S6 for details). (C) The first mode corresponds to the first shear mode for all considered models and the associated frequency scales closely as $f \propto 24/2H$.

The first eigenmode was consistently a shear mode localized near the trailing edge ($x \approx 0$), with frequency scaling as

$$f_1 \simeq \frac{c_s}{2H}, \quad c_s = \sqrt{G/\rho},$$

matching the experimentally measured flapping and opening-pulse frequencies. This confirms that the trailing-edge oscillation observed experimentally corresponds to the fundamental shear resonance of the block.

Elastodynamic simulations. To investigate the frequency selection mechanism for pulse nucleation, we perform Finite Element simulations on a 2D plane-strain model with a 1D frictional interface, as our experimental results indicate that, in the presence of ridges, the frictional interface effectively behaves as one-dimensional. In our simulations, we consider an elastomer block of length $L = 0.04$ m and height $H = 0.005 - 0.04$ m in frictional contact along the x -axis with a rigid substrate. The models are discretized using a uniform structured mesh of bilinear quadrilateral elements, each with an edge length of $H/400$ and four quadrature points.

The response of the elastomeric block is modeled as that of a viscoelastic solid. Specifically, we employ the *Standard Linear Solid* (SLS) model, which represents the material behavior using a spring in parallel with a Maxwell element (a spring and dashpot connected in series). For the Maxwell element, we set the viscosity to $\eta = 64$ Pa $\cdot$ s and the stiffness to $E_V = 0.64$ MPa, while the elastic spring in parallel has a stiffness of $E_\infty = 0.96$ MPa. At long times ($t \rightarrow \infty$), the material response approaches the equilibrium modulus E_∞ . Conversely, the instantaneous modulus is $E_0 = E_\infty + E_V$. With these parameters, the resulting shear wave speed is

$$c_s = \sqrt{\frac{E_0}{\rho 2(1+\nu)}} = 22.6 \text{ m/s}, \quad [\text{R1}]$$

when we use $\rho = 1050$ kg/m³ and $\nu = 0.45$. This value is consistent with our experimental measurements for Zhermack Elite Double 32 (see Section S2).

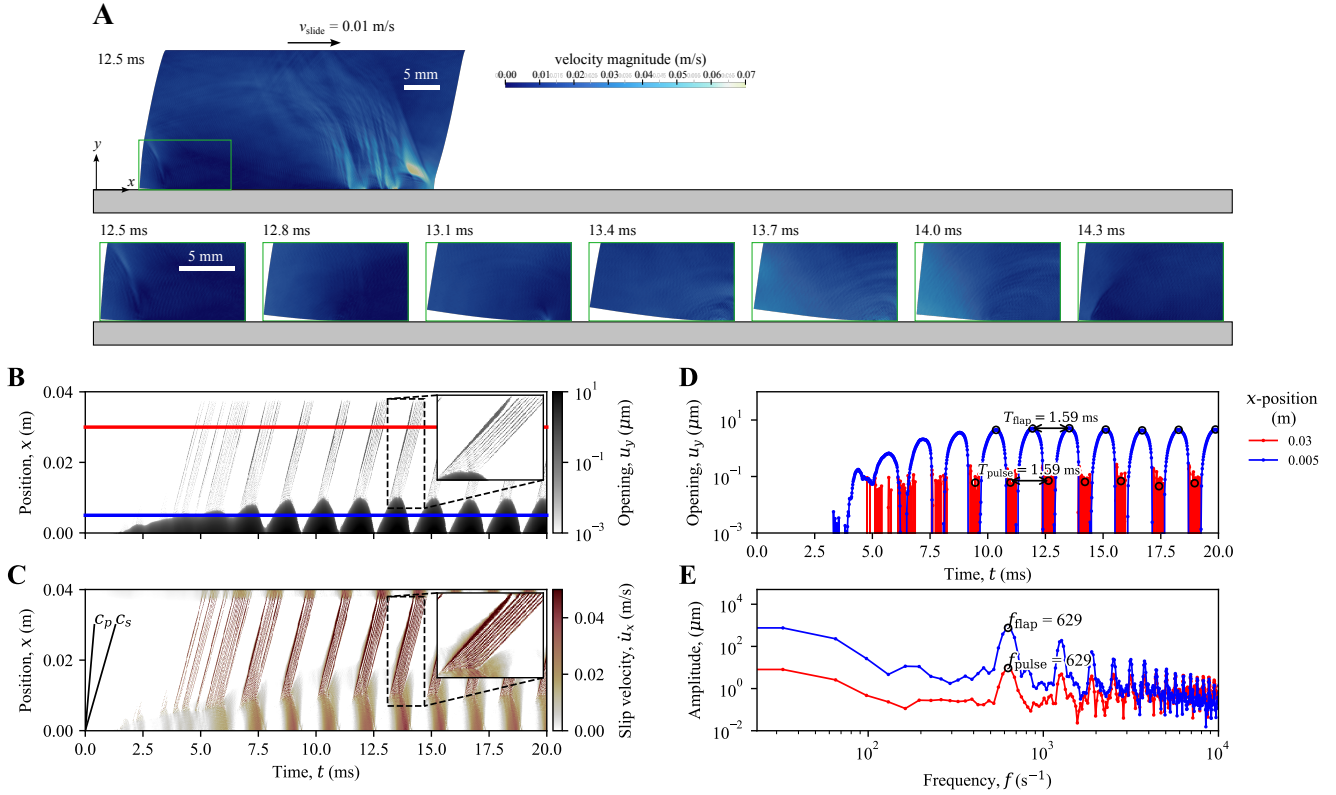


Fig. R5. Elastodynamic frictional simulation results for a block of height $H = 0.02$ m sliding at velocity $v_{slide} = 0.01$ m/s on a rigid substrate subjected to Coulomb friction. (A) Full field velocity magnitude at $t = 12.5$ ms (top), zoom-in at the trailing edge during $t \in (12.5, 14.3)$ ms (bottom). (B-C) Space-time plots of interfacial opening (B) and slip-velocity (C). (D) Temporal evolution of interfacial opening u_y at selected x -positions. Tracking local extrema of u_y at the trailing edge ($x = 0.005$ m) for $t > 9$ ms yields a flapping period $T_{flap} = 1.59 \pm 0.01$ ms. Tracking slip pulse packets yields a period $T_{pulse} = 1.59 \pm 0.03$ ms. (E) Spectrum of u_y at selected x -positions shows corresponding peaks flapping and pulse frequencies of $f_{flap} = f_{pulse} = 629 \pm 5$ Hz.

At the frictional interface, we adopt a Coulomb friction law, such that the instantaneous frictional strength τ_{fric} is given by

$$\tau_{fric} = \mu \sigma_n, \quad [R2]$$

where σ_n denotes the normal pressure at the interface and μ is the friction coefficient. In all simulations, we use $\mu = 2$.

In all simulations, the initial condition consists of a state of uniform uniaxial compressive stress $\sigma_{yy} = 600$ Pa, while all other stress components are zero $\sigma_{xx} = \sigma_{xy} = 0$. The upper boundary of the block is then driven at a velocity v_{slide} along the x -direction while being constrained in the y -direction. The imposed velocity increases linearly from rest to its final value $v_{slide} = H/2$ m/s over a ramp time of 1 ms. At the two lateral sides of the block (that is, at $x = 0$ and $x = L$) we apply a free boundary condition. Small strains are assumed, and time integration is performed with an explicit central-difference scheme in a predictor-corrector form. The time step is chosen as $\Delta t = 0.1 \Delta t_{cr}$, where $\Delta t_{cr} = \Delta x / c_p$ is the critical time step satisfying the Courant-Friedrichs-Lewy condition.

A traction-at-split-node technique is used to apply the tractions at the frictional interface, allowing independent tractions to be enforced across coincident nodes along the interface. The traction vector τ^{t+1} is computed by a forward Lagrange multiplier method (3), which uses a prediction procedure to pre-compute the slip-rate for the next time step. At frictional interfaces between elastic solids of dissimilar stiffness, $\sigma_n(x, t)$ may vary in space and time, resulting in corresponding fluctuations in τ_{fric} . This coupling can render the problem ill-posed, as studied analytically by Adams and co-workers (4–6). To make the problem well-posed, it is common practice (7–9) to apply the Prakash–Clifton regularization (10, 11), which accounts for the finite adaptation time required for frictional resistance to adjust to changes in normal stress. As such, in the simulations the effective frictional strength evolves according to (10, 11)

$$\frac{d\tau_{fric}}{dt} = \frac{1}{t^*} [\mu \sigma_n(t) - \tau_{fric}(t)] \quad [R3]$$

where t^* is the regularization timescale (in all simulations we use $t^* = 1$ ms). This treatment renders τ_{fric} a history-dependent state variable rather than an instantaneous quantity.

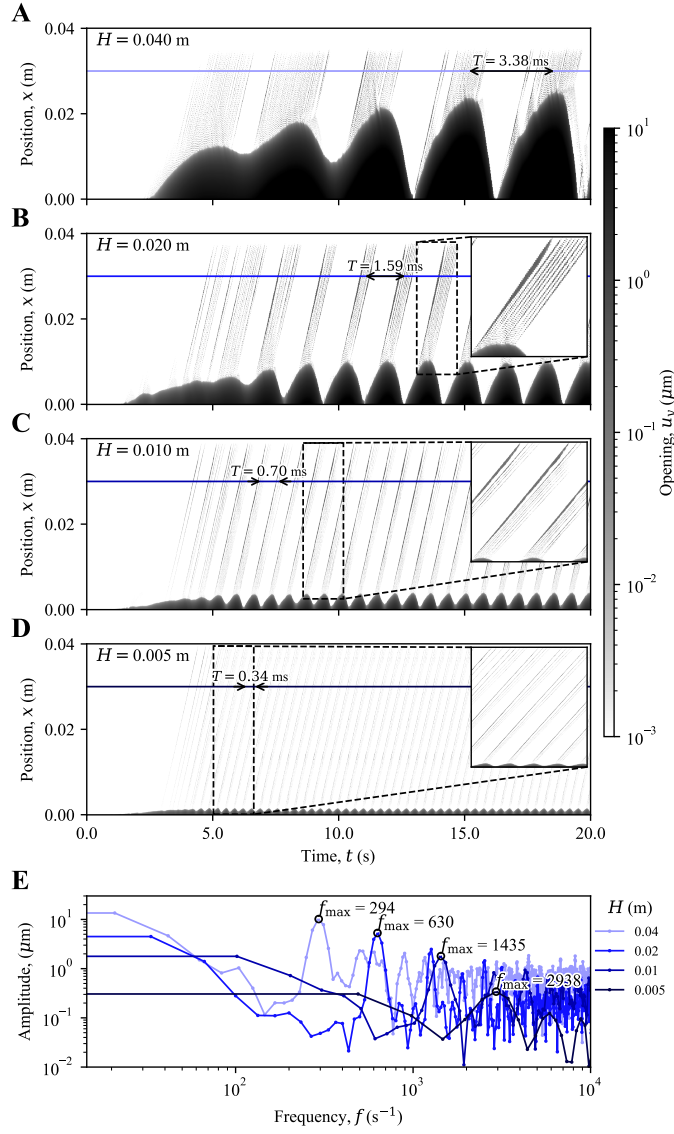


Fig. R6. Elastodynamic frictional simulations results for block of height $H = 0.005 - 0.02$ m. (A-D) Space-time plots of interfacial opening for $H = 0.005 - 0.04$ m. (E) Spectrogram of interfacial opening computed from temporal signal at $x = 0.03$ m (blue lines in panels A-D).

The Finite Element code used for all simulations is available at https://github.com/gabrielealbertini/elastodynamics_friction_squeaking.git and uses the Akantu open-source Finite Element library (<https://akantu.ch>). Results from the Finite Element simulations are shown in Section S4.

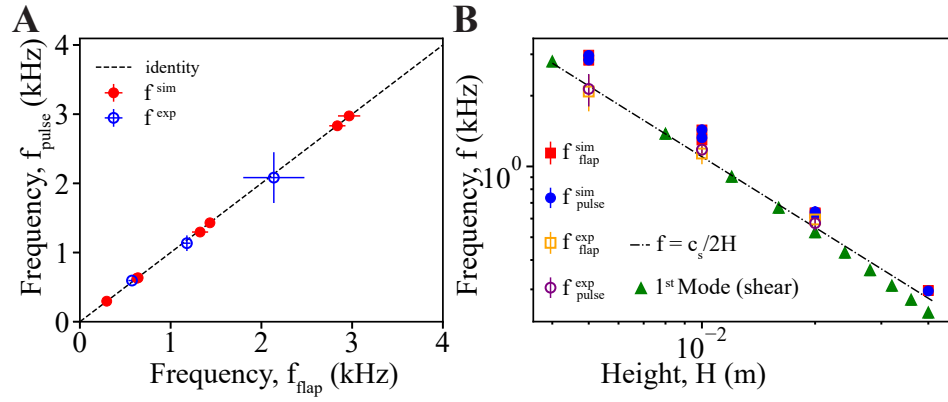


Fig. R7. Opening pulse frequency is set by the trailing edge frequency. (A) Frequency of opening pulse generation matches closely the frequency of the flapping of the trailing edge, measured both experimentally and from 2D friction simulations. (B) Frequency of flapping and frequency of pulse generation measured from the 2D friction simulations and experimentally closely match the first eigen mode computed via 2D linear perturbation analysis, which corresponds to the first shear mode. All scale closely to the proposed scaling $f = c_{\text{pulse}}/2H$.

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