

Lense-Thirring precessing magnetar engine drives a superluminous supernova

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Referee #1

(Remarks to the Author)

In this manuscript the authors discover five bumps with decreasing periods in the light curve of a super-luminous supernova (SLSN), SN 2024afav. This is a significant new discovery with important implications for revealing the central engine and energy sources of SLSNe. The manuscript presents a new extension to the magnetar-powered model, by taking into account the influence of the Lense-Thirring precessing accretion disk on the magnetar radiation and the light curve of SLSNe, and uses this model to reproduce the overall light-curve behavior and the bumps with decreasing period of SN 2024afav. The authors also consider other possible alternative models to explain SN 2024afav and find that neither the previously existing models explaining the SLSNe bumps nor the other origin models of disk precession explain the characteristic period and periodic change of the bumps in SN 2024afav.

I believe that the manuscript presents a new and important observation and provides an accompanying theoretical explanation. However, several issues with the data processing and theoretical interpretation outlined below need to be addressed before the manuscript meets the high standards for publication.

Major comments:

1) The authors identified five bumps (including five peaks and four dips, see the first paragraph of Section 2.3) in the residual plot of the magnetar-only model minus the observed data (Figure 9). I am particularly concerned about the confidence levels for five bumps (especially the fifth one) in SN 2024afav. I suggest the authors to provide the confidence level for each bump in SN 2024afav or for each dip and peak. I believe that providing the confidence level for each bump in SN 2024afav would significantly enhance the persuasiveness of the manuscript. Furthermore, I suggest the authors to provide the amplitude and duration of each bump.

2) Visual inspection shows that the first and fifth peaks in Figures 1 and 6 show little difference from the magnetar model. Furthermore, Figure 9 shows that the first bump only has a descending structure and is not successfully fitted by Equation (1). At the same time, the confidence level for the last peak in Figure 9 is not high. Therefore, it is important to demonstrate that the chirped signal derived from the five bumps in SN 2024afav is reliable. The authors can perform a power density spectrum analysis on the SN 2024afav light curve to see if there is a periodically decaying signal (chirped signal). Alternatively, they can provide other methods to demonstrate that the chirped signal found in the SN 2024afav light curve is reliable.

3) The manuscript states that the regimes of validity of the magnetar+LT model are after the diffusion time t_d and before the ejecta transitions to the nebula phase (Figure 7). However, as discussed in Hosseinzadeh et al. 2022 and Chen et al. 2023, even after the diffusion time t_d , variations originating from the central engine may be smoothed out by photon diffusion in the ejecta. Specifically, whether variations originating from the central engine after the diffusion time t_d are smoothed out by photon diffusion depends on the ratio between the duration of the bump and the instantaneous diffusion timescale. When this ratio is less than unity, variations originating from the central engine are smoothed out by photon diffusion (Chen et al. 2023). I strongly suggest that the authors discuss the relationship between the duration of each bump and the instantaneous

diffusion timescale.

4) The caption of Figure 9 indicates that the period of the chirped signal decays from $P \sim 50$ days to $P \sim 20$ days over approximately 80 days. Which 80 days after the explosion does this 80-day range correspond to? How are the values of $P \sim 50$ days and $P \sim 20$ days derived? The first paragraph of Section 3.3 and Figure 3 give the characteristic period of SN 2024afav, $P_c = 23 \pm 1.6$ days, and the period derivative, $\dot{P}_c = -0.44 \pm 0.05$. How are the values and uncertainties of P_c and $\dot{P}_c$ obtained? The caption of Figure 3 indicates that the characteristic period is defined as the average period over 80 days. However, Figure 7 shows that the valid time range for the magnetar+LT model is 37–181 days, which is greater than 80 days.

5) The manuscript does not provide a detailed description of the physical processes by which the accretion disk affects the magnetar's radiation. The authors suggest that a Lense-Thirring precessing accretion disk periodically obscures or reflects the magnetar's spin-down luminosity, but the manuscript does not clearly explain this physical process. Figure 2 only shows that the accretion disk precesses around the magnetar's rotation axis and that its radius decreases, but it does not explain how the accretion disk obscures or reflects the magnetar's spin-down luminosity. Equation (8) also fails to capture the specific process by which the accretion disk obscures or reflects the magnetar's spin-down luminosity. I suggest that the authors include a more detailed discussion of the physical processes by which a Lense-Thirring precessing accretion disk periodically obscures or reflects the magnetar's spin-down luminosity. For example, when the accretion disk obscures the magnetar's radiation along the line of sight, the amount of matter along the line of sight increases, and the diffusion timescale also increases accordingly.

6) The manuscript does not consider the influence of a Lense-Thirring precessing accretion disk on the magnetar itself. However, the accretion of matter from the surrounding accretion disk onto a magnetar may affect the evolution of the magnetar's rotation and magnetic field, thereby affecting the evolution of the magnetar's own radiation (e.g., Lin et al. 2021). I suggest that the authors include a discussion in the manuscript on whether the effects of the Lense-Thirring precessing accretion disk on the magnetar itself needs to be considered in this work.

7) As shown in Figure 10, after decomposing the LT model into envelope and precession, the precession model reproduces each peak and dip in the observed data very well. However, it can be seen that the LT model (or precession model) gives five dips in the time range of 40 to 180 days, but only the first four dips are observed.

Minor comments:

1) In the first paragraph of Section 2.3, the times for peaks and dips are reversed. The correct times should be dips at 65.3, 117.6, 139.9, and 160.4 days, and peaks at 44.9, 93.6, 128.9, 153.3, and 168.8 days.

2) I suggest that the authors add "time since explosion" to the horizontal axis of Figures 1 and 6 to make it easier for readers to compare Figures 1 and 6 with Figures 7, 9 and 10.

3) The unit should be added to the B in the corner plot in Figure 6. What is the physical meaning of sigma in Figure 6? The vertical axis title on the right side of the inset in Figure 6 should be "Absolute Magnitude + offset".

4) The reference to reference [33] in Section 1, paragraph 7 may be incorrect, as reference [33] does not provide an explanation for the bumps in SN 2017egm.

5) In the Abstract and Section 1, paragraph 3, where references [6,7] are cited, it is suggested to also cite the work of Chen et al. 2023, where they also concluded that bumps are common in the light curve of SLSNe-I.

6) As described in the third paragraph of Section 2.4.2 (see also Figure 10), the first bump is difficult to reproduce by using the magnetar + LT model. What are the possible reasons for this?

Referee #2

(Remarks to the Author)

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Too many sub, subsub, subsub, subsubsubsections! Make it simpler! Explain all acronyms, abbreviations, shorthands of facilities, observatories, methods, techniques, etc.!

Abstract

No references in the Abstract. Move them in the body of the text.

In the Abstract, after having briefly said what superluminous supernovae of type 1 are,

shorten the rest of the details about light curve and proposed explanations: they must be moved to the body of the text.

Say simply that the properties of their light curves are, at present, unexplained.

Importantly, explain in some detail how the LT precession of the disk may explain the features you observed in your light curve;

after all, the LT effect, per se, makes only the accretion disk to precess. What is the physical mechanism that would lead the LT effect to produce the observed lightcurve? Specify explicitly that the disk is intended to be tilted with respect to magnetar's equator, or, equivalently, magnetar's spin angular momentum. Otherwise, one may think that the tilt is respect to the plane of sky or some other reference plane. Just one light curve is too little: tone down and say explicitly that the conclusions are preliminary and more SLSNe1 must be studied and observed. What does it mean ($t=30$ days) in front of $\dot{M}$? Has it been inferred over a timespan 30 days long? Rephrase.

Main

What is ePESSTO+?? Make it explicit. No acronym for Global Supernova Project? Explain to readers what is MJD: I know what is, but a typical Nature's reader, say a biologist or a neuroscientist does not....

Why does the spin rate of a magnetar decrease over time? In which sense rotational energy dissipated in the spinning slowdown would go to ejecta?

Why the disk is tilted to the magnetar's spin axis? Expand and make clearer all the physical mechanisms involved by the LT effect arising to the observed luminosity curve. Why a jet from magnetar?

Are there no other physical mechanisms other than LT able to make the disk precess? The oblateness of the magnetar? A so strong magnetic field should induce a distortion in the shape of the magnetar giving it a quadrupole mass moment: in principle, it makes a tilted disk to precess.

Magnetic torques should induce a precession of the disk as well: I suspect it may be quite large.

In principle, also a distant third body. Nonstandard gravity models generally does not induce a disk precession since they impact only the pericenter

and the mean anomaly at epoch: it should be stated. In general, a section dedicated to alternative explanations expanded with respect to

the one present should be added.

To this aim, read and cite the recent book

*

L. Iorio, General Post-Newtonian Orbital Effects

From Earth's Satellites to the Galactic Centre, Cambridge University Press 2024 (Iorio CUP 2024).

*

It is relevant not only for a wealth of

modified gravity models but, especially, for the LT effect itself which was recently applied with success to the jet precession of

the M87 black hole: read and cite

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L. Iorio, Lense-Thirring effect at work in M87*, Phys. Rev. D 2025

and

*

L. Iorio, The no-hair theorems at work in M87*, MNRAS 2025

*

About the LT effect, the literature on it is exceedingly vast, but at least one or two general references about the attempts to measure it so far in other contexts should be provided. Cite

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G. Renzetti, History of the attempts to measure orbital frame-dragging with artificial satellites, Central European Journal of Physics, Volume 11, Issue 5, pp.531-544, 2013

*

L. Iorio, et al., Phenomenology of the Lense-Thirring effect in the solar system, Astrophysics and Space Science, Volume 331, Issue 2, pp. 351-395, 2011

A key point about LT and quadrupole precession, to be explicitly addressed in full details (see Iorio CUP 2024) is the following.

In general, both of them allow the orbital angular momentum of a circular orbit (the disk, in this context) to precess: this implies that not only the node but also the inclination (to the reference plane which in this case should be the plane of the sky: explicit this) do precess. Such precessions depend on the orientation of

the primary's spin axis (the magnetar's one, in this case) in the coordinate system adopted. Now, if you do not know where the spin axis is

, you cannot use the standard formulas one uses for Earth's satellites telling that only the node precesses! And, in this case, where is

the magnetar's spin axis? Are there information about it? Constraints on its orientation? Have you determined it? If not, only general

formulas must be used allowing for an arbitrary spin axis orientation implying both node and inclination precessions induced by both LT and quadrupole. So, you should estimate also three parameters: the magnitude of the magnetar's spin angular momentum, and the two polar angles determining its orientation in the coordinate system (that you must make explicit!). Most of the same points hold also for a distant companion. What if you fit with a (physically plausible) third body (I assume the disk as the orbit of a test particle)? Are there some observational hints about such a companion? These are the crucial and most important issue!! All the LT formulas used are, in principle, wrong since they use Earth's satellite-based formulas for which one knows the Earth's angular momentum direction!!

Some of the remarks already raised for the Abstract hold also in this case.

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Referee #3

(Remarks to the Author)

The authors present the discovery of a chirped modulation in the light curve of a Type I superluminous supernova (SLSNe-I) that they interpret in terms of Lense-Thirring (LT) precession of a fallback accretion disk around the spin axis of a newly born magnetar. The data are of high quality, and the modulation is clear in a way that is also consistent with modulations viewed with inferior cadence from other SLSNe-I events. The claim of Lense-Thirring precession is novel, and I believe that this result is of interest to the wider community.

I therefore believe that the decision of whether or not this paper should be published in Nature comes down to the robustness of the interpretation. In this respect, I will highlight the major positives and negatives, before giving some more detailed comments.

Positives:

It is clear that the light curves are modulated in the decaying phase of the SN, and I think I believe that the period of the modulation is robustly decreasing. The most compelling positive for me is that the magnetar spin period and magnetic field strength derived from the LT model for the oscillations and the magnetar model for the overall evolution agree so well with each other. Also the exploration, and rejection, of alternative models seems extensive to me, but the other referees may have more expertise than me on this point.

Negatives:

Although I believe that the light curve is modulated, I am not convinced by the claim in the main text that there are 5 "unambiguous" peaks. The second to fourth peaks are very clear, but the first and fifth claimed peaks are only visible after subtraction of a model that does not include a periodicity (Figs 9 and 10). I think that, for the fifth peak, the data are too noisy to claim it is unambiguous. And the first peak is too model dependent, given that it coincides with the turn over of the magnetar model.

Another negative is that the equation used for the LT precession period is appropriate for a test mass or a very thin annulus. The LT precession period of an extended disk is the surface-density-weighted average of the test mass precession period throughout the extent of the disk (e.g. equation 43 of Fragile et al 2007). I'm not sure of the disk extent in this case, but the authors argue that there is no tearing and thus that the entire disk precesses as a solid body. Precession will therefore be slower than the current model predicts, as currently the surface density is effectively assumed to be a delta-function at the disk inner radius. Moreover, this effect will also change the evolution of the precession period with time, since the fallback disk will relax outwards after forming (e.g. Fig 1 of Balbus & Mummery 2018; Fig 5.1 of Frank, King & Raine 2002). The precession frequency will therefore decrease at a slower rate than the current model assumes. Finally on this front, solid body LT precession of the entire disk only happens in the regime $H/R > \alpha$ (e.g. Section 4.2 of Ingram & Motta 2019). I suspect that the magnetar disk in this case is indeed likely to be in this regime, but I think this needs to be explicitly mentioned in the text.

Comments:

It is not clear to me if the authors perform a fit of the full magnetar + LT precession model to the data with P and B left as free parameters. At the end of 2.4.2 it is mentioned that "Incorporating estimates of the period and magnetic field from both the precession (subsubsection 2.4.1), early light curve (sub-section 2.2), and full light-curve evolution (Figure 6), we estimate $P = 4.2 \pm 0.2$ ms and $B = 1.6 \pm 0.1 \times 10^{14}$ G.". This sentence could mean that such a fit was performed, and these are the best fitting parameters, but it is ambiguous. If such a fit was performed, please make it unambiguous in the text that this was done. If such a fit was not performed, I really think it should be. This will somewhat address the issue of the peak in the magnetar model coinciding with the first candidate oscillation peak, since the fit algorithm will just optimise B and P such that the full

model best describes the full light curve. I understand the value of first treating the magnetar model and the LT oscillation model separately (and showing that they agree), but the final quoted results for P and B really should be from forward modelling that self-consistently includes both the magnetar luminosity evolution and the LT oscillations.

Throughout the manuscript, " χ^2 " is used when surely the authors mean "reduced χ^2 ". Moreover, just presenting reduced χ^2 does not convey enough information for the reader to properly evaluate the goodness of fit. For this, we need both χ^2 and the number of degrees of freedom (not just the ratio). Please therefore quote both χ^2 and the degrees of freedom.

A related point: the model shown in Fig 1 (I suppose that this is from forward fitting, which if true would make my earlier point easy to address) is quoted as having a [reduced] χ^2 of ~ 1.6 , which is claimed to be a good fit. This is not a good fit. I appreciate the difficulty of fitting a deterministic model to very complex, probably partially stochastic data, but at the very least the poor fit quality will likely mean that the parameter uncertainties are underestimated. This problem could be addressed by artificially increasing the errors on the data points by some fixed fractional value to ensure that the best fitting model has $\chi^2/\text{dof} \sim 1$.

The equations used for LT precession period include the angular momentum J, whereas the quoted model parameter is the magnetar spin period P (and assumptions for NS mass and radius are given). This means that there is a hidden assumption about the moment of inertia of the neutron star. Please specify in the text what was assumed for the moment of inertia.

A key aspect of the model is that the disk inner radius moves in with time because the equilibrium radius set by the balance of radiation pressure from the magnetar and ram pressure from the disk shifts. I think this needs to be explicitly mentioned in the main text.

For the MOSFiT fit described in Section 2.2, it is said that no burn in was used, and then the full chain was used to estimate the posteriors. This effectively assumes that the chain converged *immediately*, which surely cannot be correct.

On page 17, a diffusion correction is mentioned. I can understand the basic physics of this, but there is a lack of detail as to how this is actually implemented in the model.

Version 1:

Reviewer comments:

Referee #1

(Remarks to the Author)

The authors have thoroughly revised the manuscript, particularly for adding the confidence for each modulation, providing credible evidence for chirp in the light curves, and providing details on how the Lense-Thirring precessing accretion disk obscures and reflects radiation from the magnetar. However, my additional issues remain to be addressed before the manuscript meets the requirements for publication.

Referee #2

(Remarks to the Author)

I must commend the authors for the remarkable changes introduced to the manuscript, which has been notably improved being now ready to be accepted for publication.

Referee #3

(Remarks to the Author)

I thank the authors for so thoroughly addressing my comments and those of the other referees. I have only a few very minor remaining comments.

First, the χ^2 values. I thank the authors for clarifying that a modelling systematic is used for the purposes of extracting parameter uncertainties, and for adding the degrees of freedom to the text. I am still, however, uncomfortable with the authors claiming that a model with a reduced chi squared of 1.5 is providing an "accurate description" of, or an "excellent fit to" the data. Two such examples are:

"We find our model accurately reproduces the behavior of the transient during both the rise and the modulations (reduced $\chi^2 \approx 1.5$ for the whole fit,"

"and find an excellent fit to the data, with reduced $\chi^2 \sim 1.5$ for the whole fit"

Reduced chi squared of 1.5 is not an "excellent" fit, it is formally a terrible fit. I understand that the fit is consistent with being poor because of stochastic behaviour of the data that is not captured by the deterministic model, but it is still formally incorrect to state that the fit is excellent.

Second, I don't think it is possible for the reader (or the referees) to determine what the angles α and δ are. You say that they are two polar angles, but is this really what you mean? My best guess is that α is the angle between $\mathbf{\hat{z}}$ and $\mathbf{\hat{k}}$ (i.e. it really is a polar angle), and δ is the angle between $\mathbf{\hat{x}}$ and the projection of $\mathbf{\hat{k}}$ onto the x-y plane (i.e. it is an azimuthal angle). Please clarify.

Adam Ingram

Version 2:

Reviewer comments:

Referee #1

(Remarks to the Author)

The authors have largely addressed the comments I raised in my previous review. On the one hand, the authors cited multiple references to demonstrate that after the anisotropic energy generated by the central engine is injected into the ejecta, the light curve observed by the observer depends on the viewing angle. On the other hand, they gave their motivation for using equation (2) or equation (8). However, the manuscript still contains a few issues that need to be resolved before publication, as detailed below.

1. The authors cited the work of Zhang et al. (2025) multiple times in the manuscript, but the authors may not have correctly understood the work of Zhang et al. (2025). The precessing magnetar-powered model of Zhang et al. (2025) did not introduce a precessing jet or anisotropic energy injection. In the precessing magnetar-powered model of Zhang et al. (2025), the magnetic inclination angle undergoes quasi-periodic variations during precession, leading to quasi-periodic changes in the luminosity of the isotropic magnetic dipole radiation produced by the precessing magnetar. Therefore, in the work of Zhang et al. (2025), the energy (i.e. magnetic dipole luminosity) of the precessing magnetar is isotropically injected into the entire spherically symmetric ejecta, but the magnitude of the injected energy varies quasi-periodically.

2. In the authors' model, the modulation of SN 2024afav originates from equation (2) or equation (8). The authors cannot specify which specific physical mechanism this formula derives from, meaning that they cannot provide a first-principle derivation for this equation. The authors point out that equation (2) or equation (8) is an ansatz, noting that this ansatz has two motivations. I still have some doubts about whether these two motivations can indeed lead to this ansatz.

2.1. One motivation for equation (2) or equation (8) comes from the work of Zhang et al. (2025), but the energy injection considered in Zhang et al. (2025) is isotropic for the precessing magnetar, which is fundamentally different from the anisotropic energy injection in the magnetar-disk system analyzed here.

2.2. Another motivation for equation (2) or equation (8) is the disk occlusion model in Section 3.2, but I still have some doubts about the calculations for the disk occlusion model in Section 3.2.

2.2.1. The authors define L_{magnetar} in equation (7) as the luminosity of the magnetar along the line of sight. If so, the calculation of the second term on the right side of the equation (7), that is, the luminosity $L_{\text{reflected}}$ reflected to the line of sight, may be incorrect. This is because the luminosity L_{magnetar} of the magnetar along the line of sight is not reflected to the line of sight. In other words, $L_{\text{reflected}}$ does not depend on L_{magnetar} .

2.2.2. Why does the authors only calculate the energy injection into the ejecta along the line-of-sight direction when calculating the observer's light curve obtained from the disk occlusion model? Why can this calculation represent the results of the complete disk occlusion model? (The complete disk occlusion model involves energy injection into the ejecta not only along the line of sight but also along other directions. I understand that the authors state calculations for anisotropic energy injection are beyond the scope of this analysis, but I would like to know the reason for this treatment.)

2.2.3. Following Zhang et al. (2025), the authors substituted the injection signal obtained from equation (7) into the diffusion model of Arnett (1982) to obtain the observer's light curve. However, it should be noted that in the work of Zhang et al. (2025), the magnetar's energy is isotropically injected into the entire ejecta, thus the use of the diffusion model of Arnett (1982) is reasonable. In this analysis, however, only energy injection along the line of sight into the ejecta is considered. In this case, is the diffusion model of Arnett (1982) still applicable?

2.3. As the authors mentions, magnetar-disk systems can generate anisotropic energy injection through multiple mechanisms, and the authors cannot restrict which specific mechanism it is. Here, the authors assume that anisotropic energy injection originates from reflection/occlusion. This implies that the second motivation for equation (2) or equation (8) (i.e., the disk occlusion model) is another assumption. If another mechanism for generating anisotropic energy injection is adopted, equation (2) or (8) may not hold.

3. Although the authors point out that the envelope model is too simplistic and phenomenological, and that the physical origin of the amplitude modulation is uncertain with multiple possible interpretations, I still have doubts about the derivation of the envelope model.

The authors derived the envelope model based on equation (17). According to the physical meaning of equation (17), a

single magnetar located at the center of the ejecta (the magnetar-only model) also satisfies equation (17). However, the peak time of the magnetar-only model is $t_{\text{peak,magnetar}} \approx 50$ days (see Figure 1), while the peak time of the amplitude of the modulations obtained according to equation (17) is $t_{\text{peak,modulation}} \approx 125$ days. The inconsistency between the two requires further explanation.

4. In the left panel of Figure 5: Do the five black data points (peak/trough periods) correspond to three peaks (local maxima) and two troughs (local minima) within the range of 65 to 160 days after the explosion (see Figure 13)? How is the error for these five black data points determined?

In the right panel of Figure 5: Does blue represent the distribution of dot_P , while orange represents the distribution of the median period? How is the distribution of the median period obtained?

Does "we compute the median of the period measurements to get a characteristic period $P_c = 23 \pm 1.6$ days" refer to P_c corresponding to the third black data point in the left panel of Figure 5?

How are the medians of the linear fits obtained? For example, for a certain linear fit result (a fitted line) in the left figure of Figure 5, does the median of its period correspond to the period of the median time ($t_{\text{median}} \sim 105$ days)?

5. Are the "peaks" and "troughs" in Table 1 reversed? For example, for No. 3, $t_{\text{start}} = 80$ days and $t_{\text{end}} = 110$ days. There is a peak between 80 and 110 days (see Figure 13), so No. 3 corresponds to the peak. Therefore, in Table 1, No. 1, No. 3, No. 5, No. 7, and No. 9 are "peaks", while No. 2, No. 4, No. 6, and No. 8 are "troughs".

Referee #3

(Remarks to the Author)

I thank the authors for the further revisions that they so extensively record in their response document. I agree with their argument that the exact mechanism by which Lense-Thirring precession modulates the light curve does not need to be precisely identified in this paper for this to be an interesting result worthy of publication. I think that the changes the authors have made to emphasise this point, and to avoid leaning too heavily on one particular mechanism, have improved the paper. It seems plausible to me that an asymmetric, precessing system such as is proposed would produce a modulation of the observed flux. The ejecta from such a system would not necessarily be spherically symmetric, or sufficiently optically thick to render the escaping flux isotropic.

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Lense-Thirring precessing magnetar engine drives a superluminous supernova: *Nature* Referee Response

Joseph Farah* and Logan Prust, on behalf of the authors

October 1, 2025

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1 Introduction and summary of changes

We enthusiastically thank the referees for their thoughtful and concrete suggestions. We have revised our manuscript according to the comments provided. The primary changes to the paper and procedures consist of: (a) more physically plausible model conditions (e.g., annulus instead of ring, spin-axis agnostic model treatment, etc.); (b) significant clarifications and discussions on all questions raised by the referees; and (c) general improvements to language and precision.

Quotes from the referees are colored red, with a left vertical line of the same color; quotes from the updated manuscript are colored black, with a left vertical line and gray background. Separate referee comments (and the corresponding responses) are delineated by a horizontal line. For comments that required substantial and/or multiple repeated changes to the paper, one or more representative changes are shown. Samples from the updated draft text may have broken references (i.e., ??) to sections that can be found in the paper but are not in this document.

In [section 2](#), we address the broadest comments from the referees, which required changes to the analysis and the addition of new sections and/or figures. In [section 3](#), we address the more specific comments of the referees, which required minor to moderate changes to language or structure. In [section 4](#), we describe updates to existing figures per suggestions made by the referees. In [section 5](#), we reproduce new figures added to the paper and provide context and captions.

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2 Response to broad comments

Here, we address the broadest comments from the referees, which required changes to the analysis and the addition of new figures. To address these comments, we (i) add significant discussion on the model assumptions and physics, (ii) switch to a completely spin axis orientation-agnostic treatment following Iorio et al., and (iii) improve the model by moving to representations of an annulus and not an infinitesimally thin test particle orbit.

A key point about LT and quadrupole precession, to be explicitly addressed in full details (see Iorio CUP 2024) is the following. In general, both of them allow the orbital angular momentum of a circular orbit (the disk, in this context) to precess: this implies that not only the node but also the inclination (to the reference plane which in this case should be the plane of the sky: explicit this) do precess. Such precessions depend on the orientation of the primary's spin axis (the magnetar's one, in this case) in the coordinate system adopted. Now, if you do not know where the spin axis is, you cannot use the standard formulas one uses for Earth's satellites telling that only the node precesses! And, in this case, where is the magnetar's spin axis? Are there information about it? Constraints on its orientation? Have you determined it? If not, only general formulas must be used allowing for an arbitrary spin axis orientation implying both node and inclination precessions induced by both LT and quadrupole. So, you should estimate also three parameters: the magnitude of the magnetar's spin angular momentum, and the two polar angles determining its orientation in the coordinate system (that you must make explicit!).

The equations used for LT precession period include the angular momentum J , whereas the quoted model parameter is the magnetar spin period P (and assumptions for NS mass and radius are given). This means that there is a hidden assumption about the moment of inertia of the neutron star. Please specify in the text what was assumed for the moment of inertia.

We thank the referees for raising this important point and also for suggesting papers that present a proper treatment. To address the concern, we have moved to a fully spin axis orientation-agnostic treatment, and incorporate the quadrupole precession into our prescription. We construct our coordinate system following the citations suggested, including separate parameters for the two polar angles determining the orientation of the spin axis in the coordinate system (α and δ), and parameterize the deviation of the disk normal from the spin axis in terms of a third angle β . We present our full derivation here, but present an abbreviated version in the paper (with all citations recommended by the referee) to comply with *Nature* space constraints.

Let us begin by constructing our coordinate system. We start with the observer frame, setting $\hat{z}$ as the line of sight of the observer (with sky coordinates $\hat{x}, \hat{y}$). In the context of the orbit, we construct our unit triad $(\hat{i}, \hat{m}, \hat{h})$, following equations 2–4 of Iorio (2024b). In that formalism, we have $\hat{m} = \hat{h} \times \hat{i}$. Next, we construct our magnetar spin vector $\hat{k}$, which is constructed from two angles α and δ . Finally, the tilt between the spin axis and the disk is defined by $\cos\beta = \hat{k} \cdot \hat{h}$, since $\hat{h}$ is the unit orbital angular momentum vector, i.e., the normal of the disk.

Now that we have defined our coordinate system, we can compute the vector forms of the secular precession rates for Lense-Thirring (LT) and quadrupole (QP) precession for an arbitrary spin axis gauge $\hat{k}$. We are interested in the precessions of the inclination I (analogous to viewing angle) and the node Ω . We make two further notes. First, since we are considering orbiting fluid elements in a circularized accretion disk, the eccentricity e is by construction zero. Second, we define the following two convenience terms,

$$A = \frac{2GJ}{c^2 r^3}, \quad B = \frac{3}{2} n J_2 \left(\frac{R}{r} \right)^2 \quad (1)$$

where J is the angular momentum of the magnetar, J_2 is the magnitude of the geopotential's quadrupole deviation from spherical symmetry, R is the radius of the magnetar, $n \equiv \sqrt{GM/r}$ is the Keplerian mean motion, and r is the inner radius of the disk (we will account for the annulus structure later). We note the following subtlety: Iorio et al. constructs an eccentric orbit with semimajor axis a and therefore, instead of r in B , Iorio et al. refers to the semilatus rectum ρ . In our circularized case, $r \equiv a = \rho$. Under this construction, the rates

for the Lense-Thirring precession become ((equations I.5 and I.7),

$$\dot{\mathbf{l}}_{\text{LT}} = A(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}}), \quad \dot{\Omega}_{\text{LT}} = A \csc I(\hat{\mathbf{k}} \cdot \hat{\mathbf{m}}) \quad (2)$$

The secular precession rate for the axial quadrupole J_2 is (equations I.6 and I.8)

$$\dot{\mathbf{l}}_{J_2} = -B(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}}), \quad \dot{\Omega}_{J_2} = -B \csc I(\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}}) \quad (3)$$

We now wish to convert these precessions into a single rate. Both the Lense-Thirring and J_2 torques cause simultaneous node and inclination precession, and we wish to find the magnitude of the vector precession rate around $\hat{\mathbf{k}}$. The overall precession frequency Ω_p refers to the rotation of the normal $\vec{h}$ and therefore has evolution,

$$\dot{\hat{\mathbf{h}}} = \Omega_p \times \hat{\mathbf{h}} \quad (4)$$

In our triad $(\hat{\mathbf{i}}, \hat{\mathbf{m}}, \hat{\mathbf{h}})$ with constraint $\hat{\mathbf{m}} = \hat{\mathbf{h}} \times \hat{\mathbf{i}}$ we can expand this using equation I.4 as

$$\begin{aligned} \hat{\mathbf{h}} &= \{\sin I \sin \Omega, -\sin I \cos \Omega, \cos I\} \\ \dot{\hat{\mathbf{h}}} &= \frac{\partial \hat{\mathbf{h}}}{\partial t} = \frac{\partial I}{\partial t} \frac{\partial \hat{\mathbf{h}}}{\partial I} + \frac{\partial \Omega}{\partial t} \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \\ &= \dot{I} \frac{\partial \hat{\mathbf{h}}}{\partial I} + \dot{\Omega} \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \\ \frac{\partial \hat{\mathbf{h}}}{\partial I} &= \{\cos I \sin \Omega, -\cos I \cos \Omega, -\sin I\} \\ \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} &= \{\sin I \cos \Omega, \sin I \sin \Omega, 0\} \\ |\dot{\hat{\mathbf{h}}}|^2 &= \dot{\hat{\mathbf{h}}} \cdot \dot{\hat{\mathbf{h}}} = \left(\dot{I} \frac{\partial \hat{\mathbf{h}}}{\partial I} + \dot{\Omega} \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \right) \left(\dot{I} \frac{\partial \hat{\mathbf{h}}}{\partial I} + \dot{\Omega} \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \right) \\ &= \dot{I}^2 \left(\frac{\partial \hat{\mathbf{h}}}{\partial I} \right)^2 + \dot{\Omega}^2 \left(\frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \right)^2 + 2\dot{I}\dot{\Omega} \frac{\partial \hat{\mathbf{h}}}{\partial \Omega} \frac{\partial \hat{\mathbf{h}}}{\partial I} \\ &= \dot{I}^2 + \dot{\Omega}^2 \sin^2 I \end{aligned}$$

We will now substitute our expressions for $\dot{I}$ and $\dot{\Omega}$, beginning with the LT precession,

$$\begin{aligned} |\dot{\hat{\mathbf{h}}}|^2 &= A^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2 + A^2 \csc^2 I(\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2 \sin^2 I \\ &= A^2 [(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2 + (\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2] \\ &= A^2 \sin^2 \beta \quad \text{orthonormality; } 1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 = \sin^2 \beta \\ \implies |\dot{\hat{\mathbf{h}}}| &= A \sin \beta \end{aligned}$$

However, note now that for a unit vector $\hat{\mathbf{h}}$ rotating around a fixed axis $\hat{\mathbf{k}}$ with angular velocity $|\Omega|$, $\dot{\hat{\mathbf{h}}} = \Omega \times \hat{\mathbf{h}} \implies |\dot{\hat{\mathbf{h}}}| = |\Omega| |\hat{\mathbf{k}} \times \hat{\mathbf{h}}|$, where $|\hat{\mathbf{k}} \times \hat{\mathbf{h}}| = \sin \beta$ as defined earlier. This implies that the magnitude (or rate) of the precession

$$|\Omega_{\text{LT}}| = A = \frac{2GJ}{c^2 r^3} \quad (5)$$

We now repeat the same analysis for the quadrupole contribution J_2 . Substituting expression for $\dot{I}$ and $\dot{\Omega}$ for the quadrupole precession we find

$$\begin{aligned} |\dot{\hat{\mathbf{h}}}|^2 &= B^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 + B^2 \csc^2 I(\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 \sin^2 I \\ &= B^2 [(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 + \csc^2 I(\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 \sin^2 I] \\ &= B^2 [(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 + (\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2] \\ &= B^2 \cos^2 \beta [(\hat{\mathbf{k}} \cdot \hat{\mathbf{i}})^2 + (\hat{\mathbf{k}} \cdot \hat{\mathbf{m}})^2] \\ &= B^2 \cos^2 \beta \sin^2 \beta \quad \text{orthonormality; } 1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{h}})^2 = \sin^2 \beta \\ \implies |\dot{\hat{\mathbf{h}}}| &= B \cos \beta \sin \beta \end{aligned}$$

Using the same logic as above,

$$|\Omega_{J_2}| = B \cos\beta = \frac{3}{2} n J_2 \left(\frac{R}{r}\right)^2 \cos\beta \quad (6)$$

We combine these terms into our expression for the total precession magnitude of both the node and inclination,

$$|\Omega_p(r, \beta)| = \frac{2GJ}{c^2 r^3} + \frac{3}{2} J_2 \sqrt{\frac{GM}{r^3}} \left(\frac{R}{r}\right)^2 \cos\beta. \quad (7)$$

Note that this expression is not dependent on the α and δ polar angles; these parameters impact the amplitude of the modulations observed, but not the periodicity. We intentionally handle amplitude separately, as the amplitude of the precession due to inclination angle is degenerate with many other parameters, such as (i) mass-energy conversion fraction η , (ii) uncertainty in the assumed distance to the supernova, and (iii) spherical distribution of energy from the magnetar. We attempt a full fit varying β and J_2 . However, we were unable to place meaningful constraints on the parameters. For our system, the second (quadrupole) term is dominated by the first (Lense-Thirring) term. To validate this, we assumed unphysical upper limits for the parameters ($J_2 \sim 10^{-3}$, $\cos\beta \sim 1$; Colaiuda et al. 2008) and found that the ratio of the first term to the second term varies between 10^2 and 10^3 . This is well below the sensitivity of the data and thus none of the parameters in the second term can be effectively constrained. We add a full discussion of this investigation to the paper. Regarding the magnitude of the magnetar's spin angular momentum J : we implicitly assume $J = I\Omega = (2\pi/P_{\text{spin}})kMR^2$, with $k \approx 0.35$ per Lattimer and Prakash (2001) for our system's preferred magnetar M and R . With k reasonably determined by the M/R of the magnetar, we do not consider J to be itself a free parameter. We agree with the referee's feedback about the hidden assumption and make this assumption explicit in the text.

The phase function $\phi(t)$ is derived directly from the precession frequency magnitude (Ω_p) in the weak-field approximation, which has both a Lense-Thirring component and a quadrupole-driven component. Given that the magnetar-disk system has an unknown orientation relative to the observer, we adopt a fully general prescription, following [lorio \(2025\)](#) and [lorio \(2024b\)](#). We choose $\hat{z}$ as the line of sight of the observer (with sky coordinates $\hat{x}, \hat{y}$). In the context of the orbit, we construct our unit triad $(\hat{i}, \hat{m}, \hat{h})$, following equations 2–4 of [lorio \(2024b\)](#), and magnetar spin vector $\hat{k}$, which is constructed from two angles α and δ . Finally, the tilt between the spin axis and the disk is defined by $\cos\beta = \hat{k} \cdot \hat{h}$, since $\hat{h}$ is the unit orbital angular momentum vector, i.e., the normal of the disk. In this coordinate system, we find that the magnitude of the combined Lense-Thirring and quadrupole precession is

$$\Omega_p(r, \beta) = \Omega_{\text{LT}} + \Omega_{J_2} = \frac{2GJ}{c^2 r^3} + \frac{3}{2} J_2 \sqrt{\frac{GM}{r^3}} \left(\frac{R}{r}\right)^2 \cos\beta,$$

where J is the angular momentum of the magnetar, r is the characteristic radius of the accretion disk, R is the radius of the magnetar, and J_2 is the magnitude of the geopotential's quadrupole deviation from spherical symmetry. In theory, J_2 , J , and β are all free parameters. However, in practice, even at the theoretical upper limit of $J_2 \sim 10^{-3}$ ([Colaiuda et al., 2008](#)) and assuming $\cos\beta \sim 1$ (unlikely as the tilt of the disk relative to the spin axis is not expected to be large), the ratio $\Omega_{\text{LT}}/\Omega_{J_2} \sim 10^2\text{--}10^3$ for all system parameters. Additionally, $J = I\Omega = (2\pi/P_{\text{spin}})kMR^2$ with $k \approx 0.35$ ([Lattimer and Prakash, 2001](#)) is completely determined by the remaining free parameters (namely, M , R , P_{spin}). Thus, for simplicity, we restrict our analytic treatment to

$$\Omega_{\text{LT}} \approx \frac{2GJ}{c^2 r^3(t)},$$

We tested this simplification by performing all fits with the full expression, and found that we indeed cannot constrain β or J_2 , as the quadrupole term is indeed not sufficiently dominant to affect parameter constraints.

2.4.1. Derivation of the precession

The radius of the misaligned disk is set by the equilibrium between the radiation pressure from the magnetar-launched wind and the ram pressure of the disk ([Chashkina et al., 2019](#); [Tamilan et al., 2025](#)), which shifts inward as the accretion rate decreases, leading to the disk “infall” (??). Any misalignment between the accretion disk and the spin axis of the neutron star would induce a Lense-Thirring torque on the accretion disk (see e.g., [Mashhoon et al., 1984](#); [lorio, 2024a, 2025](#); [lorio et al., 2011](#); [Renzetti, 2013](#), for a review), causing it to precess (with increasing frequency due to the infall) around the magnetar spin axis.

Main

The manuscript does not provide a detailed description of the physical processes by which the accretion disk affects the magnetar's radiation. The authors suggest that a Lense-Thirring precessing accretion disk periodically obscures or reflects the magnetar's spin-down luminosity, but the manuscript does not clearly explain this physical process. Figure 2 only shows that the accretion disk precesses around the magnetar's rotation axis and that its radius decreases, but it does not explain how the accretion disk obscures or reflects the magnetar's spin-down luminosity. Equation (8) also fails to capture the specific process by which the accretion disk obscures or reflects the magnetar's spin-down luminosity. I suggest that the authors include a more detailed discussion of the physical processes by which a Lense-Thirring precessing accretion disk periodically obscures or reflects the magnetar's spin-down luminosity. For example, when the accretion disk obscures the magnetar's radiation along the line of sight, the amount of matter along the line of sight increases, and the diffusion timescale also increases accordingly.

We thank the multiple referees who raised this important issue; here we present a more concrete picture of the mechanism by which the magnetar's luminosity along a line of sight is altered due to disk precession. A precessing disk with misalignment angle β may periodically intersect the line of sight between the observer and magnetar. Depending on the angle between the observer and the magnetar spin axis θ_{obs} , three distinct cases are possible: there will be 0, 1, or 2 occultations per precession period. In general, observers near the spin axis of the magnetar will experience zero occultations, while one occultation per period is seen only by observers in a narrow range of θ_{obs} near $\pi/2 - \beta$. Those near the equatorial plane will experience two occultations from opposite sides of the disk. The length of the occultations relative to the disk precession period can be quantified as a function of β , θ_{obs} , and disk aspect ratio H/R :

$$\frac{T_{\text{occultation}}}{T_{\text{prec}}} = \begin{cases} 0 & \theta_{\text{obs}} < \pi/2 - \beta - \chi, \\ \frac{1}{\pi} \arccos[\cot(\beta + \chi) \cot \theta_{\text{obs}}] & \pi/2 - \beta - \chi < \theta_{\text{obs}} < \pi/2 - \beta + \chi, \\ \frac{1}{\pi} \arccos[\cot(\beta + \chi) \cot \theta_{\text{obs}}] - \frac{1}{\pi} \arccos[\cot(\beta - \chi) \cot \theta_{\text{obs}}] & \pi/2 - \beta + \chi < \theta_{\text{obs}}. \end{cases} \quad (8)$$

Here we have defined $\chi \equiv \arcsin(H/2R)$ for brevity. Additionally, in the case of 2 occultations per period, the delay between those occultations is given by

$$\frac{T_{\text{delay}}}{T_{\text{prec}}} = \frac{1}{\pi} \arccos[\cot(\beta - \chi) \cot \theta_{\text{obs}}]. \quad (9)$$

These quantities are shown in Figure 1 as a function of viewing angle, where we see that the occultation period peaks at $\approx 20\%$ of the precession period.

For an observer at a random location on the sky, the probabilities P_n for the number of occultations n depend only on β and H/R :

$$P_0 = 1 - \sin[\beta + \chi], \quad P_1 = \sin[\beta + \chi] - \sin[\beta - \chi], \quad P_2 = \sin[\beta - \chi]. \quad (10)$$

Figure 2 illustrates the dependence of P_1 and P_2 on β and H/R , showing that both are significant over some region of the parameter space. As an example, for $\beta = \pi/6$ and $H/R = 0.1$ we obtain $P_0 = 45.7\%$, $P_1 = 8.66\%$, and $P_2 = 45.6\%$. The fact that the 1-occultation case is the least likely but tends to produce the longest occultation times may explain why this mechanism is not observed in the majority of SLSNe. We assume that the disk reflects some fraction of the magnetar luminosity, and that the amount of this reflected luminosity directed toward the observer is proportional to the disk area projected along the line of sight. For a disk normal $\hat{h}$ and observer line of sight $\hat{z}$ as above, we obtain

$$L_{\text{reflected}} \propto f(\hat{h} \cdot \hat{z}), \quad (11)$$

$$\propto f(\cos \beta \cos \theta_{\text{obs}} - \sin \beta \sin \theta_{\text{obs}} \cos \Omega_{\text{prec}} t). \quad (12)$$

Here f is a free parameter. The total luminosity along a line of sight is then

$$L = L_{\text{magnetar}}[1 + f(\cos \beta \cos \theta_{\text{obs}} - \sin \beta \sin \theta_{\text{obs}} \cos \Omega_{\text{prec}} t)]. \quad (13)$$

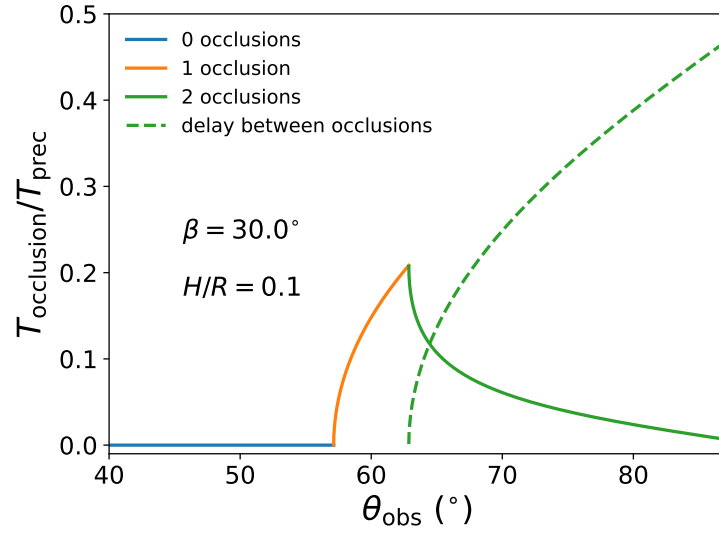


Figure 1: Fraction of the precession period during which the magnetar is occluded by the disk. The dotted line shows the delay between occlusions in the case of two occlusions per period.

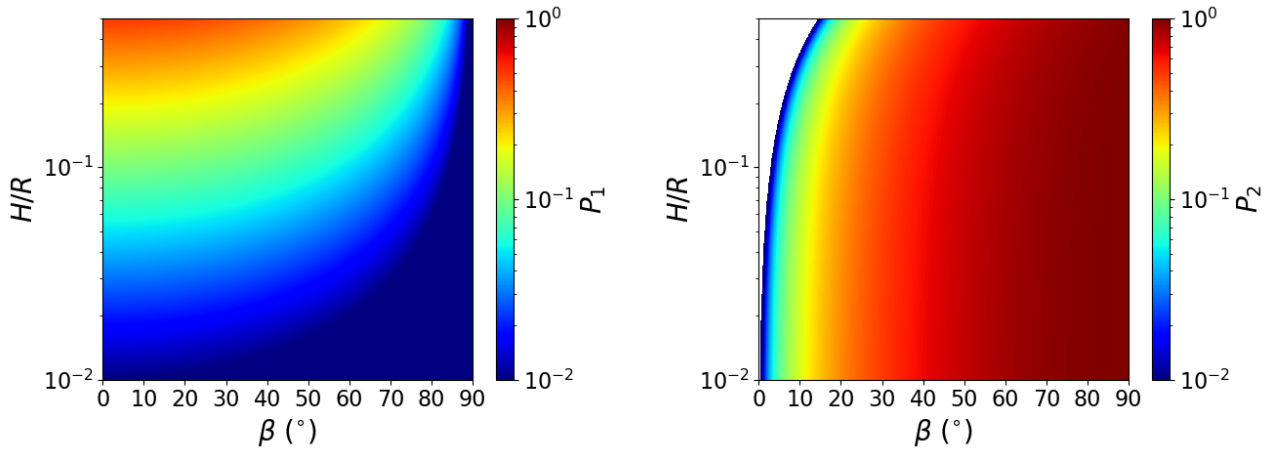


Figure 2: Heatmaps of the probabilities for 1 and 2 occlusions (P_1 and P_2 , respectively) versus β and H/R . We find that both are reasonably probable over a significant region of the parameter space.

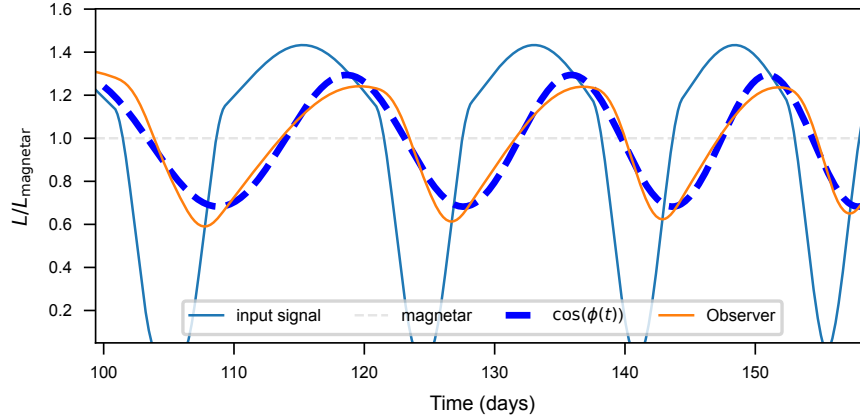


Figure 3: Simulated reflection and occlusion for an accretion disk with $f = 0.5$, $\beta = 30^\circ$, $\theta_{\text{obs}} = 60^\circ$, and Ω_{prec} producing a chirp. We compute the input signal based on the precessing accretion disk model (Equation 13) combined with the periodic occlusions (blue). We then evolve this input energy through an expanding ejecta with the same properties as SN 2024afav (orange). The resulting light curve is approximately sinusoidal and symmetric around $L/L_{\text{magnetar}} \sim 1$, as we see in the data. The blue curve is the cosine of a phase function which includes a constant phase offset parameter (i.e., $\phi(t) = \omega(t)t + \phi_0$), which motivates the use of the model we proposed in the paper.

L is assumed to be zero during an occlusion, as there is a direct line of sight from neither the magnetar nor the surface of the disk to the observer. To translate this into the observed light curve, we assume 1 occlusion per period and take as input equation (13) with an occlusion of 20% of the precession period centered on the minima. We then apply the diffusion model of Arnett (1982) (following Zhang et al. 2025) to obtain the light curve shown in Figure 3. We see that the observed light curve after accounting for photon diffusion through the ejecta is exceptionally well-reproduced by a cosine of the input signal phase function, with a constant phase offset to approximate the diffusion delay. Indeed, our reflection-occlusion model explains two interesting attributes:

1. **Symmetry around the magnetar model:** our residual analysis suggests that the modulations are roughly symmetrical around the magnetar light curve. Figure 3 shows that while the reflections increase the input energy above the magnetar model, the occlusion significantly dampens the input energy and the diffusion process smooths out the modulations to be approximately symmetric around the magnetar light curve, as we see in the data.
2. **Sinusoidal modulation structure:** the data display a very sinusoidal structure, despite our model predicting large non-sinusoidal occlusions. Figure 3 reveals that the non-sinusoidal occlusions are smoothed out by the diffusion through the ejecta and produce modulations which are well-modeled by a sinusoidal behavior.

We update the discussion of the physical processes in the draft to include the details described above, and give example text below. Outside of the example text below, we also add a short subsection in the Supplementary Information (3.2. Details of the reflection and occlusion by the disk) reproducing the details above.

The modulation of the magnetar signal by the accretion disk could occur in several ways (see ??), but for the purpose of our modeling we assume the modulation is occurring by reflection and occlusion of the magnetar energy by the disk along the line-of-sight of the observer. Reflection occurs when the surface of the disk scatters photons from the magnetar back towards the observer line of sight; occlusion occurs when the accretion disk intercepts the observer line of sight, making the diffusion timescale effectively infinite. We expand on the physical details of the modulation in ??.

2.4 Construction and fit of the Lense-Thirring modulation model

Another negative is that the equation used for the LT precession period is appropriate for a test mass or a very thin annulus. The LT precession period of an extended disk is the surface-density-weighted average of the test mass precession period throughout the extent of the disk (e.g. equation 43 of Fragile et al 2007). I'm not sure of the disk extent in this case, but the authors argue that there is no tearing and thus that the entire disk precesses as a solid body. Precession will therefore be slower than the current model predicts, as currently the surface density is effectively assumed to be a delta-function at the disk inner radius. Moreover, this effect will also change the evolution of the precession period with time, since the fallback disk will relax outwards after forming (e.g. Fig 1 of Balbus & Mummery 2018; Fig 5.1 of Frank, King & Raine 2002). The precession frequency will therefore decrease at a slower rate than the current model assumes. Finally on this front, solid body LT precession of the entire disk only happens in the regime $H/R > \alpha$ (e.g. Section 4.2 of Ingram & Motta 2019). I suspect that the magnetar disk in this case is indeed likely to be in this regime, but I think this needs to be explicitly mentioned in the text.

We thank the referee for raising this important point, and we agree with the thin vs. thick annulus distinction. The period of an annular disk undergoing LT precession was derived in Fragile et al. (2007) as

$$T_{\text{LT,annulus}} = \pi \frac{1 + 2\zeta}{5 - 2\zeta} \frac{r_o^{5/2-\zeta} r_i^{1/2+\zeta}}{aM} \frac{1 - (r_i/r_o)^{5/2-\zeta}}{1 - (r_i/r_o)^{1/2-\zeta}} \quad (14)$$

for a disk with a surface density which scales with radius as $\Sigma \propto r^{-\zeta}$. Our assumption of a steady alpha disk corresponds to $\zeta = 3/4$ (Frank et al., 2002).

We implement this expression across our model, varying r_o/r_i between 1.01 and 30. We find that the difference between the thin and thick annulus cases is absorbed by the accretion rate parameter, resulting a different best-fit accretion rate ($\dot{M}_0 \approx 3.7 \times 10^{-5} M_\odot/\text{yr}$, assuming $r_o/r_i = 25$) but approximately the same range of periods and period derivatives are reproduced, retaining the ability of the model to describe the data. As a result, the r_o/r_i parameter is effectively degenerate with the accretion rate. To report on these results in the paper, we modify our model to include the annulus scaling factor, update Figure 3 to show the distribution of P_c and $\dot{P}_c$ for the new model, and add discussion of the additional uncertainty (due to the unknown r_o/r_i) to the text. The updated figure is shown in section 4, and examples of the textual changes are highlighted below for convenience. We also explicitly clarify that we are in the regime of $H/R > \alpha$, and thus the assumption of solid body precession is a good approximation.

The annular geometry also modifies the precession periods associated with some of the alternative scenarios we explore, such as quadrupole-driven nodal precession (QDNP) and magnetic warp (MW). Using the precession of the orbit of a test particle (Morsink and Stella, 1999) and following the procedure of Fragile et al. (2007), we derive the QDNP period for an annular disk to be

$$T_{\text{QDNP,annulus}} = \frac{4\pi}{3\sqrt{GM}J_2R_{\text{mag}}^2\cos\beta} \frac{1+\zeta}{5/2-\zeta} r_o^{5/2-\zeta} r_i^{1+\zeta} \frac{1 - (r_i/r_o)^{5/2-\zeta}}{1 - (r_i/r_o)^{1+\zeta}}. \quad (15)$$

Here R_{mag} is the magnetar radius and J_2 is a dimensionless coefficient which parametrizes the oblateness of the magnetar. For magnetic warp, the precession frequency for a thin ring is given by

$$\Omega_{\text{MW,ring}} = \frac{\mu^2}{\pi^2 r^7 \Omega \Sigma} \sqrt{\frac{r_m}{2H}} \cos\beta (3\cos^2\theta - 1) \quad (16)$$

(Lai 1999). Here θ is the obliquity angle (between the spin axis and magnetic moment of the magnetar), r_m is the magnetosphere radius, and μ is the magnetic moment of the magnetar. The precession period for an annulus is then

$$T_{\text{MW,annulus}} = \frac{2\pi^2 \Sigma(r_i) \sqrt{GM}}{\mu^2 \cos\beta \sin\beta (3\cos^2\theta - 1)} \sqrt{\frac{2H}{r_m}} \frac{3}{5/2-\zeta} r_i^{11/2} (r_i/r_o)^{-5/2+\zeta} \frac{1 - (r_i/r_o)^{5/2-\zeta}}{1 - (r_i/r_o)^{-3}}. \quad (17)$$

We update the alternative scenarios models for the annulus and provide examples of the corresponding changes in the text below.

This expression is consistent with similar derivations in the literature (e.g., [Mashhoon et al., 1984](#)), and is valid for a test particle orbit or for a very thin annulus; however, the accretion disk will be composed of a range of orbits, resulting in a different precession frequency for the disk as a whole. Following [Fragile et al. \(2007\)](#), we model the disk with surface density $\Sigma = \Sigma_i(r/r_i)^{-\zeta}$, and find that for $\zeta = 3/4$ (Frank et al. 2002), the precession frequency derived above becomes

$$\Omega_{\text{LT}} = \frac{7}{5}(r_o/r_i)^{-7/4} \frac{1 - (r_o/r_i)^{7/4}}{1 - (r_o/r_i)^{5/4}} \frac{2GJ}{c^2 r^3(t)}.$$

As the outer and inner radii (r_o and r_i) of the annulus appear only as a ratio, we treat r_o/r_i as a single parameter. The accumulated phase $\phi(t)$ is the integral of the angular frequency with time. The exact form of $r(t)$ depends on the mechanism driving the infall; however, most can be approximated to first order as linear over the timescale of tens of days (see ??). We assume $r(t) \propto t$ to first order; i.e., $r(t) = r_0 - \dot{r}(t - t_0) = \dot{r}(t_{\text{infall}} - t)$ where $t_{\text{infall}} = t_0 + r_0/\dot{r}$. Under this assumption, the phase function becomes

$$\phi(t) = \int \Omega_{\text{LT}} dt \approx -\frac{7}{5}(r_o/r_i)^{-7/4} \frac{1 - (r_o/r_i)^{7/4}}{1 - (r_o/r_i)^{5/4}} \frac{GJ}{\dot{r}^3 c^2} (t_{\text{infall}} - t)^{-2} - \phi_0,$$

where the initial phase of the disk ϕ_0 is a free parameter, and t_{infall} and $\dot{r}$ are determined based on the assumed infall model.

...

We find $\alpha = 0.07 \pm 0.03$ and constrain $\dot{M} = 3.71 \pm 0.36 \times 10^{-5} M_{\odot}/\text{yr}$. This value is somewhat correlated with the choice of r_o/r_i , which introduces an additional source of uncertainty. We find that varying r_o/r_i between 1 and 30 varies the accretion rate by $\approx 1.6 \pm 10^{-5} M_{\odot}/\text{yr}$, but without a meaningful constraint as r_o/r_i is almost completely degenerate with $\dot{M}$.

2.4.1. Derivation of the precession

Additionally, in order to assume that the disk precesses as a solid body, we implicitly assume that $H/R \gtrsim \alpha$ ([Fragner and Nelson, 2010](#)). This assumption is consistent with the model we are invoking, as a relatively thick disk would be required to meaningfully occlude line-of-sight emission from the magnetar. Further, we validate this assumption by checking that best-fit solutions for α satisfy this assumption, based on the computed radial evolution (see ?? and the standard α -disk prescription, following [Shakura and Sunyaev \(1973\)](#) and using relationships summarized in [Frank et al. \(2002\)](#).

Construction and fit of the Lense-Thirring modulation model

The authors identified five bumps (including five peaks and four dips, see the first paragraph of Section 2.3) in the residual plot of the magnetar-only model minus the observed data (Figure 9). I am particularly concerned about the confidence levels for five bumps (especially the fifth one) in SN 2024afav. I suggest the authors to provide the confidence level for each bump in SN 2024afav or for each dip and peak. I believe that providing the confidence level for each bump in SN 2024afav would significantly enhance the persuasiveness of the manuscript. Furthermore, I suggest the authors to provide the amplitude and duration of each bump.

Visual inspection shows that the first and fifth peaks in Figures 1 and 6 show little difference from the magnetar model. Furthermore, Figure 9 shows that the first bump only has a descending structure and is not successfully fitted by Equation (1). At the same time, the confidence level for the last peak in Figure 9 is not high. Therefore, it is important to demonstrate that the chirped signal derived from the five bumps in SN 2024afav is reliable. The authors can perform a power density spectrum analysis on the SN 2024afav light curve to see if there is a periodically decaying signal (chirped signal). Alternatively, they can provide other methods to demonstrate that the chirped signal found in the SN 2024afav light curve is reliable.

As shown in Figure 10, after decomposing the LT model into envelope and precession, the precession model reproduces each peak and dip in the observed data very well. However, it can be seen that the LT model (or precession model) gives five dips in the time range of 40 to 180 days, but only the first four dips are observed. As described in the third paragraph of Section 2.4.2 (see also Figure 10), the first bump is difficult to reproduce by using the magnetar + LT model. What are the possible reasons for this?

Although I believe that the light curve is modulated, I am not convinced by the claim in the main text that there are 5 "unambiguous" peaks. The second to fourth peaks are very clear, but the first and fifth claimed peaks are only visible after subtraction of a model that does not include a periodicity (Figs 9 and 10). I think that, for the fifth peak, the data are too noisy to claim it is unambiguous. And the first peak is too model dependent, given that it coincides with the turn over of the magnetar model.

We thank the referees for their careful consideration of the chirped behavior, and we agree with their assessment. To implement the above comments we first will provide the amplitude, duration, and confidence level of each bump. To assess the confidence of the modulations and the chirp, we provide the following metrics:

- The statistical significance of the phenomenological model (equation 1 in the original draft) parameters from both $\Delta L = 0$ (no modulations) and $\phi(t) \propto t$ (linear phase indicating no chirp). Using likelihood-ratio tests ($\Delta\chi^2$) against nested null models, we find that both the pure-noise and constant-periodicity hypotheses are rejected at $\gg 5\sigma$. We repeat this test with the fifth bump excluded and find the same results.
- False-alarm probability versus a null hypothesis of pure noise for each bump (peak and dip computed separately, then averaged). We find that the false-alarm probability is $\ll 0.1\%$ for all except the last two modulations. The second to last modulation has false-alarm probability of $\sim 6\%$. However, the final modulation, which the referees flagged, has a false-alarm probability of $\sim 40\%$. This quantitative metric significantly strengthens the evidence for the presence and location of the remaining bumps.
- Power density spectrum (PDS) structure, which is expected to produce a sharp peak for a single periodicity and a "smeared" peak for an evolving frequency. We compute the PDS via a Lomb-Scargle periodogram and compare to a simulated stable periodic signal as well as a chirped signal, with noise comparable to our dataset. The result of this comparison is shown in Figure 4. The presence of the chirp "smears" the peak frequency in a characteristic way, visible in both the simulated chirp and the SN 2024afav data. By contrast, a stable periodicity produces a narrow peak, which is clearly inconsistent with the SN 2024afav data.

Note that the above analysis is applied to the unprocessed light curve, ignorant of any diffusion effects. Once the peaks have been robustly identified, we can correct for diffusion effects when measuring the peak times; however, assessment of the chirp is significantly more straightforward on the non-diffusion corrected light curve.

Based on the quantitative assessment of the bump confidence, we agree with the referees that the fifth bump is not "unambiguous", and have updated the text in the draft to reflect this. However, we argue that the

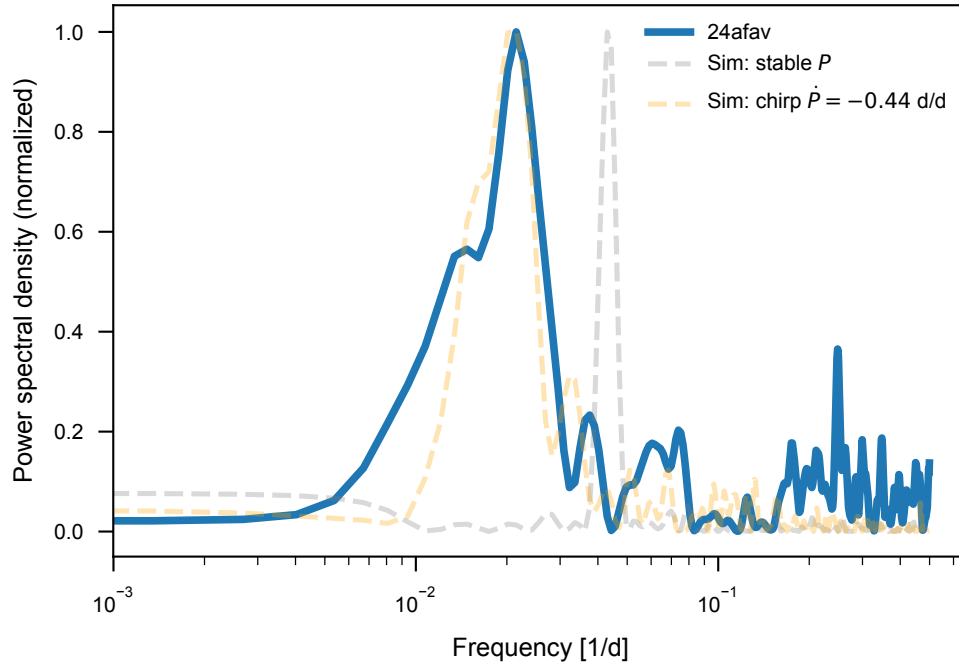


Figure 4: Power density spectrum of the SN 2024afav modulations in comparison to a simulated static period and a simulated chirp. The presence of the chirp “smears” the peak frequency in a characteristic way, visible in both the simulated chirp and the SN 2024afav data. By contrast, a stable periodicity produces a narrow peak, which is clearly inconsistent with the SN 2024afav data.

first bump is unambiguously present, though it may not be unambiguously due to our proposed model. The importance of the first bump being unambiguous is that it increases the challenge of the CSM interaction or flaring-based scenarios to explain the data. However, we acknowledge that this distinction is not made sufficiently clear in the text. To address the referees’ comments, we update the Residual Analysis section to include the above metrics (improving the confidence and robustness), demote the fifth bump as unambiguous, and clarify that the first bump is possibly degenerate in its explanation via our model or the magnetar model. We provide examples of the textual changes below (next page) for convenience.

We find that the false-alarm probability is $\ll 0.1\%$ for all except the last two modulations. The second to last dip (modulation no. 7) has false-alarm probability of $\sim 6\%$, which is acceptable. By contrast, the final modulations (modulations no. 8 and no. 9) have a false-alarm probability of $\sim 40\%$. Such a high false-alarm probability suggests that the final peak and dip are consistent with noise and not a robust detection. However, this quantitative metric significantly strengthens the evidence for the presence and location of the remaining bumps.

...

We use this phenomenological model to quantify our confidence in the presence of a chirp in the light curve. We compute the statistical significance of the model parameters from both $\Delta L = 0$ (no modulations) and $\phi(t) \propto t$ (linear phase indicating no chirp). Using likelihood-ratio tests ($\Delta\chi^2$) against nested null models, we find that both the pure-noise and constant-periodicity hypotheses are rejected at $\gg 5\sigma$. We repeat this test with the fifth bump excluded and find the same results.

To further investigate the presence of a chirp in the light curve, we compute a power density spectrum following [Lomb \(1976\)](#), and compare this spectrum to a simulated static period signal as well as a simulated chirp (contaminated with noise comparable to the SN 2024afav light curve). The results of this comparison are shown in [Figure 4](#). A static period will produce a narrow peak at a single frequency, but the presence of a chirp characteristically “smears” the highest-amplitude frequencies. We confirm that the SN 2024afav light curve indeed produces a power density spectrum profile characteristic of a chirp and visibly inconsistent with a static period. Together, the above evidence indicates the presence of four robust modulations with a clearly decreasing period.

2.3. Residual Analysis

This peak is also consistent with the expected peak of the light curve based on the magnetar model with no modulations, resulting in a minor degeneracy regarding whether the very first peak is a result of the precession, the spin-down of the magnetar, or both.

2.4.2. Modulation reproduction with magnetar parameters

3 Response to specific comments

Here, we address the more specific comments of the referees, which required minor to moderate changes to language or structure.

The manuscript states that the regimes of validity of the magnetar+LT model are after the diffusion time t_d and before the ejecta transitions to the nebula phase (Figure 7). However, as discussed in Hosseinzadeh et al. 2022 and Chen et al. 2023, even after the diffusion time t_d , variations originating from the central engine may be smoothed out by photon diffusion in the ejecta. Specifically, whether variations originating from the central engine after the diffusion time t_d are smoothed out by photon diffusion depends on the ratio between the duration of the bump and the instantaneous diffusion timescale. When this ratio is less than unity, variations originating from the central engine are smoothed out by photon diffusion (Chen et al. 2023). I strongly suggest that the authors discuss the relationship between the duration of each bump and the instantaneous diffusion timescale.

On page 17, a diffusion correction is mentioned. I can understand the basic physics of this, but there is a lack of detail as to how this is actually implemented in the model.

We agree with the points raised by the referee. In general, this is not an issue as $t_d(t) < P(t)$ in the regime of interest. For an expanding ejecta with $M \sim 7.5 M_\odot$ and velocity $v \sim 6 \times 10^3$ km/s, the timescale at $t \sim 80$ days is $t_d \sim 30$ days (vs. the observed period of ~ 50 days, ratio $\approx 1.6 > 1$). Towards the end of the modulations, the diffusion timescale at $t \sim 150$ d is $t_d \sim 15$ d (vs. the observed period of ~ 20 d, ratio ≈ 1.3). The primary effect of the diffusion timescale then is to delay the emission by $t_d(t)$. We account for this by correcting the peak locations by $\approx t_d$ before attempting to reproduce the periodicity in the extrema-only calculations (see section 2.4.1 of the original draft). We found the results are not highly sensitive to the magnitude of this correction; we could obtain acceptable results even just assuming a constant diffusion offset equal to the average $t_d(t)$ between 80 and 150 days. In the full light curve fit, this average offset is absorbed into the ϕ_0 parameter. We incorporate this discussion into the paper and provide example text below.

Finally, since the energy originates within an expanding supernova ejecta, the observed light curve is significantly affected by the diffusion timescale t_d of the ejecta. Chen et al. (2023) and Hosseinzadeh et al. (2022) found that, even after the diffusion timescale, modulations with timescales $\lesssim t_d$ are smoothed out, but modulations that occur with characteristic timescale $> t_d$ will be phase shifted. For an expanding ejecta with $M \sim 7.5 M_\odot$ and velocity $v \sim 6 \times 10^3$ km/s, the timescale at $t \sim 80$ days is $t_d \sim 30$ days (vs. the observed period of ~ 50 days, ratio $\approx 1.6 > 1$). Towards the end of the modulations, the diffusion timescale at $t \sim 150$ d is $t_d \sim 15$ d (vs. the observed period of ~ 20 d, ratio ≈ 1.3). The primary effect of the diffusion timescale then is to delay the emission by $t_d(t)$. We account for this by correcting the peak locations by $\approx t_d(t)$ before attempting to reproduce the periodicity in the extrema-only calculations (see ??). We found the results are not highly sensitive to the magnitude of this correction; we could obtain acceptable results even just assuming a constant diffusion offset equal to the average $t_d(t)$ between 80 and 160 days. In the full light curve fit, this average offset is absorbed into the ϕ_0 parameter.

2.4. Construction and fit of the Lense-Thirring modulation model

The caption of Figure 9 indicates that the period of the chirped signal decays from $P \sim 50$ days to $P \sim 20$ days over approximately 80 days. Which 80 days after the explosion does this 80-day range correspond to? How are the values of $P \sim 50$ days and $P \sim 20$ days derived? The first paragraph of Section 3.3 and Figure 3 give the characteristic period of SN 2024afav, $P_c = 23 \pm 1.6$ days, and the period derivative, $\dot{P}_c = 0.44 \pm 0.05$. How are the values and uncertainties of P_c and $\dot{P}_c$ obtained? The caption of Figure 3 indicates that the characteristic period is defined as the average period over 80 days. However, Figure 7 shows that the valid time range for the magnetar+LT model is 37–181 days, which is greater than 80 days.

We thank the referee for pointing out this ambiguity. The 80 day window corresponds to the time window from $t \approx 80$ d to $t \approx 160$ d. As the referees mentioned in comments (discussed above), the final modulation is not robust, and the first modulation may be due to the magnetar peak. Thus, to restrict to only modulations we believed were (a) robust and (b) driven by our model, we chose the above 80 day window. The precession period and period derivative are then computed by finding the average period and drift in this region. This can be computed by measuring the periods of the modulations directly and fitting a line cast in terms of a median period and period derivative. Finally, the estimates of $P \sim 50$ days and $P \sim 20$ days are based off of the visible modulations. The first modulation in our window has a dip at $t \approx 65$ days and another dip at ≈ 115 days; the last modulation in our window has its initial dip at $t \approx 140$ days and then another at $t \approx 160$ days. These are meant to be approximate. We add this clarification to the draft and provide example text below.

Based on the high-confidence peaks and troughs present, we compute a characteristic median period and period derivative for the residual modulations. To be conservative, we additionally exclude the first modulation at $t \approx 40$ days, as this may be degenerate with the magnetar model peak. Using the peak-to-peak and dip-to-dip locations to estimate periods as a function of time, we fit a linear period variation and find $P_c = 23 \pm 1.6$ days and a period derivative of $\dot{P}_c = -0.44 \pm 0.05$. We note that the choice of characteristic period and derivative estimation method does not significantly impact the results, so long as the corresponding period and period derivative are computed identically for the theoretical models.

2.3 Residual analysis

The manuscript does not consider the influence of a Lense-Thirring precessing accretion disk on the magnetar itself. However, the accretion of matter from the surrounding accretion disk onto a magnetar may affect the evolution of the magnetar's rotation and magnetic field, thereby affecting the evolution of the magnetar's own radiation (e.g., Lin et al. 2021). I suggest that the authors include a discussion in the manuscript on whether the effects of the Lense-Thirring precessing accretion disk on the magnetar itself needs to be considered in this work.

The accretion disk does not significantly influence the magnetar evolution itself. As a back-of-the-envelope check, the accretion disk inner radius computed in our model ($r_w \gtrsim 10^8$) lies entirely outside the light cone of the magnetar ($R_{LC} = cP/(2\pi) \sim 10^7$ cm $\ll r_w$), indicating that the magnetar should effectively be “unaware” of the accretion disk. More rigorously, Tong et al. (2016) laid down specific criteria for assessing the impact of an accretion disk on its host magnetar:

- Case I: Disk mass $\ll 10^{-6} M_\odot$, no effect on the magnetar;
- Case II: Large disk mass, magnetar surface dipole field $B \sim 10^{14}$ G or less, no effect on the magnetar;
- Case III: Large disk mass and surface dipole field $B \sim 10^{15}$ G, magnetar slowed down by accretion disk

Independent of the disk mass, we are definitively in Case II, as all methods identify $B \sim 1.5 \times 10^{14}$ G. Thus, we do not expect the accretion disk to significantly impact the evolution of the magnetar. We implement the referee's suggestion to add discussion of this to the text.

While the magnetar exerts a torque on the accretion disk, we do not expect the accretion disk to impact the evolution of the magnetar. First, the inner radius of the accretion disk (set by $r_w(t)$, see ??) is always outside of the light cylinder of the magnetar ($R_{LC} = cP/(2\pi) \sim 10^7 \text{ cm} \ll r_w$). More rigorously, Tong et al. (2016) found that magnetars with surface dipole strength $B \sim 10^{14} \text{ G}$ or less do not have any significant impact on the evolution of their host magnetar, regardless of disk mass. As we are in this regime (see ??), we do not expect the accretion disk to have any impact on the magnetar, and thus neglect this in our model.

2.4. Construction and fit of the Lense-Thirring modulation model

In the first paragraph of Section 2.3, the times for peaks and dips are reversed. The correct times should be dips at 65.3, 117.6, 139.9, and 160.4 days, and peaks at 44.9, 93.6, 128.9, 153.3, and 168.8 days.

We have implemented the change and corrected the error.

Using this approach, we identify dips at $t = 65.3, 117.6, 139.9$, and 160.4 days, and peaks at $t = 44.9, 93.6, 128.9, 153.3$ days as well as a low-confidence trough detection at $t = 168.8$ days.

2.3. Residual Analysis

I suggest that the authors add "time since explosion" to the horizontal axis of Figures 1 and 6 to make it easier for readers to compare Figures 1 and 6 with Figures 7, 9 and 10.

We have implemented the suggestion and thank the referee for the feedback. The updated figures are displayed in section 4 for convenience.

No example text.

The unit should be added to the B in the corner plot in Figure 6. What is the physical meaning of σ in Figure 6? The vertical axis title on the right side of the inset in Figure 6 should be "Absolute Magnitude + offset".

We implement the above feedback. The meaning of σ is an additional uncertainty added to the data points which the fit is allowed to vary to bring the reduced χ^2 to 1. We clarify this in the figure caption.

Corner plot and posterior samples (inset) for the magnetar-only model fit to the light curve of SN 2024afav. The σ parameter (magnitude units) is an additional systematic uncertainty the model is allowed to vary to bring the fit reduced χ^2 to 1. The posterior exploration achieved satisfactory convergence with minimal correlations between parameters and symmetric, unimodal uncertainties.

Figure 7

The reference to reference [33] in Section 1, paragraph 7 may be incorrect, as reference [33] does not provide an explanation for the bumps in SN 2017egm.

We thank the referee for catching this; we did indeed refer to the wrong paper (Liu et al. 2018 instead of Lin

et al. 2023). We have updated the text.

No example text.

In the Abstract and Section 1, paragraph 3, where references [6,7] are cited, it is suggested to also cite the work of Chen et al. 2023, where they also concluded that bumps are common in the light curve of SLSNe-I.

We have added the citation to the locations suggested.

No example text.

Importantly, explain in some detail how the LT precession of the disk may explain the features you observed in your light curve; after all, the LT effect, per se, makes only the accretion disk to precess. What is the physical mechanism that would lead the LT effect to produce the observed lightcurve? Specify explicitly that the disk is intended to be tilted with respect to magnetar's equator, or, equivalently, magnetar's spin angular momentum.

We appreciate the referee's clarification and we have rewritten this sentence to address the comment. We have also clarified this in other relevant areas of the draft.

Our observations are consistent with a magnetar centrally located within the expanding supernova ejecta, surrounded by an infalling accretion disk undergoing Lense-Thirring precession due to a tilt relative to the magnetar spin axis.

Abstract

A young magnetar is expected to form a small, tilted (relative to the magnetar spin axis) accretion disk from matter originating in the progenitor star that failed to eject during the supernova (??, ??) (Ogilvie and Dubus, 2001; Lin et al., 2021).

Main

What does it mean ($t=30$ days) in front of $\dot{M}$? Has it been inferred over a timespan 30 days long? Rephrase.

The intention of the $t = 30$ days is to refer to the accretion rate $\dot{M}(t) \propto t^{-C}$ at $t = 30$ days, which normalizes the proportionality. However, we agree with the referee that this is confusing in the abstract; we have modified the equation to make it clear it is an "initial" accretion rate.

Assuming standard accretion disk parameters (Frank et al., 2002), we constrain the accretion rate onto the magnetar to $\dot{M}_0 = 2.1 \pm 0.19 \times 10^{-5} M_{\odot} \text{ yr}^{-1}$.

Abstract

What is ePESSTO+?? Make it explicit. No acronym for Global Supernova Project? Explain to readers what is MJD: I know what is, but a typical Nature's reader, say a biologist or a neuroscientist does not....

We appreciate the referee's feedback to help reduce jargon, and have implemented the suggestions. Non-exhaustive examples are shown below for convenience.

It was discovered by the Liverpool – Gravitational-wave Optical Transient Observer (L-GOTO) Collaboration Dec 12.7, 2024 [Kumar et al. \(2024\)](#) and classified by the extended Public European Southern Observatory Spectroscopic Survey of Transient Objects (ePESSTO+) collaboration as an SLSN-I on Jan 24, 2025 ([de Wet et al., 2025](#)) based on spectral features (see also [??](#)). Early observations indicated a linear ~ 40 -day rise followed by a bumpy, ~ 50 -day quasiplateau phase peaking at absolute magnitude ~ -20.5 ([??](#)). Partway through the "plateau," we triggered observations using the Las Cumbres Observatory (LCO) via the Global Supernova Project (GSP) and the Fred Lawrence Whipple Observatory (FLWO) KeplerCam to obtain photometry and spectra (see [??](#) for full discussion).

Main

Why does the spin rate of a magnetar decrease over time? In which sense rotational energy dissipated in the spinning slowdown would go to ejecta?

We add text explaining why the magnetar spins down (loss of angular momentum via magnetic dipole radiation). However, the mechanism by which the energy is actually deposited into the ejecta is not well-understood. We clarify this directly in the text.

In the prevailing theory to explain superluminous supernovae, a strongly magnetized millisecond pulsar (magnetar) is created in the supernova explosion. As the magnetar radiates away energy via magnetic dipole radiation, the spin rate decreases, and it transfers some of this energy to the expanding ejecta ([Kasen and Bildsten, 2010](#); [Woosley, 2010](#)). However, while initial analytic models determined the energy budgets were sufficient to power the SLSN, they did not investigate a mechanism of energy transfer.

Main

Why the disk is tilted to the magnetar's spin axis? Expand and make clearer all the physical mechanisms involved by the LT effect arising to the observed luminosity curve. Why a jet from magnetar?

We address the referee's comment via an expanded discussion in the model construction section, which we reference from the main section (due to word limit constraints). We also cite [Soker \(2016\)](#); [Bucciantini et al. \(2008\)](#) for the presence of jets in magnetars, particularly infant magnetars.

In the minutes to hours following core collapse and the magnetar birth, an accretion disk composed of fallback material from the progenitor envelope forms and circularizes (Ogilvie and Dubus, 2001; Frank et al., 2002; Stone and Loeb, 2012; Lin et al., 2021). The inner radius of the disk is suspended by a wind launched from the magnetar (Chashkina et al., 2019; Tamilan et al., 2025) or, in more extreme cases, the magnetosphere itself. Through some process during formation, the accretion disk becomes tilted or warped, perhaps due to the presence of a binary companion (Fragile and Liska, 2024), a natal kick (Brandt and Podsiadlowski, 1995), or magnetic torques (Lai, 1999).

2.4. Construction and fit of the Lense-Thirring modulation model

The misaligned accretion disk is supported by a wind launched from the magnetar (Chashkina et al., 2019; Tamilan et al., 2025), which causes the disk to fall inward as the accretion rate in the disk decreases (??). Any misalignment between the accretion disk and the spin axis of the neutron star would induce a Lense-Thirring torque on the accretion disk (see e.g., Mashhoon et al., 1984, for a review), causing it to precess (with increasing frequency due to the infall) around the magnetar spin axis. As the accretion disk precesses, it may periodically modulate the emission from the magnetar by, for example, alternately obscuring or reflecting the magnetar luminosity (as in, e.g., Jurua et al., 2011; Romanova et al., 2014), redirecting a jet (Soker, 2016; Bucciantini et al., 2008) along the normal of the accretion disk (as in, e.g., Liska et al., 2018), redirecting wind from the disk itself (as in, e.g., Dexter and Kasen, 2013), or changing the accretion rate onto the magnetar (Nixon et al., 2012). The modulating emission from the magnetar leads to an asymmetric deposition of energy in the supernova ejecta that precesses with the accretion disk and appears to an observer to oscillate (e.g., Zhang et al., 2025) on the Lense-Thirring timescale of $P_{LT} \approx 25 - 45$ days (??).

Main

Are there no other physical mechanisms other than LT able to make the disk precess? The oblateness of the magnetar? A so strong magnetic field should induce a distortion in the shape of the magnetar giving it a quadrupole mass moment: in principle, it makes a tilted disk to precess. Magnetic torques should induce a precession of the disk as well: I suspect it may be quite large. In principle, also a distant third body. Nonstandard gravity models generally does not induce a disk precession since they impact only the pericenter and the mean anomaly at epoch: it should be stated. In general, a section dedicated to alternative explanations expanded with respect to the one present should be added. Most of the same points hold also for a distant companion. What if you fit with a (physically plausible) third body (I assume the disk as the orbit of a test particle)? Are there some observational hints about such a companion?

We address this comment via a focused section on alternative scenarios, wherein we (briefly, due to word limit constraints) discuss inclination precession driven by the oblateness of the magnetar (both jet and disk precession) and magnetic torques. We believe an in-depth discussion of non-standard theories of gravity is outside the scope of this paper. For alternatives, we considered a third body or binary companion, but in addition to their not being significant evidence of a companion, the precession frequency induced by a companion goes as $\propto (R/a)^3$ (Larwood, 1998) and therefore as the disk infalls, the frequency will decrease. We did not include discussion of this scenario due to space constraints. However, we include discussion of plausible modulation drivers, including CSM interaction, magnetic inclination angle precession, apsidal precession, and magnetic warp precession, with examples given below.

We evaluate other mechanisms that could replicate the oscillating light curve, such as different precession modes and time-variable energy injection. For each mechanism, we compute a characteristic period and average period derivative over the timescale of the modulations observed in the SN 2024afav light curve. For each mechanism, we vary model parameters to fit the characteristic period P_c and calculate from those parameters a period derivative for comparison. For accretion disk precession-based mechanisms, we invoke the wind-based radial infall to produce a $\dot{P}_c < 0$ if the mechanism is not equipped with an evolving periodicity. Finally, we vary the MOSFiT magnetar model parameters and compute the corresponding P_c and $\dot{P}_c$ values to generate a Monte Carlo sampling of their possible range. We compare these to the SN 2024afav values of $P_c = 23 \pm 1.6$ days and $\dot{P}_c = -0.44 \pm 0.05$. A visualization of the period/period-derivative parameter space of the below scenarios is shown in ???. For each scenario, we compute the precession period and period derivative exactly, and compare to an analytic prediction based on assuming linear radial infall, to validate the assumption of ???. The analytic prediction is visualized for each mechanism as the gray dashed lines in ???.

Magnetic inclination angle precession: Ellipticity of the magnetar can lead to precession of the jet, causing modulations in the light curve (Zhang et al., 2025). We assume the model of Zhang et al. (2025) and compute the period of the magnetar versus time using their Equation 6. We adopt the MOSFiT magnetar parameters and find that a high deformation parameter of $\epsilon \approx 10^{-8}$ is required to obtain $P_c \approx 23$ days. However, the combination of parameters required to reproduce the periodicity observed in the data results in $\dot{P}_c \approx 0.1$, which implies an increasing periodicity. There is no mechanism in this model to create a decreasing period, and the magnitude of the predicted $|\dot{P}_c|$ is a factor of several too small. This model is disfavored.

Quadrupole-driven nodal precession: An oblate neutron star will cause orbits in the vicinity to precess in a manner which scales with the quadrupole coefficient of the neutron star J_2 and decreases with the square of the orbital radius (Morsink and Stella, 1999). There is no inbuilt mechanism for quadrupole-driven nodal precession to produce a decreasing period, so we invoke the radial infall of ???. We find that an accretion rate of $\dot{M} = 6.4 \times 10^{-6} M_\odot/\text{yr}$ coupled with a high $J_2 \approx 10^{-3}$ (Colaiuda et al., 2008), produces $P_c = 23 \pm 0.9$ days. However, the accretion rate required to reproduce the period enforces $\dot{P}_c = -0.6$; thus, this model is disfavored.

Apsidal precession: If the disk has intrinsic ellipticity, it may experience apsidal precession due to relativistic effects (e.g., Misner et al., 2018). However, this effect is unlikely to be relevant on the timescale of the modulations observed in SN 2024afav. The disk circularizes on a timescale of roughly seconds to hours (Armitage, 2008; Strubbe and Quataert, 2010; Stone and Loeb, 2012), whereas the bumps first appear ~ 2 months into the evolution of the supernova. By this point, the accretion disk has circularized and does not experience apsidal precession.

Magnetic warp precession: A diamagnetic accretion disk experiences a precessional torque with characteristic angular frequency given by Lai (1999). This magnetic warp precession can give comparable periodicities if the disk surface density is unusually high ($\Sigma \gtrsim 10^7 \text{ g cm}^{-2}$), but the period is again likely to increase due to an induced disk threading torque (Lai, 1999), conflicting with observations. If we instead modify the period by invoking radial infall, we can obtain $P_c = 14.3 \pm 1.0$ days using an accretion rate of $\dot{M} = 3.5 \times 10^{-5} M_\odot/\text{yr}$. However, this results in a best-fit $\dot{P}_c = -0.69$, which is once again inconsistent with the observations of SN 2024afav; thus, this model is disfavored. More example text here.

Introduction

It is not clear to me if the authors perform a fit of the full magnetar + LT precession model to the data with P and B left as free parameters. At the end of 2.4.2 it is mentioned that "Incorporating estimates of the period and magnetic field from both the precession (subsubsection 2.4.1), early light curve (sub-section 2.2), and full light-curve evolution (Figure 6), we estimate $P = 4.2 \pm 0.2$ ms and $B = 1.6 \pm 0.1 \times 10^{14}$ G.". This sentence could mean that such a fit was performed, and these are the best fitting parameters, but it is ambiguous. If such a fit was performed, please make it unambiguous in the text that this was done. If such a fit was not performed, I really think it should be. This will somewhat address the issue of the peak in the magnetar model coinciding with the first candidate oscillation peak, since the fit algorithm will just optimise B and P such that the full model best describes the full light curve. I understand the value of first treating the magnetar model and the LT oscillation model separately (and showing that they agree), but the final quoted results for P and B really should be from forward modelling that self-consistently includes both the magnetar luminosity evolution and the LT oscillations.

We recognize the ambiguity and add text clarifying that what is reported in the paper is for a fit of both model components simultaneously to the data, as the referee suggests. We maintain that the separate fits are important for the confirmation aspect, but the final reported results are indeed from self-consistent forward modeling.

Incorporating estimates of the period and magnetic field from both the precession (??), early light curve (??), and full light-curve evolution (forward-modeling the light curve with both the magnetar model (??) and precession component (?? simultaneously included), we estimate $P = 4.2 \pm 0.2$ ms and $B = 1.6 \pm 0.1 \times 10^{14}$ G.

2.4.2. Modulation reproduction with magnetar parameters

We find the period determined from the rise in the standard magnetar model ($P = 5.5 \pm 1.25$ ms, ??) and the modulations ($P = 5.4 \pm 1.8$ ms, ??) are statistically self-consistent and, combined with the estimate from the full light curve (forward-modeling both the magnetar model and Lense-Thirring precession-driven modulation model simultaneously), constrain the spin of the magnetar to $P = 4.2 \pm 0.2$ ms (??)

Main

Throughout the manuscript, " χ^2 " is used when surely the authors mean "reduced χ^2 ". Moreover, just presenting reduced χ^2 does not convey enough information for the reader to properly evaluate the goodness of fit. For this, we need both χ^2 and the number of degrees of freedom (not just the ratio). Please therefore quote both χ^2 and the degrees of freedom.

A related point: the model shown in Fig 1 (I suppose that this is from forward fitting, which if true would make my earlier point easy to address) is quoted as having a [reduced] χ^2 of 1.6, which is claimed to be a good fit. This is not a good fit. I appreciate the difficulty of fitting a deterministic model to very complex, probably partially stochastic data, but at the very least the poor fit quality will likely mean that the parameter uncertainties are underestimated. This problem could be addressed by artificially increasing the errors on the data points by some fixed fractional value to ensure that the best fitting model has $\chi^2/\text{dof} \sim 1$.

We do mean reduced χ^2 ; we thank the referee for pointing out this imprecise language. To address the referee's comment, we will report the degrees of freedom when describing the dataset and the model. We do not believe it necessary to quote for every single χ^2 measurement as the degrees of freedom of the model ($k = 12$ for the magnetar-only model, $k = 15$ for the combined magnetar+LT fit) are vastly outweighed by the data ($N = 706$ for the entire dataset; even in the worst-case scenario, where we assume bands are not independent and further bin the data by day, $N \sim 170 \gg k$). Finally, we agree with the referee's point about the concern about the χ^2 . We clarify that all fits are already performed scaling the errors so that the resulting reduced $\chi^2 \sim 1$ by including an additional scatter term σ in the fit. This helps avoid underestimation of the uncertainties. However, when we report reduced χ^2 in the draft, we report the reduced χ^2 computed on the un-scaled data uncertainties, for the purpose of comparison between models. We have added text clarifying these points and reporting on N and k for the dataset and model; please see example additions below.

We model the light curve as generated by an infalling accretion disk undergoing Lense-Thirring precession ("magnetar+LT," solid line; ??, $k = 15$ model degrees of freedom) and find an excellent fit to the data, with reduced $\chi^2 \sim 1.5$ for the whole fit ($N = 706$ data points) and reduced $\chi^2 \sim 1.6$ for the modulations (40-180 days, $N = 618$ data points).

Figure 1

The model includes a parameter (σ , units of magnitudes) which scales the uncertainties so that the reduced χ^2 of the fit is approximately unity.

2.2. Magnetar-only model fits

We find that our model is able to satisfactorily explain the modulations in the data with goodness-of-fit in the range of reduced $\chi^2 \approx 1.3-1.9$ ($N \sim 75 - 105$ data points, $k = 15$), and, as shown in ??, estimates values for P_{spin} and B_{field} within normal ranges for SLSNe-I (Gomez et al., 2024).

3.4. Fits to other SLSNe-I

A key aspect of the model is that the disk inner radius moves in with time because the equilibrium radius set by the balance of radiation pressure from the magnetar and ram pressure from the disk shifts. I think this needs to be explicitly mentioned in the main text.

We implement the referee's feedback by explicitly discussing the equilibrium radius when the model is introduced in the main text.

The radius of the misaligned disk is set by the equilibrium between the radiation pressure from the magnetar-launched wind and the ram pressure of the disk (Chashkina et al., 2019; Tamilan et al., 2025), which shifts inward as the accretion rate decreases, leading to the disk "infall" (??).

Main

For the MOSFiT fit described in Section 2.2, it is said that no burn in was used, and then the full chain was used to estimate the posteriors. This effectively assumes that the chain converged *immediately*, which surely cannot be correct.

We investigated and found this was reported in error. Preliminary fits did not use a burn-in but the final fits (where maximum posterior stability was desired) used a 2000-step burn-in. We have updated the draft text to correctly reflect the procedure.

We initialize the fit with 1600 walkers and use sufficient steps (~ 8000 after a ~ 2000 -step burn-in) to achieve convergence, which we measure via chain autocorrelation length and the improved Gelman-Rubin statistic, following Farah et al. (2025) section 5.4.

2.2. Magnetar-only model fits

4 Updates to existing figures

Here, we review updates to the existing figures based on comments from the referee. Individual comments are not pointed out, but the overall changes are noted and summarized. Newest versions of the figures are provided.

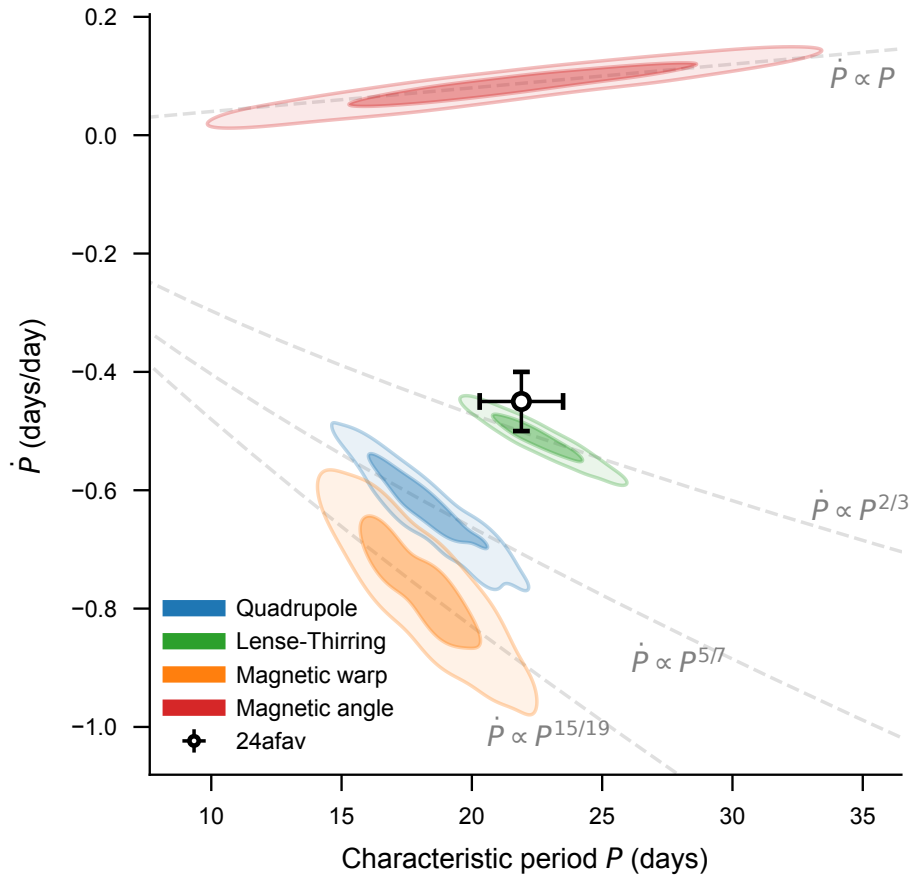


Figure 5: Ability of alternative scenarios to explain the modulation of the SN 2024afav light curve. We analyze other possible origins of the accretion disk precession to assess the uniqueness of the LT model's ability to explain the observed modulations in SN 2024afav. We characterize the candidate explanations based on their characteristic period (average period over the 80 days when the bumps are visible) and precession period derivative (see ??). The corresponding location of SN 2024afav is shown as the point with error bars. 1σ and 2σ contours are shown for the candidate explanations based on the MOSFiT uncertainties (see ??). For models not equipped with an $\dot{\Omega} > 0$ behavior, we invoked radial infall to explain the changing period. We consider magnetic angle precession, quadrupole-driven nodal precession, and magnetic warp precession. Only the Lense-Thirring effect around a magnetar produces a characteristic period and period derivative consistent with the observations. Gray dashed lines indicate the predicted $\dot{P}(P)$ behavior based on the linear radial infall assumption described in ??.

Changes: We have moved to computing quantities for an annulus instead of a thin ring. The accretion rate absorbs the difference in scaling, so the models reproduce a similar period and period derivative. For the Lense-Thirring precession, we include the contributions of the nodal precession, but as we noted this contribution is $\sim 10^3$ smaller than the Lense-Thirring contribution, so our results are largely unchanged.

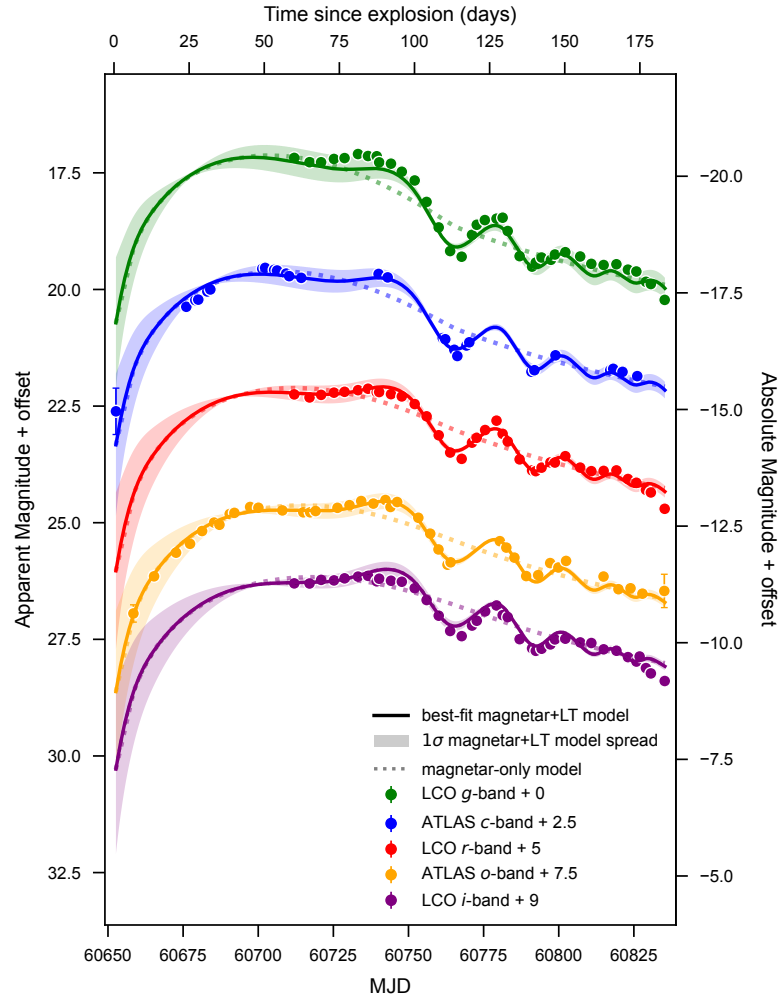


Figure 6: The combined LCO+ATLAS+KeplerCam (KC) light curves of SN 2024afav, from shortly after explosion to ~ 125 days post-peak. The magnetar model is shown as the dotted line (“magnetar-only”; see ??). While the data on average are consistent with the magnetar model, there is a chirped signal post-peak that the model fails to explain. We model the light curve as generated by an infalling accretion disk undergoing Lense-Thirring precession (“magnetar+LT,” solid line; ??, $k = 15$ model degrees of freedom) and find an excellent fit to the data, with reduced $\chi^2 \sim 1.5$ for the whole fit ($N = 706$ data points) and reduced $\chi^2 \sim 1.6$ for the modulations (40–180 days, $N = 618$ data points).

Changes: We have added a secondary time axis which converts into time since explosion. We update the caption to include the number of data points and degrees of freedom for the model to facilitate fit quality interpretation.

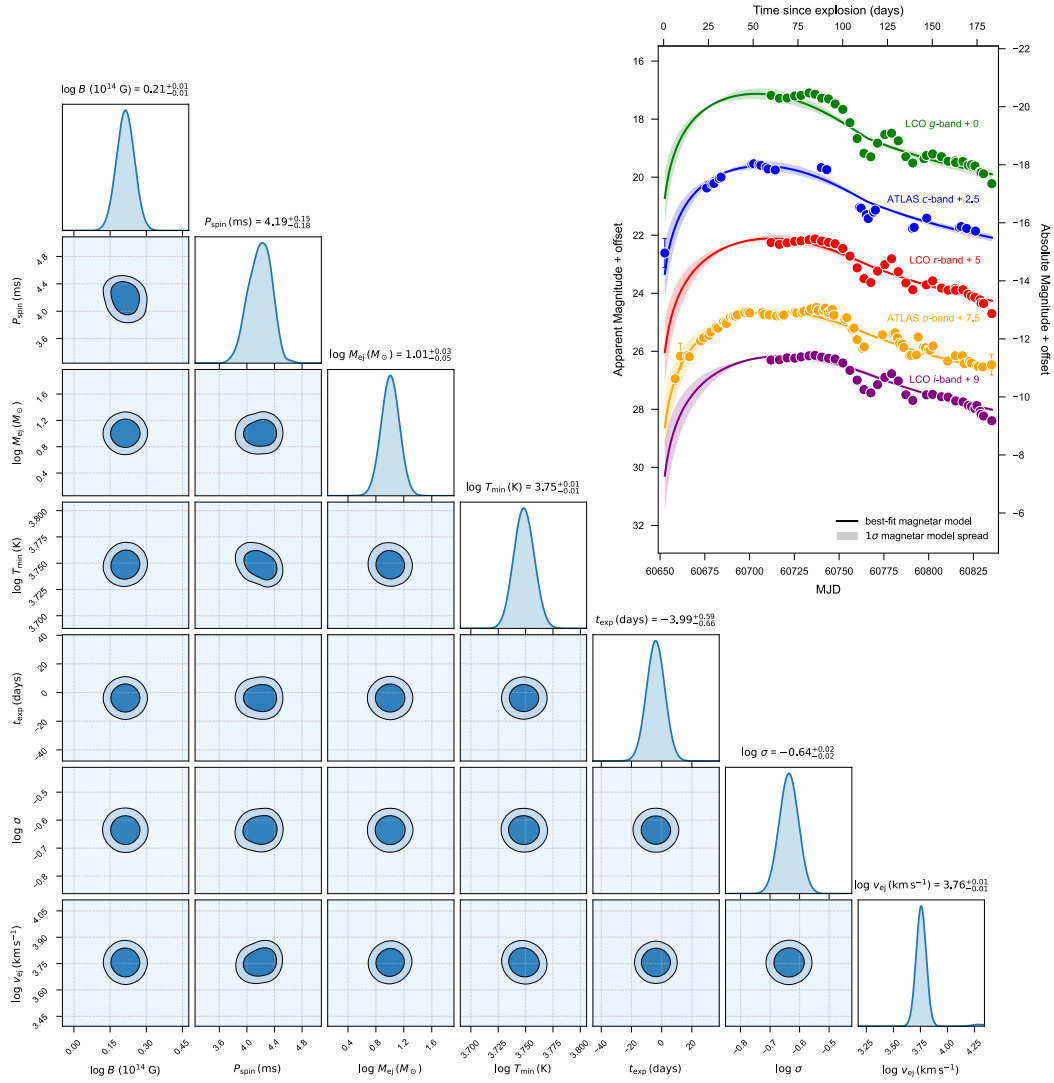


Figure 7: Corner plot and posterior samples (inset) for the magnetar-only model fit to the light curve of SN 2024afav. The σ parameter (magnitude units) is an additional systematic uncertainty the model is allowed to vary to bring the fit reduced χ^2 to 1. The posterior exploration achieved satisfactory convergence with minimal correlations between parameters and symmetric, unimodal uncertainties. The model explains the rise and overall trend of the light curve well; however, the model fails to explain the post-peak chirped modulations which appear in all bands. 1σ and 2σ regions of the joint posterior distributions are shown as contours.

Changes: We have added a secondary time axis which converts into time since explosion. We have also updated the units on the magnetic field strength B to reflect that the quantity is $\log$ of $B/(10^{14} \text{ G})$. We updated the caption to clarify the definition of σ .

5 New figures

Here, we present new figures that have been added per comments from the referee. These figures were not present in the version submitted to the journal. We also include the accompanying text added to the body of the paper designed to work in tandem with the figure.

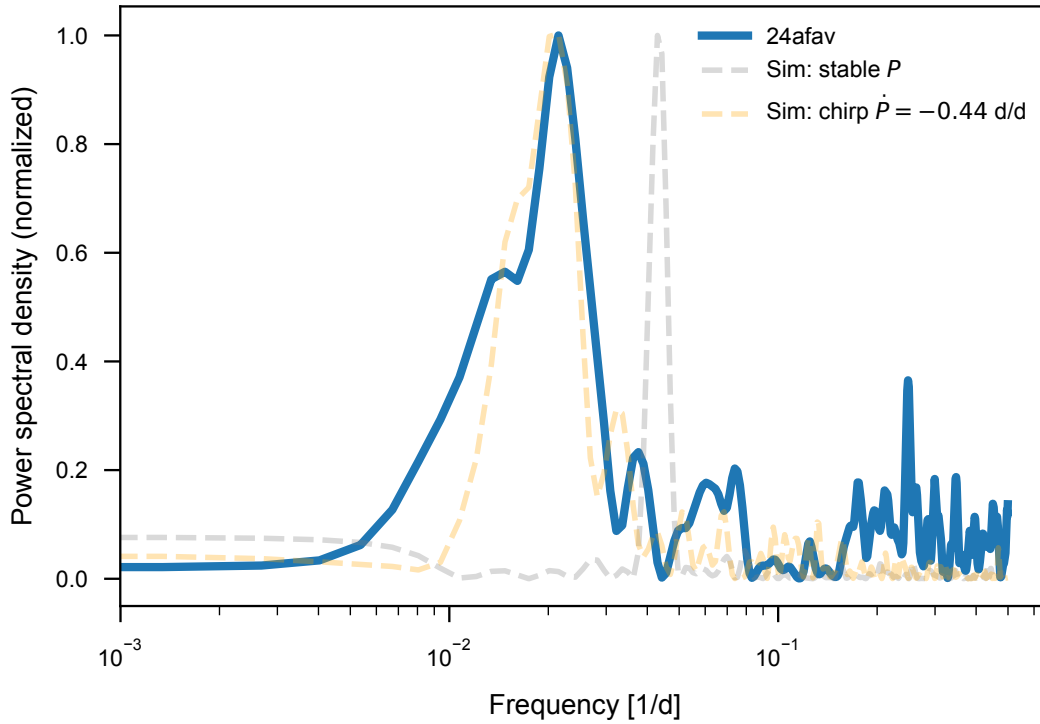


Figure 8: Power density spectrum of the SN 2024afav modulations in comparison to a simulated static period and a simulated chirp. The presence of the chirp “smears” the peak frequency in a characteristic way, visible in both the simulated chirp and the SN 2024afav data. By contrast, a stable periodicity produces a narrow peak, which is clearly inconsistent with the SN 2024afav data.

Accompanying text: To further investigate the presence of a chirp in the light curve, we compute a power density spectrum following [Lomb \(1976\)](#), and compare this spectrum to a simulated static period signal as well as a simulated chirp (contaminated with noise comparable to the SN 2024afav light curve). The results of this comparison are shown in [Figure 4](#). A static period will produce a narrow peak at a single frequency, but the presence of a chirp characteristically “smears” the highest-amplitude frequencies. We confirm that the SN 2024afav light curve indeed produces a power density spectrum profile characteristic of a chirp and visibly inconsistent with a static period. Together, the above evidence indicates the presence of four robust modulations with a clearly decreasing period.

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Lense-Thirring precessing magnetar engine drives a superluminous supernova: *Nature* Referee Response II

Joseph Farah* and Logan Prust, on behalf of the authors

November 8, 2025

Contents

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1 Introduction and summary of changes

We thank the referees for the kind words and continued feedback. We respond to the referee comments in-line below. Here we present a brief summary of our changes to the paper:

1. We provide additional citations showing that anisotropic, axisymmetric input energy from a central engine does indeed produce viewing-angle dependent light curves. This addresses the most major concern from the referee that the anisotropic energy profile produced by the disk would not be visible to the observer through the ejecta as the disk precesses.
2. We emphasize that the disk occlusion mechanism proposed in the paper is not conclusively known to be the actual mechanism, nor are we claiming it is. It is a novel but rough treatment to inspire future discussion on how disk-driven magnetar central engines can produce chirped light curves. To make this more clear, we move the disk occlusion model derivations and figures to Supplementary Information (to de-emphasize), increase focus on the other possible mechanisms in the main text, and emphasize that further theoretical work is needed, but is outside the scope of this paper. The scope of this analysis is focused on the chirp and how the timing can be explained by Lense-Thirring precession; we do not attempt to resolve the long-standing open question of how energy from the magnetar is deposited into the ejecta.
3. We address the remaining minor concerns raised by the referees.

A key change we would like to highlight is that we have moved the disk occlusion model discussion to Supplemental Information and have added this text to Methods, addressing Referee #1 Major Comment (1) and de-emphasizing the importance of the assumed reflection/occlusion model (quoted text on next page):

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The modulation of the magnetar signal by the accretion disk could occur in several ways. These include (but are not limited to) reflecting and obscuring the luminosity from the magnetar (as in, e.g., [Jurua et al., 2011](#); [Romanova et al., 2014](#)), jet redirection ([Soker, 2016](#); [Bucciantini et al., 2008](#)) along the normal of the plane of the accretion disk (as in, e.g., [Liska et al., 2018](#)), wind redirection originating in the disk (as in, e.g., [Dexter and Kasen, 2013](#)), or modulating the accretion rate onto the magnetar itself ([Nixon et al., 2012](#)). The proposed methods produce an anisotropic energy injection into the ejecta ([Jurua et al., 2011](#); [Romanova et al., 2014](#)) which evolves due to the precession of the disk and results in a modulated luminosity as seen by a distant observer ([Barnes et al., 2018](#); [Li et al., 2024](#); [Zhang et al., 2025](#)). Significant theoretical investigation into each of these scenarios will be conducted in future work but is outside the scope of this analysis. Here we adopt, for illustrative purposes, a working assumption that the modulation arises from reflection and occlusion of the magnetar energy by the disk along the observer's line of sight, while noting that the true mechanism remains uncertain and merits dedicated study.

Quotes from the referees are colored red, with a left vertical line of the same color; quotes from the updated manuscript are colored black, with a left vertical line and gray background. Separate referee comments (and the corresponding responses) are delineated by a horizontal line. For comments that required substantial and/or multiple repeated changes to the paper, one or more representative changes are shown. Samples from the updated draft text may have broken references (i.e., ??) to sections that can be found in the paper but are not in this document.

In [section 2](#), we address the broadest comments from the referee, which required sweeping changes to the paper and the addition of new sections and/or figures. Next, in [section 3](#), we address the more specific comments of the referee, which required minor to moderate changes to language or structure. In [??](#), we describe updates to existing figures per suggestions made by the referee. In [section 4](#), we reproduce new figures added to the paper and provide context and captions.

2 Response to major comments

Here, we address the major comments from Referee #1.

The calculations in Section 3.2 regarding the occlusion and reflection of radiation from the magnetar may contain some issues. Outside the magnetar-disk system, there is an outward-expanding spherically symmetric ejecta ($v_{ej} \sim 6 \times 10^8$ cm/s, non-relativistic) that surrounds the entire magnetar-disk system. The magnetar-disk system injects energy into the spherically symmetric ejecta. The radiation observed by an observer contributes from the entire spherically symmetric ejecta, not just from the portion of the ejecta along the line connecting the magnetar and the observer. The magnetic dipole radiation of a magnetar is typically isotropic. The disk obscures the magnetar's radiation, not just when the disk is along the line of sight. At any given moment, the magnetar's radiation along the disk plane is obscured by the disk, while radiation not along the disk plane is injected into the spherically symmetric ejecta. In addition, the disk reflects the magnetar's radiation in all directions, not just in the direction of the observer's line of sight, and the radiation reflected in other directions is also injected into the spherically symmetric ejecta. As a result, the occlusion and reflection of magnetar radiation by the disk will not cause modulation of the energy L_{input} injected by the magnetar-disk system into the entire spherically symmetric ejecta. The luminosity of the observer obtained by substituting this injected energy L_{input} into the diffusion model of Arnett (1982) is also not modulated. According to the above discussion, it seems unlikely that the occlusion and reflection of the magnetar spin-down luminosity by the LT precession disk will cause bumps in the light curve obtained by the observer.

Though we cannot constrain without significant theoretical investigation which specific disk-magnetar mechanism is likely generating the anisotropic energy injection (we assume reflection/occlusion, but other options include jet redirection, wind from the disk, etc.), it is well-established that anisotropic central engine emission produces viewing-angle dependent light curves, even after diffusion through ejecta. Li et al. (2024) demonstrated clearly that anisotropic emission from a rapidly rotating magnetar results in significant variations in light curve due to viewing angle. Barnes et al. (2018) explicitly shows that anisotropic energy input from a central engine into ejecta results in light curves and spectra that depend on viewing angle, particularly at early times. As a final and most relevant example, Zhang et al. (2025) directly treats an anisotropic precessing magnetar central engine diffusing through expanding ejecta on some of the same SLSNe-I discussed in this paper, and indeed identify that time-dependent precessions of the input energy are visible after diffusing through the ejecta. To address the referee's concern, we add this discussion to the paper and reference the above works.

The proposed methods produce an anisotropic energy injection into the ejecta (Jurua et al., 2011; Romanova et al., 2014) which evolves due to the precession of the disk and results in a modulated luminosity as seen by a distant observer (Barnes et al., 2018; Li et al., 2024; Zhang et al., 2025). Significant theoretical investigation into each of these scenarios will be conducted in future work but is outside the scope of this analysis. Here we adopt, for illustrative purposes, a working assumption that the modulation arises from reflection and occlusion of the magnetar energy by the disk along the observer's line of sight, while noting that the true mechanism remains uncertain and merits dedicated study.

Construction and fit of the Lense-Thirring modulation model

What specific physical process accounts for the modulation observed in SN 2024afav? In the authors' model, the modulation in SN 2024afav's observational data originates from Equation (10). This equation is a function composed of an envelope multiplied by the cosine of a phase function. What specific physical process does Equation (10) originate from? Why does this physical process yield the form of Equation (10)?

4) Although the authors provided details in Section 3.2 regarding the occlusion and reflection of radiation from the magnetar (which may be incorrect according to comment 1), the calculations presented in Section 3.2 were not incorporated when fitting the light curve data (Figure 1) using the magnetar+LT model (where the LT component is given by Equation (10)), nor when fitting the residual data (Figure 14) using Equation (10).

This is an ansatz motivated both by other treatments in the literature (e.g., Zhang et al. 2025) and the behavior of the disk occlusion model (Fig. 7 | Occluded light curve), which is well-approximated by a diffusion timescale-shifted cosine acting on the underlying arbitrary precession function. The amplitude/envelope component is a phenomenological description to assist with characterization of the chirp; however we note that description of the envelope is not required to test the precessions modeled in the analysis. We add text clarifying this to the model motivation. For Major Comment (4): The calculations presented in section 3.2 were incorporated when fitting the light curve data via the $\cos(\phi(t))$ ansatz, which was designed to reproduce the behavior seen in the literature and in our own section 3.2 analysis.

Following ??, we fit these oscillations in a two-component model, multiplying a time-evolving envelope $A(t)$ by a sinusoid,

$$\Delta L_{LT}(t) = A(t) \cos[\phi(t)].$$

The choice of a sinusoid with a phase offset to represent the observed signal is well-motivated based on the discussion in ??. The phase function $\phi(t)$ is derived directly from the precession frequency magnitude (Ω_p) in the weak-field approximation, which has both a Lense-Thirring component and a quadrupole-driven component.

Derivation of the precession

The additional signal from the precessing disk is well-approximated as the cosine of a phase function, as motivated by the investigation ??. We found that the diffusion timescale of the ejecta smooths out the geometry-based reflections and occlusions of the disk into an approximately sinusoidal signal with phase determined by the precession frequency. For our model, we assume the frequency is driven by Lense-Thirring precession and derive a phase function for the disk from first principles in ??.

Modulation reproduction with magnetar parameters

To translate this into the observed light curve, we assume 1 occlusion per period take as input equation (4) with an occlusion of 20% of the precession period centered on the minima. We then apply the diffusion model of Arnett (1982) (following Zhang et al. 2025) to obtain the light curve shown in ??. We see that the observed light curve after accounting for photon diffusion through the ejecta is exceptionally well-reproduced by a cosine of the input signal phase function, with a constant phase offset to approximate the diffusion delay.

Details of the reflection and occlusion by the disk

However, this does not affect interpretation; as demonstrated in ??, all physical quantities of interest can be estimated by fitting the extrema of our model to the dates of extrema in the light curve, without requiring a description of the amplitudes at all.

Modulation reproduction with magnetar parameters

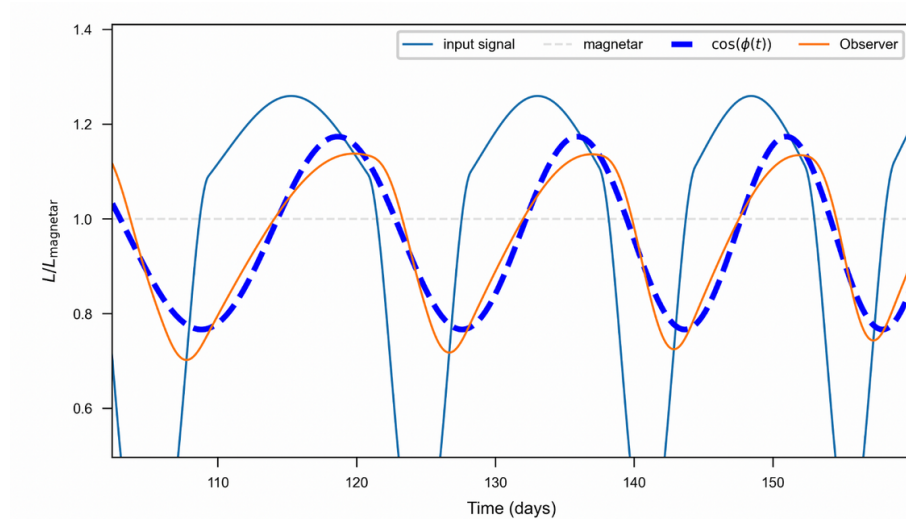


Figure 1: Demonstration of the light curve resulting from disk occlusion energy level of $0.3L_{\text{magnetar}}$ and reflection of 30% of the magnetar energy at peak. Despite varying the values from the pedagogically simpler case shown in the paper draft, it can be seen that the results are quite similar.

In addition, the authors assume that when the disk obscures the line of sight, the total luminosity L along the line of sight is zero, and therefore consider that the energy injected into the ejecta at this time is zero. However, even when the disk obscures the line of sight, the radiation from the magnetar that is not along the disk plane can still be injected into the entire spherically symmetric ejecta.

We are not particularly sensitive to the choice of occlusion percentage; we find that variations in the occlusion percentage (e.g., by changing energy injected during occlusion from 0% to 30% L_{magnetar}) can be cancelled out by varying the arbitrary reflection fraction (e.g., by lowering the reflection fraction from 50% to 30% L_{magnetar}). An example system with these parameters is shown in Figure 1; it can be seen that the results are not significantly different from the 0% occlusion/50% reflection scenario. The system with 0% occlusion energy injection was chosen for pedagogical simplicity. We thank the referee for raising this concern and add discussion to the paper explaining the relative insensitivity to those choices.

A caveat to equation (4) is that L is assumed to be zero during an occlusion, as there is a direct line of sight from neither the magnetar nor the surface of the disk to the observer. This assumption is pedagogically simple but somewhat unrealistic since some energy from the magnetar will still reach the observer through the non-shadowed portion of the ejecta; however, we find that we are not particularly sensitive to the energy level during occlusion, as a non-zero occlusion luminosity can be compensated by decreasing the fractional reflectance f while producing a similar signal.

Details of the reflection and occlusion by the disk

Equation (15) may be incorrect. L_{magnetar} is the total luminosity of the magnetar's isotropic magnetic dipole radiation, which is uniformly distributed over the entire 4π solid angle, while $L_{\text{reflected}}$ is the reflected luminosity only along the line of sight. $L_{\text{magnetar}} + L_{\text{reflected}}$ does not equal the total luminosity L along the line of sight.

We thank the referee for pointing this out; we have redefined L_{magnetar} to be the luminosity of the magnetar along the line of sight.

We assume that the disk reflects some fraction of the magnetar luminosity, and that the amount of this reflected luminosity directed toward the observer is proportional to the disk area projected along the line of sight:

$$L_{\text{reflected}} \propto \cos\beta \cos\theta_{\text{obs}} - \sin\beta \sin\theta_{\text{obs}} \cos\Omega_{\text{prec}} t. \quad (1)$$

This is in addition to the (constant) luminosity of the magnetar itself along a line of sight L_{magnetar} . The total luminosity along a line of sight is then

$$L = L_{\text{magnetar}}[1 + f(\cos\beta \cos\theta_{\text{obs}} - \sin\beta \sin\theta_{\text{obs}} \cos\Omega_{\text{prec}} t)], \quad (2)$$

where f is a free parameter. A caveat to equation (2) is that L is assumed to be zero during an occlusion, as there is a direct line of sight from neither the magnetar nor the surface of the disk to the observer.

Details of the reflection and occlusion by the disk

In addition, after accounting for disk occlusion and reflection of radiation from the magnetar in Section 3.2, the amplitude of the modulation in the light curve obtained by the observer is constant (the orange line in Figure 8), while the amplitude of the modulation in the light curve obtained by Equation (10) is Gaussian (Figure 14). This may require an explanation.

We thank the referee for raising this important issue. Indeed, we found that while our rough description of the envelope worked with SN 2024afav, it did not adequately describe the other supernovae we investigated, and we used a more phenomenological envelope expression to capture their behavior. As we mentioned in the text, this envelope model is too simple or inaccurate and does not improve interpretation of the results. To address this and the referee's feedback, we demote the envelope model to being phenomenological and do not claim a first-principles derivation, instead reframing the report to discuss with interest that the peak of the modulations seems to coincide with the expression we derived originally. We also attempt the fits with the envelope peak and spread allowed to vary and find no statistically significant deviation from the primary results, and report on this in the text. Finally, we add more discussion of possible origins of the envelope behavior. (example text below)

As demonstrated in ??, the precession behavior can be fit using only the residual peak locations, independently of the amplitude evolution or indeed the bulk behavior of the light curve as a whole. Despite this, the physical origin of the amplitude modulation in the residual is uncertain. Plausible interpretations will vary depending on the assumption of anisotropic emission generator; for example, a jet-based model may see amplitude fluctuation due to the variation of the jet power with time, while a disk wind-based model may see variation due to changes in disk size with time. Future investigation into energy injection scenarios will require significant theoretical work outside the scope of this analysis but yield quantitative predictions for the residual amplitude behavior, helping constrain the underlying mechanism. For the purposes of this analysis, we phenomenologically describe the SN 2024afav amplitude residual, and make the following observation about the peak time.

...

This ansatz satisfactorily describes the behavior of the residual amplitude, correctly reproducing the peak location and the rise and decay (reduced $\chi^2 \approx 1.6$, $N = 618$ data points). We also perform the fit allowing t_{peak} and the spread of the Gaussian to vary, and do not obtain a significantly better result (reduced $\chi^2 \approx 1.6$).

Investigation of the envelope

What is the expression for the input signal in Figure 8? Is it Equation (15) multiplied by the cosine of a phase function $\cos(\phi(t))$?

The expression for the input signal is equation (7) with simulated occlusions injected (please see the text following equation 7, we reproduce that description below for convenience).

We assume that the disk reflects some fraction of the magnetar luminosity, and that the amount of this reflected luminosity directed toward the observer is proportional to the disk area projected along the line of sight:

$$L_{\text{reflected}} \propto \cos\beta \cos\theta_{\text{obs}} - \sin\beta \sin\theta_{\text{obs}} \cos\Omega_{\text{prec}} t. \quad (3)$$

This is in addition to the (constant) luminosity of the magnetar itself along a line of sight L_{magnetar} . The total luminosity along a line of sight is then

$$L = L_{\text{magnetar}}[1 + f(\cos\beta \cos\theta_{\text{obs}} - \sin\beta \sin\theta_{\text{obs}} \cos\Omega_{\text{prec}} t)], \quad (4)$$

where f is a free parameter.

...

To translate this into the observed light curve, we assume 1 occlusion per period, and take as input equation (4) with an occlusion of 20% of the precession period centered on the minima. We then apply the diffusion model of [Arnett \(1982\)](#) (following [Zhang et al. \(2025\)](#)) to obtain the light curve shown in ???. We see that the observed light curve after accounting for photon diffusion through the ejecta is exceptionally well-reproduced by a cosine of the input signal phase function, with a constant phase offset to approximate the diffusion delay.

Details of the reflection and occlusion by the disk

3 Response to specific comments

Here, we address the more specific comments left by both Referee 1 and Referee 3.

5) I confused the usage of "dip" and "trough". The authors could replace "dip" with "trough" as used previously. "Trough" refers to a local minimum, while "peak" refers to a local maximum. There are two relevant points in the manuscript that require revision: 5.1) The sentence "we identify dips at $t = 65.3, 117.6, 139.9$, and 160.4 days, and peaks at $t = 44.9, 93.6, 128.9, 153.3$ days as well as a low-confidence trough detection at $t = 168.8$ days." should be changed to "we identify troughs at $t = 65.3, 117.6, 139.9$, and 160.4 days, and peaks at $t = 44.9, 93.6, 128.9, 153.3$ days as well as a low-confidence peak detection at $t = 168.8$ days." 5.2) In Table 1, No. 1, No. 3, No. 5, No. 7, and No. 9 are "peaks", while No. 2, No. 4, No. 6, and No. 8 are "troughs".

We implement these corrections.

6) Section 2.3, paragraph 2, gives the method for obtaining P_c and $\dot{P}_c$: "Using the peak-to-peak and trough-to-trough locations to estimate periods as a function of time, we fit a linear period variation and find $P_c = 231.6$ days and a period derivative of $\dot{P}_c = 0.440.05$ ". Can the authors provide a figure of the "periods as a function of time" mentioned here, and can they provide the results of "we fit a linear period variation", including the fitting results (figure) and fitting parameters? This is done to make it more intuitive and clear how P_c and $\dot{P}_c$ are obtained.

We add a pedagogical figure showing how P_c and $\dot{P}_c$ are obtained. This figure is reproduced for convenience in [section 4](#).

7) $t^{-5/3}$ does not appear in $L_{\text{magnetar}}(1 + t/t_p)^{-2}$. What is the origin of $t^{-5/3}$ in equation (17)?

This was a typo; previously the equation mistakenly used the power decay due to fallback accretion and not the magnetar spindown power. We thank the referee for catching this and apologize for the error, and remove the spurious term from the equation.

(Referee #3) Second, I don't think it is possible for the reader (or the referees) to determine what the angles α and δ are. You say that they are two polar angles, but is this really what you mean? My best guess is that α is the angle between $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ (i.e., it really is a polar angle), and δ is the angle between $\hat{\mathbf{x}}$ and the projection of $\hat{\mathbf{k}}$ onto the x - y plane (i.e., it is an azimuthal angle). Please clarify. (Referee #1) In Section 2.4.1, the authors use two angles α and δ to construct the spin vector $\mathbf{k}$ of the magnetar, yet in Equation (8), α is used to represent the α -disk parameter. Thus, α is assigned two distinct meanings.

We thank Referee #1 for pointing this out; α is standard for both parameters in the literature but are definitely confusing when used together here. We change the angle α to an alternative to implement the feedback. We thank Referee #3 for helping us clarify this, and update the text to include a statement explicitly saying what α and δ are.

In the context of the orbit, we construct our unit triad $(\hat{i}, \hat{m}, \hat{h})$, following equations 2–4 of [lorio \(2024\)](#), and magnetar spin vector $\hat{k}$. The spin axis $\hat{k}$ is defined by two spherical angles: the polar angle α' (primed to differentiate from α -disk characteristic parameter) between $\hat{k}$ and the observer's $\hat{z}$ axis, and the azimuthal angle δ between the projection of $\hat{k}$ onto the $(\hat{x}, \hat{y})$ plane and the $\hat{x}$ axis.

Derivation of the precession

Can the authors explain the meaning of the letters in equations (7) and (8)? For example, what do $\dot{M}$ in equation (7) and $\dot{M}_0$ in equation (8) represent?

$\dot{M}$ refers to the accretion rate through the disk. $\dot{M}_0$ is the reference accretion rate through the disk at time $t_0 = 30$ days. We add clarification of this and other parameters when the parameters are introduced.

9) The caption of Figure 14 mentions that SN 2024afav has five unambiguous bumps, but according to the analysis in Section 2.3, SN 2024afav only has four unambiguous bumps.

We have implemented this change in the text.

10) Is the power density spectrum of SN 2024afav (blue line) in Figure 5 derived from the photometric data (Figure 1) or the residual data (Figure 13)? Which time range of data was analyzed?

The power density spectrum is derived from the residual data, from day 30 to day 180. We add this clarification to the figure caption and text.

4 New figures

Here, we present new figures that have been added per comments from the referee. These figures were not present in the version submitted to the journal. We also include the accompanying text added to the body of the paper designed to work in tandem with the figure.

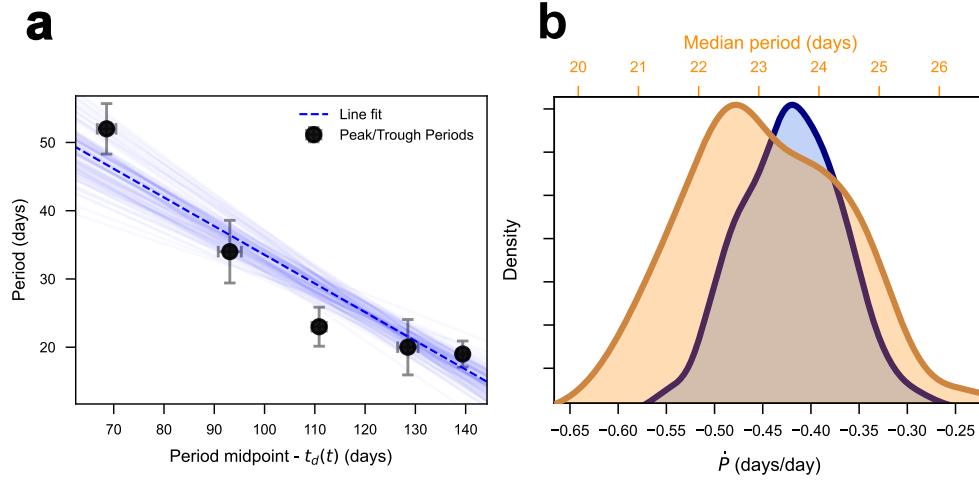


Figure 2: | **Characteristic parameter estimation.** We pedagogically demonstrate the procedure for estimating characteristic parameters $\dot{P}_c$ and P_c , as described in ???. The left panel (a) shows the periods estimated from SN 2024afav, corrected for the diffusion timescale of the ejecta. We fit a line to the periods and use a posterior sampling of the parameters to estimate the slope $\dot{P}_c = 0.44 \pm 0.05$ (right panel, b). Next, we compute the median of the period measurements to get a characteristic period $P_c = 23 \pm 1.6$ days (uncertainty is estimated from the spread in the medians of the linear fits). For our models, which generate effectively continuous time series periods, we sample them on the same days as the period midpoints shown above, and compute their characteristic quantities using that sampling. This serves as a pedagogical tool to help understand the ability of the models to describe the data.

Accompanying text: Using the peak-to-peak and dip-to-dip locations to estimate periods as a function of time, we fit a linear period variation (demonstrated in Figure 2) and find $P_c = 23 \pm 1.6$ days and a period derivative of $\dot{P}_c = -0.44 \pm 0.05$. We note that the choice of characteristic period and derivative estimation method does not significantly impact the results, so long as the corresponding period and period derivative are computed identically for the theoretical models.

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Lense-Thirring precessing magnetar engine drives a superluminous supernova: *Nature* Referee Response III

Joseph Farah* and Logan Prust, on behalf of the authors

December 17, 2025

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1 Introduction and summary of changes

We thank the referees for their continued engagement with the paper, which have and continue to make the paper significantly better and more robust. Referee #1 has raised valid criticisms of the specifics of the occlusion-reflection model which we cannot address in the detail required without significant further theoretical investigation, which is a subject of active and ongoing work. However, we have received guidance from the editor that great detail on any one particular model and first-principles (FP) justifications are unnecessary for publication. Thus, we will restrict our analysis to the timing of the chirp, and not attempt to provide an end-to-end physical scenario justified from first principles, instead relying on physical scenarios established in previous works. This will also allow us to shorten the paper to better comply with Nature's editorial guidelines.

Our overall, revised physical picture is similar to what we previously proposed but is now motivated as follows:

1. A precessing accretion disk can dynamically block or modulate line-of-sight emission (motivated by e.g., [Romanova et al. 2014](#) or any of the other references we list in the paper for disks modulating central-engine emission), casting a shadow on the ejecta that moves as the disk precesses, or modulating the emission isotropically.
2. Diffusion through the ejecta will smooth out high-frequency modes and result in approximately sinusoidal behavior even for non-sinusoidal quasiperiodic input (we motivated this earlier with a specific diffusion treatment and convincingly showed that the resulting signal is indeed sinusoidal; however, we will now only refer to general treatments of diffusion such as [Arnett 1982](#)).
3. If anisotropic, the changing emission will indeed be visible to the observer; diffusion through the ejecta does not completely remove anisotropies, so anisotropic energy injection into an expanding ejecta can produce viewing-angle-dependent light curves (motivated by, e.g., [Barnes et al. 2018](#)).
4. Finally, the form assumption of $L(t) = A(t) \cos(\phi(t))$ is purely motivated based on the citations above and the phenomenological description of the data. The timing is still related to physical mechanisms in order to explore which model best describes the data, but the description using $L(t)$ is performed only to facilitate a comparison to the full light curve and is kept phenomenological.

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Since these changes involve dropping several sections and removal of text (which is difficult to highlight), for convenience, we summarize the changes below. To implement the above re-structuring of the paper, we did the following (section by section):

- Main section: minor wording changes on the model specification
- Methods
 - Observations and data reduction 2.1: unchanged
 - Magnetar-only model fits 2.2: unchanged
 - Residual analysis 2.3: a minor note that the success of the $A(t) \cos(\phi(t))$ phenomenological model motivates the use of such a function later in the analysis
 - Construction of LT model 2.4: a minor note that the diffusion timescale also results in approximately sinusoidal behavior for even non-exactly sinusoidal quasiperiodic light curves. We showed this was true for our specific occlusion model but we will not invoke that model here, instead making the statement more broadly.
 - Modulation reproduction with magnetar parameters 2.4.1: minor wording changes; envelope form is no longer “derived”; separation from phenomenological model is de-emphasized
- Supplementary information
 - Observing instruments 3.1: unchanged
 - Details of reflection/occlusion 3.2: **removed**
 - Derivation of the precession 3.3: retained
 - Investigation of the envelope 3.4: **removed**. Discussion of the envelope will be phenomenological only with no FP derivation and restricted to 1–2 sentences in an existing relevant section.
 - Investigation of alternative scenarios 3.5: retained
 - Fits to other SLSNe-I: retained; minor model description changed. Envelope is no longer distinct from 24fav modeling.
- Figures
 - Figures 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14 retained
 - Figures 7, 8 **removed**

The effect of these changes is to avoid going into detail on any specific model. Below, we respond to the final minor comments from the report. We will specify if any of the comments are already addressed based on our treatment above, and respond in full to the remainder.

2 Response to major comments

Here, we address the major comments from Referee #1.

1. The authors cited the work of Zhang et al. (2025) multiple times in the manuscript, but the authors may not have correctly understood the work of Zhang et al. (2025). The precessing magnetar-powered model of Zhang et al. (2025) did not introduce a precessing jet or anisotropic energy injection. In the precessing magnetar-powered model of Zhang et al. (2025), the magnetic inclination angle undergoes quasi-periodic variations during precession, leading to quasi-periodic changes in the luminosity of the isotropic magnetic dipole radiation produced by the precessing magnetar. Therefore, in the work of Zhang et al. (2025), the energy (i.e. magnetic dipole luminosity) of the precessing magnetar is isotropically injected into the entire spherically symmetric ejecta, but the magnitude of the injected energy varies quasi-periodically.

2.1. One motivation for equation (2) or equation (8) comes from the work of Zhang et al. (2025), but the energy injection considered in Zhang et al. (2025) is isotropic for the precessing magnetar, which is fundamentally different from the anisotropic energy injection in the magnetar-disk system analyzed here.

The primary concern raised by the referee is the incorrect interpretation of Zhang et al. (2025) introduced into the most recent draft. The referee is right that this model does not incorporate anisotropic energy injection. We mistakenly included it as an example of an anisotropic model when attempting to address the concern from the previous round; we apologize for this error. To address this, we remove the instances where we grouped the Zhang et al. (2025) works in with anisotropic energy injection scenarios. We note that we do indeed consider the Zhang et al. (2025) model-class (precessing process causing isotropic but oscillating energy injection into the ejecta) a plausible alternative to the scenario we propose here and investigate this model in Alternative Scenarios.

2. In the authors' model, the modulation of SN 2024afav originates from equation (2) or equation (8). The authors cannot specify which specific physical mechanism this formula derives from, meaning that they cannot provide a first-principle derivation for this equation. The authors point out that equation (2) or equation (8) is an ansatz, noting that this ansatz has two motivations. I still have some doubts about whether these two motivations can indeed lead to this ansatz.

2.1. One motivation for equation (2) or equation (8) comes from the work of Zhang et al. (2025), but the energy injection considered in Zhang et al. (2025) is isotropic for the precessing magnetar, which is fundamentally different from the anisotropic energy injection in the magnetar-disk system analyzed here.

2.2. Another motivation for equation (2) or equation (8) is the disk occlusion model in Section 3.2, but I still have some doubts about the calculations for the disk occlusion model in Section 3.2.

2.2.1. The authors define L_{magnetar} in equation (7) as the luminosity of the magnetar along the line of sight. If so, the calculation of the second term on the right side of the equation (7), that is, the luminosity $L_{\text{reflected}}$ reflected to the line of sight, may be incorrect. This is because the luminosity L_{magnetar} of the magnetar along the line of sight is not reflected to the line of sight. In other words, $L_{\text{reflected}}$ does not depend on L_{magnetar} .

2.2.2. Why does the authors only calculate the energy injection into the ejecta along the line-of-sight direction when calculating the observer's light curve obtained from the disk occlusion model? Why can this calculation represent the results of the complete disk occlusion model? (The complete disk occlusion model involves energy injection into the ejecta not only along the line of sight but also along other directions. I understand that the authors state calculations for anisotropic energy injection are beyond the scope of this analysis, but I would like to know the reason for this treatment.)

We group several of the motivation-related comments here. Following guidance of editor and Referee #3, we will not attempt a first principles derivation, and instead motivate as described in the introduction, based on

plausible physical mechanisms and the functional form evident in the dataset. We make a broad set of changes to this effect (summarized in the introduction), and these include: a move away from terms like “ansatz” and first-principle derivations and towards “phenomenological” as descriptions for motivations. For example, Equations 2 and 8 are no longer motivated by an end-to-end physical model, and are instead phenomenological, motivated by the shape of the data. We examined the light curve, which contains damped oscillatory modes on top of the magnetar model light curve. The timescale of the variations should be connected to something physical, so we looked for a physical mechanism that could plausibly match that timescale. We showed that LT precession matches the timescale and nothing else worked as well. This is a significant distinction from an end-to-end physical model, which is of great interest but outside the scope of this analysis. Future theoretical investigation will help examine the broad spectrum of validity of the assumptions we have made, the plausibility of various mechanisms, and their alternatives; our analysis here provides the underlying motivation directly from the new, chirped dataset.

3 Response to specific comments

Here, we address the more specific comments left by Referee #1.

2.3. As the authors mentions, magnetar-disk systems can generate anisotropic energy injection through multiple mechanisms, and the authors cannot restrict which specific mechanism it is. Here, the authors assume that anisotropic energy injection originates from reflection/occlusion. This implies that the second motivation for equation (2) or equation (8) (i.e., the disk occlusion model) is another assumption. If another mechanism for generating anisotropic energy injection is adopted, equation (2) or (8) may not hold.

The first-principles motivations and the assumption of reflection/occlusion are removed (see Broad Comments). However, the referee has raised an interesting point here which we would like to address. The disk occlusion model is indeed an assumption. However, the motivation that a precessing disk will create an approximately sinusoidal signal is more broad. Whether the method is purely sinusoidal (e.g., a precessing jet strobing across the line of sight), quasi-sinusoidal (e.g., the occlusion/reflection model), or periodic (e.g., periodic luminosity spikes from accretion rate modulation), the diffusion through the ejecta can smooth these variations out into an approximately sinusoidal behavior.

3. Although the authors point out that the envelope model is too simplistic and phenomenological, and that the physical origin of the amplitude modulation is uncertain with multiple possible interpretations, I still have doubts about the derivation of the envelope model.

The authors derived the envelope model based on equation (17). According to the physical meaning of equation (17), a single magnetar located at the center of the ejecta (the magnetar-only model) also satisfies equation (17). However, the peak time of the magnetar-only model is $t_{\text{peak,magnetar}} \approx 50$ days (see Figure 1), while the peak time of the amplitude of the modulations obtained according to equation (17) is $t_{\text{peak,modulation}} \approx 125$ days. The inconsistency between the two requires further explanation.

The first-principles motivations and the assumption of reflection/occlusion are removed (see Broad Comments). However, we want to highlight this comment, as we appreciate the referee raising these concerns and intend to address them in future work. Following the guidance of the editor, we only describe the envelope phenomenologically, motivated based on the shape observed in the 24afav light curve. The purpose of this is to perform a comparison to the full light curve in addition to the peaks-only fit. The goal will be characterization of the behavior, not understanding of the precession mechanism (this is in contrast to the peaks-only fit, which is attempting to narrow down the precession mechanism based on timing).

4. In the left panel of Figure 5: Do the five black data points (peak/trough periods) correspond to three peaks (local maxima) and two troughs (local minima) within the range of 65 to 160 days after the explosion (see Figure 13)? How is the error for these five black data points determined?

Yes. The peaks/trough locations are determined by fitting a quadratic to each peak/trough to determine a location + uncertainty based on the extremum of the quadratic. This uncertainty is then propagated into both the period estimate (difference between pairs of peaks and troughs) and also the period midpoint. We add text clarifying the source of the error to the residuals analysis section.

Using the peak-to-peak and dip-to-dip locations to estimate periods as a function of time, we fit a linear period variation (demonstrated in ??) and find $P_c = 23 \pm 1.6$ days and a period derivative of $\dot{P}_c = -0.44 \pm 0.05$. Uncertainties are based on errors in the peak locations, estimated by fitting a quadratic function to the local region around each extrema.

Residual analysis

In the right panel of Figure 5: Does blue represent the distribution of $\dot{P}$, while orange represents the distribution of the median period? How is the distribution of the median period obtained?

Yes; we add text clarifying the legend to the figure caption. The distribution of the median period is determined by computing the spread in the median period estimates based on the linear fits. However, this distribution is offset so its mean corresponds to the actual measured median period from the data. The P_c from the data does not require a fit to measure, but using the linear fit gives a meaningful sense of the possible uncertainty in that value. The linear fit shown here is a good pedagogical visualization of how a functional form can reproduce the overall change well but not the characteristic period.

Does "we compute the median of the period measurements to get a characteristic period $P_c = 23 \pm 1.6$ days" refer to P_c corresponding to the third black data point in the left panel of Figure 5?

How are the medians of the linear fits obtained? For example, for a certain linear fit result (a fitted line) in the left figure of Figure 5, does the median of its period correspond to the period of the median time ($t_{\text{median}} \sim 105$ days)?

Yes to the questions above; we briefly clarify this in the draft. In this case (with 5 data points) P_c is simply the value of the model or the data at the time location corresponding to the third black point in the left panel of Figure 5. This corresponds to the largest (highest amplitude) modulation, which is also best-described by the sinusoidal behavior, so the meaning of "period" is most clear at that time.

Based on the high-confidence peaks and troughs present, we compute a characteristic period (median of the set of computed periods) and period derivative for the residual modulations.

Residual analysis

5. Are the "peaks" and "troughs" in Table 1 reversed? For example, for No. 3, $t_{\text{start}} = 80$ days and $t_{\text{end}} = 110$ days. There is a peak between 80 and 110 days (see Figure 13), so No. 3 corresponds to the peak. Therefore, in Table 1, No. 1, No. 3, No. 5, No. 7, and No. 9 are "peaks", while No. 2, No. 4, No. 6, and No. 8 are "troughs".

We appreciate the referee's thoughtful comments on the characterization of the light curve. There have been a few questions about this in the past reports as well so we would like to take a moment to clarify our definitions. First, an important nuance is that "peak" and "trough" refer to the conventional "brighter" and "dimmer" points on the light curve residual, which are smaller and larger magnitude, respectively, due to the construction of magnitudes. In one of the previous revisions, we mistakenly interpreted a comment as requesting we switch to a definition of "peak" and "trough" to match the numerical values and not the more intuitive interpretation, which is why they appear to be reversed in the table. However, we see from this comment that we misunderstood the referee; we apologize for the confusion. The referee's comment is correct in the definition we have laid out

above; we swap the “peaks” and “troughs” in the table to implement. Finally, we confirm that the “peaks” and “troughs” in the table are correct.

We use a standard peak-extraction algorithm ([Virtanen et al., 2020](#)) to identify candidate extrema in each band, which we then aggregate by computing a mean location for each cluster of extrema weighted by the number of bands in which the extremum appears. Using this approach, we identify troughs at $t = 65.3, 117.6, 139.9$, and 160.4 days, and peaks at $t = 44.9, 93.6, 128.9, 153.3$ days as well as a low-confidence peak detection at $t = 168.8$ days. Note that “peak” and “trough” refer to the conventional “brighter” and “dimmer” points on the light curve residual, which are smaller and larger magnitude, respectively, due to the construction of magnitudes.

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