
Supplementary information

Verification of the Outer Space Treaty with cosmic protons

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0 Supplementary Note: Orbital requirements of an Inspector Satellite

A simplified close-proximity inspection scenario was modeled using the linearized Clohessy–Wiltshire (CW) equations about an approximately circular reference orbit. The chief spacecraft is Kosmos2553 with the following orbital parameters at the time of writing of this work: perigee altitude of 2000.2 km; apogee altitude of 2006.6 km; orbital period of 127.1 minutes; inclination of 67.1°.

From the perigee and apogee radii, the semi-major axis is computed as $a = \frac{r_p + r_a}{2}$, where $r_p = 2000.2 + r_{\text{earth}}$ and $r_a = 2006.6 + r_{\text{earth}}$, and $r_{\text{earth}} = 6371$ km. The period of the mean motion is obtained from Kepler’s third law, $n = \sqrt{\mu/a^3}$, $T = 2\pi/n$, where $\mu = 398600 \text{ km}^3/\text{s}^2$ is Earth’s mean gravitational parameter.

The deputy spacecraft is an inspector satellite designed to collect data when its radial offset relative to Kosmos2553 equals -4 km (i.e., 4 km closer to Earth in the Hill frame). To maintain the same mean motion as the chief, the deputy is assigned a perigee of 1996.2 km, an apogee of 2010.6 km, and the same inclination as the chief. This results in the same exact semi-major axis as the chief and the same orbital period. Furthermore, the deputy’s phase is selected such that at $t = 0$ it is exactly 4 km radially-outward with respect to the chief. Given that both the chief and the deputy have very small eccentricities of $e = (r_a - r_p)/(r_a + r_p) \approx 0$, we neglect eccentricity effects and use the CW circular orbit framework for approximating the relative motion. The relative motion is described in the Hill frame using the planar CW solutions:

$$x(t) = A \cos(nt) + B \sin(nt), \quad y(t) = -2A \sin(nt) + 2B \cos(nt),$$

where x is the radial offset and y is the along-track offset. Choosing $A = 4$ km, $B = 0$ produces a bounded relative trajectory in which the radial separation oscillates between ± 4 km, while the along-track separation necessarily oscillates between ± 8 km. The $x(t)$ and $y(t)$ are plotted in Fig. 1.

In addition to achieving relative motion with a ± 4 km radial oscillation, the phase of the deputy needs to be chosen to achieve a -4 km position relative to the chief when the two cross the equatorial plane, where for $L = 1.3$ the Van Allen Radiation belt proton population is at maximum.

This model suggests that it is possible to design an orbit where the deputy (inspector) can position itself below the chief (Kosmos2553) once per orbit to perform neutron measurements. This analysis shows that there is one measurement opportunity per orbit; this can be doubled by having a second inspector with the same parameters but with A chosen to be $A = -4$ km. The intrinsic coupling of radial and along-track motion in the linear CW dynamics produces a bounded radial oscillation of amplitude A and an along-track oscillation of amplitude $2A$. The resulting periodic geometry defines predictable inspection windows for the deputy spacecraft during each orbit of Kosmos2553. The simple model presented above makes multiple approximations and a more detailed orbital analysis is necessary and should be part of future work.

1 Supplementary Note: Weapon composition and neutron source term

The weapon composition may significantly affect the spallation neutron source term. Unfortunately, it is impossible to make any exact assumptions about composition, as most thermonuclear weapon design information is classified and publicly inaccessible. There is some public information about the US designs that indicates that the mass of the fissionable material on a W87 warhead could be 200 kg (see Ref. [1]). According to declarations in the framework of the START treaty, the Russian R-36 missile has a throw-weight of 8800 kg and 10 multiple reentry vehicles (MIRV), making the weight of one Russian MIRV 880 kg [2]. Assuming that the fissile components are the heaviest in the weapon and make up most of its weight, this analysis implies that Russian weapons could be multiple times heavier than W87. This study assumes only 95 kg, as it is the basis of the Carpenter model used in this study. It is thus reasonable to use the Carpenter model as a basis for neutron spallation as a lower limit for the neutron source term.

Ultimately, the final goal of this study is to build an analysis framework that can be used by specialists with access to classified information to perform more detailed simulations and calculations and thus determine the actual performance of the proposed approach.

2 Supplementary Note: Shielding effects

The study presented in the main body does not include the effects of possible neutron shielding, e.g. by layers of polyethylene, which the designers of the suspected satellite could use to reduce the neutron source term. Understanding what shielding types are possible for a weapon placed in the outer space environment is not a simple task,

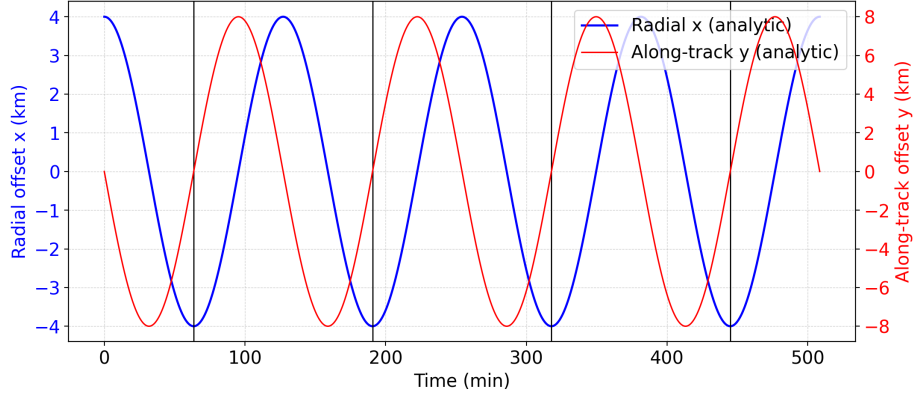


Fig. 1 The analytic CW solution plotted over four orbital periods. The radial offset is shown on the left (blue) axis and the along-track offset on the right (red) axis. Vertical black lines at half-period intervals define the inspection points where the deputy is directly 4 km below the chief. The plot shows that at the time of zero along-track offset the radial offset is ± 4 km.

which is why it has been delegated to future research efforts. While in ground-based systems neutron shielding is trivial, in outer space it is a much more complex problem. The neutron shielding will act as thermal insulation, which will make it more difficult to remove the ~ 10 W of alpha heating and any additional heating due to the increase in fission triggered by protons. Given that the weapon is expected to serve for years, any accumulation of heat due to the additional insulation may result in an increase in the weapon component temperatures, possibly affecting their reliability in the case of nuclear use.

Ignoring these circumstances, it is possible to estimate the impact of the shielding on the required size of the inspector satellite for various thicknesses of polyethylene shielding. Polyethylene, being hydrogen-rich, moderates the fast ~ 2 MeV neutrons produced by proton spallation out of the energy acceptance of the scintillator. A series of grasshopper/Geant4 simulations were performed, where an isotropic neutron source was surrounded by varying thicknesses of polyethylene, and the counts of a nominal detector with energies of $E_{\text{dep}} > 0.2$ MeV were used to estimate the transmission $T = c(x)/c(0)$, where $c(x)$ are the counts for shielding of thickness x . The neutron energies were sampled from the spectrum presented in Ref. [3].

To maintain the goal of detection in one week from a distance of 4 km in scenarios with shielding, the size of the CubeSat needs to be increased by a factor proportional to $1/T$ from 9U to XU, where X can be determined from $X = 9/T$. Table 1 lists the various possible values of shielding thickness and the corresponding required sizes of the CubeSat. For example, for 20 cm of polyethylene shielding, the inspector needs to have the form factor of at least 265U, i.e. make up a panel of approximately 1.6×1.7 m².

x (cm)	T	minimum $\lceil X \rceil$	detector size (m×m)
0	1	9	0.3×0.3
5	0.3	30	0.6×0.5
10	0.12	75	0.9×0.9
20	0.034	265	1.6×1.7

Table 1 Values of CubeSat X and its required size for various values of shielding thickness to achieve a detection from the distance of 4 km in one week. The detector size is rounded to the nearest upper decimeter.

3 Supplementary Information: False Positive Rate

In this study, the expected spallation neutron count for a 9U satellite at a distance of 4 km, assuming approximately one week of observation time, has been five. With a threshold for detection of $c = 1$ it can be determined that the probability of detection is $p = 1 - \exp(-5) = 99.3\%$. The false positive rate is driven by multiple possible sources. The greatest of these is the mixing of atmospheric neutrons arriving from below the inspector and spallation neutrons arriving from above. The main body of the manuscript describes in detail the use of the neutron scatter camera technique by

calculating θ_{error} and applying $\cos \theta_{\text{error}} > 0.95$ to achieve very efficient rejection of atmospheric neutrons. To determine the likelihood of an atmospheric neutron being tagged as a spallation, we ran a longer version of the simulation described in Fig 3(b) in the main body. The simulation sampled 1.68 bln neutrons and counted those that fell into the $\cos \theta_{\text{error}} > 0.95$, determining that the number of such counts is zero. We thus conclude that $c_{\text{error}} < 1$ per 1.68 bln incident neutrons and use this as an upper limit for the false positive rate. Analysis of the atmospheric and albedo neutron background at the altitude of 2000 km, using the model provided in Ref. [4], yields a rate of $0.033 \text{ cm}^{-2}\text{s}^{-1}$ at the altitude of 2000 km and latitude of 0° . Using the $t = 7.2$ day observation time with a 9U satellite, we can then determine the expectation of false spallation neutrons. That is $\bar{c}_{\text{false}} < (1/1.68 \times 10^9) \cdot 0.033 \cdot (30^2) \cdot 7.2 \cdot 24 \cdot 3600 = 0.011$. We can then determine the probability of a false positive during this measurement period. Using the $c_{\text{threshold}} = 1$, is $p_{\text{fp}} < 1 - \exp(-0.011) = 0.011 = 1.1\%$. We thus conclude that, based on the MC simulations performed for the primary scenario described in this study, the false positive probability is $< 1.1\%$.

Supplementary References

References

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